Article

Analysis of Electromagnetic Problems in the Presence of Non-uniform Movements

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***Abstract*–** **Electromagnetic problems with moving structures are considered. For objects moving at a uniform velocity, the most popular technique used in the literature consists of a change of the reference frame and the utilization of Lorentz transformation of Maxwell's electric and magnetic fields and Voigt-Lorentz transformations of space and time variables. In order to account for non-uniform motions, a numerical approach based on the finite-difference time-domain (FDTD) method is used. In this approach, Maxwell's equations are applied without modification and the motion of objects is implemented by changing their positions in the FDTD time loop. Three types of movements are considered: vibration, rotation, and acceleration. Full-wave numerical results are reported for different objects in motion: observer, source, and reflecting electromagnetic surface. In all the problems analyzed, interesting results are obtained and these results are discussed in detail.**

***Index Terms*–** Acceleration, FDTD method, Rotation, Vibration.

I. Introduction

The problems of electromagnetic wave propagation and radiation in the presence of moving objects have attracted a lot of attention for a long time, due to their wide application in many areas such as RF Doppler radars, astrophysics, global positioning system (GPS), and optical gyroscopes. Numerous investigations have been carried out in these domains, which are interesting from a theoretical and practical point of view [1]–[16].

In 1887, for the analysis of the Doppler effect in elastic incompressible media, Voigt imposed the convective wave equation (with moving observer) to have the same form as the wave equation with the observer at rest [17]. This made him use auxiliary variables for the three variables of space (in Cartesian coordinates) and the variable of time. Lorentz used Voigt's auxiliary variables for the investigation of different electromagnetic phenomena with moving objects, based on Maxwell's electrodynamics [18]. To account for his hypothesis of length contraction in the Michelson-Morley experiment and for the apparent mass increase of particles in particle accelerators, Lorentz multiplied these variables with the Lorentz factor [19]. Today, the investigations of electromagnetic problems with moving objects are usually based on the utilization of Voigt-Lorentz transformations.

Voigt-Lorentz transformations are applied to calculate Maxwell's electric and magnetic fields in a frame moving with uniform velocity, from the fields which are known in a reference frame. However, the problem becomes more complex if multiple objects moving at different speeds need to be considered or if non-unform motions need to be studied.

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The analysis of electromagnetic problems with objects having non-uniform motion, such as vibration, can be useful in many recent applications, for example, in time-varying waveguides [20] and Doppler radars for the detection of vital signs [21], [22].

The finite-difference time-domain (FDTD) algorithm was introduced in 1966 by Yee, based on finite differences and Yee's cell, for the numerical resolution of Maxwell's equations [23]. In 1975, the FDTD method was applied to analyze the effect of electromagnetic radiation on human eyes [24]. Nowadays, thanks to its development by numerous pioneers [25]–[27], the FDTD method is used in a wide range of applications from DC to optics.

The FDTD method has been successfully used for analyzing electromagnetic problems involving moving objects with uniform speeds [1], [28]–[42]. In some of these works, such as in [37], techniques were proposed for the implementation of Voigt-Lorentz transformations in FDTD. In other papers, such as [1], [38]–[42], interesting and meaningful results have been obtained by using a direct FDTD approach that did not include Voigt-Lorentz transformations.

The objective of this paper is to use a direct FDTD method to analyze electromagnetic problems with non-uniform movements including vibration, rotation, and acceleration. The theoretical and numerical approaches are described in detail. A series of full-wave simulations are carried out, for the following problems with non-uniform motions: vibrating metallic slab, rotating observation point, rotating line source, accelerating observation point, accelerating plane wave source, and accelerating electromagnetic reflecting surface. For all these problems, the numerical results provide insight into the physical processes involved.

The remainder of the paper is organized as follows. Section II presents and discusses the main points of the theoretical aspects: Maxwell's equations, the FDTD method, the approach based on a change of the reference frame for moving bodies, and the proposed theoretical/numerical approach. Numerical results are shown and discussed in Section III. Different electromagnetic problems are analyzed: the illumination by a plane wave of a vibrating metallic plate with different vibration frequencies, frequency spectrum for a rotating observer and for a rotating line source, electric field observed by an accelerating observer, the waves radiated by an accelerating ideal or resistive plane wave source, and reflected waves from an accelerating electromagnetic surface illuminated by a plane wave. Concluding remarks are given in Section IV.

II. Theory

A. Maxwell's equations

In an isotropic medium, Maxwell's equations can be written as:

$$\nabla \times \vec{\mathbf{E}} = -\mu \frac{\partial \vec{\mathbf{H}}}{\partial t} \tag{1}$$

$$\nabla \times \vec{\mathbf{H}} = \sigma \vec{\mathbf{E}} + \varepsilon \frac{\partial \vec{\mathbf{E}}}{\partial t} \tag{2}$$

where $\vec{\mathbf{E}}$ is the electric field and $\vec{\mathbf{H}}$ is the magnetic field. μ (permeability), ε (permittivity), and σ (conductivity) are the constitutive parameters of the medium. In Cartesian coordinates, six partial

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differential equations are obtained:

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right) \tag{3}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) \tag{4}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \tag{5}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x\right) \tag{6}$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y\right) \tag{7}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right) \tag{8}$$

B. FDTD method

FDTD is based on a discretization of Maxwell's equation in time and space:

$$F^n(i,j,k) = (i\ \delta x, j\ \delta y, k\ \delta z, n\ \delta t) \tag{9}$$

where F is an electric or magnetic field component. The application of second-order error central finite difference for a variable of space can be written:

$$\frac{\partial F^n(i,j,k)}{\partial x} = \frac{F^n(i+1/2,j,k) - F^n(i-1/2,j,k)}{\delta x} + O(\delta x^2) \tag{10}$$

For time variable, the following can be obtained:

$$\frac{\partial F^n(i,j,k)}{\partial t} = \frac{F^{n+1/2}(i,j,k) - F^{n-1/2}(i,j,k)}{\delta t} + O(\delta t^2) \tag{11}$$

The application of finite differences in (3)-(8) gives equations for the numerical updates of electric and magnetic field components. For example, the update equation for H_x in Yee's algorithm is:

$$\begin{aligned} H_x^{n+1/2}(i,j+1/2,k+1/2) &= H_x^{n-1/2}(i,j+1/2,k+1/2) + \frac{\delta t}{\mu(i,j+1/2,k+1/2)\delta} \\ &\times \{E_y^n(i,j+1/2,k+1) - E_y^n(i,j+1/2,k) \\ &+ E_z^n(i,j,k+1/2) - E_z^n(i,j+1,k+1/2)\} \end{aligned} \tag{12}$$

The Yee's cell [23] is used for the positions of the different field components. Fig. 1 shows Yee's algorithm with black curves. The algorithm presents a time loop. At each time step n, the electric field $\vec{\mathbf{E}}^n$ can be updated using magnetic field $\vec{\mathbf{H}}^{n-1/2}$, and electric field $\vec{\mathbf{E}}^{n-1}$, from previous time step. Then, $\vec{\mathbf{H}}^{n+1/2}$ is calculated by using $\vec{\mathbf{H}}^{n-1/2}$ and $\vec{\mathbf{E}}^n$. The next iteration is obtained by the incrementation $n = n + 1$. The algorithm ends when $n > Nbiteration$.

C. Method based on a change of the reference frame and Voigt-Lorentz transformations for the analysis of moving bodies

The general method considered in the literature for analyzing electromagnetic problems involving motion is based on Voigt-Lorentz transformations between a rest frame and a moving frame (with

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uniform velocity $\vec{v} = v\hat{x}$). The time variable and the space variables are transformed as:

$$t' = \gamma(t - \frac{vx}{c^2}) \tag{13}$$

$$x' = \gamma(x - vt) \tag{14}$$

$$y' = y \tag{15}$$

$$z' = z \tag{16}$$

where the $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}}$ is the Lorentz factor. The transformations of the components of the electromagnetic fields can be written [43], [44]:

$$E'_x = E_x \tag{17}$$

$$E'_y = \gamma(E_y - \frac{v}{c}H_z) \tag{18}$$

$$E'_z = \gamma(E_z + \frac{v}{c}H_y) \tag{19}$$

$$H'_x = H_x \tag{20}$$

$$H'_y = \gamma(H_y + \frac{v}{c}E_z) \tag{21}$$

$$H'_z = \gamma(H_z - \frac{v}{c}E_y) \tag{22}$$

Finally, the transformed Maxwell's equations can be expressed as follows:

$$\nabla' \times \vec{\mathbf{E}}' = -\mu \frac{\partial \vec{\mathbf{H}}'}{\partial t'} \tag{23}$$

$$\nabla' \times \vec{\mathbf{H}}' = \sigma \vec{\mathbf{E}}' + \varepsilon \frac{\partial \vec{\mathbf{E}}'}{\partial t'} \tag{24}$$

where ∇' denotes the derivatives with respect to transformed space variables.

D. Proposed approach

Lorentz aether theory [19] and Einstein's special theory of relativity [43] use both the same approach described in II-C. However, these two theories differ in the physical interpretation of Voigt-Lorentz transformations of space and time variables. In Lorentz aether theory, except for the γ factor which adds a physical effect that is not present in Maxwell's equations, the transformed space and time variables are not physical. In the special theory of relativity, Voigt-Lorentz transformations are physical.

In this work, the framework of Lorentz aether theory is preferred and the length contraction effect (γ factor) is ignored. Indeed, the γ factor can be considered equal to one for small v/c.

Furthermore, in 2004, Engelhardt raised important concerns about the relativistic equations for the transformed fields and emphasized the necessity for developing an electromagnetic theory for moving matter [44].

An alternative approach consists of using numerical solutions of Maxwell's equations and changing the positions of objects in the desired directions with time. In such a method, Maxwell's equations are applied without modification. Fig. 1 shows in red the proposed implementation of moving objects in the FDTD algorithm. Each object (observer, source, scatterer) can move at a different speed. For example, after every $mfix_i = [\frac{\delta x}{v_i \delta t}]$ iteration, an object moves by one cell in the algorithm. The electromagnetic fields can also move, as demonstrated in [1], for the implementation of the Fresnel drag effect or the

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Sagnac effect in a dielectric medium. The proposed method is not limited to uniform motion and it can be used for problems with multiple objects moving at different speeds.

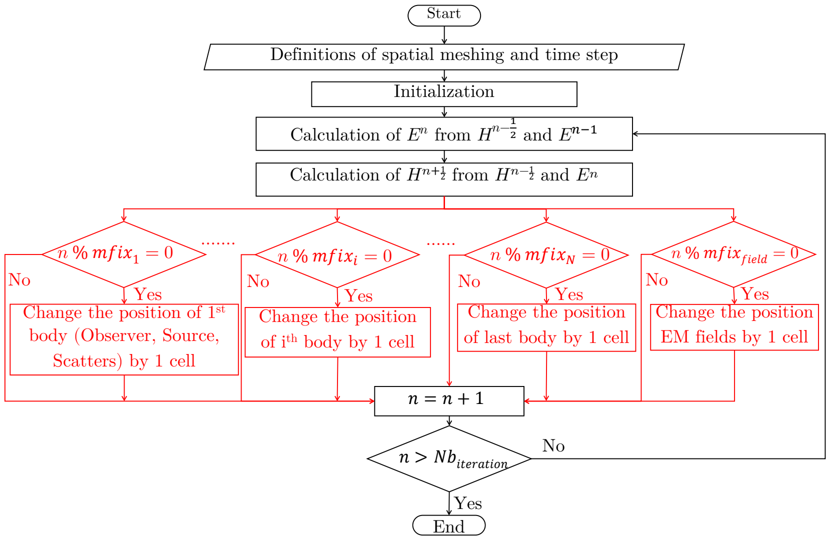


Fig. 1. Flowchart of the proposed FDTD approach for moving objects. In black: Yee's algorithm. In red: implementation of movements (source, observer, scatterer, or field) ([.%.] denotes the Modulo operator)

III. Numerical results

A. Vibratory motion

A vibrating metallic plate, as shown in Fig. 2, is considered. The boundary conditions include Absorbing Boundary Conditions (ABCs), Perfect Magnetic Conductors (PMCs), and Perfect Electric Conductors (PECs). f_0 is the frequency of the exciting source and f_m is the frequency of vibration of the metallic slab. Fig. 3a and Fig. 3b show the spectrum of the reflected wave for $f_m > f_0$ and for $f_m < f_0$, respectively. The spectrum shows the presence of the fundamental source frequency and frequencies due to intermodulation with the vibrating frequency. This is in agreement with experimental results obtained with a vibrating metallic plate and a Doppler radar [21]. As an extension of this work,

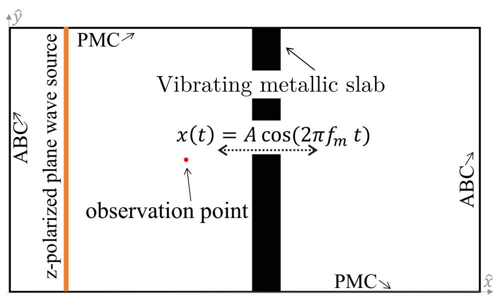
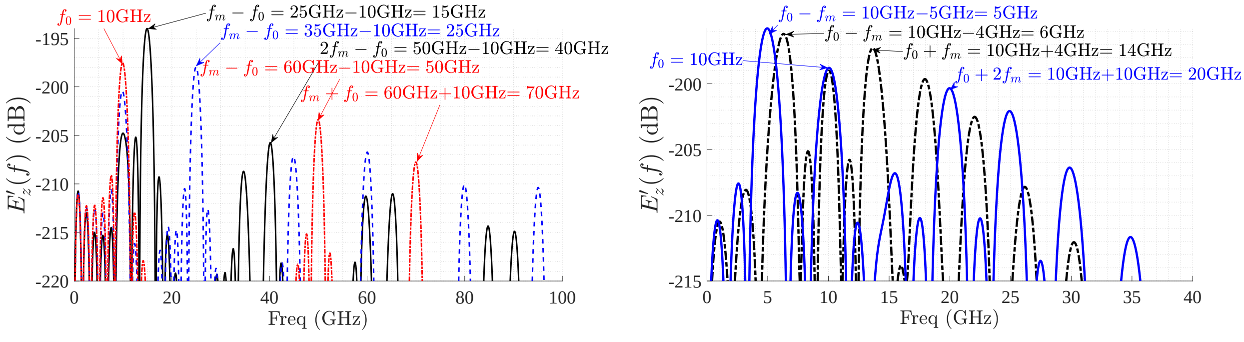


Fig. 2. Vibrating metallic plate in FDTD.

it would be possible to analyze the signal received in the context of the motion of the human chest which is due to both respiration and heartbeat [21], [22]. Instead of using a simple sinusoid for the movement of the metallic slab, one could use a more realistic motion of the human chest.



(a) (b)

Fig. 3. Simulated spectrum of the reflected wave, for vibrating metallic plate, with frequency of vibration (a) $f_m > f_0$ (f_0=10GHz; f_m=25GHz, 35GHz, or 60GHz), and (b) $f_m < f_0$ (f_0=10GHz; f_m=4GHz or 5GHz).

B. Rotatory motion

1) Rotating observation point: Fig. 4 shows a rotating observer illuminated by a plane wave. f_0 is the frequency of the exciting source and f_r is the frequency of rotation of the observer. Fig. 5a and Fig. 5b show the spectrum of the observed signal for $f_r > f_0$ and for $f_r < f_0$, respectively. In these results, the frequencies due to intermodulation can be identified.

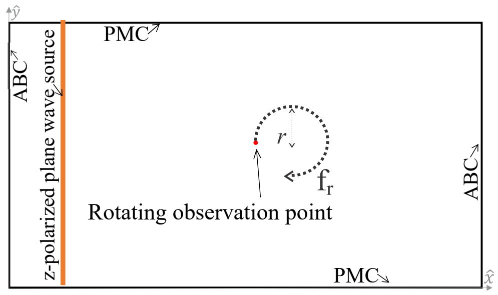
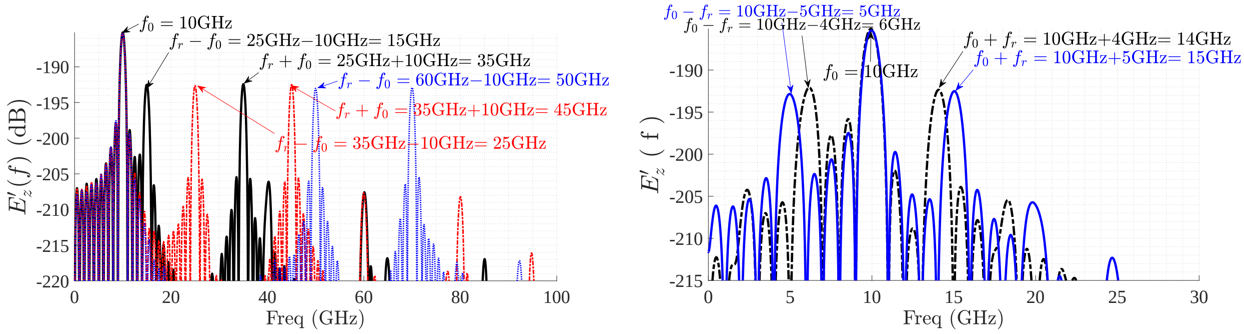


Fig. 4. Rotating observer.



(a) (b)

Fig. 5. Spectrum of observed signal, for rotating observation point, with the frequency of rotation (a) $f_r > f_0$ (f_0=10GHz; f_r=25GHz, 35GHz, or 60GHz), and (b) $f_r < f_0$ (f_0=10GHz; f_r=4GHz or 5GHz).

2) Rotating source: A line source is now rotating as shown in Fig. 6 and the observer is at rest. f_0 is the frequency of the exciting source and f_r is the frequency of rotation of the line source. Fig. 7a and Fig. 7b show the spectrum of the observed signal for $f_r > f_0$ and $f_r < f_0$, respectively. The results are again meaningful. Fig. 8a and Fig. 8b show the electric field distribution, at a time instant, for the rotating source, with $f_r > f_0$ and $f_r < f_0$, respectively.

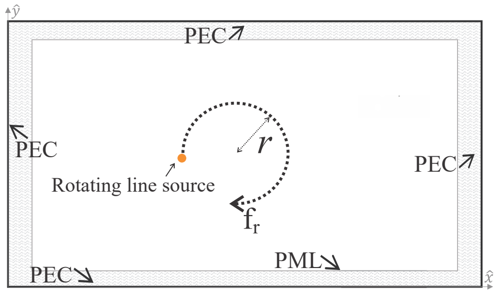
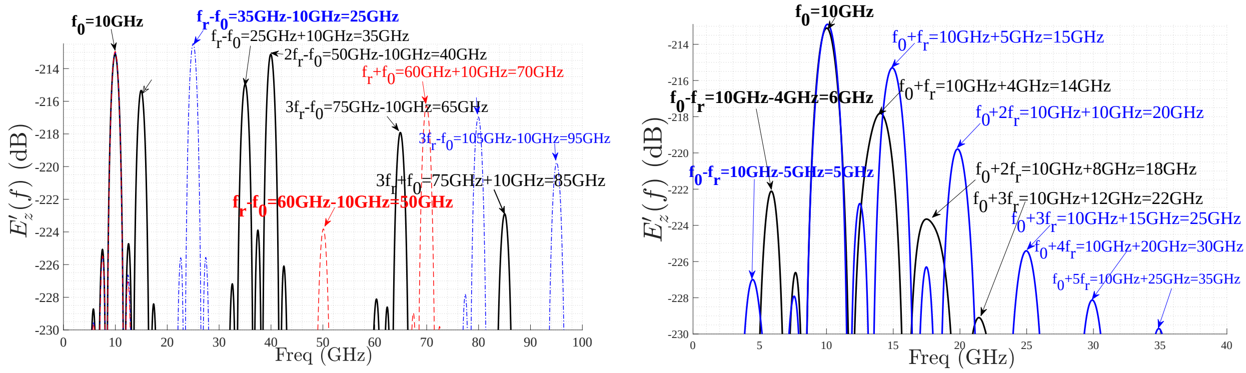
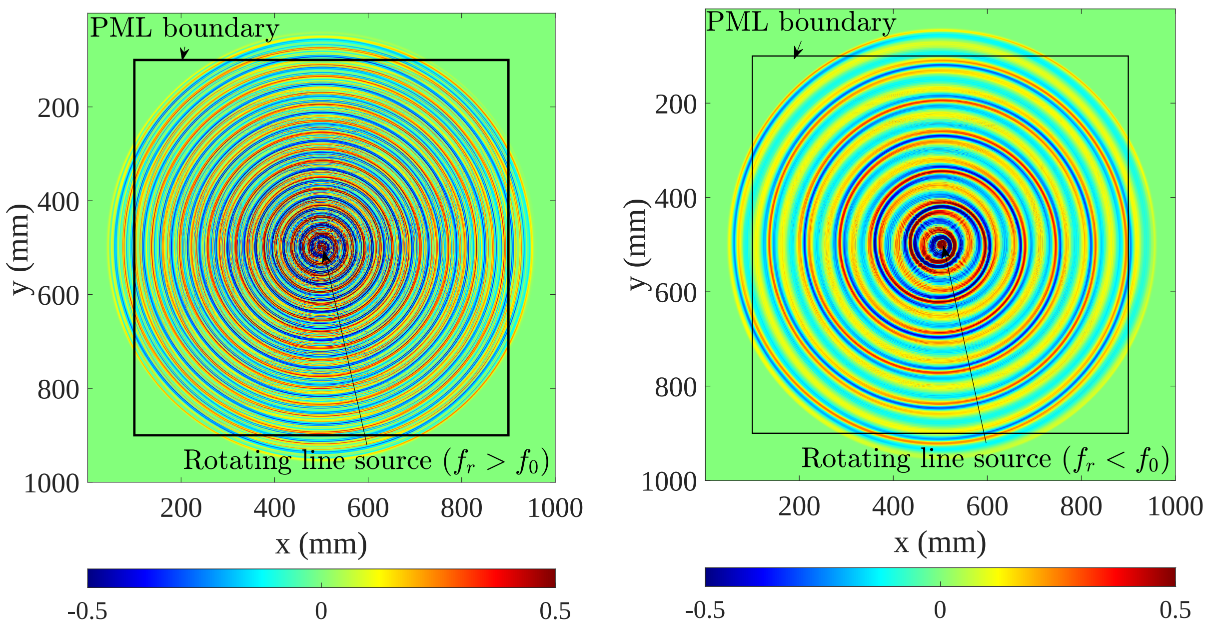


Fig. 6. Rotating line source.



(a) (b)

Fig. 7. Spectrum of observed signal, for rotating ideal line source, with frequency of rotation (a) $f_r > f_0$ (f_0=10GHz; f_r=25GHz, 35GHz, or 60GHz), and (b) $f_r < f_0$ (f_0=10GHz; f_r=4GHz or 5GHz).



(a) $f_r = 35\text{GHz}$ (b) $f_r = 4\text{GHz}$

Fig. 8. Simulated electric field distribution for a rotating ideal line source: (a) $f_r > f_0$ and (b) $f_r < f_0$.

C. Acceleration

Acceleration can be implemented in the proposed numerical approach by changing the speed of motion of an object in the FDTD time loop.

1) Accelerating observer: An observer is accelerating as illustrated in Fig. 9. As shown in Fig. 10, depending on the direction ($\pm\hat{x}$) of the observer motion (the plane wave is propagating toward $+\hat{x}$

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direction), the frequency of the signal decreases or increases with time, as one can expect. This results in clear chirp signals, which have applications in radars.

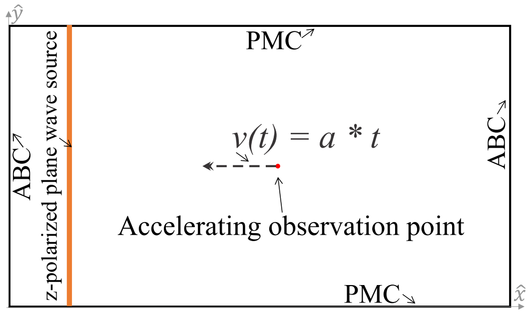

Fig. 9. Accelerating observation point.

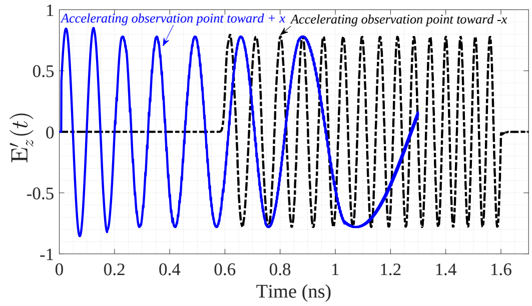

Fig. 10. Observed signals in the time domain, for accelerating observer.

2) Accelerating plane wave source: In this subsection, a monochromatic plane wave source is accelerating and the observer is at rest, as shown in Fig. 11. A distinction is made between the ideal plane wave source (made of current sources) and the resistive plane wave source (made of current sources with resistors having low resistance). The wave observed in the time domain is shown in Fig. 12 for the two cases (the source moves toward the observer). With the ideal source, which is not realistic, the field can grow without bounds if the source accelerates toward the observer. The field does not increase in the case of a resistive plane wave source. Fig. 13a, Fig. 13b, and Fig. 13c show the numerical field distributions at different time instants for an accelerating ideal plane wave source. Two chirp waves are propagating in $-\hat{x}$ and $+\hat{x}$ directions, with the speed of light c.

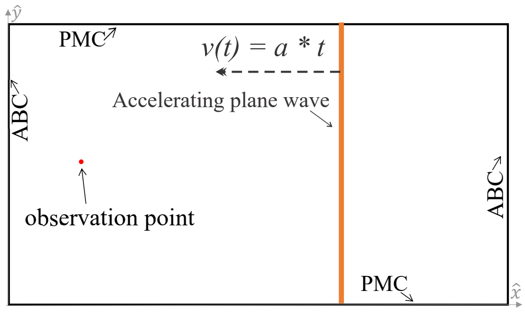

Fig. 11. Accelerating source.

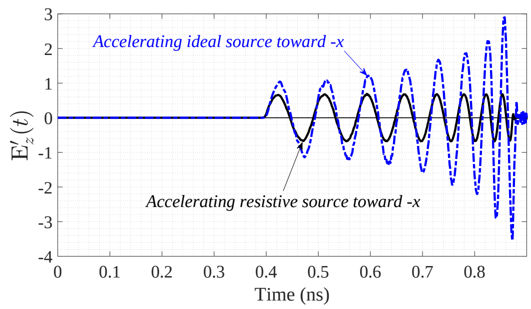

Fig. 12. Radiated waves in the time domain, for accelerating ideal or resistive plane wave sources.

3) Accelerating reflecting electromagnetic surface: A periodic reflecting electromagnetic surface, also called a partially reflecting surface (PRS), is now considered. The PRS is made of infinitely long metallic wires and it is accelerating toward the observer, as shown in Fig. 14. The observer and the monochromatic source are at rest. Fig. 15 shows the reflected wave in the time domain. One can note that the frequency of the signal increases logically with time (up-chirp).

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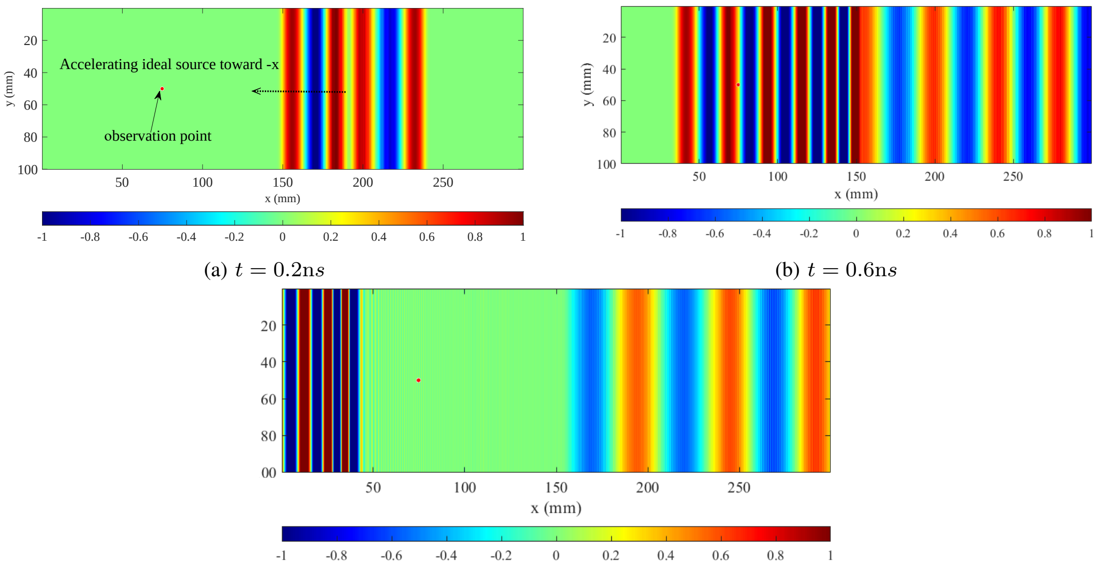

(a) $t = 0.2\text{n}s$ (b) $t = 0.6\text{n}s$

(c) $t = 1\text{n}s$

Fig. 13. Electric field distribution for accelerating ideal plane wave source toward $-x$ direction, at different time instants: (a) $t = 0.2\text{n}s$, (b) $t = 0.6\text{n}s$, and (c) $t = 1\text{n}s$. Two chirp waves are propagating in $-x$ and $+x$ directions, with the speed of light c.

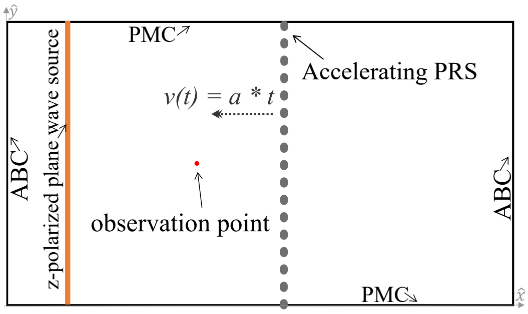

Fig. 14. Accelerating PRS in FDTD.

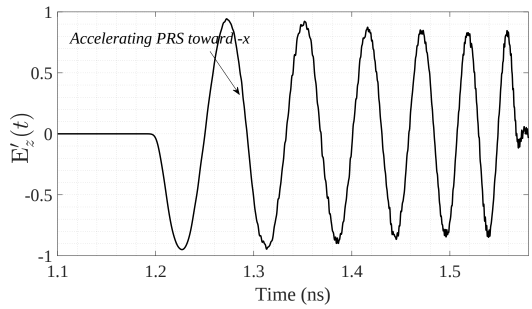

Fig. 15. Reflected plane wave from accelerating PRS.

IV. Conclusion

The concept of the change of the reference frame and the utilization of Voigt-Lorentz transformations in Maxwell's equations are usually applied in the literature for electromagnetic problems with uniform motion, where only one structure or system is moving with a single constant speed. In addition to these restrictions, important concerns have been raised about such an approach. In order to account for more non-uniform motions, a general electromagnetic approach for moving bodies has been proposed in this paper. The proposed method consists of using Maxwell's equations without modification and moving the objects numerically. With the proposed FDTD algorithm, non-uniform motions (vibration, rotation, acceleration) have been considered. Different problems have been analyzed numerically: a vibrating metallic slab, a rotating observer, and a rotating source. Physical insight has been provided by the spectrums of the reflected wave from a vibrating metallic slab, as well as the observed signal for a rotating observer or a rotating line source. This paper has also presented the analysis of accelerating observer, accelerating plane wave source, and accelerating electromagnetic reflecting surface. The ideal and resistive plane wave sources have been distinguished. It is shown that the amplitude of the field increases with an ideal (non-realistic) approaching source but does not increase with a resistive plane wave source. Furthermore, accelerating plane reflectors could be used to generate chirp signals which are used in radars.

Future work will concentrate on the utilization of the proposed method for the development of Doppler radars used in the detection of vital signs. With the FDTD method, it could be possible to model a realistic motion of the human chest based on respiration and heartbeat.

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