

Quantum Particle Swarm Optimization for Synthesis of Non-uniformly Spaced Linear Arrays with Broadband Frequency Invariant Pattern

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Abstract- This paper describes a method using Quantum Particle Swarm Optimization to obtain a broadband frequency invariant pattern for synthesis of nonuniformly spaced linear array of isotropic antennas. Two cases related to this work using QPSO have been studied, namely, in the first case, the generated frequency invariant far-field pattern is broadside in the vertical plane and in the second case, the far-field frequency invariant pattern is scanned in a particular direction. An effort is made such that the side lobe level and first null beam widths of the patterns are made similar to their related desired values. The two cases are presented in this paper to show the effectiveness of the proposed method in achieving the desired specifications. Even though, the developed method is utilized here for a linear array of isotropic antennas; it can be extended to other type of arrays. The performance of this algorithm is validated by duly comparing it with firefly algorithm.

Index Terms - Broadband array, frequency invariant pattern, first null beam width, quantum particle swarm optimization, side lobe level, firefly algorithm.

I. INTRODUCTION

Broadband antenna arrays [1] are widely used in many applications such as communication, acoustics and radio systems engineering etc. because of their ability to simultaneously broadcast multiple signals and traffic types in a wide bandwidth of transmission. Frequency invariant array pattern formation is advantageous for broadband signal acquisition and is usually designed for undistorted transmission of wideband signals [2]. Many broadband pattern synthesis techniques have been utilized

in the literature [3-6]. Array pattern synthesis techniques for designing of broadband FIR beam former with constant main lobe response and minimum side lobe level (SLL) have been discussed in [3-5]. All elements are fed coherently with a constant scan angle in phased array and phase shifters of all the elements are used to scan the beam in desired direction [6]. A technique for synthesis of narrowband and wideband array pattern using particle swarm optimization is detailed in [6].

Many methods have been proposed for the synthesis of nonuniformly spaced arrays [7-10]. Some techniques like genetic algorithm and neural-network based methodology and performance of raised power series arrays have been described in [7-8] for synthesizing wideband non-uniformly spaced arrays. The method described in [9] uses the asymptotic theory of unequally spaced arrays to produce ultra-wideband pattern with invariant main beam, but generates higher SLL. Another method for synthesis of nonuniformly spaced linear arrays with wide band frequency-invariant patterns using the generalized matrix pencil methods has been described in [10]. In this paper, a simple method is presented for broadband frequency invariant pattern using Quantum Particle Swarm Optimization (QPSO) algorithm for synthesis of nonuniformly spaced linear array antenna [11-13].

QPSO has recently enjoyed its success in the field of antenna arrays. To mention further, it was successfully employed [13] in failure correction of the radiation pattern in presence of faulty elements by newly generating the excitations. It also restored the side lobe level along with the wide null placement in the presence of faulty elements. But, in this paper, QPSO is employed in generating frequency invariant patterns and hence forth, the procedures, equations that are involved in generating these patterns are totally different from that of [13]. Moreover, these generations are tested here not only in broadside, but also in a particular direction in vertical plane for proving its versatility. In addition to it, necessary care is taken not only to control the side lobe level, but also the first null beam widths. This algorithm is used here to generate both the excitations amplitudes as well as the distance between the elements.

The reason for the choice of QPSO is because of its validity [14-16] over other algorithms. Moreover, in this paper, QPSO algorithm is compared with firefly algorithm (FA), a nature-inspired metaheuristic optimization algorithm which is described in [17-19]. Literature reports [18, 20] the superiority of FA than other basic algorithms.

II. APPROACH FOR FREQUENCY INVARIANT PATTERN SYNTHESIS

A non-uniformly spaced linear array of $2N$ isotropic antennas along Y -axis has been considered. Elements of the array are located symmetrically on each side of the origin as shown in fig.1. The equation of broadband array factor to generate free-space far-field pattern $FIP(f, \theta)$ in the vertical plane (Y - Z) with symmetric amplitude distributions is given by (1).

$$FIP(f, \theta) = \left[\sum_{n=1}^N 2I_n \cos\left[\frac{2\pi f}{c} d_n (\sin(\theta) - \sin(\theta_0))\right] \right] \quad (1)$$

where n =element number, d_n = distance from origin to centre of the n^{th} element, $\lambda=c/f$ = radiating wavelength, f = radiating frequency, θ is the polar angle of far-field measured from Z -axis (-90° to $+90^\circ$), θ_0 = scan angle, c = radiation velocity in free space, I_n = amplitude of the excitation current at n^{th} element, N is the total number of elements from one side of the origin. For a broadside pattern θ_0 will be equal to 0° . The range of frequency is represented by $[f_L, f_U]$. For synthesis of frequency invariant pattern with broadband, antenna array should maintain a constant main lobe (or first null beam width (FNBW)) in this frequency band.

Normalized absolute far-field in dB can be articulated in (2) as follows:

$$FIP_n(f, \theta) = 20 \log_{10} \left[\frac{|FIP(f, \theta)|}{|FIP(f, \theta)|_{\max}} \right] \quad (2)$$

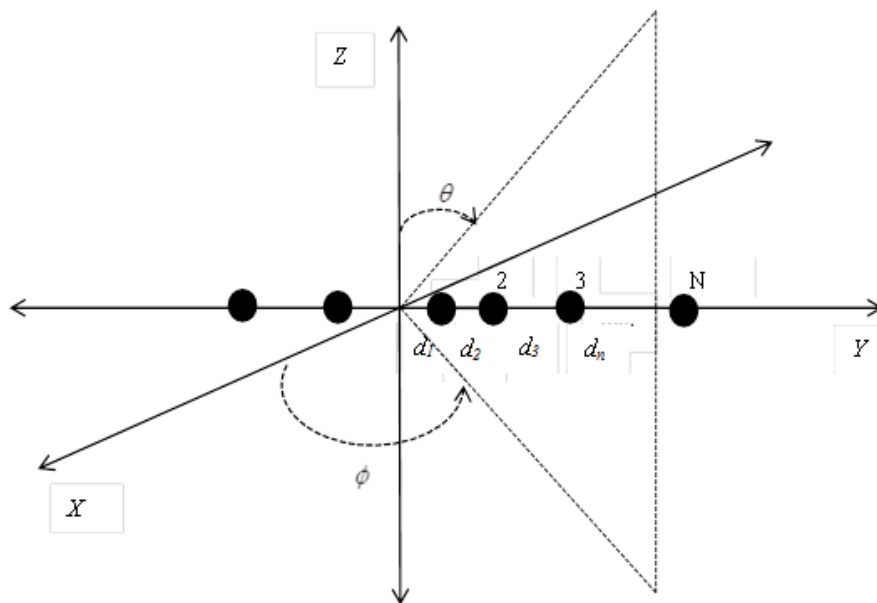


Fig. 1. Geometry of a non-uniformly spaced linear array of isotropic antennas along the Y -axis.

For scannable pattern in frequency band $[f_L, f_U]$, the response of array factor defined by the coefficients I_n at any frequency is scaled by f and shifted from θ to the desired angle θ_0 . But scaling by f results in an array pattern that becomes narrower as the frequency increases.

The problem is to find the set of parameters I_n and d_n of the array elements using QPSO that will minimize the following fitness function $Cost$, so that minimum SLL and frequency invariant beam pattern can be achieved over frequency band range for both cases.

$$Cost = \left[w_1 \times (mse|(F_1)|)^{0.5} + w_2 \times (mse(F_2))^{0.5} \right] \quad (3)$$

$$F_1 = d_p - o_p \quad (4)$$

$$F_2 = \sum_{m=1}^k F_{3,m} \tag{5}$$

$$F_3 = \begin{cases} |SLL_o - SLL_d|, & \text{if } \rightarrow SLL_o > SLL_d \\ 0, & \text{if } \rightarrow SLL_o \leq SLL_d \end{cases} \tag{6}$$

The coefficients w_1 and w_2 are the relative weights applied to each term in (3).

where k =sample points, d_p and o_p are vector of the points for desired and obtained pattern of the main beam for all the sample points in frequency band $[f_L, f_U]$. SLL_o and SLL_d are the value of SLL of obtained and desired pattern respectively. F_2 is the summation of all the sample points in frequency range $[f_L, f_U]$ for F_3 . *mse* is a network performance function. It measures the network's performance according to the mean of squared errors.

III. QUANTUM PARTICLE SWARM OPTIMIZATION ALGORITHM

A novel optimization algorithm QPSO, which is rooted on the fundamental theory of particle swarm and properties of quantum mechanics, was proposed in [11-13]. QPSO is stated only by the position vector and there is no velocity vector. The movable dealing of the particle in QPSO is different from that of the particle in standard PSO.

Several steps involved in QPSO are given below:

Step 1: Generate initial particles in the population randomly between the minimum and the maximum operating limits in the Di-dimensional space.

Step 2: Estimate the value of particle objective function.

Step 3: Current fitness value of the particle is compared with personal best (*pbest*) of every particle. If the current fitness value of the particle is superior, then assign the current fitness value to *pbest* and assign the current coordinates to *pbest* coordinates.

Step 4: Calculate the mean best position of all J particles.

$$mbest = \frac{1}{J} \sum_{i=1}^J pbest_i$$

Step 5: In the entire population determine the current best fitness and its coordinates. If the current best fitness value is superior to global best (*gbest*), then assign the current best fitness value to *gbest* and assign the current coordinates to *gbest* coordinates.

Step 6: The vector local focus of the particle is calculated using the following equation:

$$p_{id}^u = rand1_{id}^u * (pbest_{id}) + (1 - rand1_{id}^u) * (gbest)$$

Step 7: Update position (W_{id}) of the d^{th} dimension of the i^{th} particle using the following equations:

$$W_{id}^u = p_{id}^u + (-1)^{\text{ceil}(0.5 + \text{rand}2_{id}^u)} \times \sigma \\ \times |mbest - W_{id}^{u-1}| \times \log_e(1 / \text{rand}3_{id}^u)$$

If $W_{id}^u < W_{\min}^d$

then $W_{id}^u = W_{\min}^d + 0.25 * \text{rand}4_{id}^u * (W_{\max}^d - W_{\min}^d)$ (7)

If $W_{id}^u > W_{\max}^d$

then $W_{id}^u = W_{\max}^d - 0.25 * \text{rand}5_{id}^u * (W_{\max}^d - W_{\min}^d)$ (8)

where u is the current generation, $\text{rand}1$, $\text{rand}2$, $\text{rand}3$, $\text{rand}4$ and $\text{rand}5$ are uniform random numbers between 0 and 1. Equations (7) and (8) have been used along in each dimension (W_{\max}^d, W_{\min}^d) to squeeze the position. These techniques are essential to stop particles from eruption if they try to cross the required domain of interest.

Step 8: Repeat Steps from 2 to 7 till the maximum number of iterations being completed or when there is no further update of best fitness value.

The parameter p is the local attractor of each particle. To avoid untimely convergence $mbest$ is considered as the barycenter of all particles. σ is the contraction and expansion coefficient used for controlling the convergence speed and performance of the particle. This is the only one controlling parameter in QPSO, which can be tuned easily by linear variation in between minimum and maximum value or by trial and error method to control the convergence speed of the algorithm. The value of σ has been changed for different examples with frequency invariant pattern in this paper. Setting the value of σ in the interval (0.5, 0.8), can generates good results, see literature report [14].

For case 1: The value of σ with each iteration is given by for example of 20 element linear array:

$$\sigma = (\sigma_{\max} - \sigma_{\min}) \times \frac{(\max_{gen} - u)}{(\max_{gen} - 1)} + \sigma_{\min}$$

where $\sigma_{\max} = 0.8$ and $\sigma_{\min} = 0.7$ are the maximum and minimum value of σ , \max_{gen} is the maximum number of generation and u is the current generation.

For example of 26 elements the value of σ is given by:

$$\sigma = 0.71 + 0.1 * \text{rand};$$

For case 2: The value of σ for 20 elements:

$$\sigma = 0.7 + 0.1 * \text{rand}$$

The value of σ for 26 element of linear array:

$$\sigma=0.7+0.09*\text{rand}$$

where rand is the uniform random numbers between 0 and 1. Flow chart of QPSO algorithm is detailed in Fig.2.

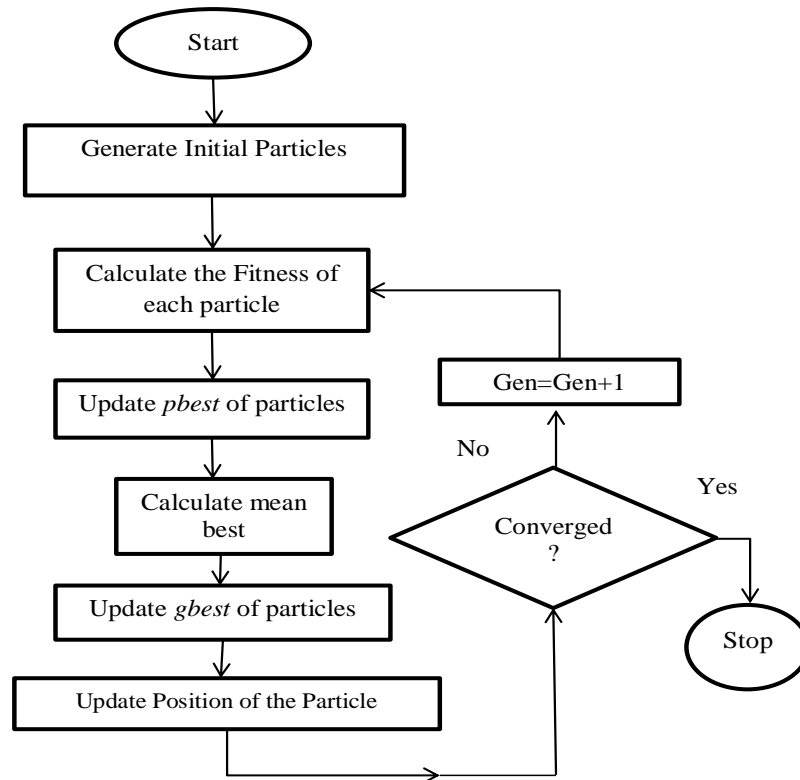


Fig. 2. Flow chart of quantum particle swarm optimization algorithm.

In addition to QPSO algorithm, Firefly Algorithm is also considered for the experiments for the sole purpose of comparisons between these two algorithms. This algorithm developed by the author Xin-She Yang is based on the idealized behaviour of the attraction characteristics of fireflies [17-19]. The flashing characteristics of the fireflies are described as follows:

- Fireflies are attracted to other ones irrespective of their sex.
- The less brighter firefly will move towards the brighter one as the attractiveness is proportional to their brightness. This attractiveness is inversely proportional to the distance as it decreases as the distance between the firefly's increases. If no one is brighter than a particular firefly, it moves randomly.
- The brightness or light intensity of a firefly is affected or determined by the cost function which is to be optimized.

The settings for the firefly algorithm parameters are: Randomization parameter = 1; Minimum value of attractiveness = 0.2 and absorption coefficient = 1.

IV. SIMULATION RESULTS

The numbers of element taken along the Y -axis for the simulation are 20 and 26 for both cases.

Operating frequency range is $f_L = 0.5$ GHz and $f_U = 1$ GHz. $\frac{f_U}{f_L} = 2$, $k=11$ (with constant increment of 0.05 GHz) sample points in frequency band and centre frequency is at 0.75 GHz.

Array elements are considered symmetric with respect to origin to reduce the number of variables in the simulation process. Desired SLL are fixed for both cases (20 elements and 26 elements) and it is -20 dB. FNBW for 20 elements linear array is fixed at 13° for both cases and for 26 elements linear array; it is 20° for case 1 and 29° for case 2.

Two cases are discussed below.

Case 1: For generation of far-field broadside frequency invariant pattern in vertical plane QPSO and FA are run for 300 iterations with a particle size of 30. Excitation current amplitude is varied in the range 0 to 1 for both elements. The spacing between the elements is varied from 0.5λ to 1.5λ (for 20 element linear array) and 0.5λ to 1.0λ (for 26 element linear array) at the highest in-band frequency of 1 GHz in order to suppress grating lobes at lower frequencies. All the current excitation phases are kept fixed at 0° . Due to symmetry, only ten excitation current amplitudes and ten distances from the origin to the centre of antenna element are to be optimized for 20 element linear array. Same thing has been done for 26 element linear array also. Algorithms has been utilized to generate a vector of 20 and 26 real values, first 10 and 13 values for current amplitude and last ten and thirteen values are for distance of antenna element from origin.

Case 2: For generation of scanned far-field frequency invariant pattern in principal vertical plane (YZ -plane) QPSO and FA are run for 300 iterations with a particle size of 30. Excitation current amplitude is varied in the range 0 to 1 for 20 and 26 elements. The spacing between the elements is varied from 0.4λ to 0.7λ (for 20 element linear array) and 0.3λ to 0.9λ (for 26 element linear array) at the highest in-band frequency of 1 GHz in order to suppress grating lobes at lower frequencies. All the current excitation phases are kept fixed at 25° (for 20 element linear array) and 45° (for 26 element linear array) for scanned far-field pattern.

Matlab software has been used here. Computational time is measured here using a PC with Intel(R) Core(TM) i5-4690 processor of clock frequency 3.50 GHz and 4 GB of RAM. Measured computational time for case 1 (20 element and 26 element for QPSO) are 633.68 seconds and 703.69 seconds and case 2 (20 element and 26 element for QPSO) are 665.43 seconds and 720.12 seconds. For firefly algorithm the computational time for case 1 are 638.16 and 667.72 and case 2 are 704.52 and 722.42 seconds.

Table I and II shows the current amplitude distribution and distance of antenna elements from origin for 20 elements and 26 elements. Fig. 3 and Fig. 4 shows the global best fitness value versus iterations obtained for both cases for 20 elements and 26 elements linear array. Fig. 5 and Fig. 6 shows the normalized current amplitude distribution using QPSO and FA for 20 and 26 elements. Fig. 7 shows the normalized frequency invariant power pattern in dB for 20 element linear arrays obtained from QPSO and FA for case 1. Fig. 8 shows the scanned normalized frequency invariant power pattern in dB for 20 element linear arrays obtained from QPSO and FA. Fig. 9 shows the normalized frequency invariant power pattern in dB for 26 element linear arrays using QPSO and FA for case 1. Fig. 10 shows the scanned normalized frequency invariant power pattern in dB for 26 element linear arrays using QPSO and FA.

It is observed from Fig. 3 and Fig. 4 that the algorithms reached to a value after which it is not converged after 200 iterations. QPSO converged well as compared to FA for both the cases. From the obtained pattern in Fig. 7 and Fig. 9, it is observed that error in obtained frequency invariant pattern and in desired pattern is less using QPSO and FA means constant frequency invariant pattern are obtained using both the algorithms; whereas in Fig. 8 the pattern obtained using FA provides higher side lobe level as compared to obtained pattern using QPSO algorithm. Moreover, in Fig.10 pattern obtained using FA provides more error in desired and obtained main beam patterns than QPSO.

TABLE I. EXCITATION CURRENT AMPLITUDE AND SPACING FROM ORIGIN

Element No	For Case 1 (20 Element)				For Case 2 (20 Element)			
	Current Excitation		Spacing (in λ)		Current Excitation		Spacing(in λ)	
	QPSO	FA	QPSO	FA	QPSO	FA	QPSO	FA
± 1	0.7128	0.8399	0.5060	0.5144	0.8475	0.8614	0.4200	0.4163
± 2	0.6067	0.9660	1.3514	1.4400	0.8319	0.9433	1.0089	0.9908
± 3	0.6473	0.8757	2.3260	2.3213	0.7244	0.6933	1.6532	1.6674
± 4	0.7282	0.6932	3.2446	3.2206	0.7975	0.8399	2.3492	2.2979
± 5	0.5867	0.4572	4.1545	4.2298	0.6036	0.6090	2.9940	2.9420
± 6	0.4717	0.4214	5.1110	5.1686	0.6606	0.7243	3.6853	3.6047
± 7	0.4254	0.5080	6.0553	6.1739	0.4740	0.4557	4.3113	4.2213
± 8	0.4308	0.4409	6.9527	7.1188	0.5150	0.5490	5.0110	4.8857
± 9	0.3216	0.1896	7.8158	8.0643	0.5374	0.6388	5.6972	5.5022
± 10	0.3663	0.4100	8.7086	9.0013	0.5754	0.6370	6.3179	6.0607

TABLE II. EXCITATION CURRENT AMPLITUDE AND SPACING FROM ORIGIN

Element No	For Case 1 (26 Element)				For Case 2 (26 Element)			
	Current Excitation		Spacing(in λ)		Current Excitation		Spacing(in λ)	
	QPSO	FA	QPSO	FA	QPSO	FA	QPSO	FA
± 1	0.8967	0.8571	0.5104	0.5115	0.6849	0.8607	0.3044	0.3045
± 2	0.9840	0.8671	1.2115	1.2744	0.7998	0.7709	0.8254	0.8256
± 3	0.7872	0.8631	1.8956	2.0477	0.6788	0.7378	1.3783	1.3888
± 4	0.7356	0.6790	2.5033	2.8339	0.6501	0.7111	1.9676	1.9542
± 5	0.6669	0.6669	3.1487	3.6280	0.5558	0.7127	2.4909	2.4472
± 6	0.6026	0.5710	3.8741	4.3514	0.5624	0.4789	3.0282	2.9982
± 7	0.4578	0.4677	4.4639	5.2408	0.4421	0.3588	3.6014	3.6629
± 8	0.3970	0.4908	5.1461	5.9887	0.4956	0.5281	4.1784	4.1673
± 9	0.3322	0.2359	5.9623	6.8003	0.4103	0.3435	4.7779	4.6534
± 10	0.0966	0.0000	6.7450	7.6297	0.5274	0.4144	5.2423	5.2569
± 11	0.6959	0.6011	7.6696	8.3301	0.3749	0.3566	6.0434	5.9064
± 12	0.8593	0.4861	8.3086	9.1841	0.0432	0.4064	6.4410	6.4595
± 13	0.9605	0.1728	8.8394	9.9364	0.0810	0.6314	7.0765	6.8373

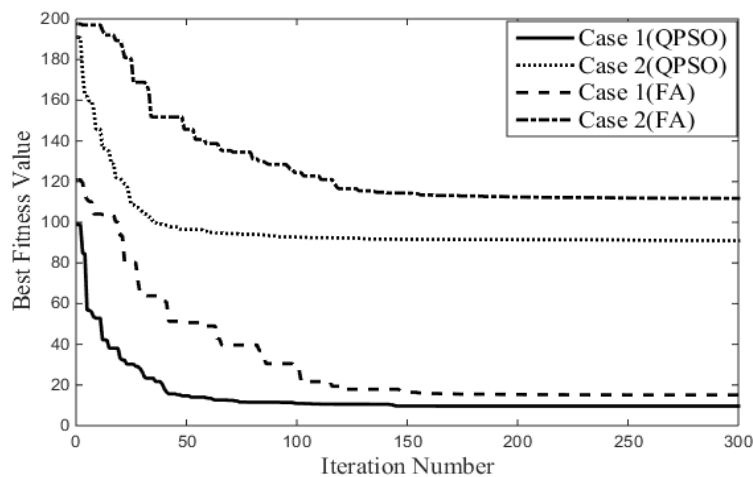


Fig. 3. Global best fitness value versus iteration number obtained from QPSO and FA for 20 element linear array.

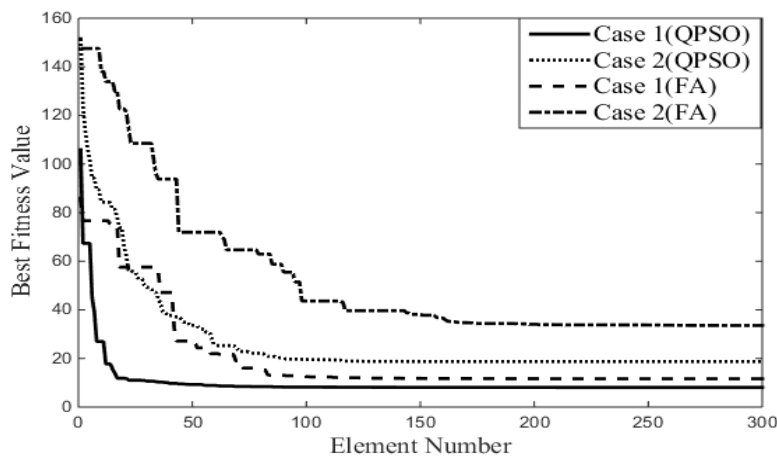


Fig. 4. Global best fitness value versus iteration number obtained from QPSO and FA for 26 element linear array.

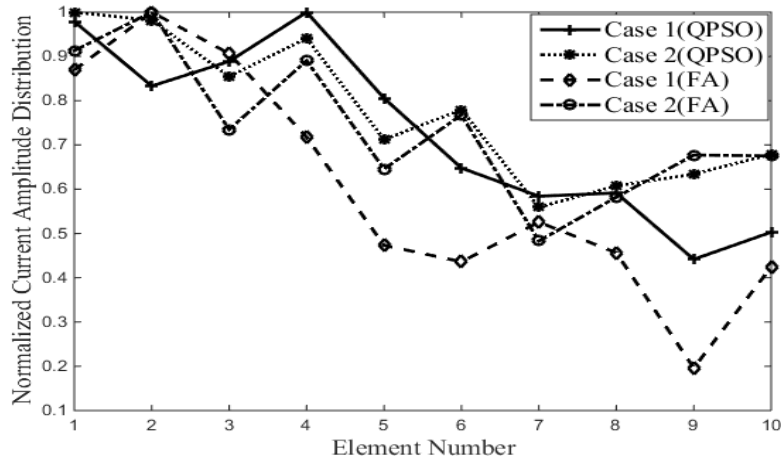


Fig. 5. Normalized current amplitude distribution for one side of the element from origin using QPSO and FA for 20 element linear array.

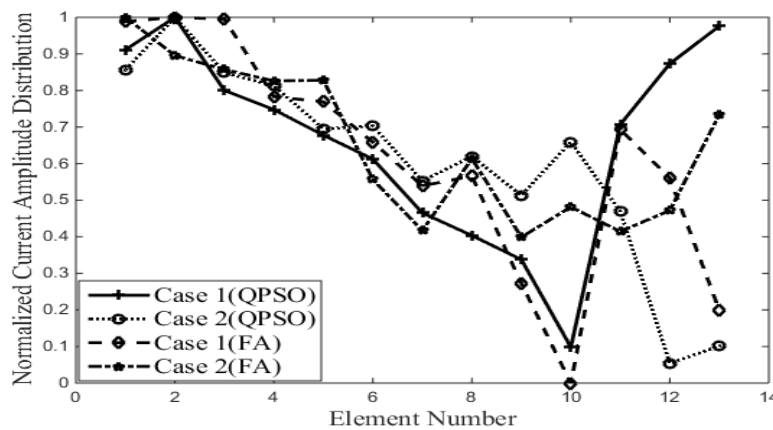
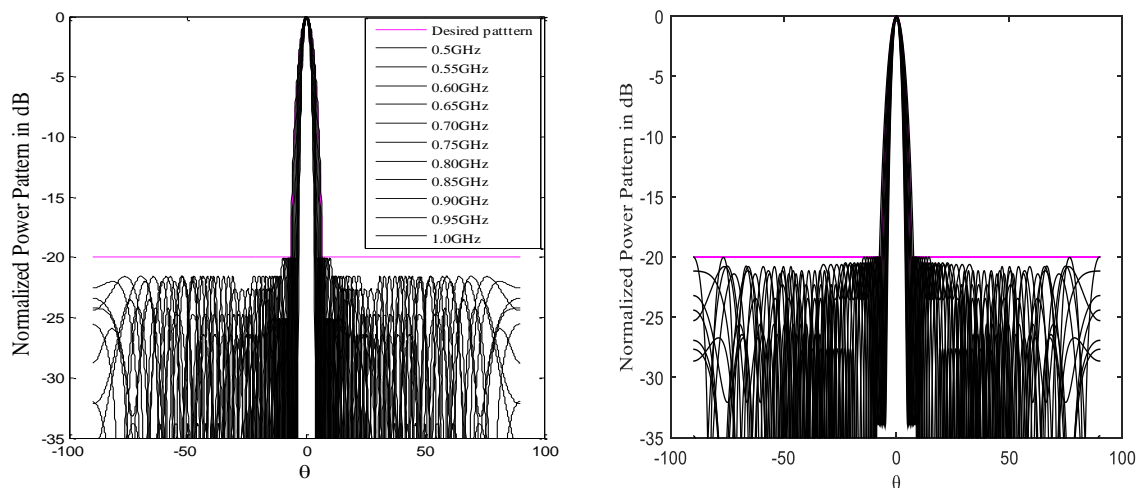


Fig. 6. Normalized current amplitude distribution for one side of the element from origin using QPSO and FA for 26 element linear array.



(a) Normalized Power Pattern in dB using QPSO

(b) Normalized Power Pattern in dB using FA

Fig. 7. Normalized Frequency invariant far-field pattern for synthesis of nonuniformly spaced broadband linear array antenna using QPSO and FA (case 1) for 20 element.

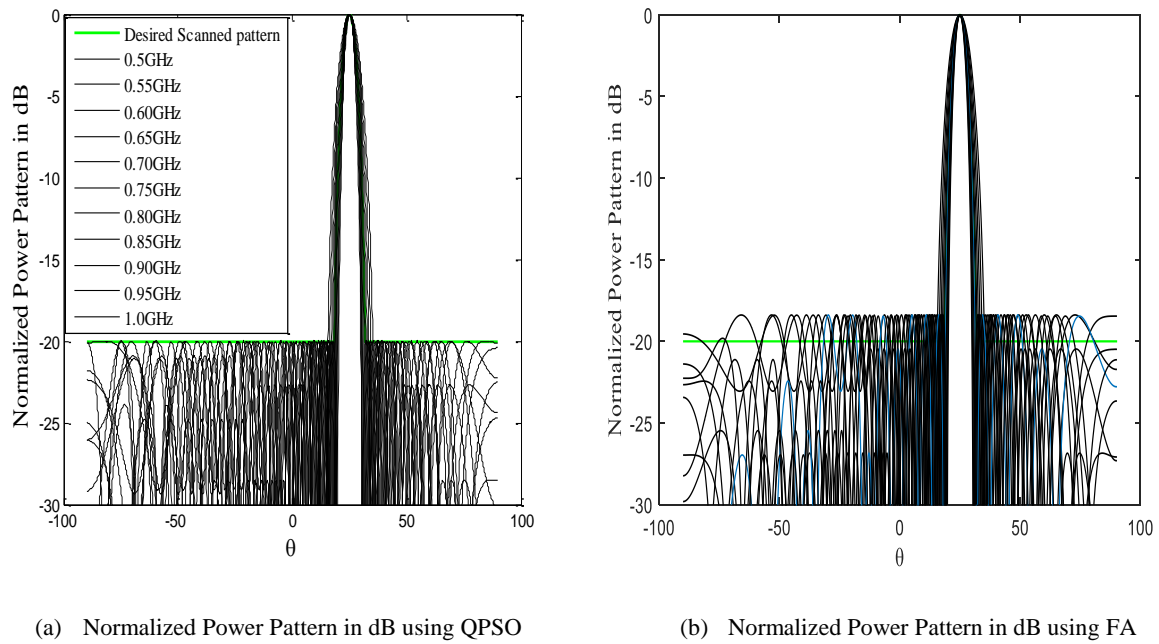


Fig. 8. Normalized Frequency invariant far-field pattern for synthesis of nonuniformly spaced broadband linear array antenna using QPSO and FA (case 2) for 20 element.

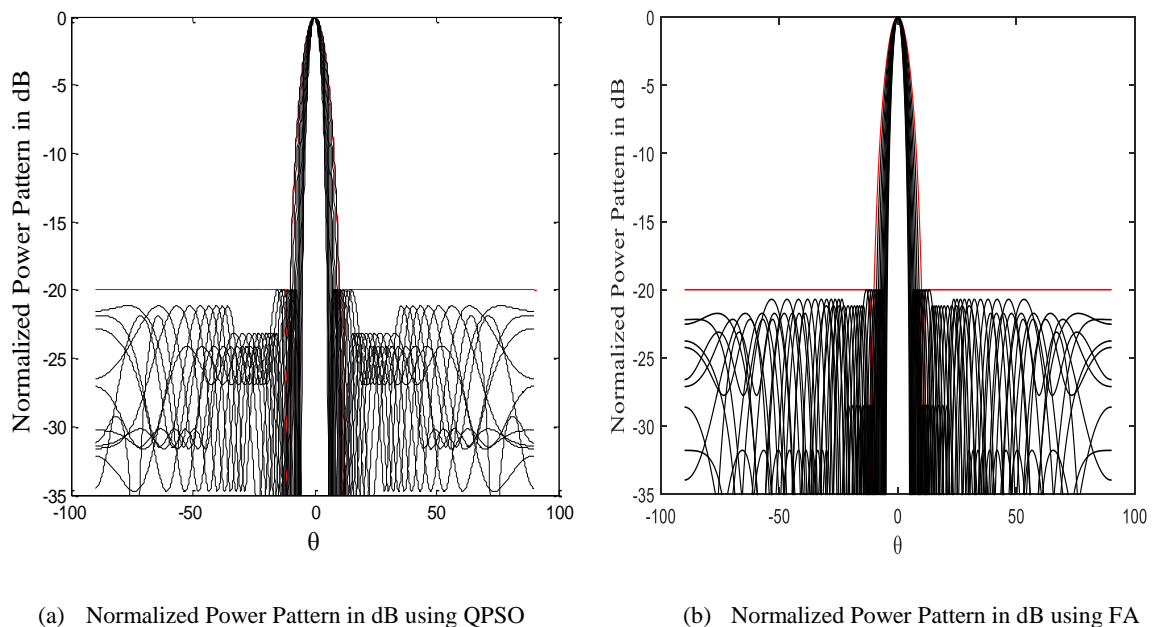


Fig. 9. Normalized Frequency invariant far-field pattern for synthesis of nonuniformly spaced broadband linear array antenna using QPSO and FA (case 1) for 26 element.

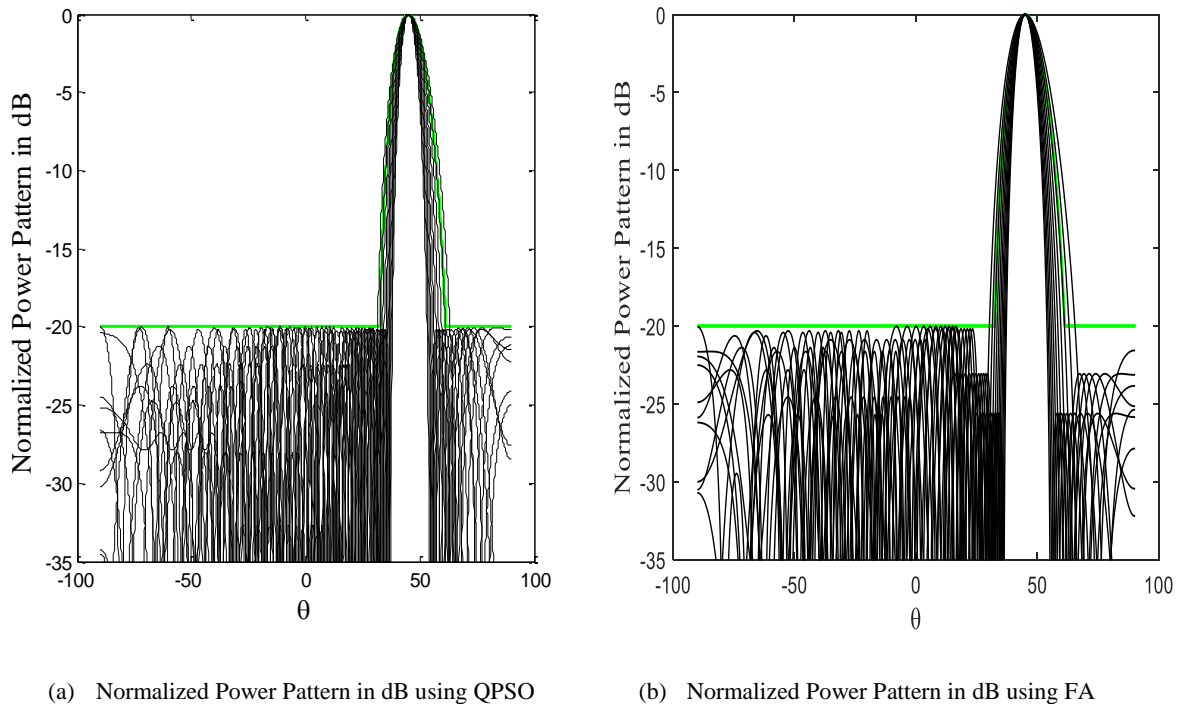


Fig. 10. Normalized Frequency invariant far-field pattern for synthesis of nonuniformly spaced broadband linear array antenna using QPSO and FA (case 2) for 26 element.

V. CONCLUSION

In this paper a method based on quantum particle swarm optimization algorithm has been successfully used for synthesis of a non-uniformly spaced linear array for broadband frequency invariant far-field pattern with fixed SLL and different constant FNBW (main beam). Simulated results obtained clearly show a great deal between the desired and obtained specifications. This technique works well for different width of main beam with different antennas for both cases. This technique is suitable to maintain constant SLL and nearly constant FNBW for both cases as SLL and FNBW are contradictory to each other. Algorithm finds suitable excitation current amplitudes and spacing between the elements to obtain the desired values. From the obtained results of the above figures it is proved that QPSO algorithm is more suitable for broadband frequency invariant pattern of linear antenna array. This work can be extended to other type of arrays. This proposed technique is suitable to generate frequency invariant pattern with different main beam shapes also.

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