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Vibration Analysis of a Magnetoelectroelastic Rectangular Plate Based on a Higher-Order Shear Deformation Theory

Abstract
Free vibration of a magnetoelectroelastic rectangular plate is investigated based on the Reddy's third-order shear deformation theory. The plate rests on an elastic foundation and it is considered to have different boundary conditions. Gauss's laws for electrostatics and magnetostatics are used to model the electric and magnetic behavior. The partial differential equations of motion are reduced to a single partial differential equation and then by using the Galerkin method, the ordinary differential equation of motion as well as an analytical relation for the natural frequency of the plate is obtained. Some numerical examples are presented to validate the proposed model and to investigate the effects of several parameters on the vibration frequency of the considered smart plate.

Keywords
Free vibration, magnetoelectroelastic smart plate, elastic foundation, Reddy's third order shear deformation theory.

Alireza Shooshtari [a]
Soheil Razavi [b]

[a] Mechanical Engineering Department, Bu-Ali Sina University, Hamedan, Iran.
Email: shooshta@basu.ac.ir

[b] Mechanical Engineering Department, Bu-Ali Sina University, Hamedan, Iran.
Email: soheilrazavi@outlook.com

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1 INTRODUCTION

Magnetoelectroelastic composite materials are a new class of smart materials which exhibit a coupling between mechanical, electric and magnetic fields and are capable of converting energy among these three energy forms. These materials have direct application in sensors and actuators, control of vibrations in structures, energy harvesting, etc.

Static and dynamic responses of piezoelectric plates have been investigated extensively in the past years (Alibeigloo and Kani, 2010; Behjat*et al.*, 2011; Rezaiee-Pajand and Sadeghi, 2013; Ghashochi-Bargh and Sadr, 2014; Rafiee *et al.*, 2014; Padoina *et al.*, 2015). Moon et al. (2007) designed a linear magnetostrictive actuator using Terfenol-D to control structural vibration. Hong (2007) studied the thermal vibration of magnetostrictive material embedded in laminated plate by using the generalized differential quadrature method. Later, the same author (2010)used the generalized differential quadrature method to compute the transient response of the laminated magnetostrictive plates under thermal vibration.

Pan (2001) studied multilayered magnetoelectroelastic plates analytically for the first time and derived exact solutions for three-dimensional magnetoelectroelastic plates. Pan and Heyliger (2002) derived analytical solutions for free vibrations of these smart plates. Pan and Heyliger (2003) studied the response of multilayered magnetoelectroelastic plates under cylindrical bending. Ramirez et al. (2006a) presented an approximate solution for the free vibration problem of two-dimensional magnetoelectroelastic laminated plates. Ramirez *et al.* (2006b) also determined natural frequencies of orthotropic magnetoelectroelastic graded composite plates by using a discrete layer model. Liu and Chang (2010) derived a closed form expression for the transverse vibration of a magnetoelectroelastic thin plate and obtained the exact solution for the free vibration of a two-layered $BaTiO_3$-$CoFe_2O_4$ composite. Single-layer approaches to static and free vibration analysis of magnetoelectroelastic laminated plates have also been introduced (Milazzo 2012, 2014a, 2014b; Milazzo and Orlando, 2012). Chen *et al.* (2014) studied the free vibration of multilayered magnetoelectroelastic plates under combined clamped/free boundary conditions. Moita *et al.* (2009) presented a higher-order finite element model for static and free vibration analyses of magnetoelectroelastic plates. Based on the nonlocal Love's shell theory, Ke *et al.* (2014) developed an embedded magnetoelectroelastic cylindrical nanoshell model to study the vibration response of these structures. Razavi and Shooshtari (2014) used Donnell shell theory to analyze the free vibration of magnetoelectroelastic curved panels.Li and Zhang (2014) studied the free vibration of a magnetoelectroelastic plate resting on a Pasternak foundation based on the Mindlin theory. Piovan and Salazar (2015) presented a one-dimensional model for dynamic analysis of magnetoelectroelastic curved beams. Based on three-dimensional elasticity theory, Xin and Hu (2015) derived semi-analytical solutions for free vibration of simply supported and multilayered magnetoelectroelastic plates. Nonlinear free and forced vibration of one-layered and multilayered magnetoelectroelastic rectangular plates based on the classical and first order shear deformation theory have also been investigated (Shooshtari and Razavi 2015a, 2015b; Razavi and Shooshtari, 2015). Li *et al.* (2014,2015) investigated dynamic response of magnetoelectroelastic nanoplate and nanobeam based on nonlocal Mindlin theory and nonlocal and Timoshenko beam theories, respectively. Ansari *et al.* (2015) developed a nonlocal geometrically nonlinear beam model for magnetoelectroelastic nanobeams subjected to external electric voltage, external magnetic potential and uniform temperature rise. Recently, Shooshtari and Razavi (2015c) investigated large amplitude vibration of laminated magnetoelectroelastic doubly-curved panels.

According to the published articles, there is not any study dealing with analytical study of free vibration of these smart plates based on a higher-order shear deformation theory. So, this study fills the gap in the analysis of magnetoelectroelastic rectangular plates. In this paper, free vibration of simply-supported, clamped and simply-supported/clamped magnetoelectroelastic rectangular plates resting on an elastic foundation is investigated based on the Reddy's third-order shear deformation theory. The Galerkin method is implemented to reduce the partial differential equation of motion to anordinary differential equation and then an analytical relation is obtained for the natural frequency. Some numerical examples are presented to validate the proposed model and to investigate the effects of several parameters such as foundation parameters, plate geometry, and the applied electric and magnetic potentials on the natural frequency of the considered smart plate.

2 THEORETICAL FORMULATION

Consider a rectangular plate resting on an elastic foundation with dimensions of $a \times b \times h$ as shown in Figure 1.

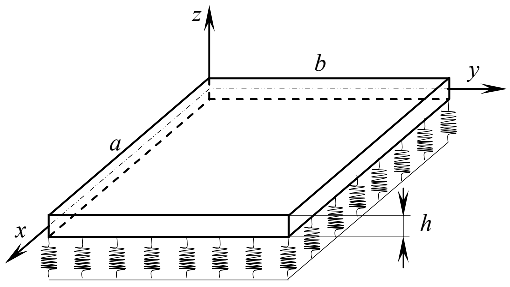

Figure 1: Schematic of a magnetoelectroelastic plate on an elastic foundation.

Based on the Reddy's third-order shear deformation theory, the displacement field of a composite plate is given as (Reddy, 2004):

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\,\theta_x(x,y,t) - \frac{4}{3h^2} z^3 \left(\theta_x + w_{0,x}\right) \\ v(x,y,z,t) &= v_0(x,y,t) + z\,\theta_y(x,y,t) - \frac{4}{3h^2} z^3 \left(\theta_y + w_{0,y}\right) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

Whereu_0, v_0, andw_0 are the displacements of the mid-surface alongx,y, and$z$ directions, respectively, andθ_x andθ_y are the rotations of a transverse normal about the$y$ and$x$ directions, respectively.

The linear strain-displacement relations based on the displacement field given in Eq. (1) are (Reddy, 2004):

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xz} \\ \gamma_{yz} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ \theta_x + w_{0,x} \\ \theta_y + w_{0,y} \\ u_{0,y} + v_{0,x} \end{Bmatrix} + z \begin{Bmatrix} \theta_{x,x} \\ \theta_{y,y} \\ 0 \\ 0 \\ \theta_{x,y} + \theta_{y,x} \end{Bmatrix} - \frac{4}{h^2} z^2 \begin{Bmatrix} 0 \\ 0 \\ \theta_x + w_{0,x} \\ \theta_y + w_{0,y} \\ 0 \end{Bmatrix} - \frac{4}{3h^2} z^3 \begin{Bmatrix} \theta_{x,x} + w_{0,xx} \\ \theta_{y,y} + w_{0,yy} \\ 0 \\ 0 \\ \theta_{x,y} + \theta_{y,x} + 2w_{0,xy} \end{Bmatrix} \tag{2}$$

Assuming that the electric and magnetic fields are applied along$z$-direction, the constitutive equations of a magnetoelectroelastic material can be written in the following form (Pan, 2001; Li and Zhang, 2014):

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xz} \\ \sigma_{yz} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{55} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xz} \\ \gamma_{yz} \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \phi_{,z} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & q_{31} \\ 0 & 0 & q_{32} \\ 0 & q_{24} & 0 \\ q_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \psi_{,z} \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xz} \\ \gamma_{yz} \\ \gamma_{xy} \end{Bmatrix} - \begin{bmatrix} \eta_{11} & 0 & 0 \\ 0 & \eta_{22} & 0 \\ 0 & 0 & \eta_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \phi_{,z} \end{Bmatrix} - \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \psi_{,z} \end{Bmatrix} \tag{4}$$

$$\begin{Bmatrix} B_x \\ B_y \\ B_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & q_{15} & 0 \\ 0 & 0 & q_{24} & 0 & 0 \\ q_{31} & q_{32} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xz} \\ \gamma_{yz} \\ \gamma_{xy} \end{Bmatrix} - \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \phi_{,z} \end{Bmatrix} - \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{22} & 0 \\ 0 & 0 & \mu_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \psi_{,z} \end{Bmatrix} \tag{5}$$

where $\{\sigma_x \ \sigma_y \ \sigma_{xz} \ \sigma_{yz} \ \sigma_{xy}\}^T$ is stress vector; $\{D_x \ D_y \ D_z\}^T$ and $\{B_x \ B_y \ B_z\}^T$ are the electric displacement and magnetic flux vectors, respectively; $[C_{ij}]$, $[\eta_{ij}]$ and $[\mu_{ij}]$ are the elastic, dielectric and magnetic permeability coefficient matrices, respectively; $[e_{ij}]$, $[q_{ij}]$ and $[d_{ij}]$ are the piezoelectric, piezomagnetic and magnetoelectric coefficient matrices, respectively; and ϕ and ψ are electric and magnetic potentials.

By neglecting in-plane inertia effects (i.e., $\ddot{u}_0 = \ddot{v}_0 = 0$) and assuming a constant value for the density of the plate, the equations of motion of a rectangular plate can be expressed in the following form (Reddy, 2004):

$$N_{x,x} + N_{xy,y} = 0 \tag{6}$$

$$N_{xy,x} + N_{y,y} = 0 \tag{7}$$

$$\begin{aligned} &\bar{Q}_{x,x} + \bar{Q}_{y,y} + \frac{4}{3h^2}\left(P_{x,xx} + 2P_{xy,xy} + P_{y,yy}\right) + \left(N_x w_{0,x} + N_{xy} w_{0,y}\right)_{,x} + \left(N_{xy} w_{0,x} + N_y w_{0,y}\right)_{,y} \\ &\quad - k_w w_0 + k_s \nabla^2 w_0 = I_0 \ddot{w}_0 - \frac{16}{9h^4} I_6 \left(\ddot{w}_{0,xx} + \ddot{w}_{0,yy}\right) + \frac{4}{3h^2} J_4 \left(\ddot{\theta}_{x,x} + \ddot{\theta}_{y,y}\right) \end{aligned} \tag{8}$$

$$\bar{M}_{x,x} + \bar{M}_{xy,y} - \bar{Q}_x = K_2 \ddot{\theta}_x - \frac{4}{3h^2} J_4 \ddot{w}_{0,x} \tag{9}$$

$$\bar{M}_{xy,x} + \bar{M}_{y,y} - \bar{Q}_y = K_2 \ddot{\theta}_y - \frac{4}{3h^2} J_4 \ddot{w}_{0,y} \tag{10}$$

where k_w and k_s are spring and shear coefficients of the elastic foundation, respectively and:

$$\begin{gathered}\bar{M}_x = M_x - \frac{4}{3h^2}P_x, \quad \bar{M}_y = M_y - \frac{4}{3h^2}P_y, \quad \bar{M}_{xy} = M_{xy} - \frac{4}{3h^2}P_{xy}, \\ \bar{Q}_x = Q_x - \frac{4}{h^2}R_x, \quad \bar{Q}_y = Q_y - \frac{4}{h^2}R_y, \quad I_0 = \rho_0 h, \quad I_2 = \rho_0 \frac{h^3}{12}, \\ I_4 = \rho_0 \frac{h^5}{80}, \quad I_6 = \rho_0 \frac{h^7}{448}, \quad J_4 = I_4 - \frac{4}{3h^2} I_6, \quad K_2 = I_2 - \frac{8}{3h^2} I_4 + \frac{16}{9h^4} I_6\end{gathered} \tag{11}$$

in which ρ_0 is the density of the material of the plate and the force and moment resultants are obtained by:

$$\begin{Bmatrix} N_{\alpha\beta} \\ M_{\alpha\beta} \\ P_{\alpha\beta} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha\beta} \begin{Bmatrix} 1 \\ z \\ z^3 \end{Bmatrix} dz, \qquad \begin{Bmatrix} Q_\alpha \\ R_\beta \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha z} \begin{Bmatrix} 1 \\ z^2 \end{Bmatrix} dz, \qquad (\alpha, \beta = x, y) \tag{12}$$

To express Eqs. (6) - (10) in terms of displacements and rotations, the resultants must be calculated from Eq. (12). To this end, $\sigma_{\alpha\beta}$ and $\sigma_{\alpha z}$ can be substituted from Eq. (3). However, since $\phi_{,z}$ and $\psi_{,z}$are unknown parameters, Eqs. (4) and(5) along with Gauss's laws for electrostatics and magnetostatics, i.e.,

$$D_{x,x} + D_{y,y} + D_{z,z} = 0, \qquad B_{x,x} + B_{y,y} + B_{z,z} = 0 \tag{13}$$

are used which results in:

$$\phi_{,zz} = \left[\lambda_1 A_3 + \lambda_2 A_1\right] z^2 + \lambda_1 A_4 + \lambda_2 A_2, \qquad \psi_{,zz} = \left[\lambda_1 A_1 + \lambda_3 A_3\right] z^2 + \lambda_1 A_2 + \lambda_3 A_4 \tag{14}$$

where

$$\lambda_1 = d_{33} \big/ \left(d_{33}^2 - \eta_{33}\mu_{33}\right), \quad \lambda_2 = -\mu_{33} \big/ \left(d_{33}^2 - \eta_{33}\mu_{33}\right), \quad \lambda_3 = -\eta_{33} \big/ \left(d_{33}^2 - \eta_{33}\mu_{33}\right) \tag{15a}$$

$$\begin{aligned} A_1 &= \frac{-4}{h^2}\left[e_{24}\left(\theta_{x,y} + w_{0,xy}\right) + e_{31}\left(\theta_{x,x} + w_{0,xx}\right) + e_{15}\left(\theta_{y,x} + w_{0,xy}\right) + e_{32}\left(\theta_{y,y} + w_{0,yy}\right)\right], \\ A_2 &= e_{24}\left(\theta_{x,y} + w_{0,xy}\right) + e_{15}\left(\theta_{y,x} + w_{0,xy}\right) + e_{31}\theta_{x,x} + e_{32}\theta_{y,y}, \\ A_3 &= \frac{-4}{h^2}\left[q_{24}\left(\theta_{x,y} + w_{0,xy}\right) + q_{31}\left(\theta_{x,x} + w_{0,xx}\right) + q_{15}\left(\theta_{y,x} + w_{0,xy}\right) + q_{32}\left(\theta_{y,y} + w_{0,yy}\right)\right], \\ A_4 &= q_{24}\left(\theta_{x,y} + w_{0,xy}\right) + q_{15}\left(\theta_{y,x} + w_{0,xy}\right) + q_{31}\theta_{x,x} + q_{32}\theta_{y,y} \end{aligned} \tag{15b}$$

Integrating the relations of Eq. (14) with respect to z, one obtains:

$$\phi_{,z} = \frac{1}{3}\left(\lambda_1 A_3 + \lambda_2 A_1\right) z^3 + \left(\lambda_1 A_4 + \lambda_2 A_2\right) z + \phi_0, \qquad \psi_{,z} = \frac{1}{3}\left(\lambda_1 A_1 + \lambda_3 A_3\right) z^3 + \left(\lambda_1 A_2 + \lambda_3 A_4\right) z + \psi_0 \tag{16}$$

$$\begin{aligned} \phi &= \frac{1}{12}\left(\lambda_1 A_3 + \lambda_2 A_1\right) z^4 + \frac{1}{2}\left(\lambda_1 A_4 + \lambda_2 A_2\right) z^2 + \phi_0 z + \phi_1, \\ \psi &= \frac{1}{12}\left(\lambda_1 A_1 + \lambda_3 A_3\right) z^4 + \frac{1}{2}\left(\lambda_1 A_2 + \lambda_3 A_4\right) z^2 + \psi_0 z + \psi_1 \end{aligned} \tag{17}$$

Where ϕ_0, ϕ_1, ψ_0 and ψ_1 are constants of the integration and are obtained by using the magnetoelectric boundary conditions on the two surfaces of the plate.

The magnetoelectroelastic body is poled along the z direction and subjected to an electric potential V_0 and a magnetic potential Ω_0 between the upper and lower surfaces of the plate. So, the magnetoelectric boundary conditions are stated as:

$$\begin{aligned} &\phi = 0, \quad \psi = 0 \qquad (z = -h/2) \\ &\phi = V_0, \quad \psi = \Omega_0 \qquad (z = h/2) \end{aligned} \tag{18}$$

Eqs. (17) and (18) give $\phi_0 = V_0/h$ and $\psi_0 = \Omega_0/h$. Then the gradients of electric and magnetic potentials are obtained from Eq. (16):

$$\phi_{,z} = \frac{1}{3}\left(\lambda_1 A_3 + \lambda_2 A_1\right) z^3 + \left(\lambda_1 A_4 + \lambda_2 A_2\right) z + \frac{V_0}{h}, \quad \psi_{,z} = \frac{1}{3}\left(\lambda_1 A_1 + \lambda_3 A_3\right) z^3 + \left(\lambda_1 A_2 + \lambda_3 A_4\right) z + \frac{\Omega_0}{h} \tag{19}$$

Now, the resultants are obtained by Eqs. (3), (12) and (19):

$$\begin{aligned} N_x &= h\left(C_{11} u_{0,x} + C_{12} v_{0,y}\right) + e_{31} V_0 + q_{31} \Omega_0, \\ N_y &= h\left(C_{12} u_{0,x} + C_{22} v_{0,y}\right) + e_{32} V_0 + q_{32} \Omega_0, \\ N_{xy} &= h C_{66}\left(u_{0,y} + v_{0,x}\right), \end{aligned} \tag{20}$$

$$\begin{aligned} Q_x &= \frac{2h}{3} C_{55}\left(w_{0,x} + \theta_x\right), \quad R_x = \frac{h^2}{20} Q_x, \\ Q_y &= \frac{2h}{3} C_{44}\left(w_{0,y} + \theta_y\right), \quad R_y = \frac{h^2}{20} Q_y, \end{aligned} \tag{21}$$

$$\begin{aligned} M_x &= \frac{h^3}{12}\left[C_{11}\theta_{x,x} + C_{12}\theta_{y,y} + e_{31}\left(\lambda_1 A_4 + \lambda_2 A_2\right) + q_{31}\left(\lambda_1 A_2 + \lambda_3 A_4\right)\right] + \\ &\frac{h^5}{80}\left[-\frac{4}{3h^2} C_{11}\left(\theta_{x,x} + w_{0,xx}\right) - \frac{4}{3h^2} C_{12}\left(\theta_{y,y} + w_{0,yy}\right) + \frac{1}{3} e_{31}\left(\lambda_1 A_3 + \lambda_2 A_1\right) + \frac{1}{3} q_{31}\left(\lambda_1 A_1 + \lambda_3 A_3\right)\right], \\ M_y &= \frac{h^3}{12}\left[C_{12}\theta_{x,x} + C_{22}\theta_{y,y} + e_{32}\left(\lambda_1 A_4 + \lambda_2 A_2\right) + q_{32}\left(\lambda_1 A_2 + \lambda_3 A_4\right)\right] + \\ &\frac{h^5}{80}\left[-\frac{4}{3h^2} C_{12}\left(\theta_{x,x} + w_{0,xx}\right) - \frac{4}{3h^2} C_{22}\left(\theta_{y,y} + w_{0,yy}\right) + \frac{1}{3} e_{32}\left(\lambda_1 A_3 + \lambda_2 A_1\right) + \frac{1}{3} q_{32}\left(\lambda_1 A_1 + \lambda_3 A_3\right)\right], \\ M_{xy} &= \frac{h^3}{15} C_{66}\left(\theta_{x,y} + \theta_{y,x}\right) - \frac{h^3}{30} C_{66} w_{0,xy} \end{aligned} \tag{22}$$

$$
\begin{aligned}
P_x &= \frac{h^5}{80}\left[C_{11}\theta_{x,x} + C_{12}\theta_{y,y} + e_{31}\left(\lambda_1 A_4 + \lambda_2 A_2\right) + q_{31}\left(\lambda_1 A_2 + \lambda_3 A_4\right)\right] + \\
&\quad \frac{h^7}{448}\left[-\frac{4}{3h^2}C_{11}\left(\theta_{x,x} + w_{0,xx}\right) - \frac{4}{3h^2}C_{12}\left(\theta_{y,y} + w_{0,yy}\right) + \frac{1}{3}e_{31}\left(\lambda_1 A_3 + \lambda_2 A_1\right) + \frac{1}{3}q_{31}\left(\lambda_1 A_1 + \lambda_3 A_3\right)\right], \\
P_y &= \frac{h^5}{80}\left[C_{12}\theta_{x,x} + C_{22}\theta_{y,y} + e_{32}\left(\lambda_1 A_4 + \lambda_2 A_2\right) + q_{32}\left(\lambda_1 A_2 + \lambda_3 A_4\right)\right] + \\
&\quad \frac{h^7}{448}\left[-\frac{4}{3h^2}C_{12}\left(\theta_{x,x} + w_{0,xx}\right) - \frac{4}{3h^2}C_{22}\left(\theta_{y,y} + w_{0,yy}\right) + \frac{1}{3}e_{32}\left(\lambda_1 A_3 + \lambda_2 A_1\right) + \frac{1}{3}q_{32}\left(\lambda_1 A_1 + \lambda_3 A_3\right)\right], \\
P_{xy} &= \frac{h^5}{105}C_{66}\left(\theta_{x,y} + \theta_{y,x}\right) - \frac{h^5}{168}C_{66}w_{0,xy}
\end{aligned}
\tag{23}
$$

Substituting Eqs. (20) – (23) into Eqs. (6) – (10) yield:

$$C_{11}u_{0,xx} + C_{66}u_{0,yy} + \left(C_{12} + C_{66}\right)v_{0,xy} = 0 \tag{24}$$

$$C_{66}v_{0,xx} + C_{22}v_{0,yy} + \left(C_{12} + C_{66}\right)u_{0,xy} = 0 \tag{25}$$

$$
\begin{aligned}
&\left\{L_1\frac{\partial}{\partial x} + L_2\frac{\partial^3}{\partial x^3} + L_3\frac{\partial^3}{\partial x^2 \partial y} + L_4\frac{\partial^3}{\partial x\,\partial y^2} + L_5\frac{\partial^3}{\partial y^3}\right\}\theta_x + \\
&\left\{L_6\frac{\partial}{\partial y} + L_7\frac{\partial^3}{\partial y^3} + L_8\frac{\partial^3}{\partial x^2 \partial y} + L_9\frac{\partial^3}{\partial x\,\partial y^2} + L_{10}\frac{\partial^3}{\partial x^3}\right\}\theta_y + \\
&\left\{L_{11}\frac{\partial^2}{\partial x^2} + L_{12}\frac{\partial^2}{\partial y^2} + L_{13}\frac{\partial^4}{\partial x^4} + L_{14}\frac{\partial^4}{\partial y^4} + L_{15}\frac{\partial^4}{\partial x^2 \partial y^2} + L_{16}\frac{\partial^4}{\partial x^3 \partial y} + L_{17}\frac{\partial^4}{\partial x\,\partial y^3} - k_w\right\}w_0 = \\
&\qquad \left\{I_0 - c_1^2 I_6\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\right\}\ddot{w}_0 + c_1 J_4\left\{\frac{\partial}{\partial x}\ddot{\theta}_x + \frac{\partial}{\partial y}\ddot{\theta}_y\right\}
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
&\left\{L_{18}\frac{\partial^2}{\partial x^2} + L_{19}\frac{\partial^2}{\partial y^2} + L_{20}\frac{\partial^2}{\partial x\,\partial y} - K_2\frac{\partial^2}{\partial t^2} + L_{21}\right\}\theta_x + \left\{L_{22}\frac{\partial^2}{\partial x^2} + L_{23}\frac{\partial^2}{\partial x\,\partial y}\right\}\theta_y = \\
&\qquad \left\{L_{24}\frac{\partial^3}{\partial x^3} + L_{25}\frac{\partial^3}{\partial x^2 \partial y} + L_{26}\frac{\partial^3}{\partial x\,\partial y^2} + L_{27}\frac{\partial}{\partial x} - c_1 J_4\frac{\partial^3}{\partial x\,\partial t^2}\right\}w_0
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
&\left\{L_{28}\frac{\partial^2}{\partial y^2} + L_{29}\frac{\partial^2}{\partial x\,\partial y}\right\}\theta_x + \left\{L_{30}\frac{\partial^2}{\partial x^2} + L_{31}\frac{\partial^2}{\partial y^2} + L_{32}\frac{\partial^2}{\partial x\,\partial y} - K_2\frac{\partial^2}{\partial t^2} + L_{33}\right\}\theta_y = \\
&\qquad \left\{L_{34}\frac{\partial^3}{\partial y^3} + L_{35}\frac{\partial^3}{\partial x^2 \partial y} + L_{36}\frac{\partial^3}{\partial x\,\partial y^2} + L_{37}\frac{\partial}{\partial y} - c_1 J_4\frac{\partial^3}{\partial y\,\partial t^2}\right\}w_0
\end{aligned}
\tag{28}
$$

where L_i (i=1,2,...,37) are constant coefficients which are functions of applied electric and magnetic potentials, foundation parameters, and material and geometrical properties of the plate and are given in Appendix A.

It can be seen that Eqs. (24) and (25) are decoupled from Eqs. (26) – (28). So, to study the transverse motion ofthe plate, it is sufficient to consider only Eqs. (26) – (28).Eqs. (27) and (28) constitute a set of linear equations in terms ofθ_x andθ_y. Algebraic solution of this equations results in:

$$\theta_x = \frac{A_3A_5 - A_2A_6}{A_1A_5 - A_2A_4} w_0, \quad \theta_y = \frac{A_1A_6 - A_3A_4}{A_1A_5 - A_2A_4} w_0 \tag{29}$$

whereA_i(i=1,...,6) are partial differential operators and are defined in Appendix B.

Substituting Eq. (29) into (26) one obtains the following partial differential equation for the transverse motion of the magnetoelectroelastic plate:

$$\begin{aligned}&\left\{\left[L_1\frac{\partial}{\partial x}+L_2\frac{\partial^3}{\partial x^3}+L_3\frac{\partial^3}{\partial x^2\partial y}+L_4\frac{\partial^3}{\partial x\,\partial y^2}+L_5\frac{\partial^3}{\partial y^3}\right]\left(A_3A_5-A_2A_6\right)+\right.\\&\left[L_6\frac{\partial}{\partial y}+L_7\frac{\partial^3}{\partial y^3}+L_8\frac{\partial^3}{\partial x^2\partial y}+L_9\frac{\partial^3}{\partial x\,\partial y^2}+L_{10}\frac{\partial^3}{\partial x^3}\right]\left(A_1A_6-A_3A_4\right)+\\&\left[L_{11}\frac{\partial^2}{\partial x^2}+L_{12}\frac{\partial^2}{\partial y^2}+L_{13}\frac{\partial^4}{\partial x^4}+L_{14}\frac{\partial^4}{\partial y^4}+L_{15}\frac{\partial^4}{\partial x^2\partial y^2}+L_{16}\frac{\partial^4}{\partial x^3\partial y}+L_{17}\frac{\partial^4}{\partial x\,\partial y^3}-k_w\right]\left(A_1A_5-A_2A_4\right)-\\&\left.\frac{\partial^2}{\partial t^2}\left[I_0-c_1^2I_6\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]\left(A_1A_5-A_2A_4\right)-c_1J_4\frac{\partial^2}{\partial t^2}\left[\frac{\partial}{\partial x}\left(A_3A_5-A_2A_6\right)+\frac{\partial}{\partial y}\left(A_1A_6-A_3A_4\right)\right]\right\}w_0=0\end{aligned} \tag{30}$$

which is expressed in terms of w_0.

Three boundary conditions are considered in the present study, which are simply-supported, clamped and combination of simply-supported and clamped edges, that is:

$$\begin{aligned}w_0 = w_{0,xx} = 0 \quad \text{at} \quad (x = 0,a),\\ w_0 = w_{0,yy} = 0 \quad \text{at} \quad (y = 0,b)\end{aligned} \quad \text{All edges are simply-supported (SSSS)} \tag{31a}$$

$$\begin{aligned}w_0 = w_{0,x} = 0 \quad \text{at} \quad (x = 0,a),\\ w_0 = w_{0,y} = 0 \quad \text{at} \quad (y = 0,b)\end{aligned} \quad \text{All edges are clamped (CCCC)} \tag{31b}$$

$$\begin{aligned}w_0 = w_{0,xx} = 0 \quad \text{at} \quad (x = 0,a),\\ w_0 = w_{0,y} = 0 \quad \text{at} \quad (y = 0,b)\end{aligned} \quad \text{Simply-supported along } x\text{-axis and clamped along } y\text{-axis (SCSC)} \tag{31c}$$

The transverse displacement for each of these boundary conditions can be obtained by:

$$w_0 = hW(t)\sin(m\pi x/a)\sin(n\pi y/b) \text{ for SSSS boundary condition} \tag{32a}$$

$$w_0 = hW(t)\{[\sin(\alpha_m x) - \sinh(\alpha_m x) - \zeta_m(\cos(\alpha_m x) - \cosh(\alpha_m x))]\times [\sin(\alpha_n y) - \sinh(\alpha_n y) - \zeta_n(\cos(\alpha_n y) - \cosh(\alpha_n y))]\} \quad \text{for CCCC boundary condition} \tag{32b}$$

$$w_0 = hW(t)\{\sin(m\pi x/a)\times [\sin(\alpha_n y) - \sinh(\alpha_n y) - \zeta_n(\cos(\alpha_n y) - \cosh(\alpha_n y))]\} \quad \text{for SCSC boundary condition} \tag{32c}$$

in which

$$\alpha_m = \frac{(2m+1)\pi}{2}, \quad \zeta_m = \frac{\sin(\alpha_m) - \sinh(\alpha_m)}{\cos(\alpha_m) - \cosh(\alpha_m)}, \qquad \alpha_n = \frac{(2n+1)\pi}{2}, \quad \zeta_n = \frac{\sin(\alpha_n) - \sinh(\alpha_n)}{\cos(\alpha_n) - \cosh(\alpha_n)}, \tag{33}$$

where (m,n) denotes the mode of vibration and $W(t)$ is unknown function in terms of time (t).

Substituting Eqs. (32a) – (32c)into Eq. (30) and employing the orthogonality of trigonometric functions, the following ordinary differential equationis obtained for each boundary condition:

$$M_{eq}\ddot{W} + K_{eq}W = 0 \tag{34}$$

in which the terms containing d^4W/dt^4 and d^6W/dt^6 are neglected. In this equation, M_{eq} and K_{eq} are the equivalent mass and stiffness of the system, respectively.

3 RESULTS

To validate the present study, some numerical examples are presentedand the results are compared with the published ones. As a first comparison, an isotropic simply-supportedsquare plate is considered and the dimensionless frequencies for different length-to-thickness ratios are obtained. The dimensionless frequencies are obtained by using $\varpi = \omega_0(a^2/h)\sqrt{\rho_0/E}$, where E is the Young's modulusof the plate and $\omega_0 = (K_{eq}/M_{eq})^{1/2}$ is the circular natural frequency.The results are shown in Table 1 and compared with the results of Vel and Batra (2004) based on the three-dimensional approach, Hosseini-Hashemi*et al.* (2011) based on the third-order shear deformation plate theory, and Kiani*et al.* (2012) based on the first-order shear deformation theory. It is seen that there is acceptable accuracy for the thick case ($a/h = \sqrt{10}$) and perfect agreements for the relatively thick ($a/h = 10$) and the thin ($a/h = 50$) plates areobserved.

Method	a/h		
	$\sqrt{10}$	10	50
Vel and Batra (2004)	4.6582	5.7769	-
Hosseini-Hashemi*et al.* (2011)	4.6225	5.7694	-
Kiani*et al.* (2012)	-	5.7693	5.9647
Present study	4.4473	5.7646	5.9647

Table 1: Comparison of dimensionless fundamental frequency of asimply-supportedsquare plate ($\nu = 0.3$).

As a second comparison, a simply-supported isotropic thin plate with different aspect ratios is considered. The dimensionless frequencies are obtained by $\omega=\omega_0 a^2\sqrt{\rho_0 h/D}$ in which D is the flexural rigidity and $D=Eh^3/(12(1-\nu^2))$. Table 2 shows the results.

Method	a/b				
	0.4	2/3	1.0	1.5	2.5
Leissa (1973)	11.4487	14.2561	19.7392	32.0762	71.5564
Present study	11.4487	14.2561	19.7391	32.0760	71.5537

Table 2: Comparison of dimensionless fundamental frequency of a simply-supportedrectangular plate (ν = 0.3, a/h=1000).

Table 3 shows first four dimensionless frequencies of clamped (CCCC) and simply-supported/clamped (SCSC) square thin plates. The frequencies are obtained by $\omega=\omega_0\left(a^2/\pi^2\right)\sqrt{\rho_0 h/D}$ and compared with the values reported by various authors. It is seen that the proposed model predicts the frequencies precisely.

Method	SCSC				CCCC			
	ω_1	ω_2	ω_3	ω_4	ω_1	ω_2	ω_3	ω_4
Kim *et al.* (1993)	2.9333	5.5466	7.0242	9.5833	3.6460	7.4362	7.4362	10.9644
Woo *et al.* (2003)	2.9306	5.5469	7.0208	9.5831	3.6448	7.4373	7.4374	10.9650
Eftekhari and Jafari (2013)	2.9333	5.5466	7.0242	9.5833	3.6460	7.4362	7.4362	10.9643
Present study	2.9219	5.5643	7.0282	9.6122	3.6315	7.4615	7.4615	11.0383

Table 3: First four dimensionless frequencies of square plates with different boundary conditions (ν = 0.3, a/h=1000).

Table 4 shows the dimensionless fundamental frequencies $\omega=\omega_0 a^2\sqrt{\rho_0 h/D}$ of a square isotropic plate with a/h = 100 resting on an elastic foundation. The dimensionless parameters of the foundation are defined as $K_w=k_w a^4/D$ and $K_s=k_s a^2/D$ .It is observed that the results are in good agreement with the accurate results reported by Hasani Baferani *et al.* (2011).It is worth noting that the dimensionless shear coefficient (K_s) has more effect on the natural frequency.Moreover, it is observed from Tables 3 and 4 that clamped edges increase natural frequencies.

(K_w,K_s)	Boundary condition	Method: Lam *et al.* (2000)	Hasani Baferani *et al.* (2011)	Present study
(0,0)	SSSS	19.74	19.7374	19.7320
	SCSC	28.95	28.9441	28.8274
(0,100)	SSSS	41.62	48.6149	48.6101
	SCSC	54.68	54.6742	55.1384
(100,0)	SSSS	22.13	22.1261	22.1209
	SCSC	30.63	30.6229	30.5123
(100,100)	SSSS	49.63	49.6327	49.6279
	SCSC	55.59	55.5811	56.0377

Table 4: Dimensionless fundamental frequency of square isotropic plates resting on elastic foundation ($\nu = 0.3$).

As the last comparison, three piezoelectric, piezomagnetic and isotropic square plates with simply-supported boundary condition are considered and two firstdimensionlessfrequencies of these plates are obtained. Table 5 shows the results. The considered piezoelectric, piezomagnetic and isotropic plates are of $BaTiO_3$, $CoFe_2O_4$ and aluminum materials, respectively. The $BaTiO_3$ (shown with B) and $CoFe_2O_4$(shown with F) plates are thick with $a = b = 1$ m and $h = 0.3$ m and their material properties are given by Wu and Lu (2009). However, the aluminum plate (shown with Al) is thin with $a = b = 300$ mm and $h = 1$ mm.The dimensionless frequencies of $BaTiO_3$ and $CoFe_2O_4$ are calculated by using $\omega = \omega_0 a\sqrt{\rho_0/C_{\max}}$ where $C_{\max}$ is the maximum value of the stiffness coefficient of the plate, whereas The dimensionless frequencies of aluminum plate are obtained by $\omega = \omega_0 a^2\sqrt{\rho_0 h/D}$ .Again, there is a good agreement between the results.

Method	Mode (m,n): (1,1) B	(1,1) F	(1,1) Al	(2,1) B	(2,1) F	(2,1) Al
Ribeiro (2005)	-	-	19.7392	-	-	49.3480
Wu and Lu (2009)	1.2523	1.0212	-	2.3003	1.9747	-
Moita *et al.* (2009)	1.2629	1.1358	-	2.4649	2.1075	-
Present study	1.2349	1.1048	19.7384	2.2857	1.9571	49.3430

Table 5: Dimensionless frequencies of several square plates.

Effects of aspect ratio, and the applied electric and magnetic potentials on the dimensionless fundamental frequencies of a magnetoelectroelastic plate with different boundary conditions are studied and the results are shown in Table 6. The dimensionless frequencies are obtained by

$\omega = \omega_0 a\sqrt{\rho_0/C_{\max}}$. The material properties of the magnetoelectroelastic plate are (Li and Zhang, 2014): $C_{11} = 226\times10^9$ Nm^{-2}, $C_{12} = 124\times10^9$ Nm^{-2}, $C_{22} = 216\times10^9$ Nm^{-2}, $C_{44} = C_{55} = 44\times10^9$ Nm^{-2}, $C_{66} = 51\times10^9$ Nm^{-2}, $e_{32} = e_{31} =$ -2.2 Cm^{-2}, $q_{32} = q_{31} = 290.2$ NA^{-1}m^{-1}, $\eta_{33} = 6.35\times10^{-9}$ C^{2}N^{-1}m^{-2}, $d_{33} = 2737.5\times10^{-12}$ NsV^{-1}C^{-1}, $\mu_{33} = 83.5\times10^{-6}$ Ns^{2}C^{-2}, and $\rho_0 = 5500$ kgm^{-3}.

Boundary Condition	a/b	V_0 (10^8 V)		Ω_0 (10^6 A)	
		0	+1	0	+1
SSSS	0.5	0.343322939	0.343322938	0.343322939	0.343322939
	1.0	0.535860885	0.535860883	0.535860885	0.535860887
	2.0	1.233226423	1.233226400	1.233226423	1.233226453
SCSC	0.5	0.380853054	0.380853053	0.380853054	0.380853055
	1.0	0.774485196	0.774485191	0.774485196	0.774485204
	2.0	2.270502531	2.270502390	2.270502531	2.270502717
CCCC	0.5	0.675570089	0.675570085	0.675570089	0.675570094
	1.0	0.962062272	0.962062261	0.962062272	0.962062287
	2.0	2.342843729	2.342843576	2.342843729	2.342843931

Table 6: Dimensionless fundamental frequencies of a magnetoelectroelastic rectangular plate($h = 1$ mm, $a/h = 10$).

It is noticed that increasingthe aspect ratio increases the dimensionless frequency of the magnetoelectroelastic plate. Moreover, Table 6 shows that increasing the electric potential decreases the dimensionless frequency of the magnetoelectroelastic plate whereas magnetic potential increases the dimensionless frequency. It is also noticeable that potentials effects on dimensionless frequency are more significant in plates with higher aspect ratios and plates with clamped edges.

Table 7 shows the effects of a/h ratio and foundation parameters on the dimensionless frequencies of a magnetoelectroelastic square plate. In this table, the dimensionless frequencies are obtained by $\omega = \omega_0 a\sqrt{\rho_0/C_{\max}}$ and dimensionless foundation parameters are obtained by $\bar{K}_w = k_w a^4/\left(C_{\max} h^3\right)$ and $\bar{K}_s = k_s a^2/\left(C_{\max} h^3\right)$. The magnetoelectric boundary condition is considered to be closed-circuit meaning that in Eq. (18), $V_0 = \Omega_0 = 0$ is substituted. It is seen that a/h ratio tends to decrease the dimensionless frequency. Foundation parameters increase the natural frequencies because the presence of elastic foundation results in the increase of the stiffness of the system. It is also obvious that the dimensionless shear coefficient ($\bar{K}_s$) has more effect on the natural frequencies.In addition, it is observed that similar to the results of Tables 3 and 4, clamped edges increase the dimensionless frequencies.

Boundary condition	$(\bar{K}_w, \bar{K}_s)$	a/h	Mode (m,n)		
			(1,1)	(1,2)	(2,2)
SSSS	(0,0)	50	0.1131	0.2792	0.4492
		100	0.0566	0.1402	0.2261
	(10,0)	50	0.1295	0.2863	0.4536
		100	0.0649	0.1437	0.2283
	(0,10)	50	0.3028	0.5244	0.7188
		100	0.1515	0.2627	0.3606
	(10,10)	50	0.3093	0.5282	0.7216
		100	0.1547	0.2646	0.3620
SCSC	(0,0)	50	0.1639	0.3909	0.5374
		100	0.0821	0.1964	0.2706
	(10,0)	50	0.1757	0.3960	0.5411
		100	0.0880	0.1989	0.2725
	(0,10)	50	0.3398	0.6146	0.7946
		100	0.1698	0.3075	0.3985
	(10,10)	50	0.3456	0.6178	0.7971
		100	0.1727	0.3092	0.3998
CCCC	(0,0)	50	0.2046	0.4158	0.6187
		100	0.1025	0.2089	0.3114
	(10,0)	50	0.2142	0.4206	0.6285
		100	0.1073	0.2113	0.3156
	(0,10)	50	0.3744	0.6383	0.8631
		100	0.1870	0.3194	0.4369
	(10,10)	50	0.3797	0.6415	0.8831
		100	0.1897	0.3210	0.4404

Table 7: Dimensionless frequencies of a magnetoelectroelastic square plate ($h = 1$ mm).

Figures 2 and 3 show the effects of shear coefficient of foundation and a/h ratio on the natural frequencies of magnetoelectroelastic plates, respectively. It can be seen that for fixed material and geometric properties, clamped plate has the most natural frequency among the considered plates. Moreover, as it was also shown above, foundation parameter increases the natural frequency whereas the a/h ratio decreases it.

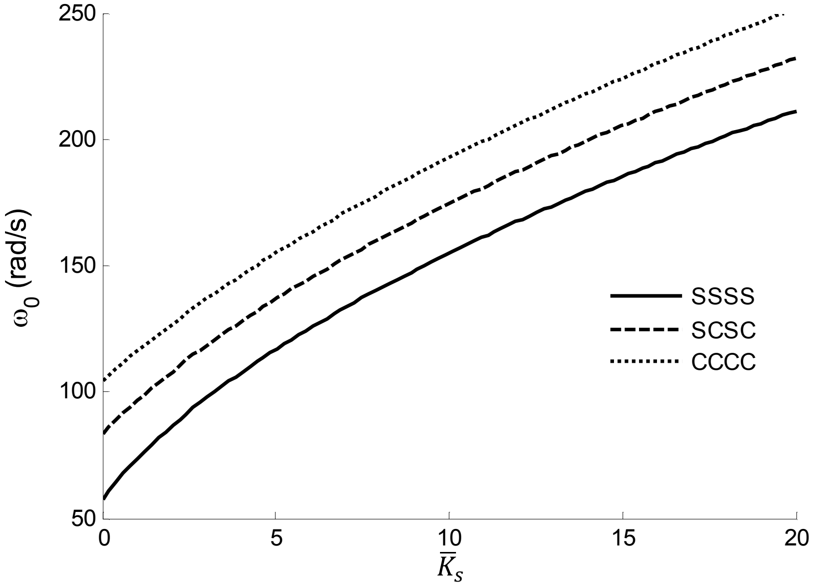

Figure 2: Effect of shear coefficient of foundation on the fundamental natural frequency of closed-circuit magnetoelectroelastic square plates ($a/h = 25,\ \bar{K}_w = 0$).

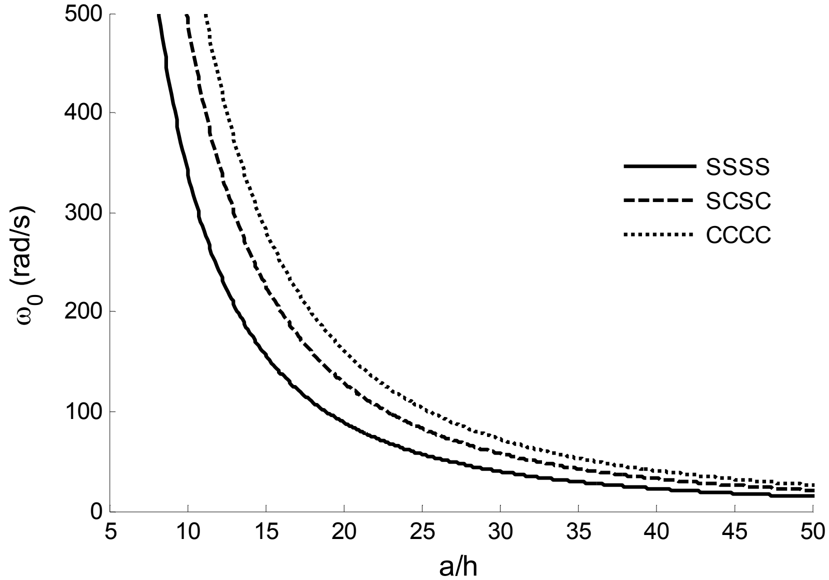

Figure 3: Effect of length-to-thickness on the fundamental natural frequency of closed-circuit magnetoelectroelastic square plates ($h = 1$ mm, $\bar{K}_w = \bar{K}_s = 0$).

4 CONCLUSIONS

In this study, free vibration of a magnetoelectroelastic rectangular plate with different edge supports was investigated analytically. To this end, Reddy's third-order shear deformation theory and Gauss's laws for electrostatics and magnetostatics were used to model the considered smart plate. Galerkin method was applied to the partial differential equation of motion to reduce it to an ordinary differential equation and then an analytical relation was obtained for the natural frequency. Some numerical examples were presented and it was shown that: (a) electric potential decreases the dimensionless natural frequency of the magnetoelectroelastic plate while the magnetic potential increases it, (b) clamped edges increase the dimensionless frequencies of magnetoelectroelastic plate so that the clamped plate has the most dimensionless frequency whereas the simply-supported plate has the least one, and (c) elastic foundation increases the stiffness of the system and consequently increases the natural frequency of the magnetoelectroelastic plate.

Appendix A

$$L_1 = 8hC_{55}/15\,,\ L_2 = 4h^3C_{11}/315 + \beta_4\,,\ L_3 = \beta_1\,,\ L_4 = 4h^3\left(C_{12} + 2C_{66}\right)/315 + \beta_{11}\,,\ L_5 = \beta_8 \quad (A.1)$$

$$L_6 = 8hC_{44}/15\,,\ L_7 = 4h^3C_{22}/315 + \beta_{10}\,,\ L_8 = 4h^3\left(C_{12} + 2C_{66}\right)/315 + \beta_3\,,\ L_9 = \beta_9\,,\ L_{10} = \beta_2 \quad (A.2)$$

$$\begin{aligned} &L_{11} = 8hC_{55}/15 + k_s + e_{31}V_0 + q_{31}\Omega_0, L_{12} = 8hC_{44}/15 + k_s + e_{32}V_0 + q_{32}\Omega_0, L_{13} = -h^3C_{11}/252 + \beta_6 \\ &L_{14} = -h^3C_{22}/252 + \beta_{12}, L_{15} = -h^3\left(C_{12} + 2C_{66}\right)/126 + \beta_{13}, L_{16} = \beta_7, L_{17} = \beta_{14} \end{aligned} \quad (A.3)$$

$$\begin{aligned} &L_{18} = 17h^3C_{11}/315 + \alpha_4, L_{19} = 17h^3C_{66}/315, L_{20} = \alpha_1, L_{21} = -8hC_{55}/15, L_{22} = \alpha_2, \\ &L_{23} = 17h^3\left(C_{12} + C_{66}\right)/315 + \alpha_3, L_{24} = 4h^3C_{11}/315 - \alpha_6, L_{25} = -\alpha_7, \\ &L_{26} = 4h^3\left(C_{12} + 2C_{66}\right)/315 - \alpha_5,\ L_{27} = 8hC_{55}/15 \end{aligned} \quad (A.4)$$

$$\begin{aligned} &L_{28} = \alpha_8, L_{29} = 17h^3\left(C_{12} + C_{66}\right)/315 + \alpha_{11}, L_{30} = 17h^3C_{66}/315, \\ &L_{31} = 17h^3C_{22}/315 + \alpha_{10}, L_{32} = \alpha_9, L_{33} = -8hC_{44}/15, \\ &L_{34} = 4h^3C_{22}/315 - \alpha_{12}, L_{35} = 4h^3\left(C_{12} + 2C_{66}\right)/315 - \alpha_{13}, L_{36} = -\alpha_{14}, L_{37} = 8hC_{44}/15 \end{aligned} \quad (A.5)$$

where

$$\begin{aligned}
\alpha_1 &= 17h^3\left[\lambda_2 e_{24}e_{31} + \lambda_1\left(e_{24}q_{31} + e_{31}q_{24}\right) + \lambda_3 q_{24}q_{31}\right]/315, \\
\alpha_2 &= 17h^3\left[\lambda_2 e_{15}e_{31} + \lambda_1\left(e_{15}q_{31} + e_{31}q_{15}\right) + \lambda_3 q_{15}q_{31}\right]/315, \\
\alpha_3 &= 17h^3\left[\lambda_2 e_{31}e_{32} + \lambda_1\left(e_{31}q_{32} + e_{32}q_{31}\right) + \lambda_3 q_{31}q_{32}\right]/315, \\
\alpha_4 &= 17h^3\left[\lambda_2 e_{31}^2 + 2\lambda_1 e_{31}q_{31} + \lambda_3 q_{31}^2\right]/315, \\
\alpha_5 &= -4h^3\left[\lambda_2 e_{31}e_{32} + \lambda_1\left(e_{31}q_{32} + e_{32}q_{31}\right) + \lambda_3 q_{31}q_{32}\right]/315, \\
\alpha_6 &= -4h^3\left[\lambda_2 e_{31}^2 + 2\lambda_1 e_{31}q_{31} + \lambda_3 q_{31}^2\right]/315, \\
\alpha_7 &= 17h^3\left[\lambda_2\left(e_{15}e_{31} + e_{24}e_{31}\right) + \lambda_1\left(e_{15}q_{31} + e_{31}q_{15} + e_{24}q_{31} + e_{31}q_{24}\right) + \lambda_3\left(q_{15}q_{31} + q_{24}q_{31}\right)\right]/315, \\
\alpha_8 &= 17h^3\left[\lambda_2 e_{24}e_{32} + \lambda_1\left(e_{24}q_{32} + e_{32}q_{24}\right) + \lambda_3 q_{24}q_{32}\right]/315, \\
\alpha_9 &= 17h^3\left[\lambda_2 e_{15}e_{32} + \lambda_1\left(e_{15}q_{32} + e_{32}q_{15}\right) + \lambda_3 q_{15}q_{32}\right]/315, \\
\alpha_{10} &= 17h^3\left[\lambda_2 e_{32}^2 + 2\lambda_1 e_{32}q_{32} + \lambda_3 q_{32}^2\right]/315, \\
\alpha_{11} &= 17h^3\left[\lambda_2 e_{31}e_{32} + \lambda_1\left(e_{31}q_{32} + e_{32}q_{31}\right) + \lambda_3 q_{31}q_{32}\right]/315, \\
\alpha_{12} &= -4h^3\left[\lambda_2 e_{32}^2 + 2\lambda_1 e_{32}q_{32} + \lambda_3 q_{32}^2\right]/315, \\
\alpha_{13} &= -4h^3\left[\lambda_2 e_{31}e_{32} + \lambda_1\left(e_{31}q_{32} + e_{32}q_{31}\right) + \lambda_3 q_{31}q_{32}\right]/315, \\
\alpha_{14} &= 17h^3\left[\lambda_2\left(e_{15}e_{32} + e_{24}e_{32}\right) + \lambda_1\left(e_{15}q_{32} + e_{32}q_{15} + e_{24}q_{32} + e_{32}q_{24}\right) + \lambda_3\left(q_{15}q_{32} + q_{24}q_{32}\right)\right]/315
\end{aligned} \tag{A.6}$$

$$\begin{aligned}
\beta_1 &= h^5\left[\lambda_2 e_{24}e_{31} + \lambda_1\left(e_{24}q_{31} + e_{31}q_{24}\right) + \lambda_3 q_{24}q_{31}\right]/105, \\
\beta_2 &= h^5\left[\lambda_2 e_{15}e_{31} + \lambda_1\left(e_{15}q_{31} + e_{31}q_{15}\right) + \lambda_3 q_{15}q_{31}\right]/105, \\
\beta_3 &= h^5\left[\lambda_2 e_{31}e_{32} + \lambda_1\left(e_{31}q_{32} + e_{32}q_{31}\right) + \lambda_3 q_{31}q_{32}\right]/105, \\
\beta_4 &= h^3\left[\lambda_2 e_{31}^2 + 2\lambda_1 e_{31}q_{31} + \lambda_3 q_{31}^2\right]/105, \\
\beta_5 &= -h^5\left[\lambda_2 e_{31}e_{32} + \lambda_1\left(e_{31}q_{32} + e_{32}q_{31}\right) + \lambda_3 q_{31}q_{32}\right]/336, \\
\beta_6 &= -h^5\left[\lambda_2 e_{31}^2 + 2\lambda_1 e_{31}q_{31} + \lambda_3 q_{31}^2\right]/336, \\
\beta_7 &= h^5\left[\lambda_2\left(e_{15}e_{31} + e_{24}e_{31}\right) + \lambda_1\left(e_{15}q_{31} + e_{31}q_{15} + e_{24}q_{31} + e_{31}q_{24}\right) + \lambda_3\left(q_{15}q_{31} + q_{24}q_{31}\right)\right]/105, \\
\beta_8 &= h^5\left[\lambda_2 e_{24}e_{32} + \lambda_1\left(e_{24}q_{32} + e_{32}q_{24}\right) + \lambda_3 q_{24}q_{32}\right]/105, \\
\beta_9 &= h^5\left[\lambda_2 e_{15}e_{32} + \lambda_1\left(e_{15}q_{32} + e_{32}q_{15}\right) + \lambda_3 q_{15}q_{32}\right]/105, \\
\beta_{10} &= h^5\left[\lambda_2 e_{32}^2 + 2\lambda_1 e_{32}q_{32} + \lambda_3 q_{32}^2\right]/105, \\
\beta_{11} &= h^5\left[\lambda_2 e_{31}e_{32} + \lambda_1\left(e_{31}q_{32} + e_{32}q_{31}\right) + \lambda_3 q_{31}q_{32}\right]/105, \\
\beta_{12} &= -h^5\left[\lambda_2 e_{32}^2 + 2\lambda_1 e_{32}q_{32} + \lambda_3 q_{32}^2\right]/336, \\
\beta_{13} &= -h^5\left[\lambda_2 e_{31}e_{32} + \lambda_1\left(e_{31}q_{32} + e_{32}q_{31}\right) + \lambda_3 q_{31}q_{32}\right]/336, \\
\beta_{14} &= h^5\left[\lambda_2\left(e_{15}e_{32} + e_{24}e_{32}\right) + \lambda_1\left(e_{15}q_{32} + e_{32}q_{15} + e_{24}q_{32} + e_{32}q_{24}\right) + \lambda_3\left(q_{15}q_{32} + q_{24}q_{32}\right)\right]/105,
\end{aligned} \tag{A.7}$$

Appendix B

$$A_1 = L_{18}\frac{\partial^2}{\partial x^2} + L_{19}\frac{\partial^2}{\partial y^2} + L_{20}\frac{\partial^2}{\partial x\,\partial y} - K_2\frac{\partial^2}{\partial t^2} + L_{21} \tag{B.1}$$

$$A_2 = L_{22}\frac{\partial^2}{\partial x^2} + L_{23}\frac{\partial^2}{\partial x\,\partial y} \tag{B.2}$$

$$A_3 = L_{24}\frac{\partial^3}{\partial x^3} + L_{25}\frac{\partial^3}{\partial x^2\partial y} + L_{26}\frac{\partial^3}{\partial x\,\partial y^2} + L_{27}\frac{\partial}{\partial x} - c_1 J_4\frac{\partial^3}{\partial x\,\partial t^2} \tag{B.3}$$

$$A_4 = L_{28}\frac{\partial^2}{\partial y^2} + L_{29}\frac{\partial^2}{\partial x\,\partial y} \tag{B.4}$$

$$A_5 = L_{30}\frac{\partial^2}{\partial x^2} + L_{31}\frac{\partial^2}{\partial y^2} + L_{32}\frac{\partial^2}{\partial x\,\partial y} - K_2\frac{\partial^2}{\partial t^2} + L_{33} \tag{B.5}$$

$$A_6 = L_{34}\frac{\partial^3}{\partial y^3} + L_{35}\frac{\partial^3}{\partial x^2\partial y} + L_{36}\frac{\partial^3}{\partial x\,\partial y^2} + L_{37}\frac{\partial}{\partial y} - c_1 J_4\frac{\partial^3}{\partial y\,\partial t^2} \tag{B.6}$$

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