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Parametric Resonance in Concrete Beam-Columns

Abstract

A dynamic instability, called parametric resonance, is exhibited by undamped elastic beam-columns when under the action of pulsating axial force. The scope of the existing theory of parametric resonance is restricted to physically linear beam-columns undergoing finite lateral displacements. In this Paper, the dynamic behaviour of physically nonlinear elastic cracked concrete beam-columns under pulsating axial force and constant lateral force is investigated. The constitutive equations derived earlier by Authors in the form of force-displacement relations are employed here to formulate equations of motion of the SDOF cantilever with mass lumped at its free end. The expected phenomenon of parametric resonance is exhibited in the form of regular subharmonic resonance at about the frequency ratio of two. Resonance peaks broaden with increase in pulsating force. Like damping, physical nonlinearity is also predicted to stabilize the dynamic response at resonance frequencies. In some particular statically unstable conditions, the loss of dynamic stability is shown to occur by divergence. Unexpectedly, similar phenomenon of parametric resonance is exhibited by these physically nonlinear beam-columns undergoing even small lateral displacements. The contribution made to the theory of parametric resonance and the potential relevance of the proposed theory to design of concrete beam-columns is discussed.

Keywords
Concrete beam-columns, pulsating force, parametric resonance, divergence, dynamic stability, small deformations

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1 INTRODUCTION

Euler method of carrying out stability analysis of structures involves establishing alternate states of static equilibrium adjacent to the given equilibrium state. This method is known to be equivalent to the energy methods, like the method of minimum potential energy, for the determination of the critical loads. Obviously, the validity of these two methods is restricted to those mechanical systems for which the strain energy exists as a unique function of the state variables. Such is the case of conservative structures, i.e., the structure under the action of conservative loads. The problem of stability analysis of nonconservative structures cannot be solved using these methods. For example, Euler method predicts infinitely high value of the critical load for a column under the action of a tangential follower load. Methods of dynamic stability analysis have to be used to investigate the stability of such nonconservative mechanical systems. Their loss of dynamic stability occurs at definite critical values of the loads by a phenomenon called flutter (Bazant and Cedolin, 2010; Bolotin, 1964; Chen and Atsuta, 1976; Leipholz, 1976).

Parametric resonance is another type of dynamic instability suffered by columns and beam-columns with distributed mass and under pulsating axial forces. For such structures undergoing finite lateral displacements, the lateral stiffness depends upon the magnitude of the axial force. Temporal variation of the axial force results in the corresponding variation of the stiffness parameter of the system. Under periodic forcing, a resonance-like phenomenon is expected to occur. For example, for an undamped pinned-pinned elastic column with distributed mass, such parametric resonance is known to occur when the forcing frequency is twice its natural frequency. Even an infinitesimally small magnitude of peak sinusoidal axial force can also destabilize the system. For damped structures, such dynamic instability occurs only at sufficiently high peak sinusoidal axial force, even though the system is rendered vulnerable to parametric resonance at wider range of frequency ratios. Energy audit of the system under pulsating load provides an explanation for the phenomenon of parametric resonance. Dynamic instability of these structures is associated with the unbounded solutions of the Mathieu-Hill type second order partial differential equation of motion with time-dependent coefficients (Bazant and Cedolin, 2010; Kounadis and Belbas, 1977).

Concrete structures are known to be cracked in axio-flexural tension even under service loads. The flexural rigidity of framed column sections depends upon the ratio of the flexural moment to the axial force. Thus, the cracked concrete sections, members and structures are physically nonlinear mechanical systems. In contrast, the existing theory of elastic columns and beam-columns is based upon the assumption of their physical linearity. Static and dynamic stability of such nonlinear elastic beam-columns under conservative loads (Sharma, Singh and Benipal, 2012a, 2012b) and under tangential follower load (Sharma, Singh and Benipal, 2013) has recently been investigated by the Authors. With the intention of obviating the need to incorporate the inelasticity associated with crack formation, the concrete beam-columns are assumed by Authors to be fully cracked a priori. Their physical nonlinearity is associated with the closing and reopening of the extant cracks. To the best of Authors' knowledge, dynamic stability of concrete beam-columns has not yet been investigated (Chopra, 1998; Dutta, 2010; Hoyer and Hansen, 2002).

The objective of the present Paper is to investigate the phenomenon of parametric resonance of fully cracked flanged concrete beam-column under pulsating axial load in the presence of some constant lateral load. The beam-column is in the form of a massless vertical cantilever with mass lumped at its free end. Following existing theories, beam-columns undergoing finite lateral displacements but small rotations are studied. The physically linear beam-columns undergoing small lateral displacements are not expected to exhibit parametric resonance phenomenon. However, the lateral stiffness of the physically nonlinear elastic concrete beam-columns also does depend upon the axial force. In view of this, their dynamic behaviour under pulsating axial force is explored as well. The theoretical significance and the practical relevance of the Paper are also discussed.

2 FINITE AMPLITUDE LATERAL VIBRATIONS

Consider a vertical flanged concrete cantilever, as shown in Fig.1, undergoing finite lateral displacements 'w' under axial load P and lateral load Q_0. The following expression for the lateral stiffness K of the expressions for the completely uncracked structures has been derived earlier by the Authors (Sharma, Singh and Benipal, 2012a).

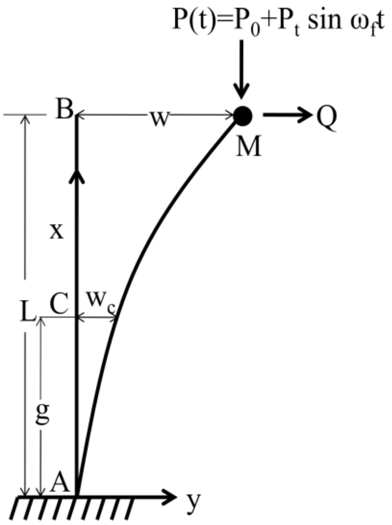

Figure 1: Concrete beam-column.

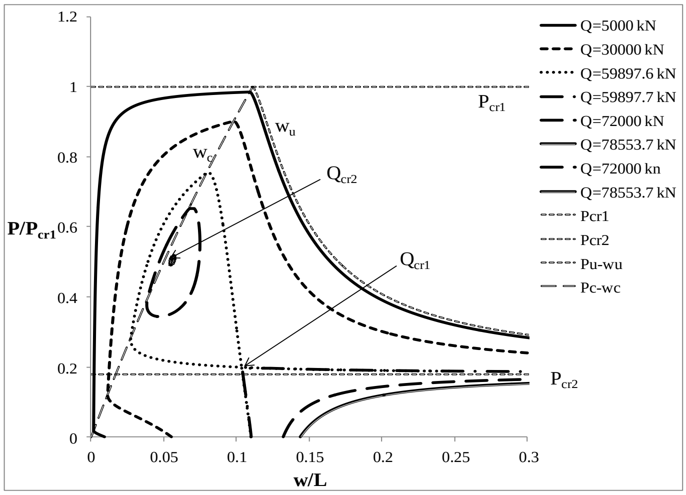

Figure 1: Concrete beam-column. **Figure 2:** P-w curves for constant lateral force.

$$K = \frac{k_1 P}{\tan k_1 L - k_1 L} \tag{1}$$

When the structure is partly cracked, the expression for K turns out to be

$$K = \frac{Q}{w} = \frac{k_2 P Z_1}{\left(\frac{1-B}{\sqrt{B}}\right)\tan k_1 (L-g) + Z_2 - k_2 L\, Z_1} \tag{2}$$

Here, subscripts 1 and 2 refer to the uncracked and cracked sections. The symbols I and r denote the section moment of inertia and the distance of the centroid from the compression face respectively. Here, $k_1 = \sqrt{\frac{P}{EI_1}}$, $k_2 = \sqrt{\frac{P}{EI_2}}$ and $I_2 = BI_1$. The length of g of the cracked segment depends upon the loads P and Q as per the following relation:

$$Q = \frac{P k_1 r_1 Z_1}{\tan k_1 (L-g)}$$

$$Z_1 = \cos k_2 g - \sqrt{B} \sin k_2 g \; \tan k_1 (L-g)$$

$$Z_2 = \sin k_2 g + \sqrt{B} \cos k_2 g \; \tan k_1 (L-g) \tag{3}$$

The cracking load Q_c is given by $\frac{P k_1 r_1}{\tan k_1 L}$. Static and dynamic stability of such cracked concrete beam-columns is investigated in detail and reported elsewhere (Sharma, Singh and Benipal, 2012a, 2012b). Here, for ready reference, certain characteristic aspects of their static stability are presented in Fig. 2 in the form of $P-w$ curves for different constant values of Q. This study pertains to concrete beam-columns with following specific details:

Length: 20m $M = 450000$ kg $E_c = 25000 \text{N/mm}^2$

$I_1 = 1.6167 \text{ X } 10^{13}\ \text{mm}^4$ $\quad\quad B = 0.18006$

The two critical values each for the axial and lateral loads turn out to be as follows:

$P_{cr1} = 2.4885 \text{ X } 10^6\ \text{kN}$ $\quad\quad P_{cr2} = BP_{cr1} = 4.4808 \text{ X } 10^5\ \text{kN}$

$Q_{cr1} = 59897.65\ \text{kN}$ $\quad\quad Q_{cr2} = 78553.7\ \text{kN}$

For the partly cracked concrete beam-columns, the lateral stiffness is known to depend upon both the axial load and the lateral load (or equivalently on lateral displacement). As the axial load reaches either of the two critical loads, P_{cr1} and P_{cr2}, the stiffness vanishes. Such also is the case when, for $P_{cr2} < P < P_{cr1}$, the lateral displacement reaches a limited value w_u.

For $Q < Q_{cr1}$, the lateral displacement first decreases with increase in axial load before it start increasing with further increase in axial load. The $P - w$ curve shows a maximum axial load P_{max} and a post peak response. In contrast, where $Q_{cr1} < Q < Q_{cr2}$, the lateral displacement asymptotically reaches infinitely high values as axial load reaches P_{cr2}. In addition, another segment of $P - w$ curve lying between P_{min} and P_{max} is predicted in the form of a closed loop for $P_{cr2} < P < P_{cr1}$. In both the cases, two equilibrium lateral displacements are admissible for some specific sets of axial and lateral loads. For $P_{cr2} < P < P_{cr1}$, the lateral stiffness vanishes as a certain value w_u given by $r_1 + r_2(\sec k_2\, g - 1)$ of lateral displacement is reached.

The massless beam-column carries a lumped mass M at its free end. The following equation of motion is derived for these SDOF structures with damping ratio ξ (Sharma, Singh and Benipal, 2012b).

$$M\ddot{w} + C(t)\dot{w} + K(t)w = Q_0 \qquad C(t) = \xi\sqrt{MK(t)} \tag{4}$$

Here, $K(t)$ and $C(t)$ represent the instantaneous values of stiffness and damping coefficients. The beam-column is under the action of a constant lateral load Q_0 and a pulsating axial force $P(t)$ given as $P_0 + P_t \sin \omega_f t$. The symbols P_0 and P_t denote the constant component and the peak sinusoidal component respectively and ω_f is the forcing frequency. Irrespective of the presence or extent of cracking, the lateral stiffness K of the concrete beam-columns undergoing finite lateral displacements depends upon the axial force and so on time. In the absence of cracking, the lateral force does not affect the stiffness. For cracked beam-columns, both the lateral and axial forces affect the lateral displacement as well as stiffness. In statics, it is the magnitude of the applied lateral force Q_0 that is relevant. For beam-columns executing lateral vibrations, it is the net lateral force Q which determines the stiffness. Its instantaneous value is determined as

$$Q(t) = Q_0 - M\ddot{w} - C\dot{w} \tag{5}$$

For plotting the frequency domain response, the frequency ratio r_f is the ratio of the forcing frequency (ω_f) to the equilibrium state natural frequency (ω_n) corresponding to the sustained load set (P_0, Q_0). Frequency domain response of the concrete beam-columns is determined for different values of P_0 relative to P_{cr1}, P_{cr2}, P_{min} and P_{max} for constant Q_0 and for different values of P_t relative to P_0. Here, α represents the ratio $\frac{P_t}{P_0}$. The effect of presence and level of damping is also investigated.

Particular concrete beam-columns ($B = 1$) are indistinguishable from conventional beam-columns. The frequency domain response expected from such beam-columns under constant lateral

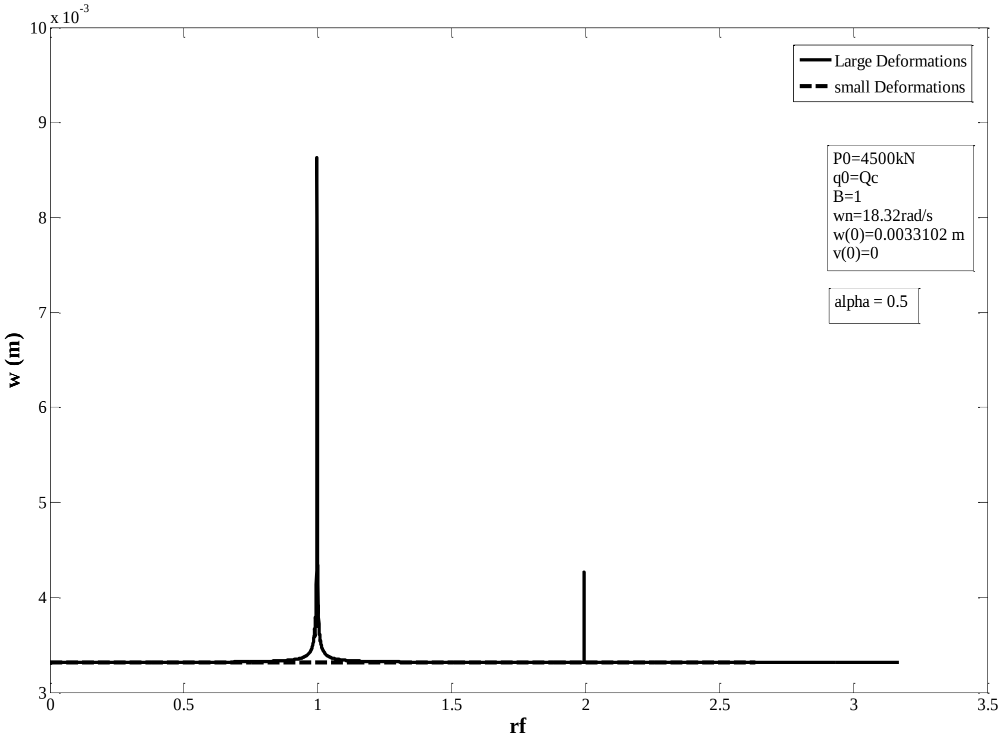

Figure 3: Frequency domain response at B = 1 for small and large deformations.

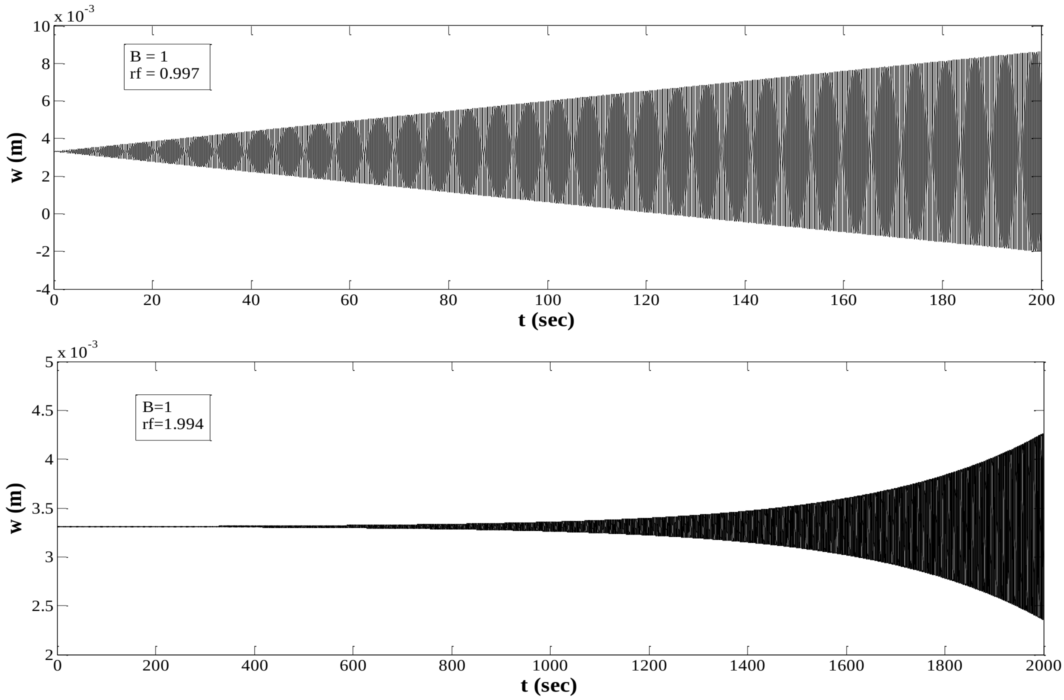

Figure 4: Time domain response at fundamental and subharmonic resonance peaks (B = 1).

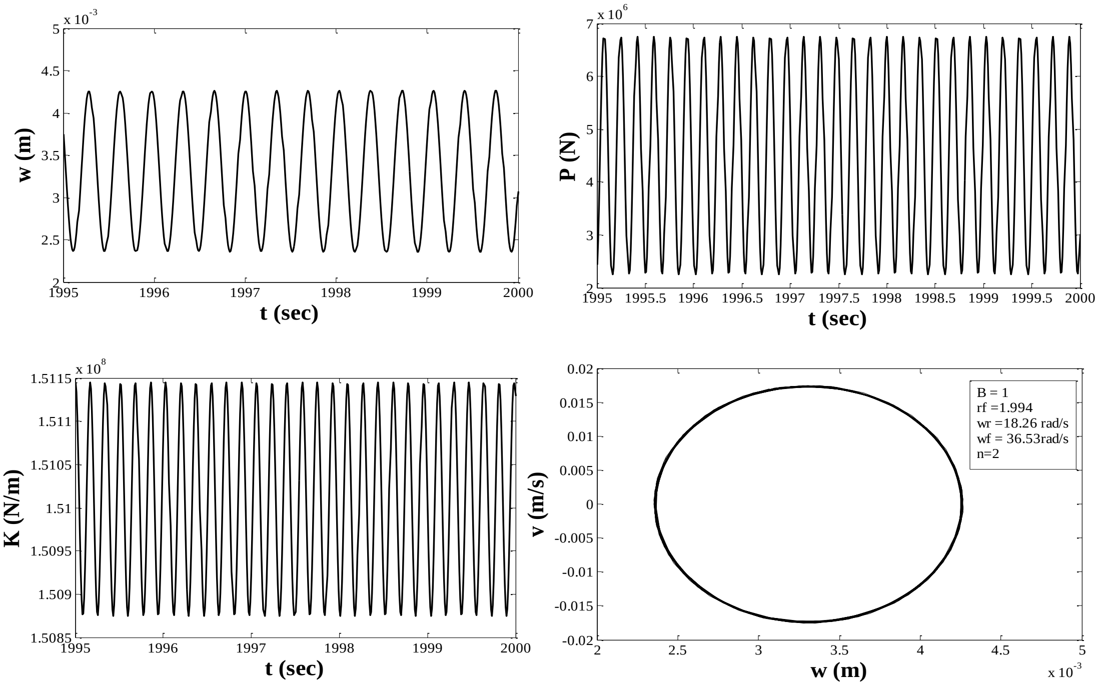

Figure 5: Typical time and phase response at fundamental resonance peak ($B = 1, r_f = 1.994$).

force is shown in Fig. 3. For the particular chosen magnitudes of Q_0, P_0 and P_t, the parametric resonance is predicted to occur at frequency ratios of 0.997 and 1.994. For vanishing Q_0 and P_0 and small P_t, the corresponding frequency ratios are 1 and 2 respectively. It is well known that the T2 regular subharmonic resonance corresponds to a wave number, n, (ratio of forcing frequency to response frequency) of two. Thus, the phenomenon of parametric resonance occurs in the form of T2 regular subharmonic resonance.

From the displacement waveforms in Fig. 4 obtained at the resonance frequencies in the present case, the vibration amplitudes can be observed to increase with time in an unbounded manner. Thus, the dynamic instability at parametric resonance is of divergence type. Fig. 5 depicts the temporal variation of w, P and K as well as phase plot for a very short duration at frequency ratio of 1.994. The variation of lateral stiffness is synchronous with that of axial force. The parametric resonance at this frequency is in fact a subharmonic resonance (wave number, n=2). In contrast, the parametric resonance not shown here at frequency ratio of 0.997 is fundamental resonance.

The dynamic response of concrete beam-columns under constant lateral force and pulsating axial force depends upon their corresponding static axial force-lateral displacement $(P - w)$ curves shown in Fig. 2. Here, the response under pulsating axial force is investigated for the following different combinations of P_0 and Q_0:

a. $P_0 = 4500kN \ll P_{cr2}$ and $Q_0 = Q_c$
b. $P_0 = 0.9P_{cr2}$ and $Q_0 = Q_c$
c. $P_0 = 0.65Pcr1 = P_{max}$ and $Q_0 = 72000kN$
d. $P_{min} < P_0 = 0.45Pcr1 < P_{max}$ and $Q_0 = 72000kN$

Frequency domain response of undamped concrete beam-columns is shown in Fig. 6 for different values of the parameter α (0.01, 0.1, 0.25, 0.5. 0.75, 1). The magnitudes of constant axial and lateral forces are 4500kN ($P_0 \ll< P_{cr2}$) and Q_c respectively and correspond to the equilibrium lateral displacement as 0.003317 m. This equilibrium displacement is chosen as the initial displacement. The natural frequency ω_n of small vibrations about the equilibrium state is estimated as 18.32 rad/s. At all α, the frequency domain plots exhibit both fundamental and subharmonic resonances. Only one regular subharmonic resonance peak is predicted for some values of α (0.25, 0.5, 0.75) while many such peaks appear in frequency domain plots of other values of α.

In all the cases, many irregular subharmonic peaks appear with increase in α, the system experiences large excursions away from the equilibrium state. Though not shown here, the lateral stiffness has been predicted earlier by the Authors to decrease with increase in lateral displacements (Sharma, Singh and Benipal, 2013). Thus, the stiffness of the vibrating structures is expected to vary with time. The frequency ratios and peak amplitudes corresponding to the fundamental and various subharmonic resonances for different α are presented in Table 1. The fundamental and the T2 regular subharmonic resonance frequencies are predicted to decrease with increase in α. As expected, the corresponding irregular subharmonic resonance frequencies also register a decrease. Also, these resonance peaks are observed to occur to a broader range of frequency ratios with increase in α. This observation is substantiated in Fig. 7 where the range of the frequency ratios for fundamental and

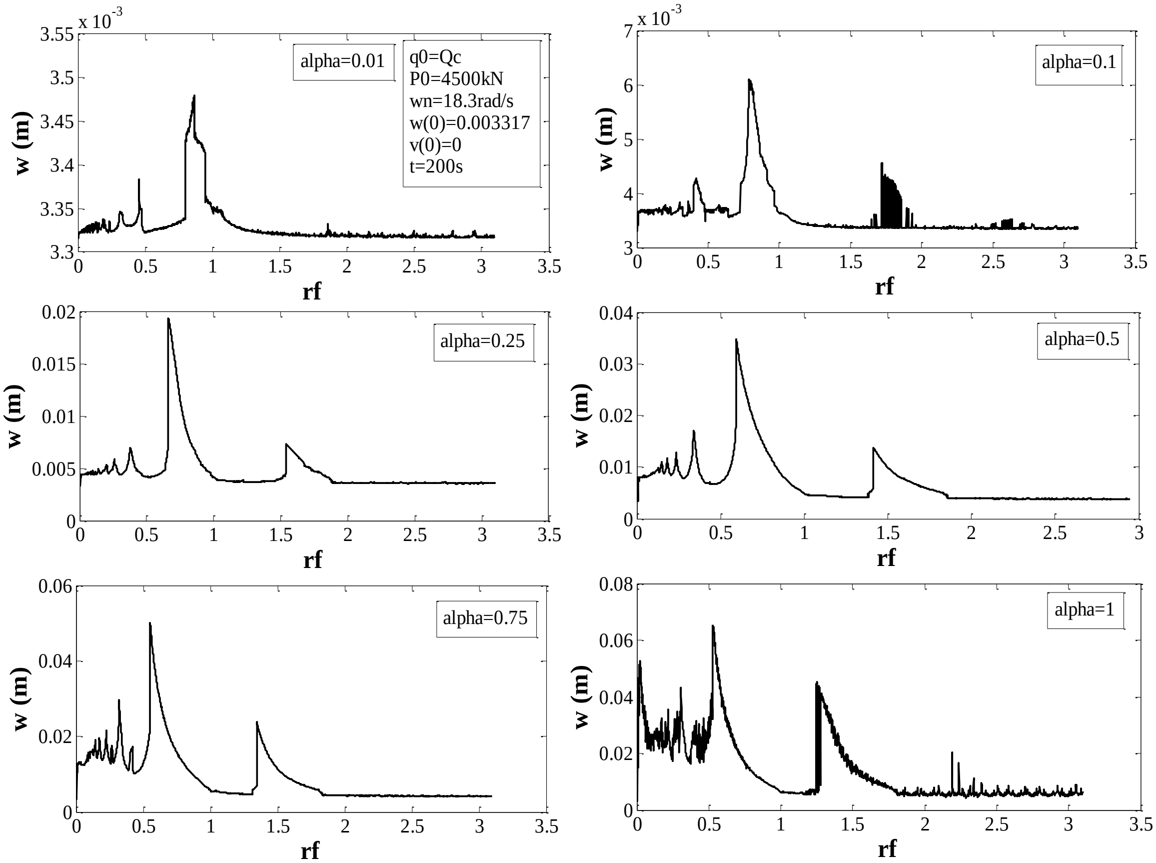

Figure 6: Effect of magnitude of pulsating load.

	Fundamental Peak		Regular subharmonics		Irregular subharmonics
α	r_f	w (m)	r_f	w(m)	r_f
0.01	0.866	0.00348	1.856	0.003332	0.157, 0.236, 0.312, 0.456
0.1	0.789	0.006092	1.719	0.004561	0.2, 0.235, 0.30, 0.363, 0.416
0.25	0.663	0.0193	1.543	0.007278	0.141, 0.2, 0.26, 0.377
0.5	0.595	0.03467	1.414	0.0138	0.127, 0.149, 0.182, 0.236, 0.34
0.75	0.553	0.05014	1.346	0.02374	0.145, 0.174, 0.224, 0.264, 0.319, 0.421
1.0	0.529	0.06531	1.248	0.04467	0.024, 0.171, 0.218, 0.415, 0.306

Table 1: Resonance frequency ratios and amplitudes.

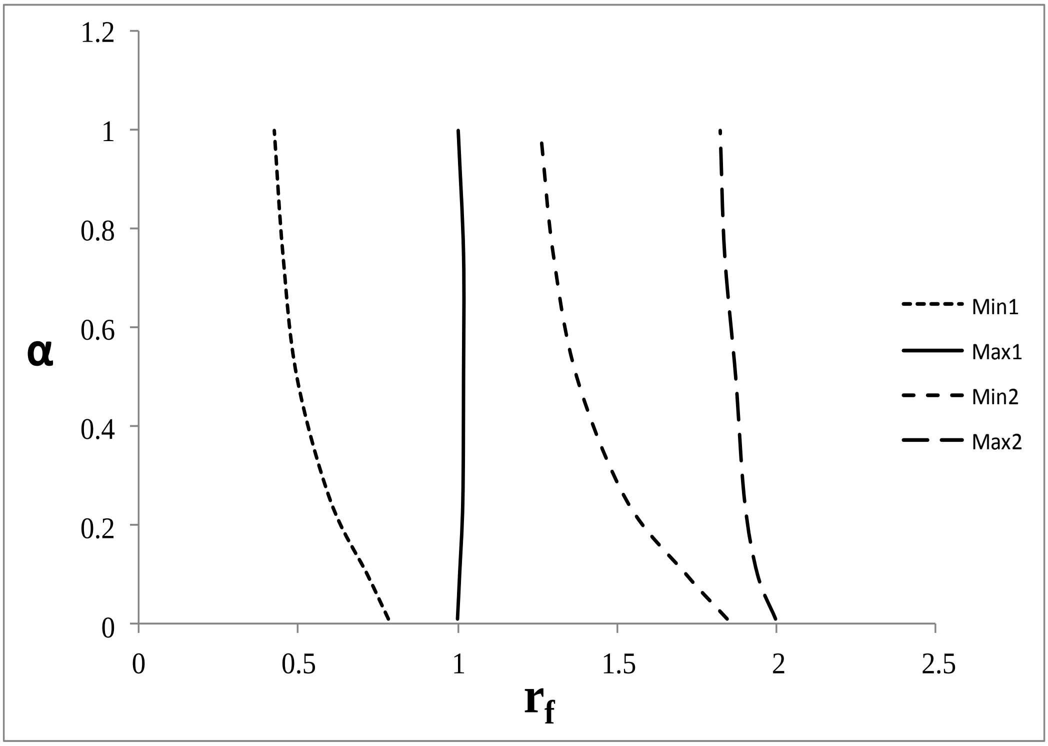

Figure 7: Ranges of parametric resonance frequencies for finite deformations.

regular T2 subharmonic resonance is presented. For a particular case ($\alpha = 0.5$), the waveforms for P, w and K alongwith phase plot for the fundamental resonance are shown in Fig. 8. The response frequency equals the forcing frequency. The temporal variation of lateral stiffness is more complex. As the lateral displacement changes sense in every cycle, the stiffness changes twice. The displacement waveform lags behind that of axial force. The regular subharmonic resonance response presented in Fig. 9 shows forcing frequency to be double the response frequency (wave number, n=2).

To recapitulate, P_{cr1} and P_{cr2} represent the Euler critical loads for the uncracked and fully cracked concrete beam-columns respectively. When the axial force P_0 approaches P_{cr2}, the system response depends considerably on Q_0. The predicted effect of α on the frequency domain response in one such case ($P_0 = 0.9P_{cr2}$ and $Q_0 = Q_c$) is presented in Fig. 10. Apart from fundamental resonance peak, one regular subharmonic peak and many irregular subharminc peaks are observed. Increase in α results in shifting and broadening of resonance peaks in a manner similar to that discussed earlier $P_0 \lll P_{cr2}$.

For a particular value (72000kN) of Q_0 in the range $Q_{cr1} < Q < Q_{cr2}$, the P_{max} and P_{min} turn out to be $0.65P_{cr1}$ and $0.34P_{cr1}$. These values represent the demarcation between statically and

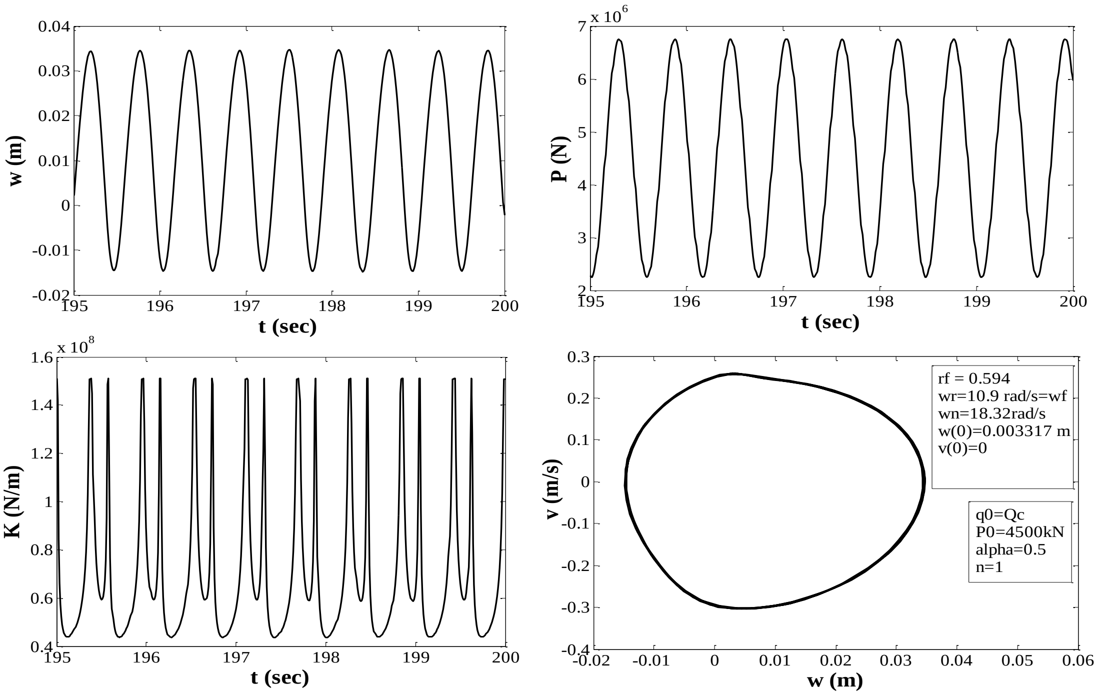

Figure 8: Typical time and phase response at fundamental peak ($B = 0.18006, r_f = 0.594$).

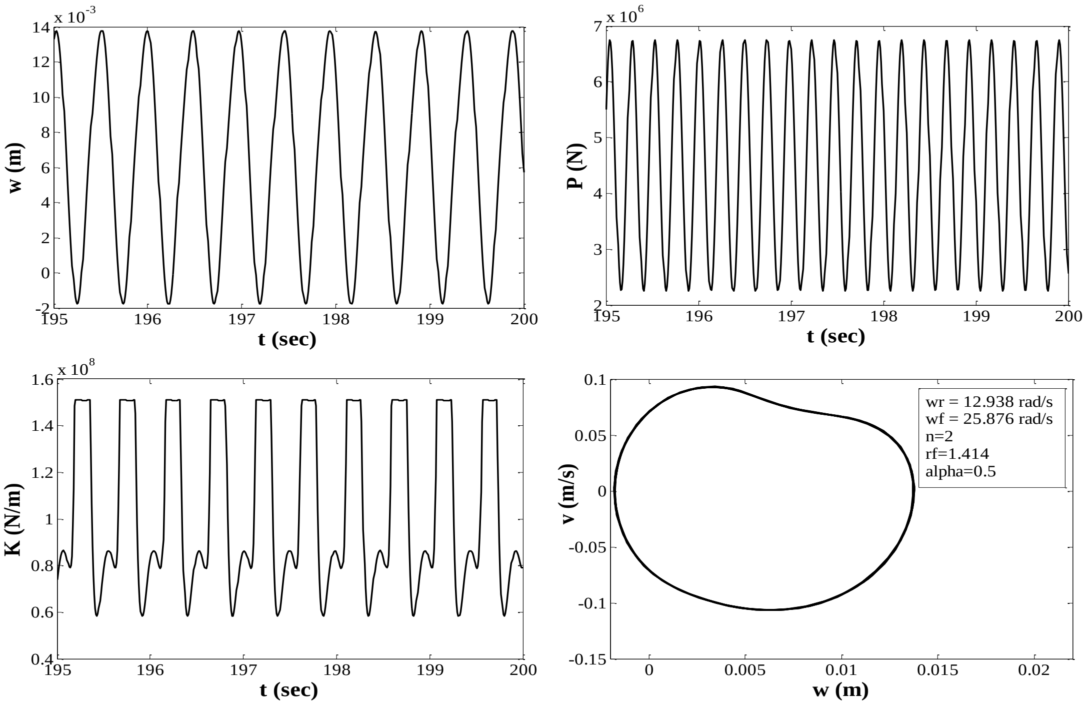

Figure 9: Typical time and phase response at subharmonic peak ($B = 0.18006, r_f = 1.414$).

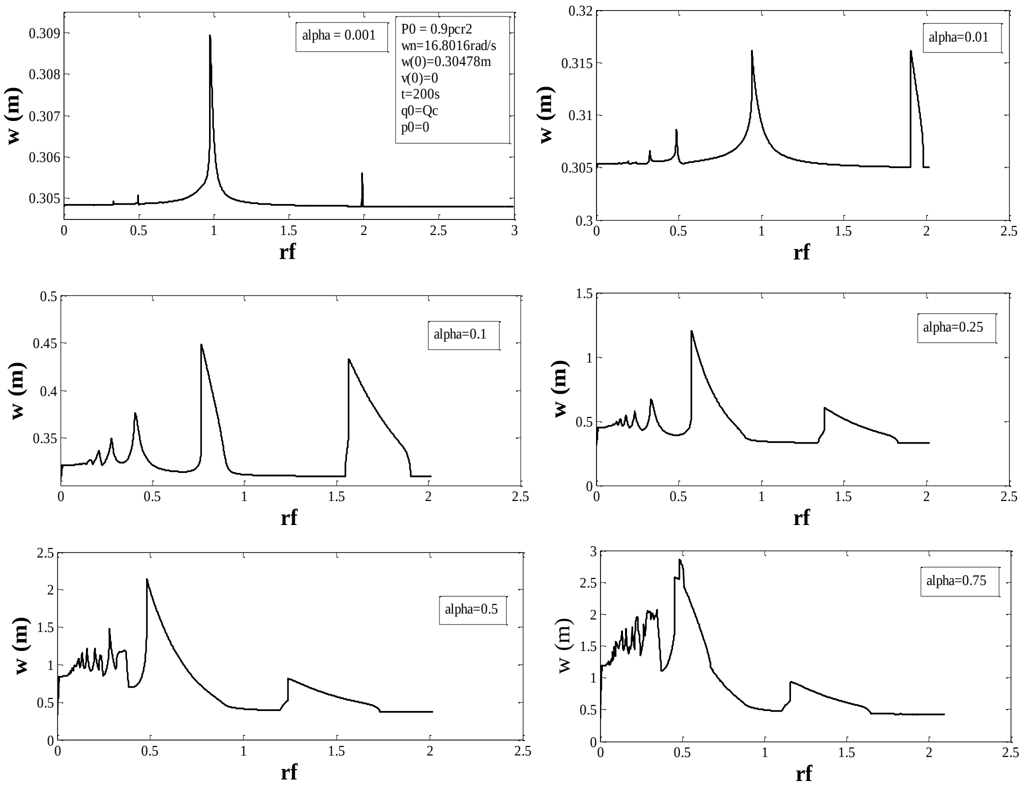

Figure 10: Effect of magnitude of pulsating load on frequency domain response ($P = 0.9P_{cr2}$, $Q << Q_{cr1}$).

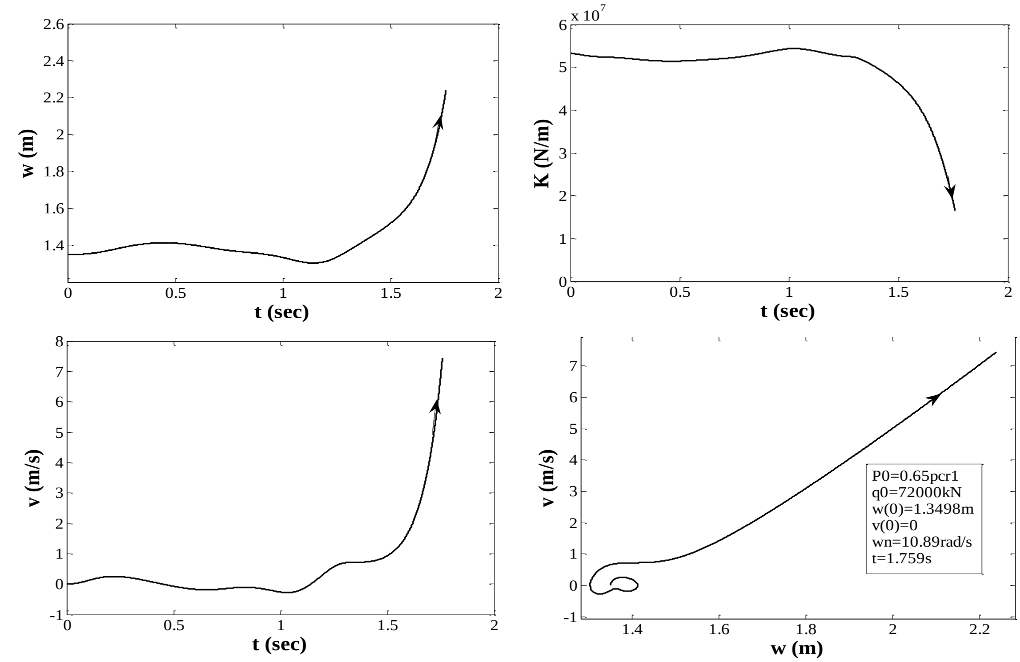

Figure 11: Typical divergence phenomenon at $r_f = 1$, $P = 0.65P_{cr1}$.

stable and unstable regions. In one such case ($P_0 = 0.65P_{cr1}$ and $r_f = 1$), the concrete beam- columns under pulsating axial force exhibit dynamic instability by divergence as shown in Fig. 11. This displacement waveform and the corresponding phase plot imply on unbounded increase in the lateral displacement and velocity with time. Correspondingly, the lateral stiffness can be observed to decrease with increase in lateral displacement.

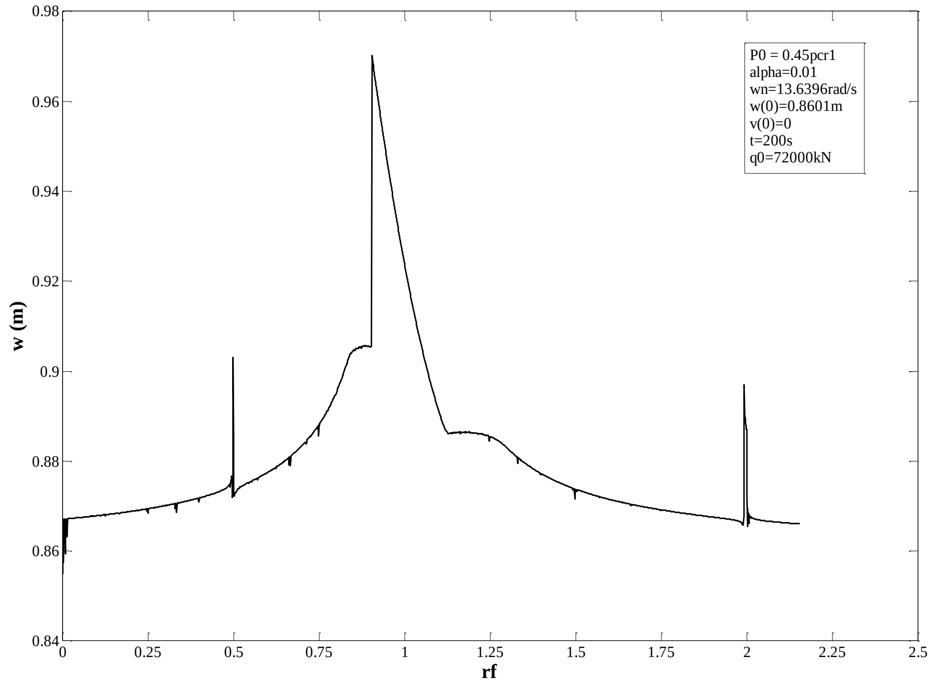

Figure 12: Frequency domain response for $P_{max} < P = 0.45P_{cr1} > P_{min}$.

When the concrete beam-columns happen to be in the stable regime ($P_0 = 0.45P_{cr1}$ and $Q_0 = 72000kN$), the frequency domain response plotted in Fig. 12 exhibits one regular ($r_f = 1.993$) and one irregular subharmonic resonance ($r_f = 0.499$) apart from the fundamental resonance ($r_f = 0.905$). In contrast to the quite broad-based fundamental peak, both the regular and irregular subharmonic peaks are very sharp. The corresponding waveforms and plots are shown in Fig. 13a, 13b and Fig.13c respectively. Fundamental peak response as well as lateral stiffness in Fig.13a seems to be out of phase with axial force. Regular T2 subharmonic response occurs at half the forcing frequency. However, relative to the axial force, Fig.13b the temporal variation of lateral stiffness is to be more complex. As expected, the irregular subharmonic response frequency equals the forcing frequency, despite the complexity of displacement waveform and corresponding phase plot shown in Fig. 13c. However, the variation of stiffness seems to be out of phase with that of axial force.

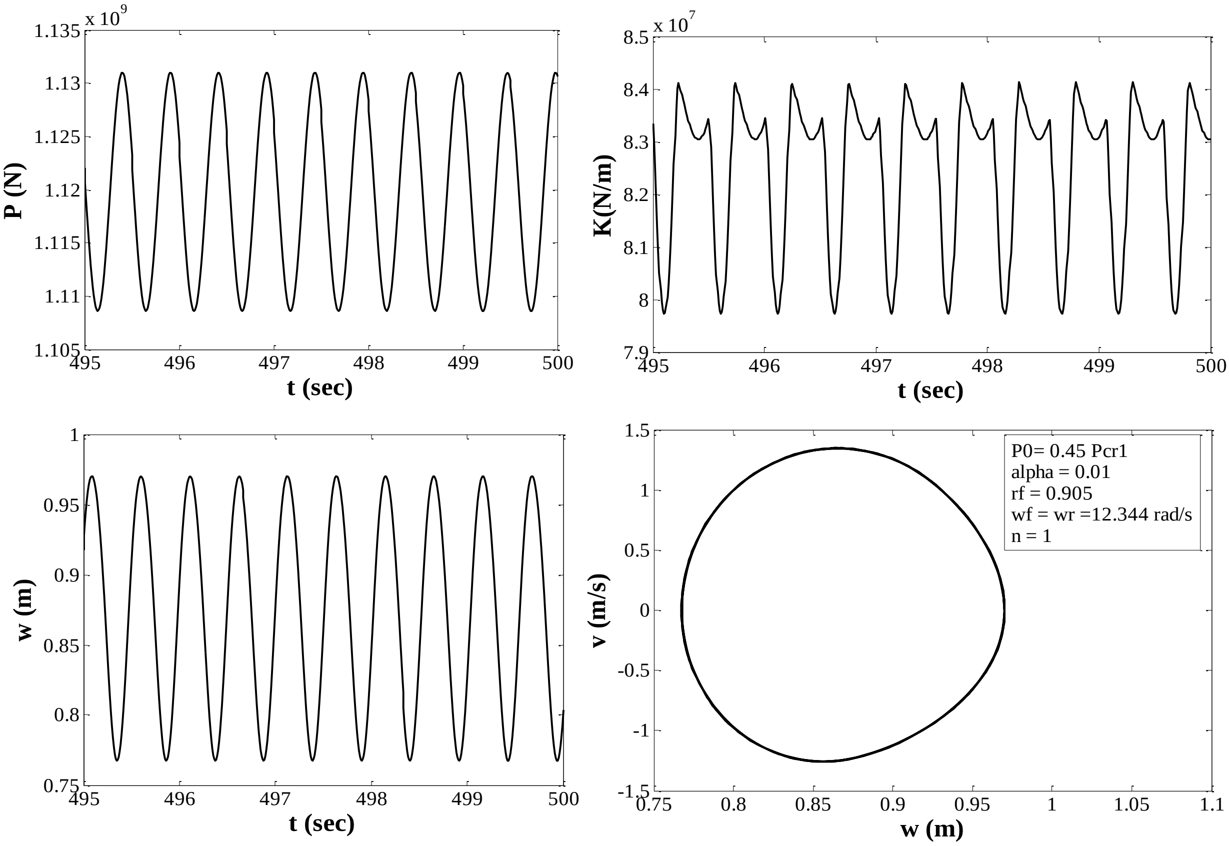

Figure 13a: Typical time domain and phase response at fundamental resonance peak ($r_f = 0.905$, $P = 0.45P_{cr1}$).

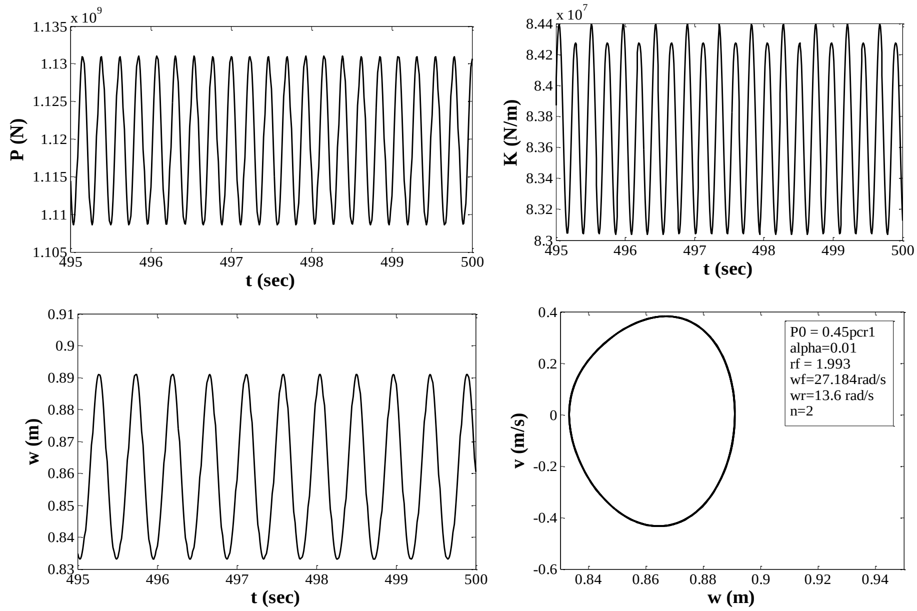

Figure13b: Typical time domain and phase response at subharmonic resonance peak ($r_f = 1.993$ for $P = 0.45P_{cr1}$).

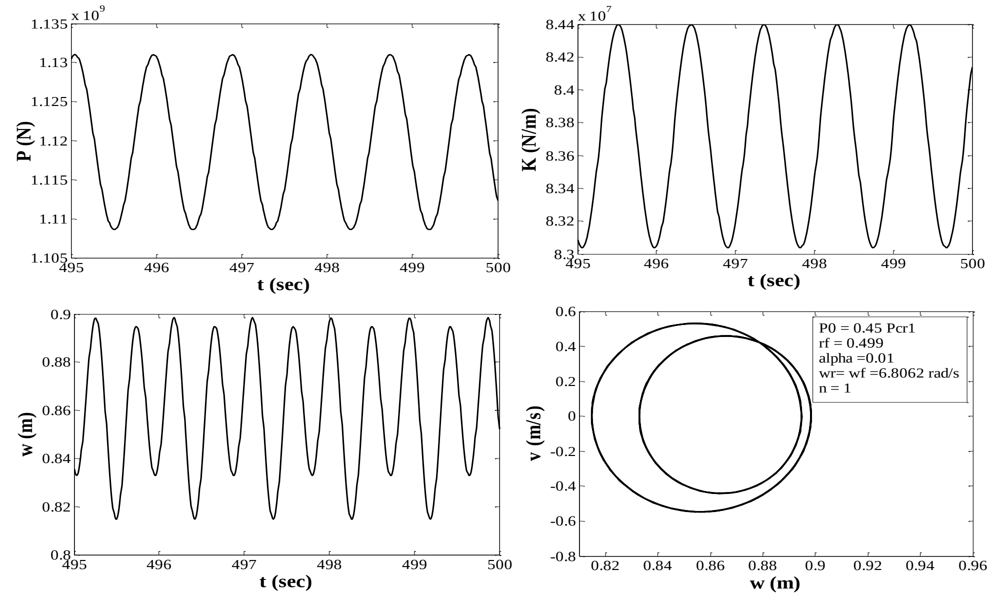

Figure 13c: Typical time domain and phase response at irregular subharmonic resonance peak ($r_f = 499$ for $P = 0.45P_{cr1}$).

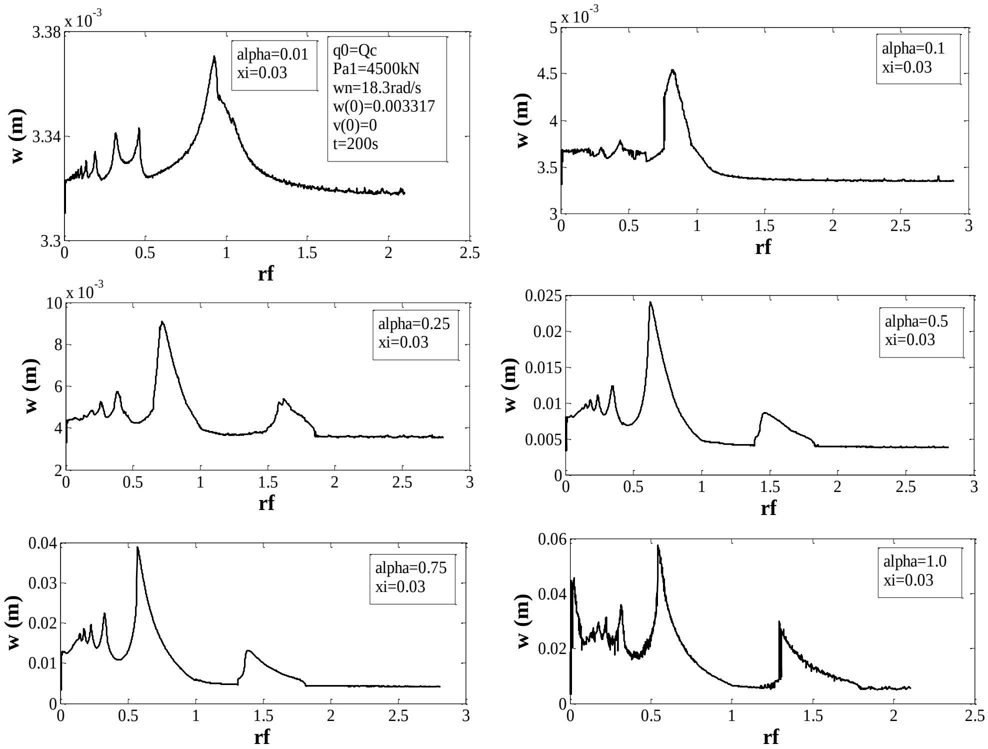

Figure 14: Effect of damping on frequency domain response for $\xi = 0.03$.

The predicted parametric resonance behaviour presented above pertains to undamped systems. It is well known (Bazant and Cedolin, 2010) that, for damped systems, parametric resonance occurs only for sufficiently high pulsating force. This observation is confirmed for concrete beam-columns as well. Typical frequency domain response is presented in Fig. 14 for damped beam-columns for different values of α. A comparison with frequency domain response for undamped beam-columns reveals that the smaller values of α (0.01, 0.1) the phenomenon of parametric resonance does not occur. Also, the peak amplitude for fundamental subharmonic resonances is suppressed by damping.

3 SMALL AMPLITUDE LATERAL VIBRATIONS

The following expressions for the lateral displacements and lateral stiffness of partly cracked beam-columns undergoing small lateral displacements have been derived earlier by the Authors (Sharma, Singh and Benipal, 2013):

$$w = \frac{(1-B)Qg^2(3l-g)}{6EI_2} + \frac{Ql^3}{3EI_1} \qquad (6)$$

$$K = \frac{Q}{w} = \frac{3EI_2}{l^3}\left[\frac{2}{2B+(1-B)\left\{3\left(\frac{g}{L}\right)^2-\left(\frac{g}{L}\right)^3\right\}}\right] \qquad (7)$$

where $g = L - \frac{Pr_1}{Q} \geq 0$. The cracking loads P_c and Q_c (g=0) are related as

$$Q_cL = P_cr_1 \qquad (8)$$

In contrast, in the absence of cracking, the relevant expressions are as follows

$$K = \frac{3EI_1}{L^3} \qquad\qquad w = \frac{QL^3}{3EI_1} \tag{9}$$

To recapitulate, the phenomenon of parametric resonance of concrete beam-columns undergoing small lateral displacements is investigated in this section under constant lateral force and pulsating axial force. Their lateral stiffness being independent of the applied forces, the uncracked concrete beam-columns do not exhibit parametric resonance. This fact is confirmed in Fig. 3 by their frequency domain response without any peaks. However, the stiffness of the partly cracked beam-columns depends of the magnitude of the axial force relative to the constant lateral force and their $P - w$ relation is nonlinear.

Such physically nonlinear elastic beam-columns are expected to experience parametric resonance. As shown in Fig. 15 undamped cracked concrete beam-columns undergoing even small lateral displacements indeed exhibit the phenomenon of parametric resonance for different magnitudes of the pulsating force. Apart from fundamental peak, one T2 regular subharmonic and many irregular subharmonic peaks are observed. The corresponding resonance frequencies and peak amplitudes are presented in the Table 2. Comparison of Fig. 15 and Table 2 with the corresponding figure Fig. 6 and Table1 plotted for finite deformation case reveals higher fundamental and subharmonic resonance frequencies for small deformation case, the peak amplitudes case, the peak amplitudes

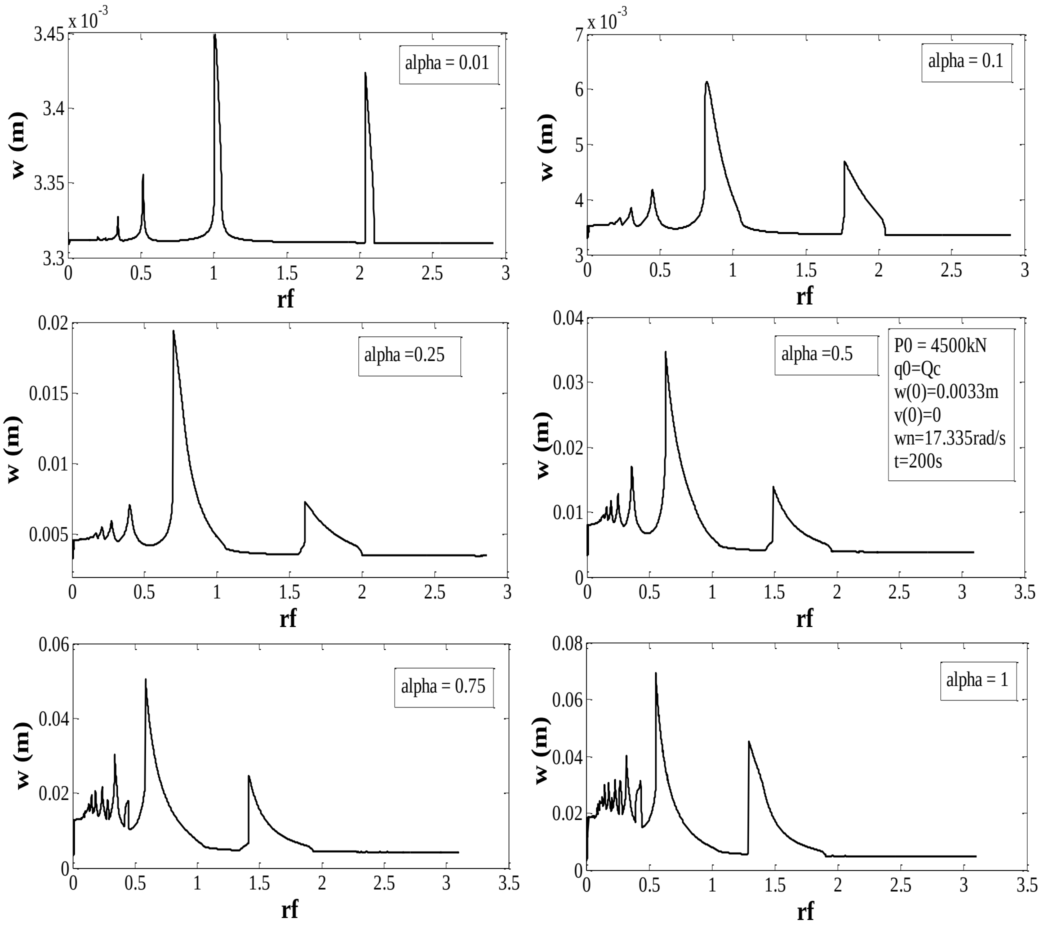

Figure15: Effect of magnitude of pulsating load on frequency domain response.

are substantially similar. Also, the effect of α on the resonance frequency ranges is presented in Fig. 16. Comparing with the corresponding Fig. 7 for finite deformations, lead to the conclusion that fundamental resonance peaks are broader at lower pulsating forces for small deformations. Subharmonic resonance peaks is broader for small deformations at the lowest pulsating force. No such definite conclusion could be drawn for higher pulsating forces. Typical wave forms (α=1) for P, w and K alongwith phase plots for fundamental and T2 regular subharmonic resonances are presented in Fig. 17 and Fig. 18 respectively. As expected, the response frequencies in these two cases are equal to and half the forcing frequencies. In both the case, the temporal variation of lateral stiffness occurs at the response frequency. When partly cracked, the concrete beam-columns are physically nonlinear. In the case of finite lateral displacements, geometric nonlinearity is presented as well. As such these structures under pulsating axial force are also expected to exhibit extreme sensitivity to initial conditions, system parameters and loading details. Such indeed happens to

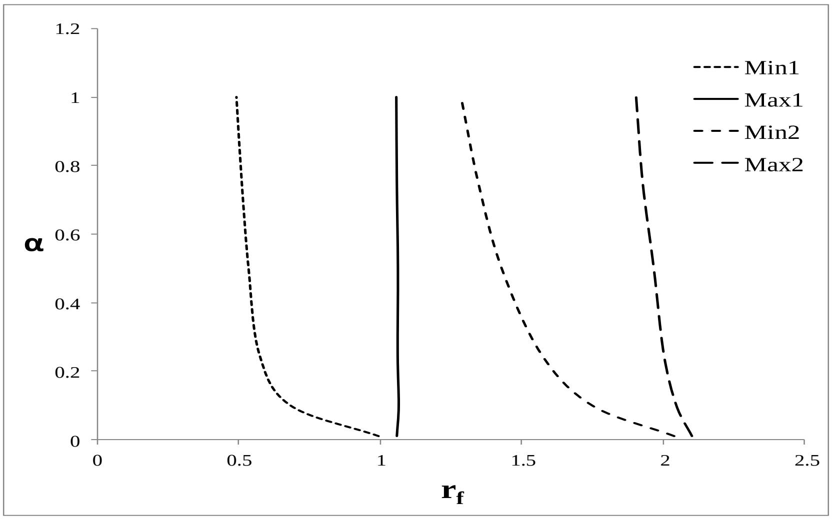

Figure 16: Range of parametric resonance frequencies for small deformations.

$q_0=Q_c$	Fundamental Peak		Regular subharmonics		Irregular subharmonics
α	r_f	w (m)	r_f	w(m)	r_f
0.01	1.009	0.003449	2.041	0.003423	0.344, 0.515
0.1	0.824	0.00614	1.767	0.004693	0.230, 0.304,0.449
0.25	0.698	0.01942	1.609	0.007327	0.166,0.209,0.273,0.398
0.5	0.629	0.03459	1.49	0.01382	0.158,0.192,0.249,0.359
0.75	0.584	0.05056	1.415	0.02468	0.131,0.151,0.183, 0.236,0.282
1.0	0.554	0.06907	1.294	0.04509	0.129,0.147,0.178,0.206,0.228,0.272,0.318,0.433

Table 2: Resonance frequency ratios and vibration amplitudes.

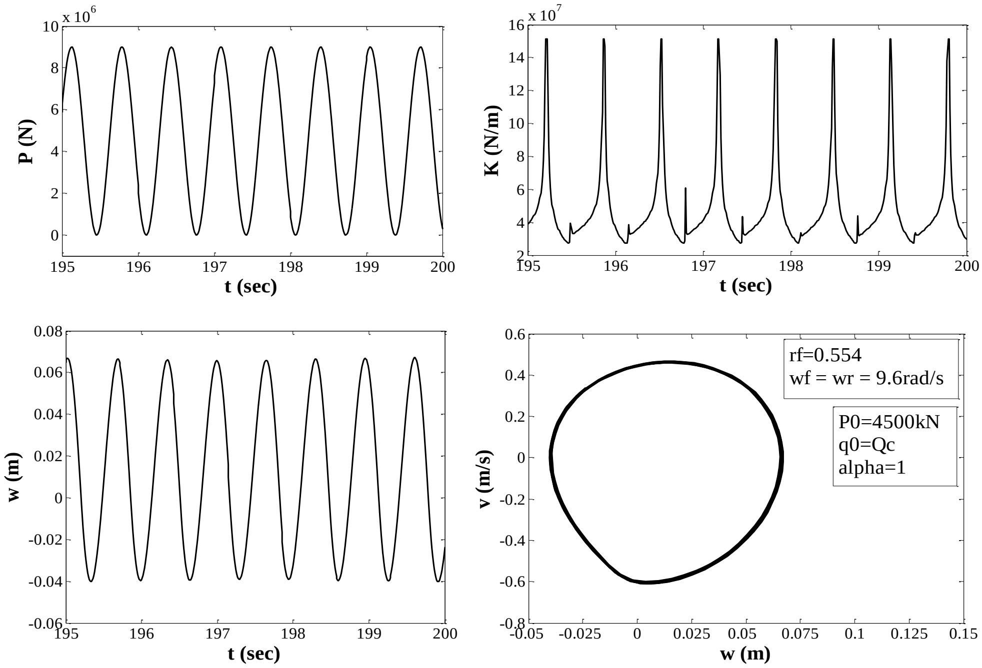

Figure 17: Typical time and phase response at fundamental resonance peak ($B = 0.18006, r_f = 0.554$).

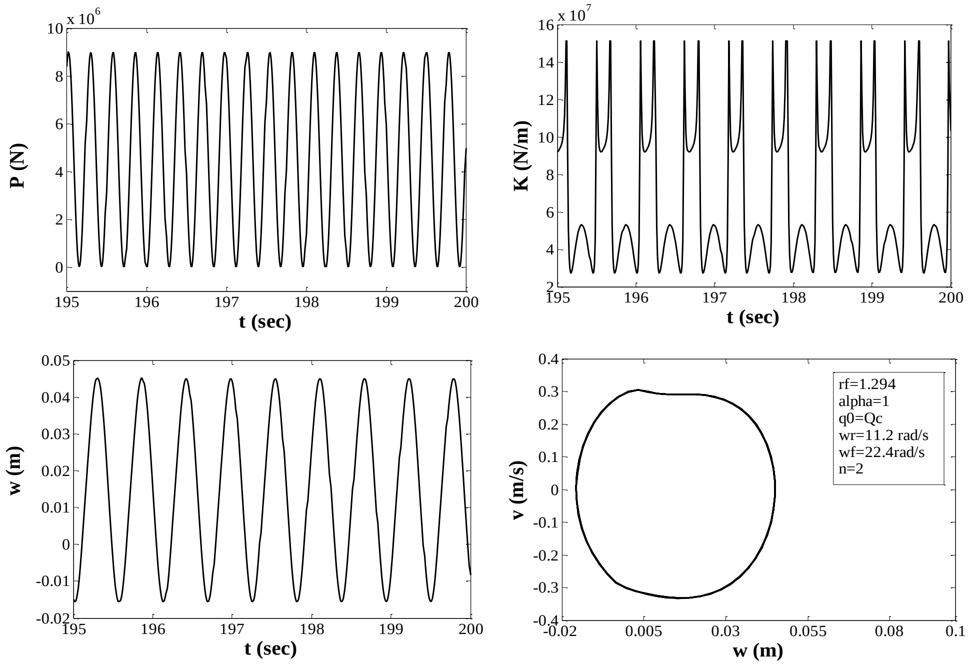

Figure 18: Typical time and phase response at subharmonic resonance peak ($B = 0.18006, r_f = 1.294$).

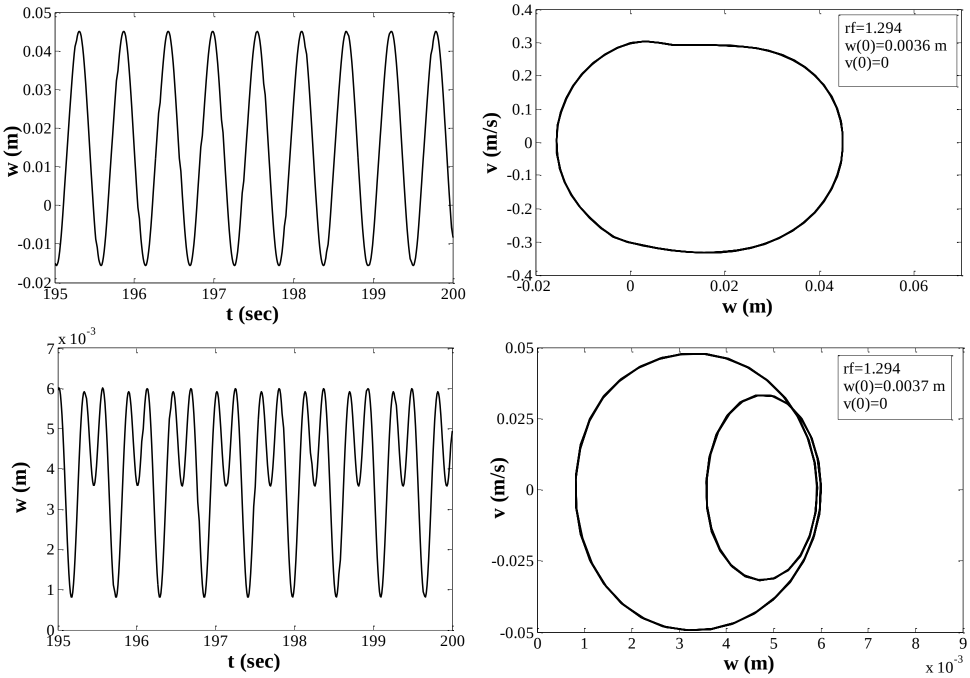

Figure 19: Effect of initial conditions on resonance amplitude at $r_f = 1.294$.

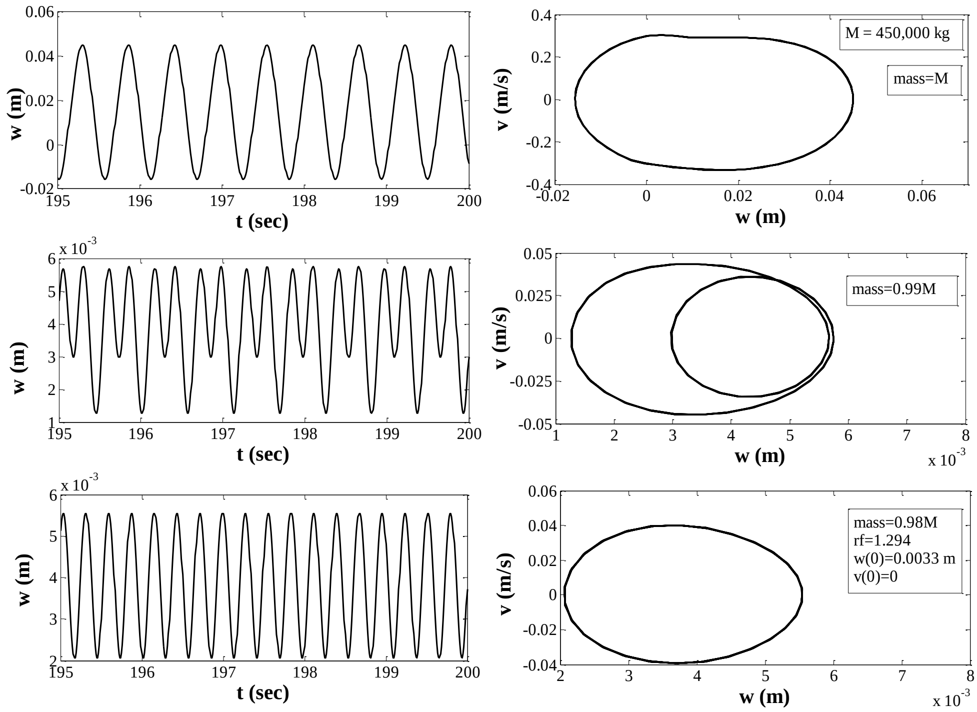

Figure 20: Effect of mass on resonance amplitude at $r_f = 1.294$.

be the case. As shown in Fig. 19 an infinitesimally small difference (0.0001m) in initial displacement results in an altogether different response. Similarly, as shown in Fig. 20, one percent decrease in the lumped mass results in considerably smaller response. Similar further reduction in lumped mass results in similar displacement response but entirely different phase plot.

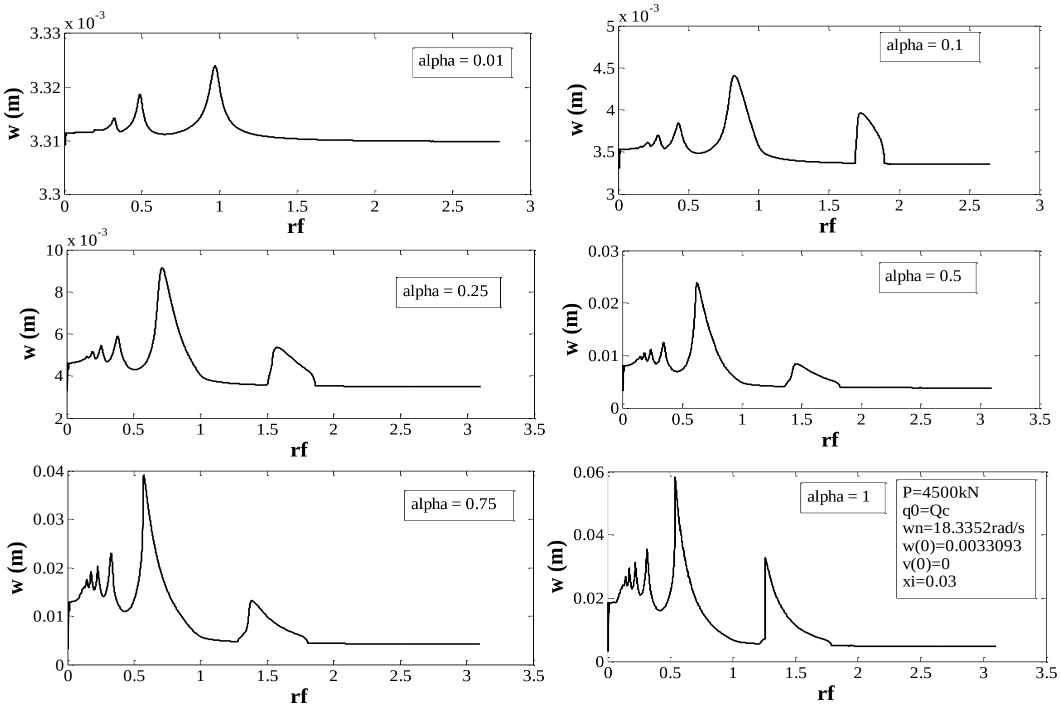

Figure 21: Effect of damping on frequency domain response for $\xi = 0.03$.

Typical effect of damping (ξ=0.03) on small deformation frequency domain response for different α is depicted in Fig. 21. As expected, the parametric resonance in the form of T2 regular subharmonic does not at very small pulsating force (α=0.01). Both fundamental and subharmonic peak responses are suppressed by the presence of damping.

4 DISCUSSION

In this Paper, the phenomenon of parametric resonance of cracked concrete beam-columns in the form of a massless vertical cantilever with mass lumped at the free end is investigated. The second order inhomogeneous linear differential equation of motion with variable coefficients is formulated both for the finite and the small lateral vibration cases. For ready reference, the dynamic stability of uncracked physically linear concrete beam-columns undergoing finite lateral displacements is presented. As expected, parametric resonance occurs at a frequency ratio of two in the form of T2 regular subharmonic peak. At the resonance frequencies, the system is predicted to exhibit unbounded divergent response. In other words, the dynamic instability called parametric resonance is of divergence type. In contrast, the physically nonlinear undamped cracked concrete beam-columns exhibit both regular and irregular subharmonics in addition to fundamental resonance under various combinations of sustained axial and lateral loads. The resonance peaks get broader with increase in the magnitude of the peak sinusoidal force. For statically stable sustained load sets ($P_0 < P_{cr2}$, $P_{min} < P_0 < P_{max}$), the vibration amplitudes at these resonance frequencies are bounded. This implies that, in such cases the parametric resonance does not constitute dynamic instability. Loss of dynamic stability by divergence is predicted to occur at statically unstable sustained load sets ($P_0 = P_{max}$ or $P_0 = P_{min}$).

To recapitulate, the physically linear undamped beam-columns undergoing small lateral displacements do not exhibit the phenomenon of parametric resonance. However, the physically nonlinear cracked concrete beam-columns, even when executing small lateral vibrations do so. Such happens to be the case because of the explicit dependence of the lateral stiffness on axial force and hence on time. Apart from the fundamental resonance, both regular and irregular subharmonic resonances are predicted. These beam-columns exhibit stable dynamic response even at resonance frequencies. This happens because concrete beam-columns undergoing small lateral displacements are always statically stable.

Irrespective of the magnitude of the lateral displacements, the phenomenon of parametric resonance has been predicted not to occur at smaller pulsating loads for typically damped concrete beam-columns. Even at high pulsating forces, the resonance peaks are suppressed by damping. As expected, the steady state resonance of nonlinear dynamical systems executing small or finite amplitude vibrations is extremely sensitive to initial conditions and system parameters like lumped mass.

The most significant contribution of this Paper to the theory of parametric resonance involves its extension to the physically nonlinear beam-columns. In some cases, the physical nonlinearity has been shown to stabilize the beam-columns experiencing parametric resonance as well as fundamental resonance. The phenomenon of parametric resonance is predicted for these physically nonlinear beam-columns undergoing even small lateral displacements.

Seismic design of structures aims at ensuring their safety against the dynamic forces introduced by strong ground motions. These base excitations include horizontal and vertical accelerations as well as angular accelerations. Seismic design Codes recommend that peak vertical acceleration be taken as two-thirds of the peak horizontal acceleration. Generally, the structures are designed for separately for the peak (horizontal) ground acceleration and checked for vertical vibrations. Elastic response obtained by using linear theory of vibrations is modified by using response reduction factor. These horizontal and vertical seismic forces are considered to be conservative. When the lateral vibration amplitudes are of finite magnitude, geometrically nonlinear response is incorporated in the form of second order effects like P-δ and P-Δ effects respectively at the local and global scale (Chopra, 1998; Dutta, 2010).

Free and forced lateral vibrations and stability nonlinear elastic concrete beam-columns under constant axial forces have been investigated earlier by the Authors (Sharma, Singh and Benipal, 2012a, 2012b). In this Paper, the lateral force has been assumed to remain constant and the horizontal dynamic response of concrete beam-columns under pulsating axial force has been predicted. Effect of the pulsating vertical force on the small and finite amplitude lateral vibrations of concrete beam-columns is quantified. To the best of Authors' knowledge such an exercise has not yet been under-taken in the field of concrete structural dynamics (Chopra, 1998; Dutta, 2010).

Concrete structures supporting pulsating or rotating machinery constitute another example. When the machine platform remains horizontal, the dynamic forces imposed on the structure are conservative. Specifically, the pulsating machines impose vertical periodic force, while the structures supporting rotating machines experience both horizontal and vertical sinusoidal forces. The particular case of two rotating machines with opposite direction of rotation results only in the vertical sinusoidal force. As a matter of fact these so-called vertical and horizontal forces respectively act in a direction tangential and normal to the deformed centroidal axis. Thus, in the case of finite rotations, these dynamic forces turn out to be simultaneously-acting nonconservative pulsating tangential and normal follower forces. Stability of concrete beam-columns subjected to constant tangential follower force has been investigated by the Authors (Sharma, Singh and Benipal, 2013). These aspects of seismic response of concrete structures and dynamic behaviour of machinery-supporting concrete structures are of great relevance to structural designers. However, the scope of the present investigation is restricted to vibration response of cracked concrete beam-columns subjected to a constant lateral force and a pulsating vertical conservative force. Obviously, the proposed theory has to be extended and validated with relevant experimental data. Only then will it be of some practical relevance to designers of such concrete structures.

5 CONCLUSIONS

Concrete beam-columns are rendered physically nonlinear by the closing and reopening of the existing transverse cracks. An attempt has been made in this Paper to incorporate the effect of such physical nonlinearity in the classical theory of parametric resonance. The phenomenon of parametric resonance is predicted to occur in the form of T2 regular subharmonic resonance. This dynamic instability related to parametric resonance occurs only for the statically unstable sustained load sets (P_0, Q_0). Otherwise, the presence of physical nonlinearity stabilizes these beam-columns experiencing resonance. Also, physically nonlinear concrete beam-columns, even when undergoing small lateral displacements, are predicted to exhibit parametric resonance. When fully developed and empirically validated, the theory of parametric resonance purposed in this Paper is expected to be relevant for the design of concrete beam-columns subjected to pulsating axial forces due to earthquakes and machinery.

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