Latin American Journal of Solids and Structures
www.lajss.org

Thermoelastic interactions in an isotropic unbounded medium due to moving heat source using GNIII model

Abstract

In this work, we study a problem of thermoelastic interaction due to moving heat source in an isotropic infinite medium under Green and Naghdi model of type III (GNIII). The form of vector-matrix differential equation in the Laplace transform domain, the basic equations have been written, which is then solved by an eigenvalue technique. The analytical solution in the Laplace transforms domain with eigenvalue approach has been obtained. Numerical results for the displacement, temperature and the stress distributions are represented graphically. Some comparisons have been shown in figures to estimate the effect of heat source velocity and time in the physical quintets.

Keywords

Laplace transform; Green and Naghdi theory; thermoelasticity; Eigenvalue approach.

Ibrahim A. Abbas[a,b,c]

[a]Department of Mathematics, Faculty of Science and Arts - Khulais, University of Jeddah, Saudi Arabia.
[b]Nonlinear Analysis and Applied Mathematics Research Group (NAAM), Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia
[c]Department of mathematics, Faculty of Science, Sohag University, Sohag, Egypt.

Corresponding author:
ibrabbas7@yahoo.com

http://dx.doi.org/10.1590/1679-78251553

Received 06.09.2014
In revised form 18.02.2015
Accepted 11.03.2015
Available online 10.04.2015

1 INTRODUCTION

Lord and Shulman (1967) formulated an important generalized thermoelasticity theory with one relaxation time (LS model). After five years of L-S Model, Green and Lindsay (1972) introduced another generalized thermoelasticity theory with two relaxation time (GL model). In both of these theories, the basic equations of thermoelasticity are modified to eliminate the paradox of infinite velocity of heat propagation. These theories have practical importance in problems involving high heat fluxes for minor intervals.

Green and Naghdi (1991; 1992; 1993) proposed three new thermoelastic theories based on entropy equality rather than the usual entropy inequality. The heat-flux vector are different in each theory in the constitutive assumptions. These theories called thermoelasticity of type I, type II, and type III, where we obtain the classical system of thermoelasticity when the theory of type I is linearized.

Many investigators have treated the non-isothermal problems of the theory of elasticity, and so it become important. This is due to their many applications in widely diverse fields. In the extremely high temperature, the nuclear field and temperature gradients originating inside nuclear reactors influence their design and operations. In the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure. The counterparts of our problem in the contexts of the thermoelasticity theories have been considered by using numerical and analytical methods (Mukhopadhyay, 2006; Mukhopadhyay and Kumar, 2009; Jiangong and Tonglong, 2010; Kumar and Chawla, 2010; Abbas et al., 2011; Abo-Dahab and Abbas, 2011; Abbas, 2011; 2012; 2013; 2014a; 2014b; 2014c; Abbas and Abo-Dahab, 2014; Abbas and Zenkour, 2014). In which, Abbas solved different problems by eigenvalue approach in the Laplace transformation domain. Abbas and his collogues solved one and two-dimension problems by finite element method. Kumar and Chawla studied the wave propagation at the boundary surface of elastic layer overlaying a thermoelastic without energy dissipation half-space. Mukhopadhyay and his collogues used the state space approach for several problems.

Chandrasekharaiah and Srinath (1998a, 1998b) studied the Thermoelastic interactions without energy dissipation due to a point and line heat source. He and Cao (2009) considered generalized magneto-thermoelastic problem in thin slim strip subjected to a moving heat source. Youssef (2009, 2010) established the thermoelastic interactions in a unbounded medium with cylindrical and spherical cavity subjected to moving heat source.

In the present paper we have applied the technique of eigenvalue approach developed in Das et al. (1997) to solve generalized thermoelastic interaction problem subjected to a moving heat source using GNIII model. The eigenvalue approach gives exact solution in the Laplace domain without any assumed restrictions on the actual physical quantities. The governing equations of the mathematical model is presented when the beam is quiescent first. Laplace transforms techniques with eigenvalue approach are used to get the general solution for any set of boundary conditions. Numerical results are represented graphically. The moving heat source velocity have a significant effect on all distributions.

2 BASIC EQUATION AND FORMULATION OF THE PROBLEM

Following Othman and Abbas (2012), the system of equations that include the displacement, the stress, the strain and the temperature for a linear, homogenous and isotropic thermoelastic continuum without body forces take the following form:

The equations of motion

$$\sigma_{ji,j} = \rho \frac{\partial^2 u_i}{\partial t^2} \tag{1}$$

The equation of heat conduction

$$K^* T_{,ii} + K \dot{T}_{,ii} = \frac{\partial}{\partial t}\left(\rho c_e \dot{T} + \gamma T_0 \dot{e} - Q\right). \tag{2}$$

The constitutive equations are given by

$$\sigma_{ij} = 2\mu e_{ij} + \left[\lambda e - \gamma\left(T - T_0\right)\right]\delta_{ij}, \tag{3}$$

where λ, μ are the Lame's constants; K is the thermal conductivity; ρ is the density of the medium; $\gamma = \left(3\lambda + 2\mu\right)\alpha_t$ and α_t is the coefficient of linear thermal expansion; c_e is the specific heat at constant strain; t is the time; T_0 is the reference temperature; K^* is the material constant T is the temperature; characteristic of the theory; δ_{ij} is the Kronecker symbol; u_i are the components of displacement vector and Q is the moving heat source; σ_{ij} are the components of stress tensor. Let us consider a homogeneous isotropic thermoelastic solid at a uniform reference temperature T_0 occupying the region $x \geq 0$ where the x-axis is taken perpendicular to the bounding plane of the half-space pointing inwards. For one-dimensional problem the displacement vector u and temperatures field T can be expressed in the following form:

$$u_x = u\left(x,t\right),\ \ u_y = 0,\ \ u_z = 0,\ \ T = T\left(x,t\right). \tag{4}$$

Then the equations (1) to (3) take the following form

$$\left(\lambda + 2\mu\right)\frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{5}$$

$$K^*\frac{\partial^2 T}{\partial x^2} + K\frac{\partial^3 T}{\partial t\partial x^2} = \frac{\partial}{\partial t}\left(\rho c_e\frac{\partial T}{\partial t} + \gamma T_0\frac{\partial^2 u}{\partial t\partial x} - Q\right), \tag{6}$$

$$\sigma_{xx} = \left(\lambda + 2\mu\right)\frac{\partial u}{\partial x} - \gamma\left(T - T_0\right), \tag{7}$$

For convenience, we introduce the following non-dimensional variables

$$(x', u') = c_1\eta(x,u),\ \ T' = \frac{\gamma\left(T - T_0\right)}{\lambda + 2\mu},\quad t' = c_1^2\eta t,\ \ \sigma'_{xx} = \frac{\sigma_{xx}}{\lambda + 2\mu},\quad \mathrm{Q}' = \frac{\gamma Q}{K\rho c_1^4\eta^2},$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}\ \text{ and }\ \eta = \frac{\rho c_e}{K}.$$

Equations (5)-(7), and after suppressing the primes, we obtain

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{8}$$

$$\varepsilon_1\frac{\partial^2 T}{\partial x^2} + \frac{\partial^3 T}{\partial t\partial x^2} = \frac{\partial}{\partial t}\left(\frac{\partial T}{\partial t} + \varepsilon_2\frac{\partial^2 u}{\partial t\partial x} - Q\right), \tag{9}$$

$$\sigma_{xx} = \frac{\partial u}{\partial x} - T, \tag{10}$$

where

$$\varepsilon_1 = \frac{K^*}{\rho c^2 c_e}, \quad \varepsilon_2 = \frac{T_0 \gamma^2}{\rho^2 c^2 c_e}.$$

We consider that the medium is subjected to a moving heat source in the following non-dimensional form

$$Q = Q_o \delta\left(x - vt\right) \tag{11}$$

where Q_o is constant and δ is the delta function.

3 APPLICATION

We assume that the medium is initially at rest. The undisturbed state is maintained at reference temperature. Then we have

$$u\left(x,0\right) = \frac{\partial u\left(x,0\right)}{\partial t} = 0, \quad T\left(x,0\right) = \frac{\partial T\left(x,0\right)}{\partial t} = 0. \tag{12}$$

We consider boundary conditions of two types:
Case (I)

$$\sigma_{xx}\left(0,t\right) = 0, \; T\left(0,t\right) = T_1 \mathrm{H}\left(t\right), \tag{13}$$

Case (II)

$$u\left(0,t\right) = 0, \; \frac{\partial T\left(0,t\right)}{\partial x} = 0, \tag{14}$$

where $H(t)$ denotes the Heaviside unit step function and T_1 is a constant.

Applying the Laplace transform define by the formula

$$\overline{f}(s) = L\left[f(t)\right] = \int_0^\infty f(t) e^{-st} \mathrm{d}t. \tag{15}$$

Hence, we obtain the following system of differential equations

$$\frac{d^2\overline{u}}{dx^2} - \frac{d\overline{T}}{dx} = s^2 \overline{u}, \tag{16}$$

$$\varepsilon_1 \frac{d^2\overline{T}}{dx^2} + s\frac{d^2\overline{T}}{dx^2} = s^2\overline{T} + s^2 \varepsilon_2 \frac{d\overline{u}}{dx} - Q_o \frac{s}{v} e^{-sx/v}, \tag{17}$$

$$\overline{\sigma}_{xx} = \frac{d\overline{u}}{dx} - \overline{T}, \tag{18}$$

$$\bar{\sigma}_{xx}\left(0,s\right) = 0, \quad \bar{T}\left(0,s\right) = \frac{T_1}{s}, \tag{19}$$

$$\bar{u}\left(0,s\right) = 0, \frac{d\bar{T}\left(0,s\right)}{dx} = 0, \tag{20}$$

Equations (16) and (17) can be written in a vector-matrix differential equation as follows Das et al. (1997)

$$\frac{d\vec{V}}{dx} = A\vec{V} + \vec{f}, \tag{21}$$

where

$$\vec{V} = \left[\bar{u}\ \bar{T}\ \frac{d\bar{u}}{dx}\ \frac{d\bar{T}}{dx}\right]^T, \quad A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{31} & 0 & 0 & a_{34} \\ 0 & a_{42} & a_{43} & 0 \end{bmatrix} \text{ and } \vec{f} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ ge^{-sx/v} \end{bmatrix},$$

with

$$a_{31} = s^2, \ a_{34} = 1, \ a_{42} = \frac{s^2}{\varepsilon_1 + s}, \ a_{43} = \frac{s^2\varepsilon_2}{\varepsilon_1 + s}, \ g = -\frac{Q_o}{\varepsilon_1 + s}\frac{s}{v}.$$

Now, the eigenvalue approach are used to solve the equation (21) as Das et al. (1997)
The characteristic equation of the matrix A takes the form

$$a_{31}a_{42} - \left(a_{31} + a_{42} + a_{34}a_{43}\right)R^2 + R^4 = 0. \tag{22}$$

The roots of the characteristic equation (22) which are also the eigenvalues of matrix A are of the form $\pm R_1, \pm R_2$. The eigenvector $\vec{X} = \left[X_1, X_2, X_3, X_4\right]^T$, corresponding to eigenvalue R can be calculated as:

$$X_1 = a_{34}R\ , \ X_2 = R^2 - a_{31}\ , \ X_3 = RX_1\ , \ X_4 = RX_2. \tag{23}$$

From equations (23), we can easily calculate the eigenvector $\vec{X}_j$, corresponding to eigenvalue $R_i, \ i = 1,2,3,4$. For further reference, we can write

$$\vec{X}_1 = \left[\vec{X}\right]_{R=-R_1}, \quad \vec{X}_2 = \left[\vec{X}\right]_{R=-R_2}, \quad \vec{X}_3 = \left[\vec{X}\right]_{R=R_1}, \quad \vec{X}_4 = \left[\vec{X}\right]_{R=R_2}. \tag{24}$$

Thus, the complementary solution of equation (21) take the form

$$\vec{V}_c = \sum_{j=1}^{4} B_j\vec{X}_j e^{R_i x} = B_1\vec{X}_1 e^{-R_1 x} + B_2\vec{X}_2 e^{-R_2 x}, \tag{25}$$

where the terms containing exponentials of growing nature in the space variable x have been discarded due to the regularity condition of the solution at infinity, B_1 and B_2 are constants to be determined from the boundary condition of the problem.

The general solutions $\vec{V}$ of the nonhomogeneous system (21) are the sum of the complementary solution $\vec{V}_c$ of the associated homogeneous system and a particular solution $\vec{V}_p$ of the nonhomogeneous system. The inhomogeneous terms in (21) contain the exponential function $e^{-sx/v}$, therefore, the particular solution $\vec{V}_p$ should be sought in the form of a vector quasi-polynomial

$$\vec{V}_p = \vec{A}e^{-sx/v}, \tag{26}$$

where $\vec{A}$ is a constant vector. From (25), (26) and (21), the general solutions of the field variables can be written for x and s as:

$$\overline{u}(x,\mathrm{s}) = B_1 x_3^1 e^{-R_1 x} + B_2 x_3^2 e^{-R_2 x} + \frac{a_{34}\beta g}{D} e^{-\beta x} \tag{27}$$

$$\overline{T}(x,\mathrm{s}) = B_1 x_4^1 e^{-R_1 x} + B_2 x_4^2 e^{-R_2 x} + \frac{\left(a_{31} - \beta^2\right) g}{D} e^{-\beta x} \tag{28}$$

$$\overline{\sigma}_{xx}(x,\mathrm{s}) = -\left(R_1 x_3^1 + x_4^1\right) B_1 e^{-R_1 x} - \left(R_2 x_3^2 + x_4^2\right) B_2 e^{-R_2 x} - \left(\left(a_{34} - 1\right)\beta^2 + a_{31}\right)\frac{g}{D} e^{-\beta x}, \tag{29}$$

where

$$D = -a_{31}a_{42} - \left(a_{31} + a_{42} + a_{34}a_{43}\right)\beta^2 - \beta^4 \quad \text{and} \quad \beta = \frac{s}{v}.$$

To complete the solution we have to know the constants B_1 and B_2, by using the boundary conditions (19) for case (I) while the boundary conditions (20) for case (II).

4 NUMERICAL INVERSION OF THE LAPLACE TRANSFORMS

For the final solution of the temperatures, the displacement, the concentration, the stress and chemical potential distributions in the time domain, we adopt a numerical inversion method based on the Riemann-sum approximation method is used to obtain the numerical results. In this method, any function in the Laplace domain can be inverted to the time domain as

$$f(x,t) = \frac{\mathrm{e}^{mt}}{t}\left[\frac{1}{2}\mathrm{Re}[\overline{F}(x,m)] + \mathrm{Re}\sum_{n=0}^{N}\left((-1)^n \overline{F}\left(x, m + \frac{in\pi}{t}\right)\right)\right], \tag{30}$$

where Re is the real part and i is the imaginary number unit. For faster convergence, numerical experiments have shown that the value that satisfies the above relation is $m = 4.7/t$ (Tzou, 1996).

5 NUMERICAL RESULTS AND DISCUSSION

In the present work, the thermoelastic interactions due to moving heat source under Green and Naghdi of type III model is analyzed by considering an isotropic unbounded medium. The material parameters are given as following Abbas (2009)

$$\lambda = 7.76 \times 10^{10}(\mathrm{kg})(\mathrm{m})^{-1}(\mathrm{s})^{-2}, \quad \mu = 3.86 \times 10^{10}(\mathrm{kg})(\mathrm{m})^{-1}(\mathrm{s})^{-2}, \quad T_0 = 293\left(\mathrm{K}\right),$$

$$K = 3.68 \times 10^{2}(\mathrm{kg})(\mathrm{m})(\mathrm{K})^{-1}(\mathrm{s})^{-3}, \quad c_e = 3.831 \times 10^{2}(\mathrm{m})^{2}(\mathrm{K})^{-1}(\mathrm{s})^{-2},$$

$$\rho = 8.954x10^{3}(\mathrm{kg})(\mathrm{m})^{-3}, \quad \alpha_t = 17.8 \times 10^{-6}(\mathrm{K})^{-1}, \; t = 0.5, \;\; T_1 = 1.$$

Using this data set, the temperature T, displacement u and stress σ_{xx} are numerically computed for different values of the distance x and their graphical representation is presented in figures 1-12. As expected, in both the cases, the velocity of moving heat source has a great effect on the distribution of field quantities. The time have a great effect on all distributions.

Case (I): The figures 1-6 is investigating the variation of the non-dimensional temperature, displacement and stress when the traction free and subjected to a thermal shock on the surface $x = 0$. Figure 1-3 display the effects of velocity of moving heat source $\left(v = 0.2,\, 0.4,\, 0.6\right)$ when $t = 0.5$. It can be found that the temperature, magnitude of displacement and the magnitude of the stress decreases as the velocity increases before the intersection of the three curves. However, after the intersection, its increases as the velocity increases. Figure 4-6 show the variations of non-dimensional temperature, displacement and stress with distance for different value of time $\left(t = 0.2,\, 0.6,\, 1.0\right)$ when the moving heat source velocity $\left(v = 0.2\right)$ remains constant. It can be found that the temperature, magnitude of displacement and the absolute of the stress increases as the time increases. From figures 1-6, the temperature starts with $\left(T = T_1 = 1\right)$ at the origin and increases due to the moving heat source then decreases until attaining zero beyond a wave front for the generalized theory, which agree with the boundary conditions. The displacement component attains maximum negative values and gradually increases until it attains a peak value at a particular location in close proximity to the surface and then continuously decreases to zero. The stress, always starts from the zero value and terminates at the zero value to obey the boundary conditions.

Case (II): The figures 7-12 is investigating the variation of the non-dimensional temperature, displacement and stress when the surface thermally insulation and fixed. Figures 7-9 show the effects of velocity of moving heat source when time remains constant while, figures 10-12 show the effects of time when the velocity of moving heat source remains constant.

6 CONCLUSIONS

Two cases have been considered in our application. The first one for the traction free and subjected to a thermal shock on the surface while the second case for the surface thermally insulation and fixed. The eigenvalue approach gives exact solution in the Laplace domain without any assumed restrictions on the actual physical quantities. The velocity of moving heat source have a significant effect in on all distributions.

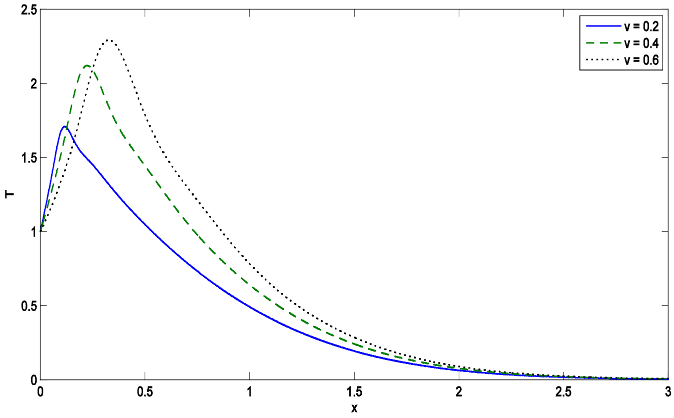

Figure 1: The variation of temperature with distance for different values of v (Case I).

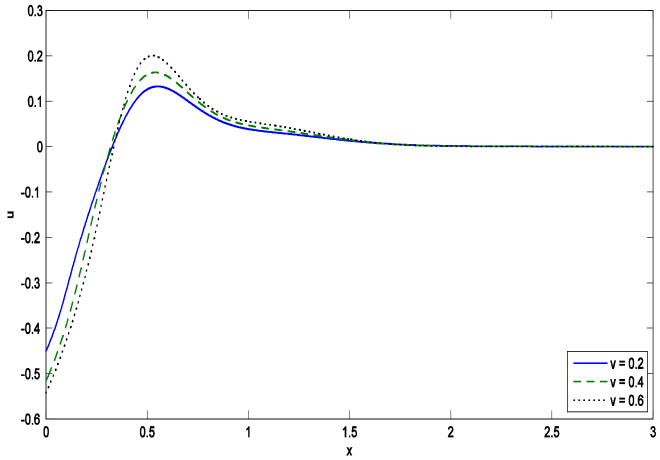

Figure 2: The variation of displacement with distance for different values of v (Case I).

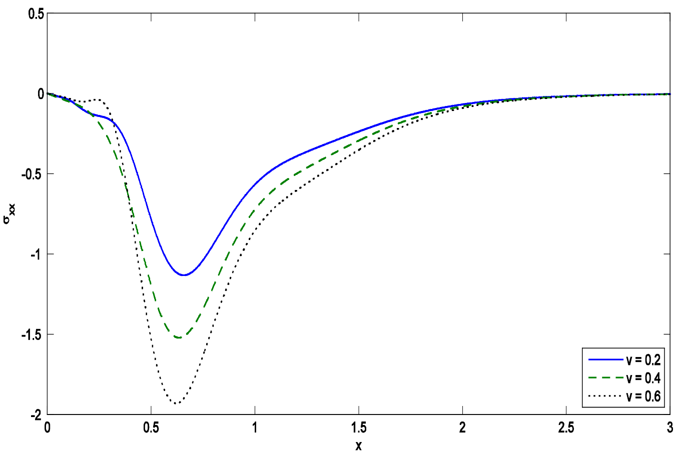

Figure 3: The variation of stress with distance for different values of v (Case I).

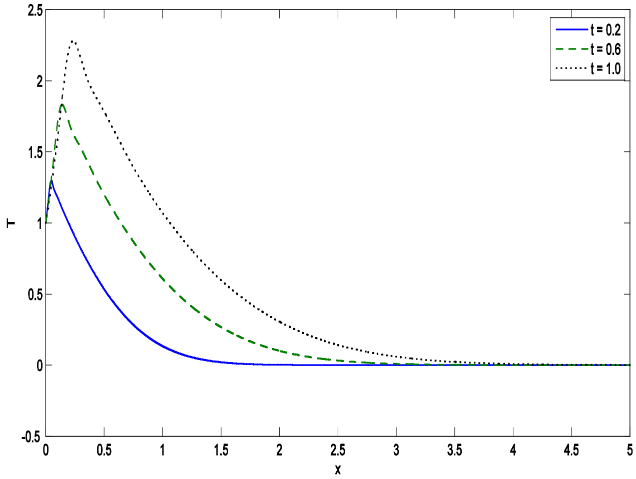

Figure 4: The variation of temperature with distance for different values of t (Case I).

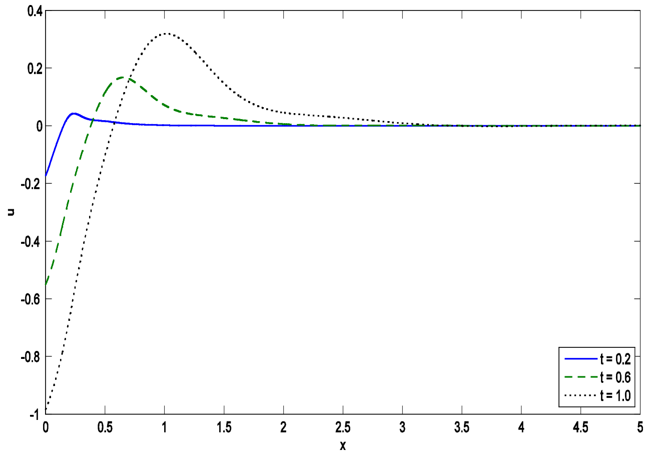

Figure 5: The variation of displacement with distance for different values of t (Case I).

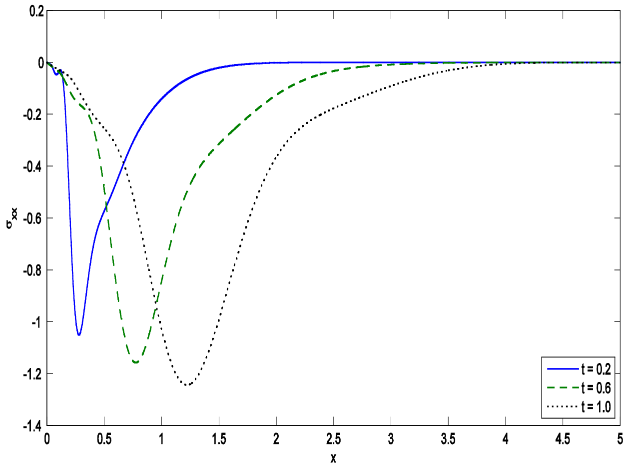

Figure 6: The variation of stress with distance for different values of t (Case I).

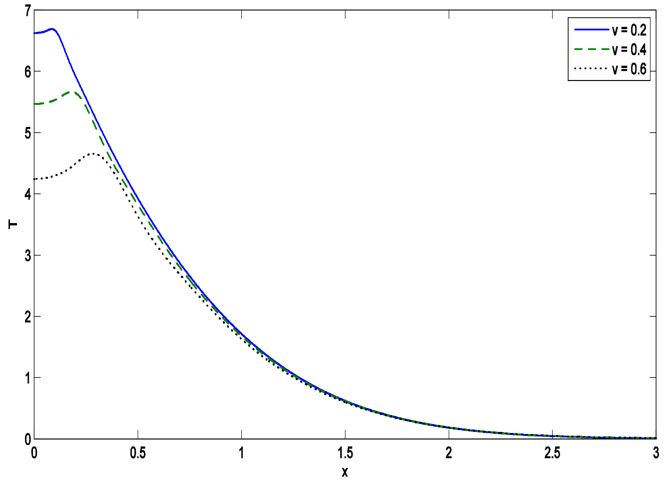

Figure 7: The variation of temperature with distance for different values of v (Case II).

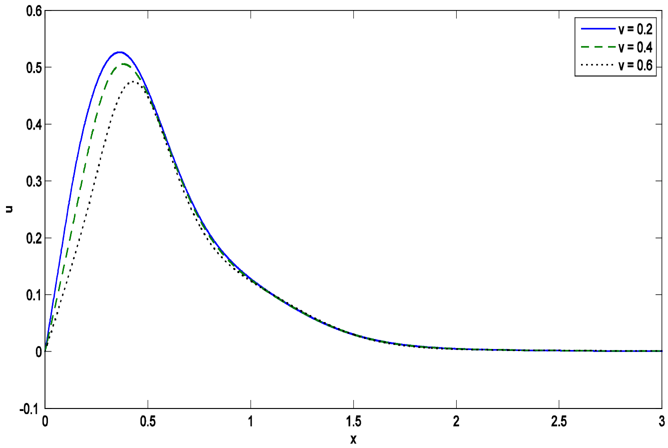

Figure 8: The variation of displacement with distance for different values of v (Case II).

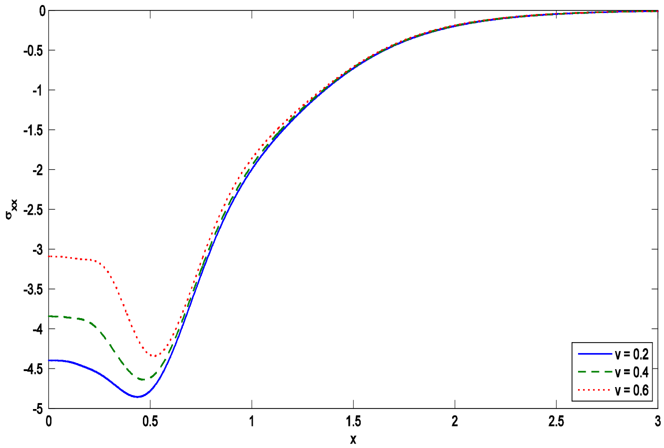

Figure 9: The variation of stress with distance for different values of v (Case II).

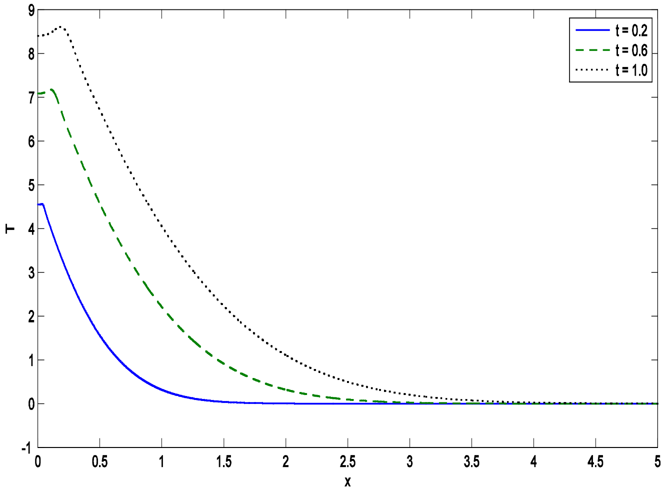

Figure 10: The variation of temperature with distance for different values of t (Case II).

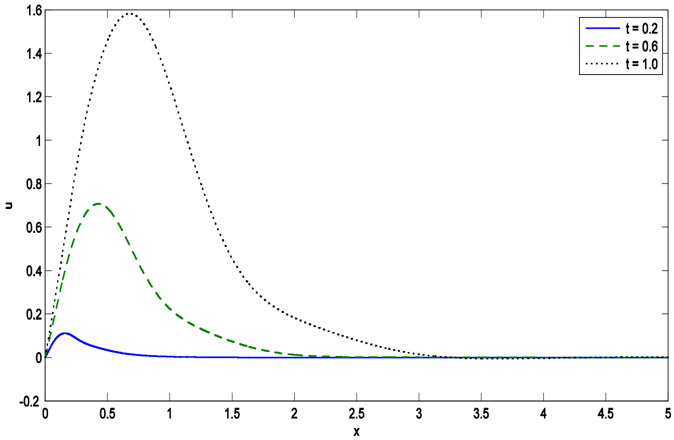

Figure 11: The variation of displacement with distance for different values of t (Case II).

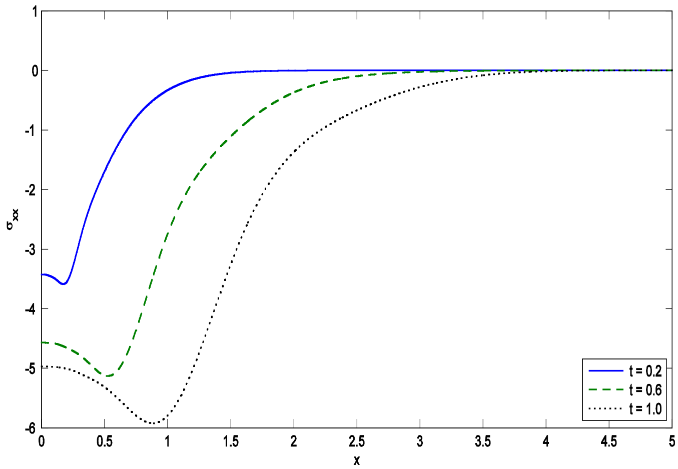

Figure 12: The variation of stress with distance for different values of t (Case II).

References

Abbas, I.A., (2009). Generalized magneto-thermoelasticity in a nonhomogeneous isotropic hollow cylinder using the finite element method. Archive of Applied Mechanics 79(1): 41-50.

Abbas, I.A., (2011). A two-dimensional problem for a fibre-reinforced anisotropic thermoelastic half-space with energy dissipation. Sadhana-Academy Proceedings in Engineering Sciences 36(3): 411-423.

Abbas, I.A., (2012). Generalized magneto-thermoelastic interaction in a fiber-reinforced anisotropic hollow cylinder. International Journal of Thermophysics 33(3): 567-579.

Abbas, I.A., (2013). A GN model for thermoelastic interaction in an unbounded fiber-reinforced anisotropic medium with a circular hole. Applied Mathematics Letters 26(2): 232-239.

Abbas, I.A., (2014a). A GN model based upon two-temperature generalized thermoelastic theory in an unbounded medium with a spherical cavity. Applied Mathematics and Computation 245: 108-115.

Abbas, I.A., (2014b). Eigenvalue approach for an unbounded medium with a spherical cavity based upon two-temperature generalized thermoelastic theory. Journal of Mechanical Science and Technology 28(10): 4193-4198.

Abbas, I.A., (2014c). Eigenvalue approach in a three-dimensional generalized thermoelastic interactions with temperature-dependent material properties. Computers & Mathematics with Applications 68(12): 2036-2056.

Abbas, I.A., Abd-alla, A.N., Othman, M.I.A., (2011). Generalized magneto-thermoelasticity in a fiber-reinforced anisotropic half-space. International Journal of Thermophysics 32(5): 1071-1085.

Abbas, I.A., Abo-Dahab, S.M., (2014). On the numerical solution of thermal shock problem for generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity. Journal of Computational and Theoretical Nanoscience 11(3): 607-618.

Abbas, I.A., Zenkour, A.M., (2014). The effect of rotation and initial stress on thermal shock problem for a fiber-reinforced anisotropic half-space using Green-Naghdi theory. Journal of Computational and Theoretical Nanoscience 11(2): 331-338.

Abo-Dahab, S.M., Abbas, I.A., (2011). LS model on thermal shock problem of generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity. Applied Mathematical Modelling 35(8): 3759-3768.

Chandrasekharaiah, D.S., Srinath, K.S., (1998a). Thermoelastic interactions without energy dissipation due to a line heat source. Acta Mechanica 128(3-4): 243-251.

Chandrasekharaiah, D.S., Srinath, K.S., (1998b). Thermoelastic interactions without energy dissipation due to a point heat source. Journal of Elasticity 50(2): 97-108.

Das, N.C., Lahiri, A., Giri, R.R., (1997). Eigenvalue approach to generalized thermoelasticity. Indian Journal of Pure and Applied Mathematics 28(12): 1573-1594.

Green, A.E., Lindsay, K.A., (1972). Thermoelasticity. Journal of Elasticity 2(1): 1-7.

Green, A.E., Naghdi, P.M., (1991). A re-examination of the basic postulates of thermomechanics. Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences 432(1885): 171-194.

Green, A.E., Naghdi, P.M., (1992). On undamped heat waves in an elastic solid. Journal of Thermal Stresses 15(2): 253-264.

Green, A.E., Naghdi, P.M., (1993). Thermoelasticity without energy dissipation. Journal of Elasticity 31(3): 189-208.

He, T., Cao, L., (2009). A problem of generalized magneto-thermoelastic thin slim strip subjected to a moving heat source. Mathematical and Computer Modelling 49(7-8): 1710-1720.

Jiangong, Y., Tonglong, X., (2010). Generalized thermoelastic waves in spherical curved plates without energy dissipation. Acta Mechanica 212(1-2): 39-50.

Kumar, R., Chawla, V., (2010). Wave propagation at the boundary surface of elastic layer overlaying a thermoelastic without energy dissipation half-space. Journal of Solid Mechanics 2(4): 363-375.

Lord, H.W., Shulman, Y., (1967). A generalized dynamical theory of thermoelasticity. Journal of the Mechanics and Physics of Solids 15(5): 299-309.

Mukhopadhyay, S., (2006). One dimensional state space approach to thermoelastic interactions without energy dissipation. Indian Journal of Pure and Applied Mathematics 37(3): 151-166.

Mukhopadhyay, S., Kumar, R., (2009). A problem on thermoelastic interactions without energy dissipation in an unbounded medium with a spherical cavity. Proceedings of the National Academy of Sciences India Section A - Physical Sciences 79(1): 135-140.

Othman, M.I.A., Abbas, I.A., (2012). Generalized thermoelasticity of thermal-Shock problem in a non-homogeneous isotropic hollow cylinder with energy dissipation. International Journal of Thermophysics 33(5): 913-923.

Tzou, D.Y., (1996). Macro-to micro-scale heat transfer: the lagging behavior, CRC Press.

Youssef, H.M., (2009). Generalized thermoelastic infinite medium with cylindrical cavity subjected to moving heat source. Mechanics Research Communications 36(4): 487-496.

Youssef, H.M., (2010). Two-temperature generalized thermoelastic infinite medium with cylindrical cavity subjected to moving heat source. Archive of Applied Mechanics 80(11): 1213-1224.