ORIGINAL ARTICLE

Latin American Journal of Solids and Structures

www.lajss.org

Nonlinear H∞ control scheme for a flying robot

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http://dx.doi.org/10.1590/1679-78255467

Abstract

In this paper, a nonlinear H∞ state feedback control is designed for both orientation and altitude of a flying robot system in the presence of external disturbance. An analytical solution is proposed for Hamilton-Jacobi-Isaac (HJI) equation. According to the quadrotor's orientation and altitude, a suitable storage function is considered and the appropriate robust control law is derived. The controller coefficients are tuned from Hamilton-Isaac-Jacobi inequality. The closed-loop nonlinear system with the proposed controller has L_2–gain less than or equal to γ, and guarantee its asymptotic stability closed-loop nonlinear system with external disturbance. Simulations are provided with the model uncertainties and external disturbance to verify the robustness of the proposed controller. Simulation results confirm the effectiveness of the desired robust controller.

Keywords

L_2-gain, Quadrotor, Nonlinear H∞, External disturbances, parametric uncertainties

1 INTRODUCTION

Flying robots have been become more and more important in the last few years. These kinds of robots are used in a variety of scopes, namely, film making, rescue mission, and crop spraying. Recently quadrotors have been attracted by researchers in different areas. Quadrotors have the ability of vertical landing and take-off and they are capable of hovering in the fixed location, which make them more efficient and safer to the fixed –wing aircrafts in usages (Min et al., 2009).

These kinds of flying robots are underactuated nonlinear systems and they are affected by external disturbances and parametric uncertainties. Therefore, they need robust controllers to attenuate the effect of external disturbance and overcome to parametric uncertainty.

Different linear control strategies have been developed for a flying robots. For instance, PID controller and LQR in Bouabdallah et al. (2004) and Budiyono and Wibowo (2007), have been developed for flying robot, but they have limited to a special range of flight. A feedback linearization was designed for spacecraft attitude control with hardware-in-the-loop simulations (Navabi et al. (2017), Navabi and Hosseini (2017)). In Bouabdallah (2005), two nonlinear control schemes developed for a quadrotor: sliding mode control and backstepping control. The model of quadrotor consists of two subsystems: the rotational dynamics and translational dynamics. These control strategies are used for rotational or translational dynamics.

Linear H∞ controller has been developed for linearized model of quadrotor in some researchers. In Chen and Huzmezan (2003), a linear H∞ controller was used for stabilization of angular and vertical velocities. Then a similar

Received: January 05, 2019. In Revised Form: February 25, 2019. Accepted: March 10, 2019. Available online: March 12, 2019.
http://dx.doi.org/l10.1590/1679-78255467

strategy was applied to the outer loop for yaw movement and altitude. In the last part, a predictive control was developed for tracking control. In Mokhtari and Benallegue (2006), a linear H_∞ controller was designed for a desired trajectory following of a rotorcraft UAV involving model uncertainties and aerodynamics disturbances. For this purpose, a robust adaptive controller was developed by a Lyapunov-like energy function (Islam et al., 2015). A robust optimal adaptive trajectory tracking control was designed for a quadrotor helicopter (Navabi and Mirzaei, (2016), Navabi and Mirzaei, (2017)). A composite nonlinear robust controller scheme, variable structure control and backstepping approach has been developed for the position and yaw angle of a quadrotor (Chen et al., 2016). In Xu et al. (2017), a robust set-point tracking controller has been applied for a quadrotor aircraft involving uncertainties.

Researchers have been developed many types of controllers on the quadrotor, however, most of the papers ignoring external disturbances and model uncertainties (Raffo et al., 2008). Nonlinear H_∞ control has the ability of attenuating the influence of external disturbances and un-modeled dynamics.

Nonlinear H_∞ schemes successfully have been developed for many nonlinear systems. For instance, power converter (Kugi and Schlacher, 1999), rigid spacecraft (Kang,1995), robot manipulator (Chen et al.,1994), and chemical processes (Li and Zhang,1999). A nonlinear robust scheme using nonlinear H_∞ was proposed for attitude stability of a flying robot and a backstepping control scheme was developed for path tracking (Guilheme et al., 2008). In (Jasim and Gu, 2014), a candidate storage function V(x) was considered for the rotational dynamics and developing a nonlinear H_∞ scheme for the stability of the orientation of quadrotor. In Raffo et al. (2008), a composite control approach using backstepping technique and nonlinear H_∞ were used to perform the robust tracking problem of a quadrotor. However, this scheme was able to provide robustness only in the rotational dynamics.

In this paper, a nonlinear H_∞ control scheme has been developed in the sense of L_2 – gain for both orientation and altitude stabilizing of a flying robot. As far as our belief, the nonlinear H_∞ control approach with an analytical solution of the HJI equation has not been introduced yet. The major contributions of the suggested controller in this paper are as follows:

1. It is robust against parametric uncertainties in both altitude and orientation. Compared with the variable structure control which has the same property, but its switching logic cause the chattering phenomenon.
2. Compared with the feedback linearization techniques which are effective in some specific cases, it guarantees the performance for a variety of operating conditions.

The paper structured as follows. In the next section, a flying robot model is expressed based on Euler angles. The nonlinear H_∞ controller of both orientation and altitude is developed in Section 3 along with mathematical proof of stability. The results of simulation are described in Section 4. At last, the paper is concluded in Section 5.

2 MATHEMATICAL MODELING

The quadrotor has four rotors in a cross configuration. The directions of rotation of diagonal rotors are clockwise while the directions of the other diagonal rotors are counter-clockwise to eliminate gyroscopic effects. This quadrotors controlled by the rotors' speed to produce the desired lift force. Any increase or decrease of four rotors' speed will vary the lift force and create the vertical motion of the system. Different speed in diagonal rotors causes yaw angle. Different speed in diagonal rotors along y-axis causes roll angle and different speed in diagonal rotors along x-axis causes pitch angle.

The mathematical modeling of a quadrotor has been described by many researches such as Castillo et al. (2004a), Czyba and Szafranski (2013), and Fernando et al. (2013). The origin of the body frame (the moving frame) of the flying robot is attached to the centroid of the flying robot (See Figure 1).

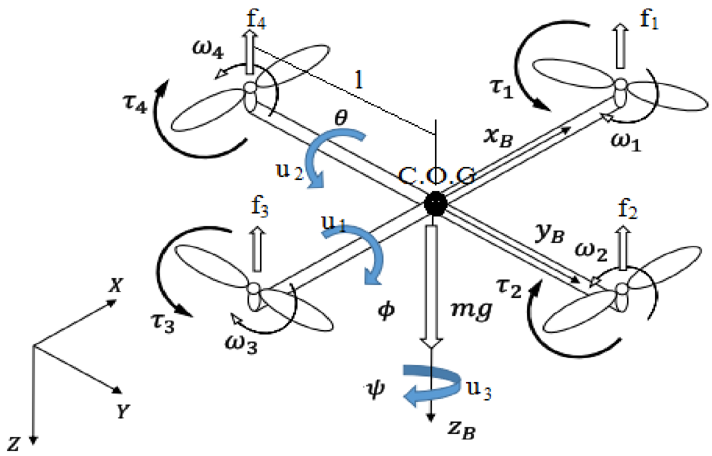

Figure 1: Flying robot coordinates

With the three components of Euler angles in the form $\eta = [\phi \quad \theta \quad \psi]^T$, the orientation of the quadrotor is obtained. The pitch and roll angles are restricted by $\left(-\frac{\pi}{2} \quad \frac{\pi}{2}\right)$, while the yaw angle is restricted by $(-\pi \quad \pi)$. The position of flying robot in the reference frame is shown by $\xi = \begin{bmatrix} x & y & z \end{bmatrix}^T$. We will consider the following assumptions:

1) The flying robot has a rigid and symmetric structure.
2) Neglecting air force because of low speed of the air.
3) The flying robot has rigid propellers.

The vector ξ measures the position of the flying robot in the moving frame. To transform ξ to the reference frame, the standard aeronautical matrix R is used:

$$\mathrm{R}(\eta) = \begin{bmatrix} c\theta c\psi & c\psi s\theta s\phi - c\phi s\theta & c\phi c\psi s\theta + s\phi s\psi \\ c\theta s\psi & s\theta s\phi s\psi + c\phi c\psi & c\phi s\theta s\psi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

where $c.$ and $s.$ stand for $\cos(.)$ and $\sin(.)$, respectively.

The kinematics for translational subsystem can be obtained as:

$$V_I = R(\eta) V_B \tag{1}$$

where V_I and V_B are the centroid translational velocities in the reference frame and the moving frame, respectively. According to the relationship between $R(\eta)$ and its derivatives (Olfati-Saber, 2001), the rotational kinematics is given by:

$$\dot{\eta} = T(\eta)\omega$$

$$\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & \frac{s\phi}{c\theta} & \frac{c\phi}{c\theta} \\ 0 & c\phi & -s\phi \\ 1 & s\phi t\theta & c\phi t\theta \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{2}$$

where $\omega = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T$ represents the three components of rotational velocity respect to moving frame and $t\theta$ is stand for $\tan(\theta)$. The thrust and moment generated by each propeller is described by:

$$f_i = b\omega_i^2$$

$$\tau_i = d\,\omega_i^2, i = 1,2,3,4$$

where ω_i is the angular speed of *i*th motor, *b* is the positive thrust constant, and *d* is the positive drag constant. The moments created by the motors along the three body axes, x_B, y_B, and z_B, can be written as

$$u_1 = l(f_4 - f_2) = bl(\omega_4^2 - \omega_2^2)$$

$$u_2 = l(f_3 - f_4) = bl(\omega_3^2 - \omega_1^2)$$

$$u_3 = \tau_2 + \tau_4 - \tau_1 - \tau_3$$

$$= d(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2)$$

$$u_4 = f_1 + f_2 + f_3 + f_4$$

where *l* is the distance from the motor to the center of mass.

The dynamical model, ignoring aerodynamics effects and gyroscopic moments is given by (Castillo et al., 2004b):

$$\begin{cases} \dot{\eta} = T(\eta)\omega \\ \dot{\omega} = -J^{-1}s(\omega)J\omega + J^{-1}u_r + J^{-1}d \\ \dot{\xi} = \nu \\ \dot{\nu} = ge_3 - m^{-1}R(\eta)u_t \end{cases} \tag{3}$$

where

$$e_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T, u_r = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T, u_t = \begin{bmatrix} 0 & 0 & u_4 \end{bmatrix}^T, \nu = \begin{bmatrix} \nu_x & \nu_y & \nu_z \end{bmatrix}^T, \xi = \begin{bmatrix} x & y & z \end{bmatrix}^T,$$

$\eta = \begin{bmatrix} \phi & \theta & \varphi \end{bmatrix}, s(\omega) = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$ and J is the inertia matrix given by $J = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$, *d* is the disturbance input vector and $u = \begin{bmatrix} u_r & u_t \end{bmatrix}$.

By defining $x = \begin{bmatrix} \eta & \omega & \xi & \nu \end{bmatrix}^T$, equation (3) is expressed via the following form:

$$\dot{x} = f(x) + g_1(x)u + g_2(x)d \tag{4}$$

where

$$f(x) = \begin{bmatrix} T(\eta)\omega \\ -J^{-1}s(\omega)J\omega \\ \nu \\ 0_{3\times1} \end{bmatrix}, g_1(x)\begin{bmatrix} 0_{3\times3} \\ J^{-1} \\ 0_{3\times3} \\ m^{-1}R(\eta) \end{bmatrix}, g_2(x) = \begin{bmatrix} 0_{3\times3} \\ J^{-1} \\ 0_{3\times3} \\ 0_{3\times3} \end{bmatrix}$$

3 THE PROPOSED CONTROL STRUCTURE FOR THE FLYING ROBOT

3.1 Nonlinear H∞ control theory

Nonlinear H∞ scheme provides a method for driving robust controllers for nonlinear dynamics with external disturbances and parametric uncertainties. The formulation of nonlinear H∞ control is derived from the dissipativity concept. Dissipativity indicates the way energy is stored and dissipated in a nonlinear system around on equilibrium point.

In spite of fully development of the nonlinear H∞ control theory, the HJI equation has not any general analytical solution and usually difficult to solve for a special nonlinear system. So, this is the major problem for practical applications.

Consider nonlinear affine system with external disturbance in the form (Isidori and Astolfi (1992), Van der Schaft (1992), Bianchini et al. (2004)):

$$\begin{aligned} \dot{x} &= f(x) + g_1(x) + g_2(x)d \\ z &= \begin{bmatrix} h(x) & u(x) \end{bmatrix}^T \end{aligned} \tag{5}$$

where $x \in R^n$ and $u \in R^m$ are the state vector and the control input respectively, while d is the external disturbance and $z \in R^q$ is the regulated output and $h(x)$ is state weighting function and f, g_1, g_2, h are approximately dimensioned smooth function of states with $f(0) = 0, h(0) = 0$.

The goal of sub-optimal H∞ problem is to obtain a C^1 controller, $u(x)$ which can satisfy the following inequality:

$$\begin{aligned} &\int_0^T \|z\|^2 dt \le \gamma^2 \int_0^T \|d\|^2 dt + V(x(0)) \\ &\forall d \in L_2[0 \quad T], \forall T > 0 \end{aligned} \tag{6}$$

where $\gamma > 1$ and a nonnegative storage function, $V(x)$ should be constructed which $x(0)$ shows the initial states. The following Lemma helps us to design the $u(x)$.

Lemma 3.1 (Van der Schaft, 2017). Define the following HJI partial differential inequality

$$H_v = L_f V(x) - \frac{1}{2}\left\|L_{g_1}^T V(x)\right\|^2 + \frac{1}{2\gamma^2}\left\|L_{g_2}^T V(x)\right\|^2 + \frac{1}{2}h(x)^T h(x) < 0 \tag{7}$$

where $L_i, i = f, g_1, g_2$ represent the Lie derivatives.

If a C^1 storage function with $V(0)=0$, $V(x)>0, x\neq 0$ exists which satisfies (7), then we have an L_2-gain <γ for the closed-loop system.

Moreover, when (5) is zero-state detectable, its asymptotic stability is satisfied and $u(x)$ is obtained by:

$$u(x)=-g_1^T(x)V_X(x) \tag{8}$$

to satisfy the L_2-gain.

It is obvious that finding of the storage function $V(x)$ is the hardest stage of this control approach.

In summary, for the system was described in (4), we want to find a controller such that satisfying

$$\frac{\|z\|_2}{\|d\|_2}\leq\gamma$$

where γ is a positive scalar.

3.2 Controller design

The nonlinear H∞ control tries to keep both orientation and altitude of the quadrotor aligned with the reference frame involving external disturbance and parametric uncertainty. Hence the cost function is defined as follows:

$$z=\begin{bmatrix} h(x) & u(x)\end{bmatrix}^T \tag{9}$$

Which contains two parts: the first part pertaining to the attitude and altitude control performance, and the other part is the amount of control input. The $h(x)$ is considered as the following form:

$$h(x)=\left\{\frac{a_1}{2}T(\omega)+\frac{a_2}{2}N^2(R(\eta),I)+\frac{a_3}{2}(\nu+\xi)^T(\nu+\xi)\right\}^{\frac{1}{2}} \tag{10}$$

where the first part of (10) shows the amount of rotational kinetic energy, the second part shows that how two frames are far from each other and the third part is a measure of linear momentum and potential energy.

$N(R(\eta),I)$ is the geodesic metric on $SO(3)$ (Samson, 1991):

$$N(R(\eta),I)=2\cos^{-1}\left(\frac{\sqrt{1+Tr(R(\eta))}}{2}\right) \tag{11}$$

where $R(\eta)$ is the rotation matrix given in Section 2. The third part is a combination of translational kinetic energy and linear momentum and potential energy. It can be seen that $h(x)=0$ yields $x=0$ which guarantees the coincidence of the reference frame and the body frame.

Let us define (Kang, 1995):

$$Q(\eta)=3-Tr(R(\eta)) \tag{12}$$

and

$$Y\ (\eta)=\begin{bmatrix} s\psi s\theta c\phi - s\phi(c\psi + c\theta) \\ -s\theta(1+c\psi c\phi) - s\psi s\phi \\ c\psi s\theta s\phi - s\psi(c\phi + c\theta) \end{bmatrix} \tag{13}$$

Moreover:

$$T\ (v)=\frac{1}{2}mv^{T}v \tag{14}$$

$$K\ (\xi)=\frac{1}{2}\xi^{T}\xi \tag{15}$$

Where $Q(\eta)$ represents the distance between the rotation matrix and identity matrix in a different appearance, $Y\ (\eta)$ is a vector that will be used in H∞ feedback.

Lemma 3.2 (AliAbbasi et al., 2002). We have the following relations:

$$Y^{T}Y\ \leq 6Q \tag{16}$$

$$Q \leq Y^{T}Y \tag{17}$$

$$N^{2}\left(R\left(\eta\right), I\right)\leq 4Y^{T}Y \tag{18}$$

Lemma 3.3. Consider (5), the following equation hold:

$$(a)L_{f}T\ (\omega)=0$$

$$(b)L_{g_1}T\ (\omega)=\omega^{T}$$

$$(c)L_{g_2}T\ (\omega)=\omega^{T}$$

$$(d)L_{f}T\ (v)=0$$

$$(e)L_{g_1}(v)=v^{T}R$$

$$(f)L_{g_2}T\ (v)=v^{T}R$$

$$(g)L_{f}Q\ (\eta)=-Y^{T}\omega$$

$$(h)L_{g_1}Q\ (\eta)=0$$

$$(i)\, L_{g_2} Q(\eta) = v^T R$$

$$(j)\, L_f \left(\omega^T JY\right) = Y^T s(\omega) J\omega + \omega^T J \frac{\partial Y}{\partial\psi\partial\theta\partial\phi} T(\eta)\omega$$

$$(k)\, L_{g_1} \left(\omega^T JY\right) = Y^T$$

$$(l)\, L_{g_2} \left(\omega^T JY\right)$$

$$(m)\, L_f K(\xi) = \xi^T v$$

$$(n)\, L_{g_1} K(\xi) = 0$$

$$(o)\, L_{g_2} K(\xi) = 0$$

$$(p)\, L_f \left(mv^T \xi\right) mv^T v$$

$$(q)\, L_{g_1} \left(mv^T \xi\right) = \frac{1}{2}\xi^T R$$

$$(r)\, L_{g_2} \left(mv^T \xi\right) = 0$$

$$(s)\, \left\| \frac{\partial Y}{\partial\psi\partial\theta\partial\phi} T(\eta) \right\| \leq 4$$

$$(t)\, \|Y\| = 3, \|s(\omega)\| = \|\omega\|$$

where ω and v are rotational and translational velocities in the moving frame respectively, m and J are the mass and the inertia matrix of the flying robot respectively and R is the rotation matrix which defined in the second section. The lemma can be proved by direct calculations and omitted for the sake of brevity.

As it pointed out in the previous subsection, the construction of $V(x)$ is a crucial step in the nonlinear H∞ control approach. Because HJI equation is a nonlinear first order PDE which has not a general solution. To this end, the candidate storage function is considered as follows:

$$V(x) = \frac{1}{2}a_1\omega^T J\omega + \frac{1}{2}a_2 mv^T v + \frac{1}{2}a_3 Y^T Y + \frac{1}{2}a_4 \xi^T \xi - a_5 \omega^T JY - a_6 mv^T \xi \tag{19}$$

The following relation can be found:

$$V(x) \geq \frac{1}{2}a_1\omega^T J\omega + \frac{1}{2}a_2 mv^T v + \frac{1}{2}a_3 Y^T Y + \frac{1}{2}a_4 \xi^T \xi - a_5 \omega^T JY - a_6 mv^T \xi$$

Hence, the storage function can be written as:

$$\frac{1}{2}\begin{bmatrix} Y^T & \omega^T & \xi^T & v^T \end{bmatrix}\begin{bmatrix} a_3I & a_5J & 0 & 0 \\ a_5J & a_3J & 0 & 0 \\ 0 & 0 & a_4I & -a_6mI \\ 0 & 0 & -a_6mI & a_2mI \end{bmatrix}\begin{bmatrix} Y \\ \omega \\ \xi \\ v \end{bmatrix} \tag{20}$$

The conditions for positive definiteness of (20) can be easily written as:

$$a_1a_3I > a_5^2J \tag{21}$$

$$a_2a_4 > a_6^2m \tag{22}$$

where I is an (3×3) identity matrix.

Now, we have to show that the candidate storage function satisfies the HJI inequality (7):

$$\begin{aligned} H_v &= L_fV(x) - \frac{1}{2}\left\|L_{g_1}^TV(x)\right\|^2 + \frac{1}{2\gamma^2}\left\|L_{g_2}^TV(x)\right\|^2 + \frac{1}{2}h(x)^T h(x) = \\ &\left\{-3a_3Y^T\omega - a_5Y^Ts(\omega)J\omega - a_5\omega^TJ\frac{\partial Y}{\partial\psi\partial\theta\partial\phi}T(\eta)\omega + \frac{1}{2}\left(\frac{1}{\gamma^2}-1\right)\left\|a_1\omega - a_5Y\right\|^2\right\} \\ &+\left\{\frac{a_1}{2}T(\omega) + \frac{a_2}{2}N^2(R,I) + \frac{a_3}{2}m(v+\xi)^T(v+\xi)\right\} = A_1 + A_2 \end{aligned} \tag{23}$$

where

$$\begin{aligned} A_1 &= \left\{-3a_3Y^T\omega - a_5Y^Ts(\omega)J\omega - a_5\omega^TJ\frac{\partial Y}{\partial\psi\partial\theta\partial\phi}T(\eta)\omega\right\} \\ &+\frac{a_1}{2}T(\omega) + \frac{1}{2}\left(\frac{1}{\gamma^2}-1\right)\left\|a_1\omega - a_5Y\right\| + \frac{a_2}{2}N^2(R,I) \end{aligned}$$

and

$$\begin{aligned} A_2 &= a_4\xi^Tv + a_3m\xi^Tv - a_6m\left\|v\right\|^2 + \frac{1}{2}\left(\frac{1}{\gamma^2}-1\right)\left\|a_2R^Tv\right\|^2 - \frac{a_6^2}{2m^2}\left\|R^T\xi\right\|^2 \\ &+\frac{a_3}{2}m\left\|v\right\|^2 + \frac{a_3}{2}m\left\|\xi\right\|^2 \end{aligned}$$

Now, by choosing

$$a_3 = \frac{a_2a_1}{3}\left(1-\frac{1}{\gamma^2}\right) \tag{24}$$

The condition for non-positiveness of A_1 are:

$$\frac{a_5^2}{2}\left(\frac{1}{\gamma^2}-1\right)+2a_2<0 \tag{25}$$

For A_2 to be non-positive, the following relations should be hold:

$$a_4+a_3m=0 \tag{26}$$

$$a_3<0 \tag{27}$$

Hence, to make H_v non-positiveness, the inequalities (24), (25), (26), and (27) should be hold. Now, according to lemma 3.1 and employing (8) the control law can be obtained as:

$$u(x)=-g_1^T(x)V_x(x)=-\begin{bmatrix}0_{3\times3} & J^{-1} & 0_{3\times3} & m^{-1}R\end{bmatrix}\begin{bmatrix}-a_3Y \\ a_1J\omega-a_5JY \\ a_4\xi+a_6mv \\ a_2mv+a_6\xi\end{bmatrix} \tag{28}$$
$$=a_1\omega+a_5Y+a_2Rv+a_6m^{-1}R\xi$$

where R , ξ and v are defined in the second section and Y is defined in (13). Control input, $u(x)$ is considered as:

$$u(x)=[u_r \quad u_t]^T$$

where u_r and u_t are defined in the second section. Hence, the control inputs for $u_r=[u_1 \quad u_2 \quad u_3]^T$ are given by:

$$\begin{aligned} u_1&=-a_1\omega_x+a_5\left(s\psi c\theta c\phi-s\phi\left(c\psi+c\theta\right)\right) \\ u_2&=-a_1\omega_y+a_5\left(-s\theta\left(1+c\psi c\phi\right)-s\psi s\phi\right) \\ u_3&=-a_1\omega_z+a_5\left(c\psi s\theta s\phi-s\psi\left(s\phi+c\theta\right)\right) \end{aligned} \tag{29}$$

and u_4 in $u_t=[0 \quad 0 \quad u_4]^T$ is given by:

$$u_4=-\left[\left(c\phi c\psi s\theta+s\phi s\psi\right)\left(v_x+a_6m^{-1}x\right)+\left(c\phi s\theta s\psi-s\psi s\phi\right)\left(v_y+a_6m^{-1}y\right)+\left(c\theta c\phi\right)\left(v_z+a_6m^{-1}z\right)\right] \tag{30}$$

4 SIMULATION RESULTS

Through computer simulation with model uncertainties and external disturbances, we demonstrate that the proposed approach is effective. The closed-loop system (4) and (28) was simulated using MATLAB. The numerical values for the parameters of flying robot are used for simulation are adopted from (Raffo et al., 2010):

$$m=0.74kg, g=9.81\frac{m}{s^2}$$

$$J = \begin{bmatrix} 0.004 & 0 & 0 \\ 0 & 0.004 & 0 \\ 0 & 0 & 0.004 \end{bmatrix} kg.m^2$$

The initial conditions of flying robot are:

$$\eta_0 = \begin{bmatrix} 0.25 & 0.25 & 0.25 \end{bmatrix}^T \ rad$$

$$\dot{\eta}_0 = \begin{bmatrix} 10.8982 & -1.6821 & 0.1 \end{bmatrix}^T \ rad.s^{-1}$$

$$\xi_0 = \begin{bmatrix} 0 & 0.5 & 0.5 \end{bmatrix}^T \ m$$

$$\dot{\xi}_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \ m.s^{-1}$$

The proposed controller gains have been tuned with the following parameters:

$$\gamma = \sqrt{2}, a_1 = 2.1708, a_2 = 1, a_5 = 20, a_6 = 0.74$$

The following moment disturbances were used:

$$d1(t) = 0.005 + 0.005\sin(0.024\pi t) + 0.01\sin(1.32\pi t) \ N.m \tag{31}$$

$$d2(t) = 0.01 + 0.01\sin(0.024\pi t) + 0.05\sin(1.32\pi t) \ N.m \tag{32}$$

The proposed controller was tested for different disturbances consist of the model uncertainties (mass and moment of inertia) and moment disturbances. The simulation for PID controller obtained in (Comert and Kasnakoglu (2017)) was also tested for comparative study. Figure 2 shows the performance of Euler angles (orientation) and altitude using nonlinear H∞ controller with that of nonlinear H∞ controller under action of the designed control law compared with ±40% model parameter uncertainties and the moment disturbances are shown in (31) and (32). The performance of angular velocities under the use of nonlinear H∞ controller and nonlinear H∞ controller with the effect of disturbances and model parameter uncertainties is shown in Figure 3.

The result of simulation using PID controller and PID controller with ±40% model parameter uncertainties and the moment disturbances are depicted in Figure 4. Figure 5 shows the angular velocities performance using PID controller and PID controller with the effect of disturbances and parameter uncertainties.

It can be seen that the proposed controller can reject disturbances and cover the changes in parameters uncertainties and remains asymptotically stable, while the PID controller cannot reject the disturbances and has a big noise.

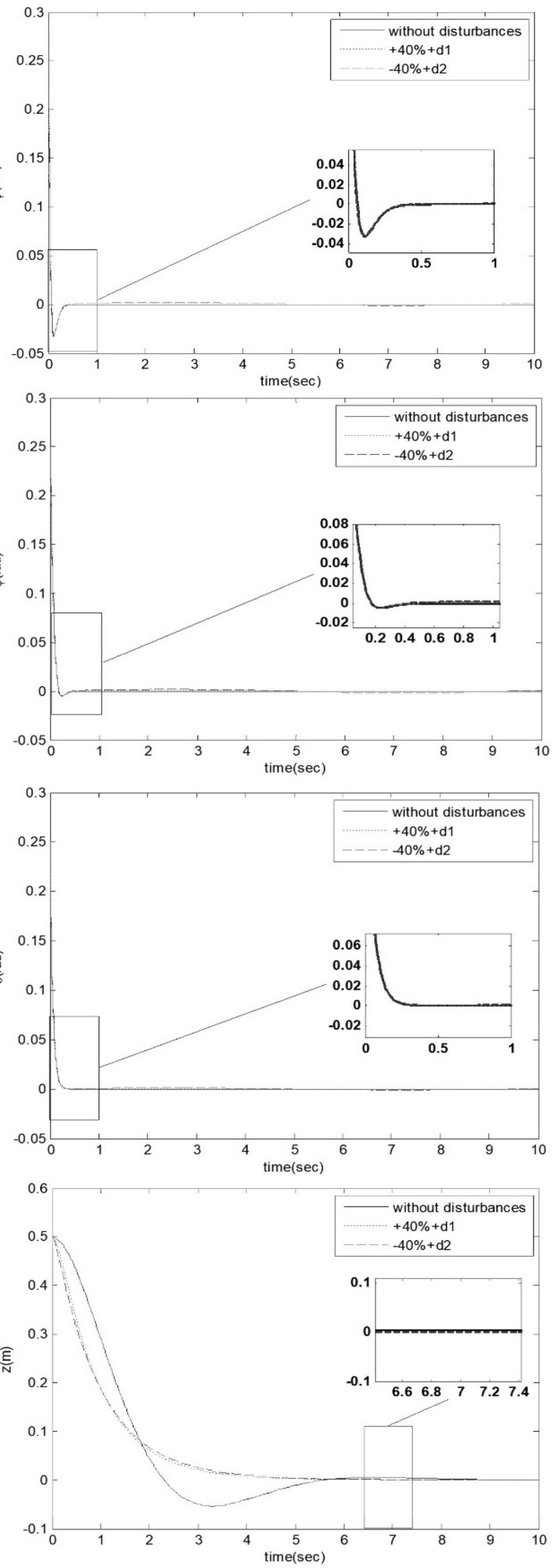

Figure 2: Orientation and altitude, nonlinear H_∞ controller.

The comparisons between Figure 3 and Figure 5 illustrates that the angular performance using the proposed controller achieves the stability conditions faster than that of using PID controller.

In general, with nonlinear H_∞ controller obtained a good result compared with that of the PID controller in terms of time-consuming, disturbance rejection and model parameter uncertainties change cover.

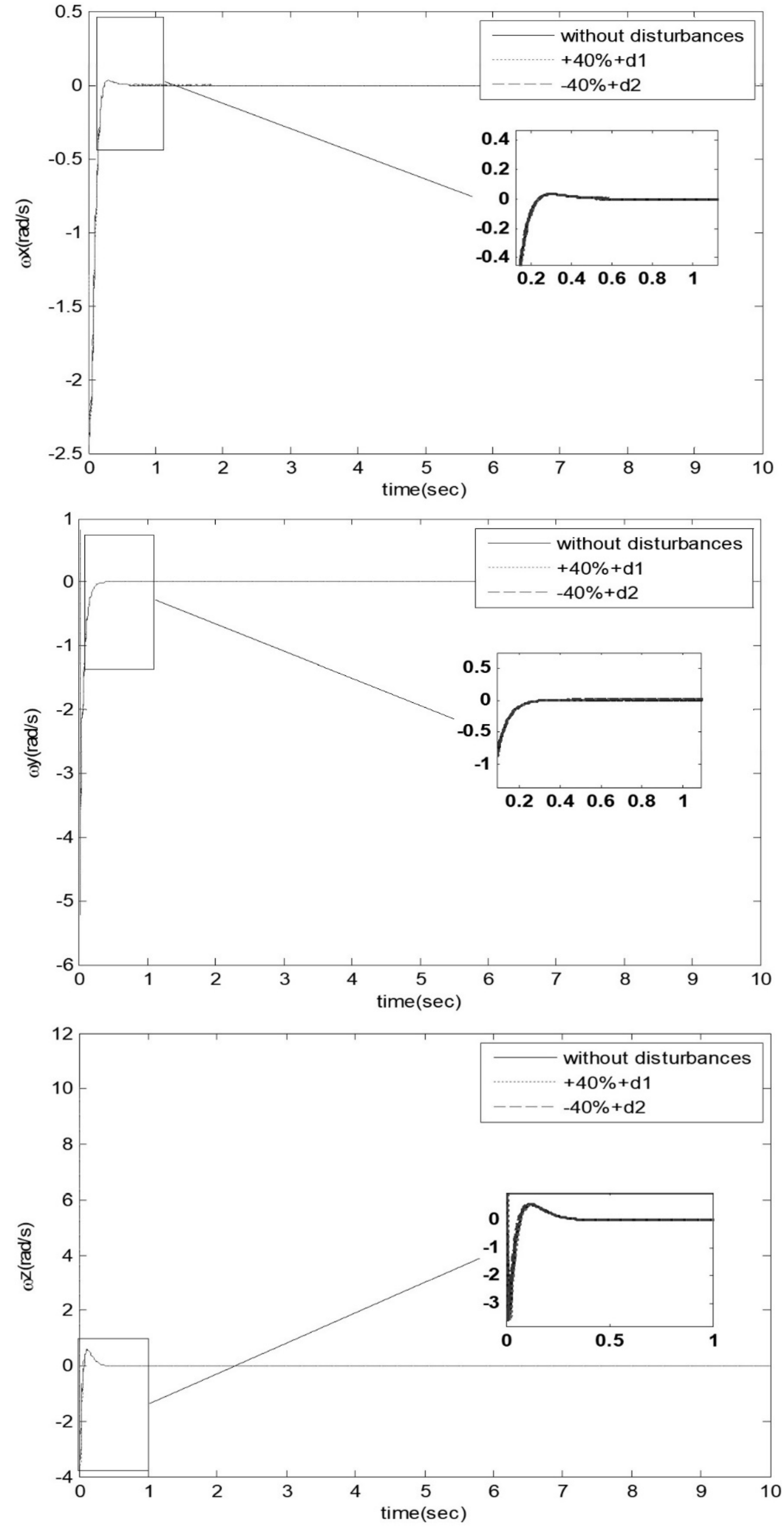

Figure 3: Angular velocities, nonlinear H_∞ controller

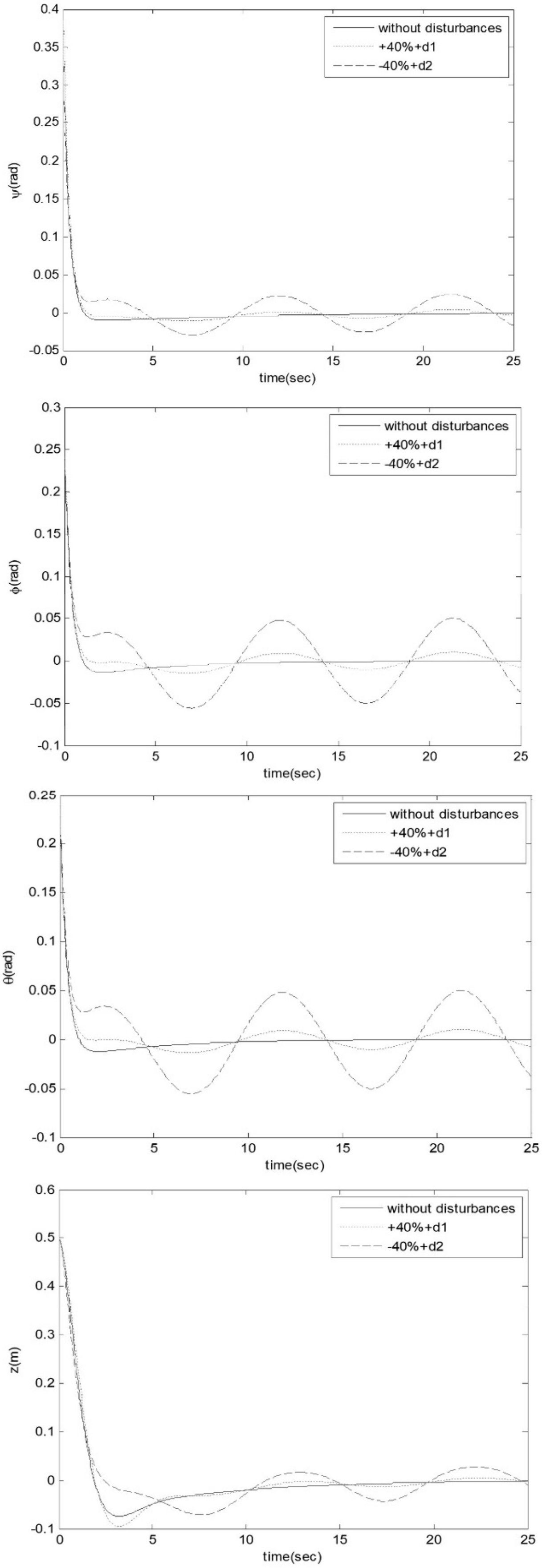

Figure 4: Orientation and altitude, PID controller.

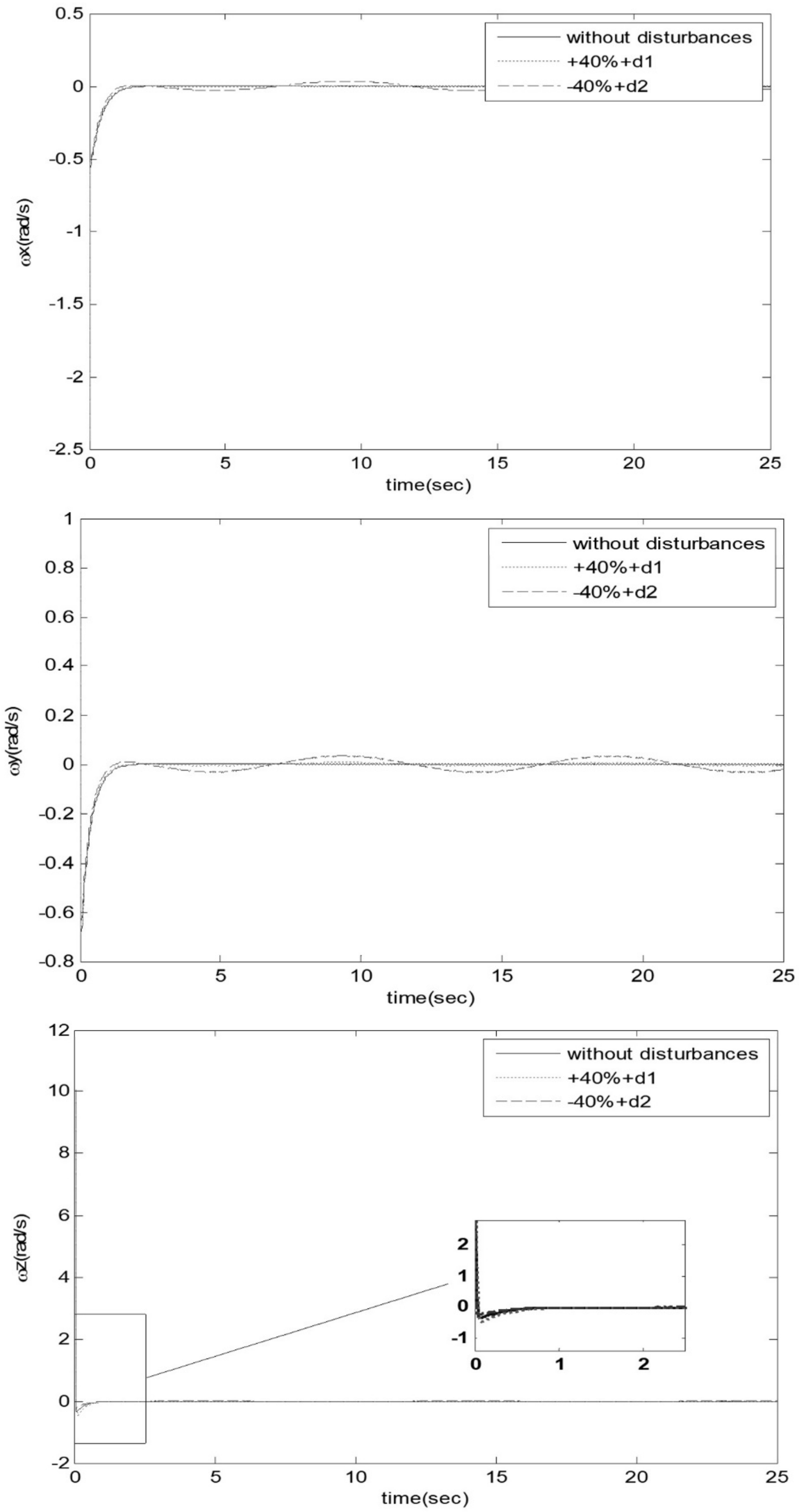

Figure 5: Angular velocities, PID controller

5 CONCLUSIONS

In the present study, a nonlinear H∞ approach is applied for both orientation and altitude stabilization problem in the presence of disturbance. An analytical solution of HJI inequality was presented for a quadrotor. By considering a candidate storage function, $V(x)$ for both orientation and altitude dynamics and using L_2-gain analysis, the nonlinear

H∞ control scheme was obtained and applied. Finally, the proposed controller has been evaluated by simulation results to show the stabilization of orientation and altitude, in the presence of the moment of inertia uncertainty and external disturbances.

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