BOOK REVIEW

Walter Carnielli & Claudio Pizzi, *Modalities and Multimodalities*. Springer, 2008, p. 320. ISBN 978-9048137626.

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Many philosophers are taking advantage of modal logic in order to approach their problems. This happens especially because the development of modal tools to formalize philosophical issues has been shown to be decisive to understand deep topics, for instance, in metaphysics, epistemology, ethics and their connections. This is evident when we read articles on philosophical logic. Modal tools play a role in building concepts, modelling theories and solving paradoxes, just to mention a few examples. In this sense, modal logic has been a necessary (but not sufficient) condition for philosophical research.

The book under review contains several of these modal tools. It has nine chapters dealing with a great plurality of aspects of modal logics. By the end of each chapter, there are exercises (but answers are not provided) and also a brief history of the main concepts introduced through the text. The book is accessible to anyone with a background in classical logic and it is of interest to all those studying logic. This is the English version of a book published years ago in Italian. Comparing this actual version with the old one, we see many improvements but no crucial changes.

The reader finds in chapter one an overview of classical propositional logic - called by the authors standard *propositional calculus*

(PC) - because all modal logics studied in the book are extensions of this logic. The most important topic explored in this chapter is the constructive completeness proof of PC by Kalmár's method. Moreover, other metalogical properties of PC are studied.

The syntactical aspects of modal logics besides their proof theory are introduced in chapter two. Beginning with Aristotle's modal square of oppositions, this chapter goes up to the hierarchy of normal modal systems ranging from **K** to **S5.** Many properties of **S5** are examined. In order to prove its consistency, they use the concept of *translation* of logics. The proof of the reduction theorem for modalities in **S5** is presented in detail**.** In this chapter, the reader starts to understand the fundamental role of induction proofs in modal logics: this kind of proof is used to demonstrate the reduction theorem and the syntactical deduction theorem for modal logics. One very positive aspect of this book is that theorems are proved in detail and this is really helpful in order to precisely follow the whole proof.

After exploring syntactical aspects in chapter two, chapter three deals with semantics, and the famous Dugundji's theorem which banned modal logic from the many-valued scenario is proved in detail. This theorem, although very important in modal logic, is almost never mentioned in modal logic textbooks. Afterwards, the fundamental distinction between Carnapian models (without accessibility relations) and Kripke models (with accessibility relations) is addressed. This lead us to correspondence theory (a branch of modal logic which investigates the connections between modal and first-order logic) and more fashionable concepts such as bissimulations, p-morphisms, and the Goldblatt-Thomason theorem. This dense chapter ends with *tableaux* for several modal systems. The authors show how to get different kinds of *tableaux* depending on the kind of accessibility of a given system.

The essential notion of (modal) *completeness* is analyzed in chapter four. It contains a detailed completeness proof of the general modal system **K+G**∞ which has as instantiations a great variety of

modal logics. In this sense, proving completeness for this general system allows immediate proofs for all systems of the hierarchy from **K** to **S5**. This completeness proof is realized by what is called Henkin's method. Thus, results such as the Lindenbaum theorem and the fundamental theorem of canonical models are presented. The chapter ends with a study of the logic of provability (linking modal logic with arithmetic), because this logic does not fall under the **K+G**∞ schema and, therefore, another strategy is required to prove completeness.

Following the same line, and developing metalogical properties of modal systems, chapter five shows how some modal logics are, indeed, incomplete. Notions like *modal algebra* and *finite model property* are presented. The latter is connected with the problem of proving decidability of modal logics.

From now on, the book, despite still examining a plurality of technical results, touches modal notions which are richer from the philosophical viewpoint. The theme of chapter six is that of temporal/tense logics. Temporal logics are studied from the syntactical but also from the semantical viewpoint. *Tableaux* are used as the main proof-theoretical strategy and completeness and incompleteness proofs of these temporal logics are also based on this technique. Tense logics are very useful in order to formalize models of time such as, for instance: branching, linear and circular time. Each temporal logic reflects some properties of these readings of time. The reader finds here a broad discussion on these subjects and a discussion of other interesting systems as, for example, hybrid logics.

Still going into modal notions with an intense philosophical flavour, the theme of chapter seven is that of epistemic logics. These are responsible for the mathematics of notions like knowledge and belief which are extremely important in computer science (and, needless to say, epistemology). Single and multi-agent epistemic logics are also studied from the proof and semantical theoretical viewpoints. There is a special session on doxastic logics as well as on common and

distributed knowledge. Standard discussions on epistemic logic like positive and negative introspection are also examined.

It is rare to find a book dealing with multimodalities. These are exactly the topic of chapter eight. Indeed, it is important to note that central philosophical problems are formulated in sentences containing interactive concepts like, for instance, combined notions from metaphysics and epistemology. Thus, in order to reason about philosophy, multimodalities are the rule. This part of the book shows how to treat these multimodalities. Some multimodal systems are discussed: epistemic doxastic logics, deontic temporal logics, epistemic temporal logics etc. There is also a completeness proof of a very general multimodal system.

The last chapter (that is, chapter nine) enters into the landscape of complicated quantified modal logics which have a strong expressive power. Here we find a discussion on necessary and contingent identities, on rigid designation, and on the wild topic of quantification and multimodalities.

Despite the fact that the word "multimodalities" appears in the title, there is only one chapter dedicated to the theme. It is difficult to write a perfect book. Authors should prepare an *errata* correcting typos and other problems. There are no big errors, only minor and peripheral mistakes. However, some general negative aspects should be pointed out: 1) There is almost no mention of non-classical modal logics which are also pretty much investigated nowadays (this is surprising given that one the authors – W. A. Carnielli – works on non-classical logic); 2) Multimodal logics are introduced without appeal to methods like fusions and products (this is also surprising given that W. A. Carnielli does intensive research on combining logics); 3) Many-dimensional modal logics are not touched; 4) There is almost nothing about deontic logic; 5) Complexity of modal logics is not studied. This is a negative point, given that exploring concepts from the realm of computational complexity such as *NP-completeness* would make the book more attractive to the computer science community. Of course, some of

these broad negative aspects would be, in a certain sense, too much for the purposes of this introductory guide to modal logic. This is the reason why I hope authors write a second volume covering all or some of these topics with the same elegance and simplicity.

I have used *Modalities and Multimodalities* to teach at the undergraduate level and the experience has been very good: after studying it, undergraduate students were able to develop *creative* research, going from non-abstract and primitive reasoning about the actual world to the richness and beauty of limitless modal inference.