

# Study of Mechanical Stresses in Thin-Walled Pressure Vessels Using Ultrasonic Methods

Yonka Ivanova<sup>a,b,\*</sup> 

<sup>a</sup>Sofia University "St.Kliment Ohridski", Faculty of Physics, Sofia, Bulgaria.

<sup>b</sup>Institute of Mechanics at Bulgarian Academy of Sciences, Sofia, Bulgaria.

Received: September 24, 2021; Revised: March 3, 2022; Accepted: March 6, 2022

This paper presents an experimental study of the propagation of ultrasonic Rayleigh waves in the walls of a pressure vessel in order to estimate mechanical stresses. The ability to assess stresses using ultrasonic methods is based on the acoustoelastic effect, i.e. the change of velocity of propagation of the ultrasonic Rayleigh waves (URW) in stressed media. The experiments were carried out using a hydraulic test conducted in a pressure vessel. Measurements of the travel time of the URW over the walls of the vessel in the axial and circumferential direction were carried out with a pressure change of up to 7 MPa at a constant temperature. Relations between the relative changes in the travel time of waves and the change in pressure were found. The influence of temperature and thermal stresses on the velocities of ultrasonic waves was not taken account. The conducted experiments confirmed the finding that the difference in the relative changes in the travel time of URW in circumferential and axial directions changes linearly with the change in the pressure in the vessel.

**Keywords:** *thin-walled pressure vessel, biaxial stress state, ultrasonic Rayleigh waves, acoustoelastic effect, Momentless Shell theory.*

## 1. Introduction

Pressure vessels are widely used in industry and energy production. Monitoring of the mechanical stress state is required in order to ensure their safe and accident-free operation<sup>1-3</sup>. Strain gauges are traditionally used for determining mechanical stresses in pressure vessels<sup>4</sup>. Non-destructive methods (such as X-ray, thermoelectric, magnetic noise method and ultrasonic method) are considered to be promising as they allow indirect assessment of the stress state of structural components of the equipment<sup>5</sup>.

Ultrasonic methods used for assessing mechanical stresses are based on the acoustoelastic effect which is how the velocity of ultrasonic waves changes when they propagate in media subjected to stress. Bulk longitudinal and transverse waves<sup>6-11</sup>, subsurface, surface or Lamb waves are usually used. The longitudinal and transverse waves are mainly used to assess stresses along the thickness of the wall in structures (such as pipelines, tanks, etc.)<sup>8,9</sup>.

Landa et al.<sup>6</sup> describe various ultrasonic measurement techniques to find ultrasonic wave velocity changes during compression tests. The effect of temperature changes on the estimation of biaxial stresses in pipelines by ultrasonic method is considered<sup>7</sup>. Nikitina et al.<sup>8-10</sup> propose a method for estimation of the uniaxial and biaxial stressed states using the changes in the propagation times of longitudinal and shear elastic waves along the normal to the surface of pipes. Li et al.<sup>11</sup> propose a new non-destructive approach for determination of the uniaxial stress of structural steel members using spectrum analysis of ultrasonic shear waves. Belyaev et al.<sup>12</sup> present a review of the main publications on theoretical and experimental scientific results obtained

by researchers in the field of the acoustic anisotropy of metallic materials under uniaxial and complex stress states and plastic deformation.

Most of the damages in pressure vessels are caused by cracks in the subsurface or surface layers. In the literature, there are articles regarding the determination of residual or applied mechanical stresses by a longitudinal critically refracted (LCR technique), which is excited at the first critical angle of incidence of the longitudinal wave at the plexiglas-steel border and propagates beneath the surface at certain depths depending on the wave frequency<sup>13-15</sup>. Bray<sup>13</sup> describes the principles of ultrasonic stress measurement and specific applications of LCR waves in rolled and welded steel and aluminum plate, pressure vessels, turbine rotors and so on. Javadi et al.<sup>14,15</sup> present a study for estimation of welding residual stress in a pressure vessel. Hoop and axial residual stresses are evaluated using different frequency range of ultrasonic LCR transducers. An applications of LCR wave method on residual stress testing of oil pipeline weld joint, vehicle's torsion shaf and so on are studied in<sup>16</sup>. Gandhi et al.<sup>17</sup> develop the theory of acoustoelastic Lamb wave propagation in isotropic media subjected to a biaxial, homogeneous stress field. Theoretical investigations are compared to experimental results for several Lamb wave modes and frequencies for uniaxial loads applied to an aluminum plate. The ultrasonic Rayleigh wave (URW) is created when the angle of incidence of a longitudinal wave is equal to the second critical angle, and it propagates on the surface at a depth of 1 to 1.5 times the wavelength<sup>18</sup>. Its use makes it possible to study the average surface stress of materials and products<sup>19</sup>. The acoustoelastic effect of URW is generalized by Hirao et al.<sup>20</sup> and proved experimentally

\*e-mail: [yonivan@phys.uni-sofia.bg](mailto:yonivan@phys.uni-sofia.bg)

for different alloys by Husson,<sup>21</sup> Zeiger et al.<sup>22</sup>, Jassby and Saltoun<sup>23</sup>, Hu et al.<sup>25</sup>. The URW are used to estimate applied<sup>20,23,24</sup> and residual stresses<sup>26,27</sup>. Akhshik et al.<sup>27</sup> study residual stresses of circumferential welds in thin walled pipes.

The URW acoustoelastic coefficients (AEC) for biaxial stressed plate are theoretically and experimentally established for an orthotropic material<sup>28</sup>. Zeiger et al.<sup>22</sup> study the acoustoelastic effect in biaxial stressed steel samples by time of flight of waves. Jassby and Saltoun.<sup>23</sup> estimate biaxial surface stresses in aluminum plates. Experiments for determination of the AEC are presented in papers<sup>29,30</sup>. A cross correlation method is used to determine the difference in the time of flight of URW in a sample under the biaxial stresses<sup>31</sup>.

Zhang et al.<sup>32</sup> show that the URW velocity in the walls of thin-walled pressure vessels is influenced by mechanical stresses, temperature and thermo-stresses. The propagation distance of ultrasonic wave is influenced by strain and thermal deformation. Considering these important factors, the authors propose a model for measuring the pressure in thin-walled vessels. The relations between the delay time of the ultrasonic wave and the pressure is established. Two vessels made of different materials are used as specimens to estimate the pressure with ultrasound and validate the model and the method.

Assessing the mechanical stresses in pressure vessels by the use of ultrasonic methods is a promising approach<sup>32</sup> which at present is not used widely in industry and energy production. One of the reasons is that the acoustoelastic effect is less pronounced. According to the literature sources, the relative change of the ultrasonic waves in media subject to stress of 100 MPa is about 0.1% for aluminum and around 0.01% for steel<sup>22,31</sup>. Despite the high accuracy of the method, the lack of automated devices for recording temperature, pressure and ultrasonic wave velocities slows down its wide practical application. The method for assessing stresses by using URW is suitable for testing thin-walled cylindrical vessels under uniform internal pressure<sup>32</sup>.

This work is a continuation and extension of the research<sup>19</sup> on the acoustoelastic effect of Rayleigh waves in stressed media. The purpose of the current paper is to study propagation of

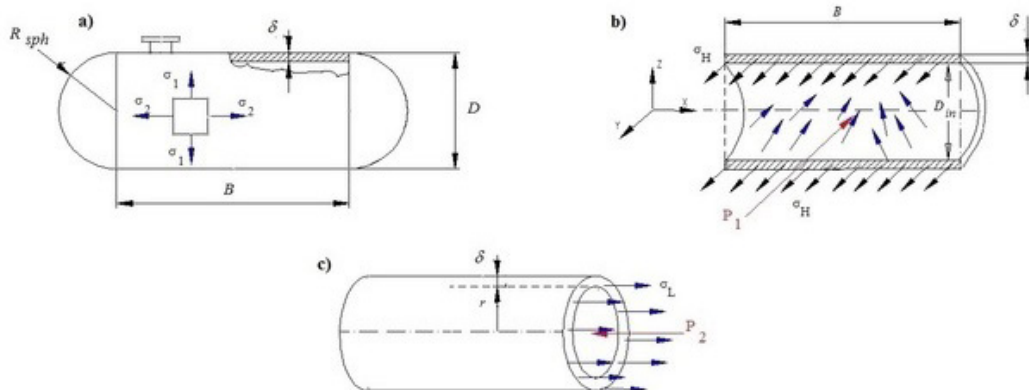
URW in the walls of pressure vessel at constant temperature in order to estimate the mechanical stresses.

The remainder of the paper is organized as follows. After the introduction, which gives a brief overview of ultrasonic methods for estimation of the mechanical stresses, the section “Thin-walled pressure vessels” describes the basic concepts of the Momentless Shell theory for determination of the mechanical stresses in thin-walled vessels under uniform internal pressure. The section “Acoustoelastic Effect of Ultrasonic Rayleigh Wave” presents the main dependencies related to the acoustoelastic effect in propagation of URW in stressed media. The experimental part describes the experimental setup and the measurement procedure, preparatory considerations and calculations, parameters of ultrasonic Rayleigh waves. The results are followed by a discussion and conclusion, which notes the future directions in our study.

## 2. Thin-Walled Pressure Vessels

Pressure vessels are usually used as tanks for storing liquids, gases, etc. Depending on the thickness of their walls, they are divided into thin-walled and thick-walled pressure vessels. The cylindrical pressure vessel with closed ends contains a fluid at uniform pressure  $p$  is shown in the Figure 1a. The outer diameter is  $D$ , the inner diameter is  $D_{in}$  and the wall thickness is  $\delta$ . Thin-walled vessels have a wall thickness-to-inner diameter ratio of no more than 0.05. When studying the stress-strain state, this class of vessels is schematized to an axisymmetric thin-walled shell. Due to the small thickness of the wall, it is assumed that normal stresses are evenly distributed and that no bending moments exist. The internal pressure acts on an area determined by the inner diameter ( $D_{in}$ ) of the vessel<sup>1,3</sup>. The theory based on this premise is called the Momentless Shell Theory<sup>1,3</sup>.

In the Figure 1a an element with stresses parallel and perpendicular to the axes (Figure 1b and 1c) of the vessel is given. The normal mechanical stresses  $\sigma_1$  and  $\sigma_2$  act on the side surfaces of the element. Tangential stresses do not act on these surfaces due to the load and the symmetry of the vessel. Normal stresses are the principal stresses. The stress acting in the circumferential direction is called a circumferential stress (hoop stress). Longitudinal stress



**Figure 1.** a). Scheme of a thin-walled pressure vessel. b) Circumferential stresses in a circular cylindrical pressure vessel. c) Longitudinal stresses in a circular cylindrical pressure vessel.

in axial direction along the length of the vessel is called a longitudinal or axial stress.

Figure 1b shows a section of the cylindrical part of the vessel perpendicular to the longitudinal axis, where the circumferential stresses  $\sigma_H = \sigma_1$  and the internal pressure  $p$  are marked. The stresses  $\sigma_1$  have a resultant force equal to  $\sigma_1 \cdot 2B\delta$ , where  $B$  is the length of cylindrical part of vessel. The resultant force  $P_1$  of the internal pressure is equal to  $P_1 = 2prB$ , where  $r$  is the inner radius of the cylinder.

The section in the vertical plane through the longitudinal axis of the tank is given in Figure 1c. The stresses  $\sigma_L = \sigma_2$  act longitudinally and have a resultant force equal to  $\sigma_2 \cdot 2\pi r\delta$ . The resultant force of the internal pressure is  $P_2 = p\pi r^2$ . The stress formulas are derived from the equilibrium conditions of the forces due to the internal pressure and the stresses in the walls of the pressure vessel<sup>1,3</sup>:

Circumferential stress

$$\sigma_1 = \sigma_y = \frac{pD_{in}}{2\delta} \quad (1)$$

Longitudinal stress:

$$\sigma_2 = \sigma_x = \frac{pD_{in}}{4\delta} \quad (2)$$

as  $p$  is the internal pressure that the contents of the vessel exert on the walls of the vessel;  $D_{in}$  is the inner diameter;  $\delta$  is the thickness of the vessel.

### 3. Acoustoelastic Effect of Ultrasonic Rayleigh Waves

The propagation of elastic waves in materials depends on the magnitude and the direction of the mechanical stresses that occur in them. In a uniaxial stress state, the change in the velocity of propagation of the surface wave is proportional to the mechanical stress. The coefficient of proportionality is named an acoustoelastic coefficient that depends on the elastic properties of the material<sup>21-23</sup>. The acoustoelastic dependence for the biaxial stress state in the case of an isotropic medium can be written as<sup>23,25,31</sup>:

$$\frac{\Delta V}{V_0} = \beta_1 \sigma_1 + \beta_2 \sigma_2 \quad (3)$$

Here  $\sigma_1$  and  $\sigma_2$  are the principal mechanical stresses ( $\sigma_1 > \sigma_2$ ) and  $\beta_1$  and  $\beta_2$  are the AEC dependent on the second and third order elastic constants of the media<sup>21,25</sup>.

Based on the theoretical considerations<sup>28</sup>, the dependences between the relative change of the URW velocity and the stresses in pressure vessel walls are presented<sup>32</sup>:

$$\frac{\Delta v_x}{v_x^0} = \frac{v_x^\sigma - v_x^0}{v_x^0} = A_{R_x}^x \sigma_x + A_{R_x}^y \sigma_y \quad (4)$$

$$\frac{\Delta v_y}{v_y^0} = \frac{v_y^\sigma - v_y^0}{v_y^0} = A_{R_y}^x \sigma_x + A_{R_y}^y \sigma_y \quad (5)$$

where  $\sigma_y = \sigma_1$  is the circumferential stress,  $\sigma_x = \sigma_2$  is the longitudinal stress;  $v_y^0$  and  $v_x^0$  are the velocities of URW propagating in unstressed vessel along the principal stress

directions,  $v_y^\sigma$  and  $v_x^\sigma$  are the velocities of URW propagating in stressed media at constant temperature,  $\Delta v_x / v_x^0$  and  $\Delta v_y / v_y^0$  are the relative changes in the ultrasonic wave velocities.

The coefficients  $A_{R_x}^x, A_{R_x}^y, A_{R_y}^x, A_{R_y}^y$  are the Rayleigh wave AEC, which depends on the propagation and polarization directions of the waves as well as on the stress directions. The superscripts of the AEC denote the direction of the applied stress, while the subscripts denote the direction of URW.

Zhang et al.<sup>32</sup> developed a model for the acoustoelastic effect of URW propagating in vessel walls subjected to internal pressure ( $p$ ), taking into account not only the influence of mechanical stresses  $\sigma_x^P, \sigma_y^P$  but also thermal stresses  $\sigma_x^T, \sigma_y^T$ :

$$\sigma_x = \sigma_x^T + \sigma_x^P \quad (6)$$

$$\sigma_y = \sigma_y^T + \sigma_y^P \quad (7)$$

$$\sigma_x^T = \sigma_y^T = \frac{\beta \cdot E \cdot \Delta t_w}{2(1-\mu)} \quad (8)$$

where  $\sigma_x^T, \sigma_y^T$  are the thermal stresses in the axial and circumferential directions,  $\beta$  is the thermal expansion

coefficient,  $\Delta t_w = t_{w1} - t_{w2}$ ,  $t_{w1}$  and  $t_{w2}$  are the temperatures of the inner and outer wall of the vessel, respectively, and  $E, \mu$  are Young's modulus and Poisson's ratio of material.

The velocity of ultrasonic wave is influenced by stresses, temperature and thermal stresses, consequently the relative changes of URW velocities is written as<sup>32</sup>.

$$\frac{\Delta v_x}{v_x^0} = K_x^T \cdot \Delta t_w + K_x^P \cdot \frac{D_{in} \cdot P}{4\delta} \quad (9)$$

$$\frac{\Delta v_y}{v_y^0} = K_y^T \cdot \Delta t_w + K_y^P \cdot \frac{D_{in} \cdot P}{2\delta} \quad (10)$$

where  $K_y^T, K_y^P, K_x^T, K_x^P$  depend on the acoustoelastic coefficients  $A_{R_x}^x, A_{R_x}^y, A_{R_y}^x, A_{R_y}^y$  and thermal expansion coefficient<sup>32</sup>:

$$K_y^T = \frac{-\beta \cdot E \cdot (A_{R_y}^x + A_{R_y}^y)}{2(1-\mu)} \quad (11)$$

$$K_y^P = \frac{1}{2} \cdot A_{R_y}^x + A_{R_y}^y \quad (12)$$

$$K_x^T = \frac{-\beta \cdot E \cdot (A_{R_x}^x + A_{R_x}^y)}{2(1-\mu)} \quad (13)$$

$$K_x^P = A_{R_x}^x + 2 \cdot A_{R_x}^y \quad (14)$$

Considering that the velocity of ultrasonic wave is affected by stress and temperature, as well as the distance of ultrasonic propagation is influenced by strain and thermal deformation, a complex dependence on the change of propagation time of URW is found<sup>32</sup>. Neglecting the influence of temperature,

a reference model on the URW time delay is obtained due to the applied pressure ( $p$ )<sup>32</sup>:

$$\Delta t_y - \Delta t_x = \left\{ \left[ \frac{2-\mu}{2E} - K_y^P \right] \frac{D_{in} t_y^0}{2\delta} - \left[ \frac{1-2\mu}{E} - K_x^P \right] \frac{D_{in} t_x^0}{4\delta} \right\} \cdot p \quad (15)$$

where  $\Delta t_y$  and  $\Delta t_x$  are the time delay of the URW in the circumferential and axial directions, and  $t_y^0$  and  $t_x^0$  are the propagation times of URW in unstressed state in the main stresses directions.

In order to estimate the internal pressure, the delay times of the ultrasonic wave in the two principal directions of pressure vessel have to be measured<sup>32</sup>. Following (15), it is assumed that the difference in the travel time of the URW in the circumferential and axial direction should be proportional to the internal pressure in the vessel. In the present work, an experiment is set up, which aims to confirm this relation.

## 4. Experiments

### 4.1. Experimental setup

The vessel used in the setup is made of P275NH steel. The vessel has a cylindrical shape and the following dimensions: length of 500 mm; outer diameter of 250 mm and wall thickness of 4 mm.

A hydraulic test was performed during the experiments, and the pressure of the fluid in the vessel was raised to up to 7-8 MPa using a water pump. The pressure was measured by a manometer with an accuracy class of 1, which means that the error limits at a temperature of 20 °C are  $\pm 1\%$  of the used scale range of the manometer, i.e. the accuracy of the measured pressure was 1 bar. The experimental setup for studying the stresses in a pressure vessel by ultrasonic Rayleigh waves is shown in Figure 2. The main components of the setup are: a pressure vessel (1), which is used as a prototype; water pressure pump (2), pressure gauge (3); pulser-receiver unit (4), ultrasonic transducers for URW (5); ultrasonic system for collecting and processing the results (Ultrasonic box -6) and computer (7).

### 4.2. Experimental procedure

To study the propagation of ultrasonic surface waves, an ultrasonic US-expert device (a pulser/receiver, an A/D converter and a computer) with time measurement accuracy of 1 ns was used. URW were excited by an angular transducer with variable angle at an angle of refraction bigger the second critical one when the amplitude of signal is visibly maximal. The nominal frequency of the transducers is 3.5 MHz. A through transmission method, implemented by the means of two URW transducers was used to generate and receive the waves. The signal was transmitted by transducer T, received by transducer R and recorded. The wavelength  $\lambda_r$  was approximately 0.8 mm for the used frequency. The propagation depth of the URW was of the order of  $1,5 \cdot \lambda_r$ <sup>18</sup>, i.e. about 1.2 mm .

The transmitting and receiving transducers were rigidly connected which allowed the distance between them to remain the same when arbitrarily positioning the sensors on the vessel walls. Viscous grease was used as a couplant in ultrasonic measurements. The ultrasonic sensors were placed away from the welded seams of the vessel, thus avoiding the influence of stress concentration.

Before the experiments, pressure was applied to the sensors placed on the vessel. With each pressure change, the pressure increment of 1 MPa was adjusted. Measurements of the travel time of the URW over the walls of the vessel were made in axial and circumferential directions without pressure as well as with pressure in the vessel increasing to up to 7 MPa. The wave propagation distance was adjusted so it remained the same when taking measurements in both directions and was 150 mm.

During the experiment, the water temperature remained constant. Due to the fact that the temperature of the outer and inner surfaces of the vessel wall was the same, the experiments did not take into account the effect of temperature on the velocities of ultrasonic waves, as well as the change in the ultrasonic propagation distance. The ultrasound signals were recorded, visualized by the use of an oscilloscope and stored on a computer. During the recording process, in order to reduce accidental noise, an averaging technique was used.

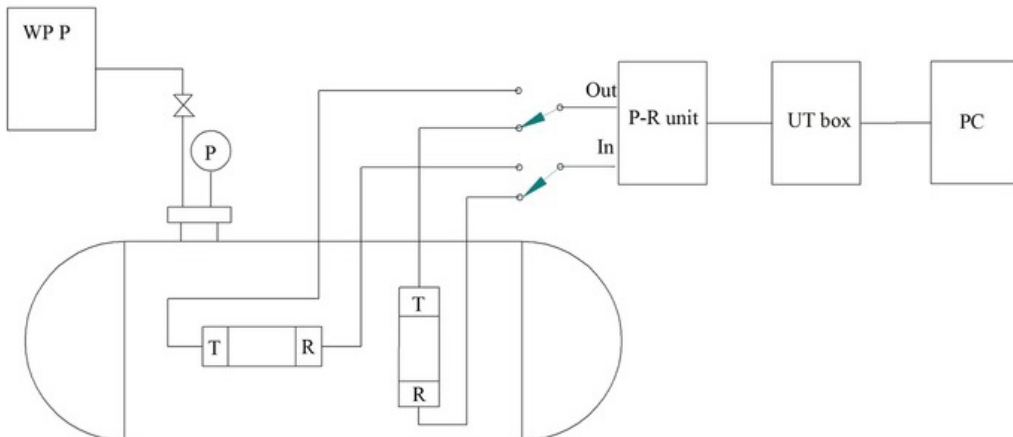


Figure 2. Experimental setup.

### 4.3. Preparatory considerations

#### 4.3.1. Mechanical stresses and deformations

The mechanical stresses calculated according to dependencies (1) and (2) on the vessel walls during the hydraulic test are in the elastic region and are lower than the yield strength of the steel from which the vessel is made. Figure 3 shows the changes of the circumferential and axial stresses with values of the pressure ( $p$ ) of the water in the vessel in the range of 0 to 7 MPa. At the maximum pressure of 7 MPa, the mechanical stress in the circumferential direction was 212 MPa, and the stress in the axial direction was 106 MPa.

In a biaxial stress state, the absolute elongation ( $\Delta L$ ) caused by the mechanical stresses in the elastic region in the walls of the thin-walled vessel due to the pressure can be determined using the Hooke's law equation<sup>3</sup>:

$$\Delta L_1 = \Delta L_y = (\sigma_1 - \mu\sigma_2) \frac{L_1}{E} \quad (16)$$

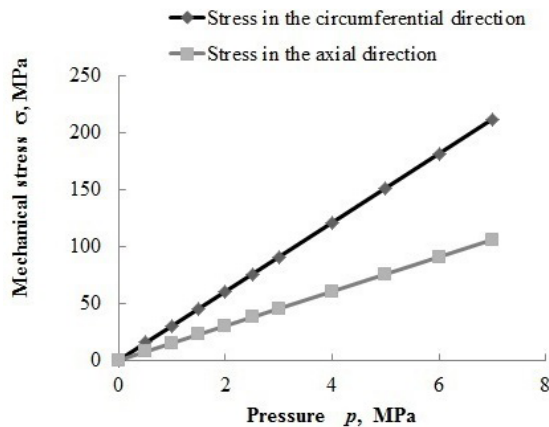
$$\Delta L_2 = \Delta L_x = (\sigma_2 - \mu\sigma_1) \frac{L_2}{E} \quad (17),$$

as  $\Delta L_1$  and  $\Delta L_2$  are the absolute elongations caused by the main stresses  $\sigma_1$  and  $\sigma_2$ ,  $\mu$  is the Poisson's ratio,  $E$  - Young's modulus,  $L_1$  and  $L_2$  are the initial lengths in the two directions before the load is applied. Figure 4 shows the calculated change in the absolute elongation under the action of the created mechanical stresses in the vessel. The specific values of Young's modulus and Poisson's ratio are:  $E = 210 \text{ GPa}$ ,  $\mu = 0.3$ . The maximum values of  $\Delta L_2$  and  $\Delta L_1$  at maximum pressure are 0.03 mm and 0.13 mm, respectively.

## 5. Ultrasonic Rayleigh Waves Parameters

The travel time of the URW in the initial unstressed state in circumferential and axial directions of the walls in the vessel can be written as:

$$t_{R01} = \frac{L_1}{v_{Ry}^0} \quad (18)$$



**Figure 3.** The variations of the circumferential and axial stresses in the water vessel.

$$t_{R02} = \frac{L_2}{v_{Rx}^0} \quad (19)$$

where  $t_{R01}$  and  $t_{R02}$  are the propagation times of URW in unstressed state that are the same;

$L_1$  and  $L_2$  are the initial propagation distances in the two principal directions before the load is applied. The velocities of URW propagating on the vessel walls at zero pressure are denoted by  $v_{Ry}^0$  and  $v_{Rx}^0$ .

The travel times of the URW propagating in the walls of the vessel under internal pressure in the circumferential and in the axial directions can be expressed by (20) and (21), respectively:

$$t_{R1}^{\sigma} = t_{Ry}^{\sigma} = \frac{L_1 + \Delta L_1}{v_{Ry}^{\sigma}} \quad (20)$$

$$t_{R2}^{\sigma} = t_{Rx}^{\sigma} = \frac{L_2 + \Delta L_2}{v_{Rx}^{\sigma}} \quad (21)$$

as  $v_{Ry}^{\sigma}$  and  $v_{Rx}^{\sigma}$  are the velocities of the Rayleigh waves in the studied directions.

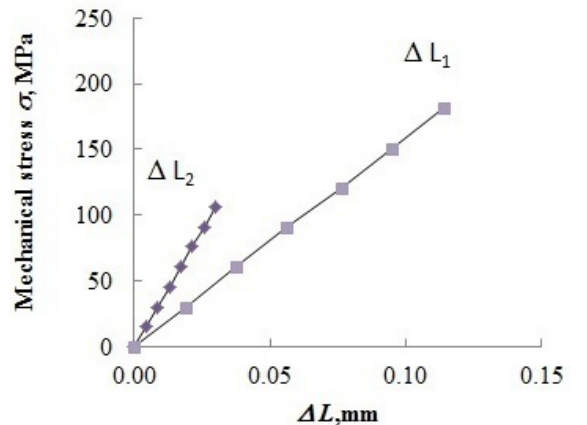
Since the maximum absolute elongations  $\Delta L_1$  and  $\Delta L_2$  are very small compared to the URW propagation distance in both directions, the error in the travel time due to them is less than 0.08%. The delay in the travel time of the URW in the vessel walls, observed in the experiments can be written as (22) and (23) :

$$\Delta t_{R1} = t_{R1}^{\sigma} - t_{R01} \quad (22)$$

$$\Delta t_{R2} = t_{R2}^{\sigma} - t_{R02} \quad (23)$$

## 6. Results and Discussion

Signals from ultrasonic surface waves before applying pressure and at a pressure  $p = 6 \text{ MPa}$  in the axial direction are shown in Figure 5. The ultrasonic propagation distance is 150 mm. The ultrasonic pulse of the stressed vessel at pressure is obtained with a delay compared to the signal



**Figure 4.** The change in the absolute elongation under the action of mechanical stresses in the pressure vessel.



without pressure. Relatively weak signals can be seen due to the curvature of the vessel and the not very smooth surface of its walls.

The results of the current study are presented using the ultrasonic parameter - the relative change in travel time of URW, obtained by measuring in the principal stress directions:

$$\frac{\Delta t_{R1}}{t_{R01}} = \frac{t_{R1}^{\sigma} - t_{R01}}{t_{R01}}, \% \quad (24)$$

$$\frac{\Delta t_{R2}}{t_{R02}} = \frac{t_{R2}^{\sigma} - t_{R02}}{t_{R02}}, \% \quad (25)$$

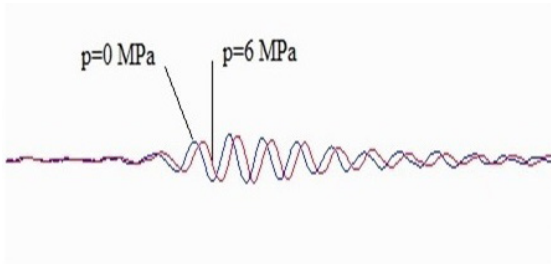


Figure 5. Signals at pressure  $p=0$  MPa and  $p=6$  MPa in the axial direction of water vessel.

Table 1 shows the results of ultrasonic measurements. It can be seen that the parameters  $\frac{\Delta t_{R2}}{t_{R02}}$  and  $\frac{\Delta t_{R1}}{t_{R01}}$  when measured in axial direction at a pressure of 7 MPa was approximately 0.089%, and 0.19% in circumferential direction (Table 1).

The relative changes in URW travel time in circumferential and axial directions are shown in Figures 6a and 6b at ultrasonic propagation distance of 150 mm. The results show that the dependencies are not linear. The linear approximation is not appropriate due to the large scatter between the experimental data and the predicted values.

The difference in the relative changes in the travel time of the URW in the two main directions of the pressure vessel should be proportional to the internal pressure and may be expressed as:

$$\frac{\Delta t_{R1}}{t_{R01}} - \frac{\Delta t_{R2}}{t_{R02}} = K \cdot p \quad (26)$$

where  $K$  is the proportionality coefficient.

The dependence of the difference in travel time changes of the URW on the pressure is presented in Figure 7. From the obtained results a better linear dependence on the pressure  $p$  can be seen. The results from regression statistics are:  $R^2 = 0.9551$ ; error = 0.0087. The coefficient  $K$  in relation (26) is obtained:  $K = 0.0166$  [1/MPa].

The measurement model of the water vessel may be written as:

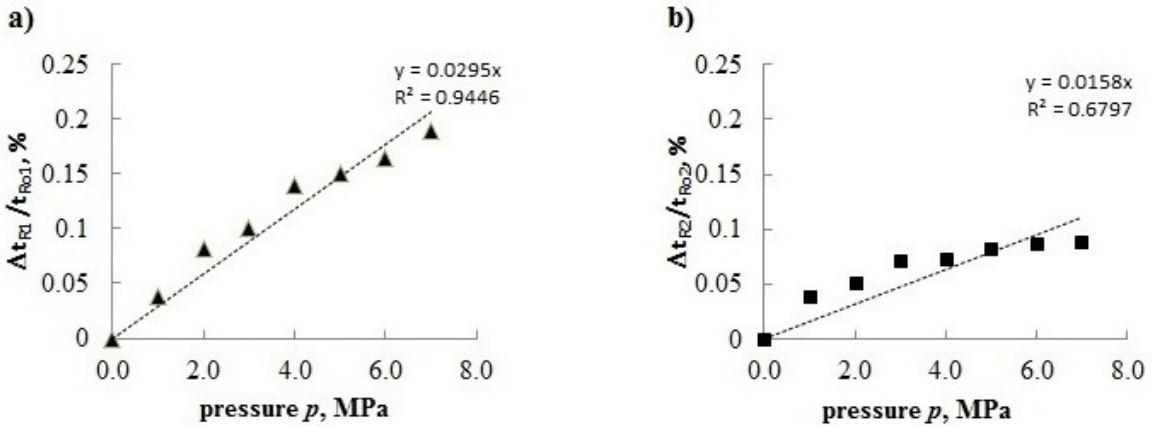
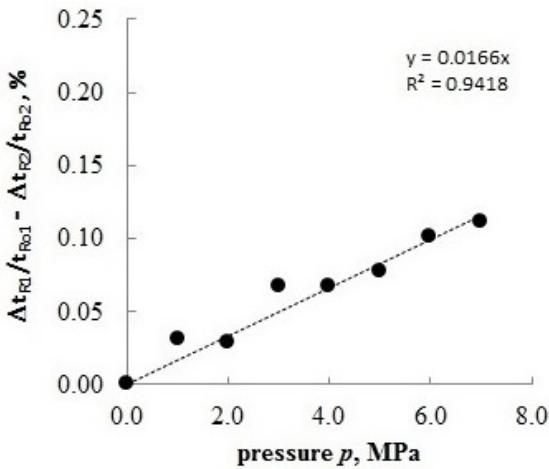


Figure 6. a) The relative changes in the travel time of URW in circumferential direction. b) The relative changes in the travel time of URW in axial direction.

Table 1. Measurement results

Pressure $p$ , MPa	Estimated Hoop stress $\sigma_1$ , MPa	$\frac{\Delta t_{R1}}{t_{R01}}$ , %	Estimated axial stress $\sigma_2$ , MPa	$\frac{\Delta t_{R2}}{t_{R02}}$ , %
0	0	0	0	0
1.0	30	0.0320	15	0.0390
2.0	61	0.0820	30	0.0510
3.0	91	0.1000	45	0.0710
4.0	121	0.1400	61	0.0730
5.0	151	0.1500	76	0.0830
6.0	182	0.1650	91	0.0878
7.0	212	0.1900	106	0.0891



**Figure 7.** The difference in the relative changes in the travel time of URW in circumferential and axial directions.

$$\frac{\Delta t_{R1}}{t_{R01}} - \frac{\Delta t_{R2}}{t_{R02}} = 0,0166 \cdot p \quad (27)$$

Dependence (27) is the experimental verification of equation (15). The difference in the relative changes in the travel time of the URW in the circumferential and axial directions of the vessel changes linearly with increasing the internal pressure. The nonlinearities of the dependencies in the Figure 6 are probably due to a complex factors when determining the time delay of ultrasonic waves. The most important possible reasons are: the influence of the curvature of the vessel on the accuracy of the measurement, the change in the thickness of the couplant. In addition, the anisotropy of the material was not taken into account in the work. The accuracy in estimating the time delay of ultrasonic wave can be significantly increased by using signal processing methods, such as cross-correlation technique.

## 7. Conclusion

In this work, an experimental study of propagation of ultrasonic Rayleigh waves in the walls of a pressure vessel was performed using a hydraulic test at a constant temperature.

Due to the fact that the temperature of the outer and inner surface of the vessel wall was the same, the effect of temperature on the velocities of ultrasonic waves was not taken into account in the present study.

Dependencies of the relative changes in the travel time of URW on the internal pressure in the circumferential and axial directions were obtained. Nonlinearity has been identified, which needs to be further investigated.

Experimental confirmation of the linear dependence between internal pressure and difference of relative changes in travel time of URW in the two principal directions of pressure vessel walls was obtained.

Future research efforts can be focused on the following important points:

1. Improving the accuracy of measuring the time delay of ultrasonic waves by using signal processing methods.
2. Considering the effect of the anisotropy of the material on ultrasonic measurements.
3. Investigation of the factors influencing the nonlinearity of the relations between the relative changes in travel time of the URW in the two main directions of the pressure vessel wall.
4. Study on the influence of temperature and thermal stresses on the ultrasonic waves velocities.
5. Study on the effect of deformations on the ultrasonic propagation distance.
6. Estimation of the residual stresses in the places of the welded joints in pressure vessel.

## 8. Acknowledgements

This work was implemented with the financial support of Project No. BG05M20P001-1.002-0011 "Creation and Development of Center of Competence in Mechatronics and Clean Technologies MIRACle (Mechatronics, Innovation, Robotics, Automation, Clean Technologies)" which is funded by the Operational Programme "Science and Education for Smart Growth" and co-financed by the European Regional Development Fund (ERDF).

## 9. References

1. Harvey J, editor. Theory and design of pressure vessels. New York: Van Nostrand Reinhold; 1985.
2. Megson TNG, editor. Structural and stress analysis. 4th ed. Oxford: Butterworth-Heinemann; 2019.
3. Gere JM. Mechanics of materials. 6th ed. Australia: Thomson Learning Academic Resource Center; 2004.
4. Hoffmann K. An introduction to stress analysis and transducer design using strain gauge [Internet]. HBM Test and Measurement; 2012 [cited 2021 Sept 24]. Available from: <https://www.hbm.com>
5. Schajer GS, editor. Practical residual stress measurement methods. Chichester: John Wiley & Sons; 2013.
6. Landa M, Plešek J. Ultrasonic techniques for nondestructive evaluation of internal stresses. In: Proceedings of the 15th World Conference on Non-Destructive Testing; 2000; Rome. Proceedings. Rome: ICNDT; 2000.
7. Nikitina NE, Kamyshev AV, Kazachek SV. Ultrasonic testing of the stressed state of pipelines with consideration of the temperature factor. Russ J Nondestr Test. 2012;48(5):272-6. <http://dx.doi.org/10.1134/S1061830912050087>.
8. Nikitina NY, Kamyshev AV, Kazachek SV. The application of the acoustoelasticity method for the determination of stresses in anisotropic pipe steels. Russ J Nondestr Test. 2015;51(12):171-8. <http://dx.doi.org/10.1134/S1061830915030079>.
9. Nikitina NY, Kamyshev AV, Smirnov VA, Borshchevskii AV, Sharygin YM. Determination of axial and circumferential stresses in the wall of a closed tube via an ultrasonic method using the acoustoelasticity effect. Russ J Nondestr Test. 2006;42(3):185-9. <http://dx.doi.org/10.1134/S1061830906030065>.
10. Nikitina NY, Ostrovsky LA. An ultrasonic method for measuring stresses in engineering materials. Ultrasonics. 1998;35(8):605-10. [http://dx.doi.org/10.1016/S0041-624X\(97\)00154-6](http://dx.doi.org/10.1016/S0041-624X(97)00154-6).
11. Li Z, He J, Teng J, Huang Q, Wang Y. Absolute stress measurement of structural steel members with ultrasonic shear wave spectral analysis method. Struct Health Monit. 2019;18(1):216-31. <http://dx.doi.org/10.1177/1475921717746952>.

12. Belyaev AK, Polyanskiy VA, Tretyakov DA. Estimating of mechanical stresses, plastic deformations and damage by means of acoustic anisotropy. *PNRPU Mech. Bulletin*. 2020;4(4):130-51. <http://dx.doi.org/10.15593/perm.mech/2020.4.12>.
13. Bray DE. Current directions of ultrasonic stress measurement techniques. In: *Proceedings of the 15th World Conference on Non-Destructive Testing; 2000; Rome. Proceedings. Rome: ICNDT; 2000*.
14. Javadi Y, Pirzaman HS, Raeisi MH, Najafabadi MA. Ultrasonic stress evaluation through thickness of a stainless steel pressure vessel. *Int J Press Vessels Piping*. 2014;123-124:111-20. <http://dx.doi.org/10.1016/j.ijpvp.2014.08.006>.
15. Javadi Y, Pirzaman HS, Raeisi MH, Najafabadi MA. Ultrasonic inspection of a welded stainless steel pipe to evaluate residual stresses through thickness. *Mater Des*. 2013;49:591-601. <http://dx.doi.org/10.1016/j.matdes.2013.02.050>.
16. Xu C, Song W, Pan Q, Li H, Liu S. Nondestructive testing residual stress using ultrasonic critical, refracted longitudinal wave. *Phys Procedia*. 2015;70:594-8. <http://dx.doi.org/10.1016/j.phpro.2015.08.030>.
17. Gandhi N, Michaels JE, Lee SJ. Acoustoelastic Lamb wave propagation in biaxially stressed plates. *J Acoust Soc Am*. 2012;132(3):1284-93. <http://dx.doi.org/10.1121/1.4740491>.
18. Viktorov A. *Rayleigh and lamb waves: physical theory and application*. New York: Plenum Press; 1967.
19. Ivanova Y, Partalin T, Pashkuleva D. Acoustic investigations of the steel deformation during the tensile. *Rus J NDT*. 2017;53(1):39-50. <http://dx.doi.org/10.1134/S1061830917010077>.
20. Hirao M, Fukuoka H, Hori K. Acoustoelastic effect of Rayleigh surface wave in isotropic material. *J Appl Mech*. 1981;48(1):119-24. <http://dx.doi.org/10.1115/1.3157553>.
21. Husson D. A perturbation theory for the acoustoelastic effect of surface waves. *J Appl Phys*. 1985;57(5):1562-8. <http://dx.doi.org/10.1063/1.334471>.
22. Zeiger A, Jassby K. Measurement of acoustoelastic coefficients of Rayleigh waves in steel alloys. *J Nondestruct Eval*. 1982;3:115-24.
23. Jassby K, Saltoun D. Use of ultrasonic Rayleigh waves for the measurement of applied biaxial surface stresses in aluminum 2024-T351 alloy. *J Mater Eval*. 1982;40(2):198-205.
24. Husson D, Bennett SD, Kino GS. Measurement of surface stresses using Rayleigh waves. In: *IEEE Ultrasonics Symposium; 1982; San Diego. Proceedings. New York: IEEE; 1982. p. 1-4*.
25. Hu E, He Y, Chen Y. Experimental study on the surface stress measurement with Rayleigh wave detection technique. *Appl Acoust*. 2009;70(2):356-60. <http://dx.doi.org/10.1016/j.apacoust.2008.03.002>.
26. Duquennoy M, Ouafouh M, Qian ML, Jenot F, Ourak M. Ultrasonic characterization of residual stresses in steel rods using a laser line source and piezoelectric transducers. *NDT Int*. 2001;34(5):355-62. [http://dx.doi.org/10.1016/S0963-8695\(00\)00075-X](http://dx.doi.org/10.1016/S0963-8695(00)00075-X).
27. Akhshik S, Moharrami R. Improvement in accuracy of the measurement of residual stress due to circumferential welds in thin-walled pipe using Rayleigh wave method. *J Nucl Eng Des*. 2009;239(10):2201-8. <http://dx.doi.org/10.1016/j.nucengdes.2009.03.021>.
28. Duquennoy M, Ouafouh M, Ourak M, Jenot F. Theoretical determination of Rayleigh wave acoustoelastic coefficients: comparison with experimental values. *Ultrasonics*. 2002;39(8):575-83. [http://dx.doi.org/10.1016/S0041-624X\(02\)00262-7](http://dx.doi.org/10.1016/S0041-624X(02)00262-7).
29. Gartsev S, Köhler B. Direct measurements of Rayleigh wave acoustoelastic constants for shot-peened superalloys. *NDT Int*. 2020;113(102279):1-7. <http://dx.doi.org/10.1016/j.ndteint.2020.102279>.
30. Takahashi S, Motegi R. Measurement of thirdorder elastic constants and applications to loaded structural materials. *Springerplus*. 2015;4(325):1-20. <http://dx.doi.org/10.1186/s40064-015-1019-2>.
31. Hu E, Shao Y. Travel-time difference extracting in experimental study of Rayleigh wave acoustoelastic effect. *ISRN Mech Eng*. 2014;2014:349020.
32. Zhang H, He Q, Yan Y. A new nondestructive technique for measuring pressure in vessels by surface waves. *Appl Acoust*. 2008;69(10):891-900. <http://dx.doi.org/10.1016/j.apacoust.2007.05.011>.