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# On the Reduction of Stress Concentration Factor Around a Notch Using a Functionally Graded Layer

***Sadek Gouasmi***[a,b]***, Abdelkader Megueni***[a,b]****, Aboubakar Seddik Bouchikhi***[a,b]***,***

***Kamel Zouggar***[a,b]***, Abderahmane Sahli***[a]

[a]*Faculty of Technology, University of Sidi Bel Abbès, Sidi Bel Abbes, BP 89, Cité Ben M'hidi, 22000, Algeria*
[b]*Laboratory of Structures and Solids Mechanics – LMSS, Faculty of Technology, University of Sidi Bel Abbès, Sidi Bel Abbes, BP 89, Cité Ben M'hidi, 22000, Algeria*

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The aim of this paper is to study the effect of functionally graded material (FGM) layer around a notch in a plate in three dimensions. Using exponential law of variation of Young's modulus and a coefficient of Poisson constant, the stress concentration factor (SCF) depends on the gradation direction of the constituent materials. The finite element method is used to study the performance of FGM layer around a notch in a ceramic plate under in tensile load. A parametric study is performed for several geometric and mechanical parameters such that width of the FGM layer and the ratio of FGM layer components. The effect of notch radius is also studied.

**Keywords:** *FGM layer, notch, plate, exponential law, SCF, finite element method*

## 1. Introduction

Since the beginning of the century, the use of composite materials in the form of plate and beam has expanded considerably to this day whether in the automotive, construction, and more recently in aeronautic. The composite materials have significant advantages over traditional materials. In conventional multilayer structures, layered composite materials are used to improve the performance (mechanical, thermal, acoustic...) of the structure (plate). The disadvantage of this type is to create stress concentrations at the interfaces due to the change of mechanical and thermal properties. In the late 80s, a team of Japanese researchers has proposed to overcome these difficulties by developing new materials known as functionally graded materials (FGM). Functionally graded materials are composites in which the material properties vary continuously as a known function of the spatial position; these materials are usually associated with particulate composite where the volume fraction of particles varies in one or several directions. The initial development of FGMs is designed to serve as a thermal barrier[1]. Today, there have been more and more numerous modern engineering applications of FGM, like the spacecraft, rocking engine casings and packaging materials in the microelectronics industry, biomaterials (dental implants) and others[2,3].

The metallurgical field has also been the subject of recent research work dealing with the behavior of functionally graded steels[4,5]. The effect of a band in FGM around a hole in a homogeneous plate was analyzed for the reduction of the SCF under biaxial loading[6]. Many researchers were interested in calculating the stress concentration factor (SCF) in FGM plates for two types of holes (circular, elliptical)[7-12]. In the present work, we used the finite element method to calculate the stress concentration factor at the edge of a notch in a homogeneous ceramic plate and TiB FGM plate for different combinations ceramic-metal. The calculation of FGM's stress concentration SC involves three directions (x, y) and the direction of the radius of the notch. To reduce the SCF, we used a FGM layer around a notch in a homogeneous ceramic under uniaxial load. The graded finite elements are implemented in the FE software Abaqus[13] to verify the UMAT used subroutine (Appendix).

*e-mail: a_megueni@yahoo.fr

## 2. Problem Formulation

A Functionally graded material plate with a semicircular lateral notch is subjected to uniaxial tensile load equal to $\sigma_0$ =100 MPa (Figure 1). Geometric characteristics of the plate are:

Width W = 20mm; height H = 2W and thickness denoted t=2 mm. The lateral notch has a radius r = a.

The origin of the coordinates coincides with the center of the notch lateral. For reasons of symmetry of loading and geometry, half of the plate has been studied (Figure 1). The boundary conditions of the model were imposed by constraining the y-displacements (U2 = 0) at y = 0 and rotations around the axes ox and oz (UR3 = 0 and UR2= 0) at y = 0 in the Oxz plane of symmetry of the full model. The properties of the different constituents of the FGM (metal, ceramic) used are indicated in Table 1.

The Figure 2 shows the law graph for different combinations of metal-ceramic. The FGM material is expressed by the following exponential law:

$$E(\xi) = E_2 e^{\beta\xi} \tag{1}$$

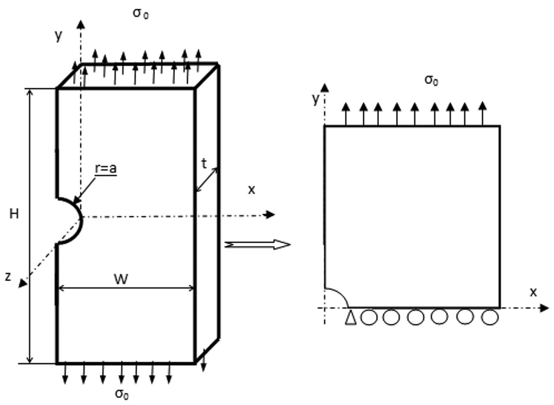


**Figure 1.** Geometric model of the plate with a lateral notch circular, (a) the entire plate, (b) half of the plate.

**Table 1.** Material properties of the FGM used in calculations.

| Nature | Young's modulus (GPa) | Poisson ratio |
|---|---|---|
| Al | 72 | 0.3 |
| Ti | 110 | 0.3 |
| Cu | 124 | 0.3 |
| Ni | 215 | 0.3 |
| TiB | 375 | 0.3 |

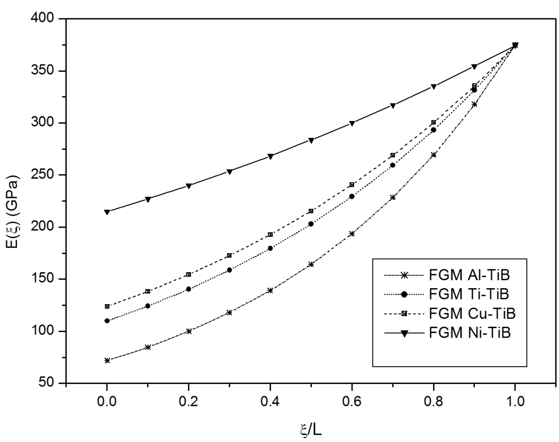


**Figure 2.** Exponential variation of Young's modulus for different combinations of the constituents.

With:

$$\beta = \frac{1}{L} log\left(\frac{E_1}{E_2}\right) \tag{2}$$

Where $E_2$ = E(0) is the Young's modulus of the metal and $E_1$ the Young's modulus of ceramic.

Different functionally graded material configurations can be obtained by varying the gradation direction of the constituent materials as:

ξ= x and L=W for x-FGM (Figure 3a).

ξ= y and L=W for y-FGM (Figure 3b).

ξ= $\sqrt{x^2+y^2}$ and L=W $\sqrt{2}$ for r-FGM (Figure 3c).

## 3. Finite Element Modeling

For modeling the Young's modulus in a desired direction, the subroutine UMAT Abaqus was used[14]. The routine is written in FORTRAN language and runs in parallel with the Abaqus solver. It allows us to establish an algorithm to calculate the variables used by the solver Abaqus. The routine was coded so that the material stiffness matrices are established with appropriate material properties, namely, the Young's modulus. In this study, the Poisson's ratio is assumed constant, since it has been shown that variations in the Poisson ratio are much less important than the Young's modulus[15]. Calculating the stiffness matrix requires the use of the Gaussian quadrature[16].

A half symmetric model of a 20 × 20 mm² plate and a thickness t with 2 mm radius center notch was used for verification as shown in Figure 4. The mesh was refined around the center notch and was graded in the direction moving away from the notch toward the outer boundaries. The initial mesh consisted of 13810 C3D20R quadratic brick, full integration stress elements with 276200 nodes. The base Young's modulus is TiB with $E_1$ =375 GPa and $E_2$ is the metal Young's modulus. The Poisson's ratio was held constant equal to 0.3.

## 4. Results and Analyses

The finite element method has allowed us to calculate the normalized stress concentration of a notched plate noted Kt for different directions of FGM and a constant normalized thickness (t/W=0.1). The stress concentration allows increasing

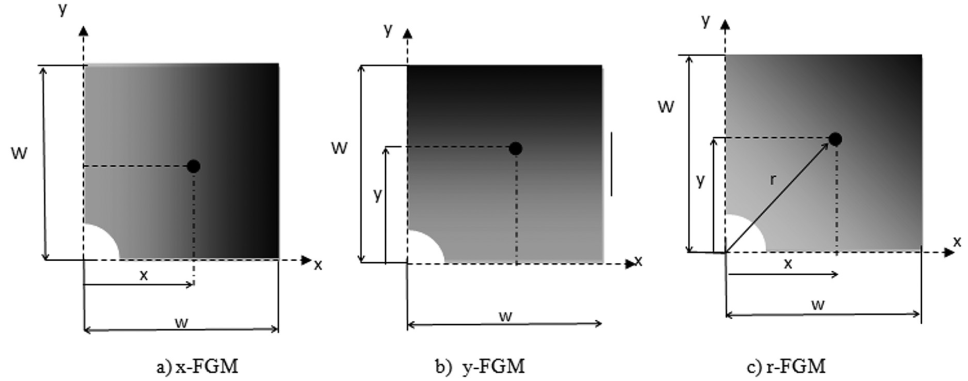


**Figure 3.** Direction of the FGM Young's modulus FGM (a) x-FGM variation, (b) y-FGM variation, (c) r-FGM variation.

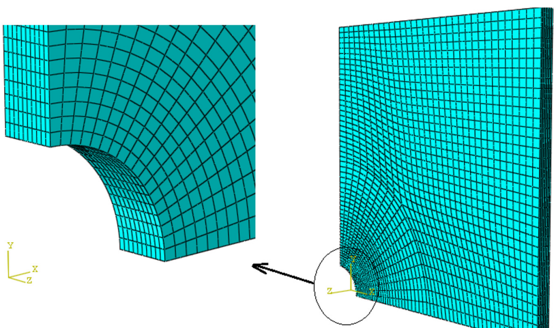

**Figure 4.** Model verification mesh with 13810 C3D20R elements.

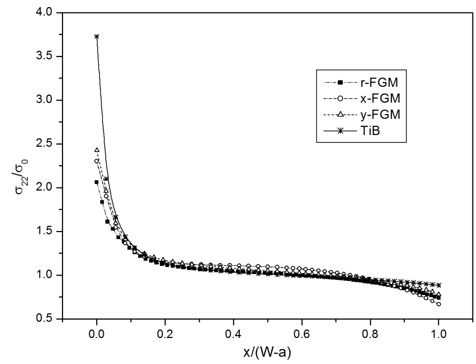


**Figure 5.** Variation of normalized normal stresses vs normalized x for different gradation direction of Ti-TiB FGM.

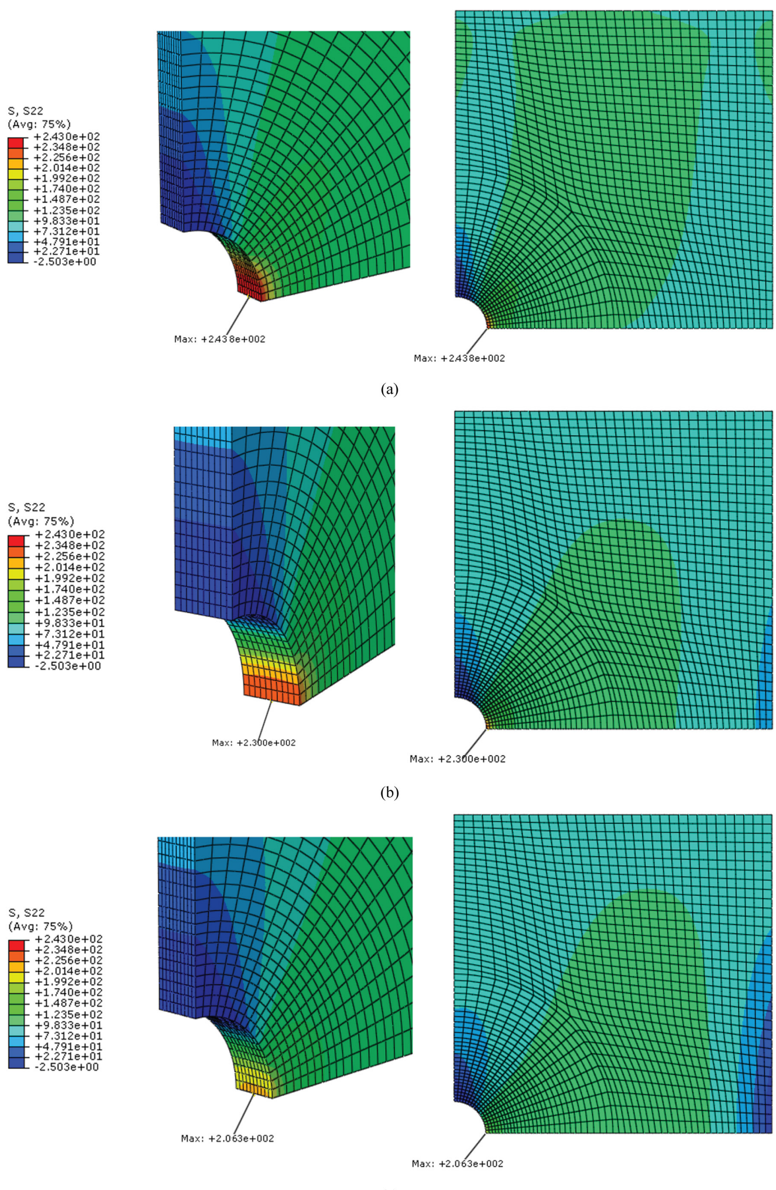


**Figure 6.** Strain max contours for different gradation direction distribution of FGM (a) x-FGM variation, (b) y-FGM variation, (c) r-FGM variation.

the stress at the notch root. The factor Kt is defined by the ratio of the maximum stress to the applied stress $\sigma_0$:

$$K_t = \frac{\sigma_{max}}{\sigma_0} \tag{3}$$

In the case of a homogeneous and isotropic plate, Kt is called stress concentration factor. The Figure 5 shows the normalized distribution of $\sigma_{22}$ stress versus x from the edge of the notch for the tree direction of FGM and the normalized distribution of $\sigma_{22}$ stress of ceramic.

In our case, the stress $\sigma_{max}$ represents the normal stress along the y axis at the edge of the notch at y = 0 and noted $\sigma_{22max}$. The value of $\sigma_{22max}$ is taken in the middle of the thickness where it is maximum (Figure 6). The maximum values are naturally obtained at notch edge (Figure 6). We deduce that the direction of FGM along r gives the minimum value. We note that FGM distribution management for the rest of the study. The gain of normalized stress concentration (Kt) of an r-FGM plate at the edge of the notch is 44% compared to a homogeneous ceramic plate.

The field of deformation shape of the plate is similar to the graduated FGM modulus for the three directions (Figure 7). The maximum deformation is lower in the case of r-FGM.

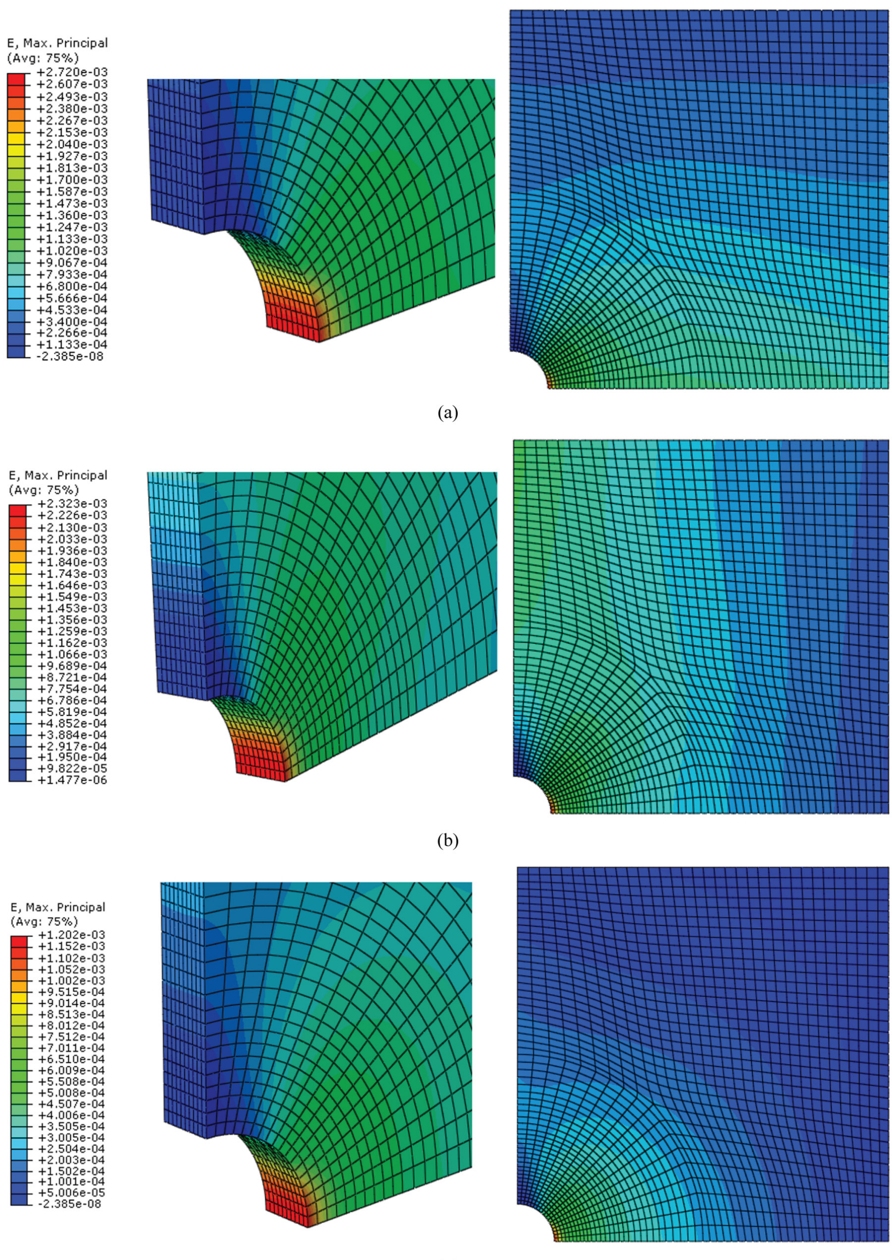


**Figure 7.** Stresses contour for different gradation direction distribution of FGM (a) x-FGM variation, (b) y-FGM variation, (c) r-FGM variation.

## *4.1. Parametric study*

The profile of the stress distribution $\sigma_{22}$ normalized along the x-axis is the same for the various combinations of metal-ceramic. The decrease of the stress concentration factor

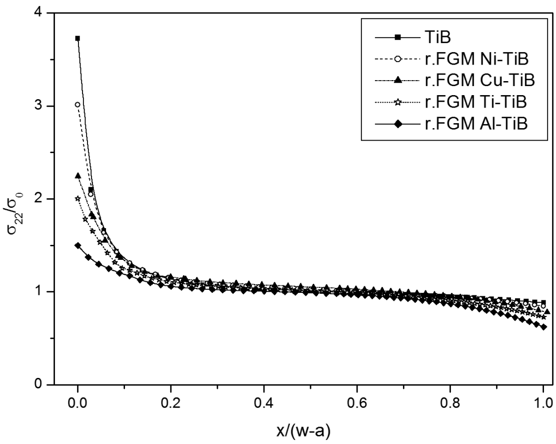


**Figure 8.** Variation of normalized normal stresses vs normalized x for different combinations of the FGM.

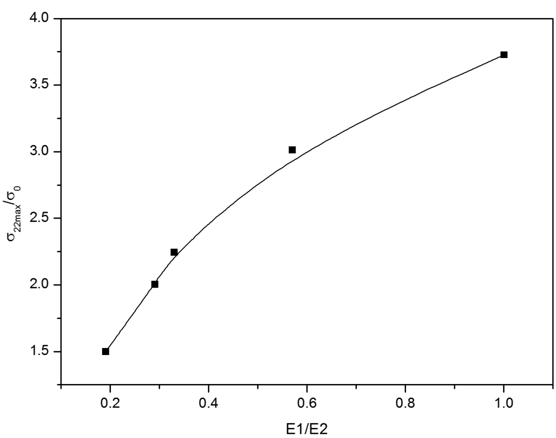


**Figure 9.** Variation of the stress concentration factor Kt based on the normalized x for different ratios E1 / E2.

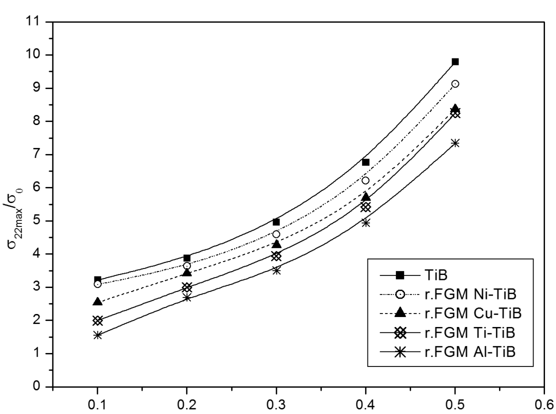


**Figure 10.** Variation of stress concentration factor as a function of normalized radius of notch for different ratios E1 / E2.

at the edge of the notch is proportional to the decrease in the $E_1/E_2$ report. For a normalized depth graduation of FGM between 0.4 and 0.7, the normalized stress tend towards the value 1, for a standardized depth graduation of FGM superior to 0.7, the stresses decrease slightly (Figure 8). The greatest reduction of the stress concentration factor at the edge of the notch is obtained for the combination AL-TiB and is about 60% compared to the TiB ceramics (Figure 9).

Figure 10 shows the evolution of $K_t$ as a function of the normalized notch radius for different combinations of the FGM. $K_t$ increases with increasing radius of the notch for all combinations. $E_2$ is the Young modulus of ceramic.

## *4.2. Effect of the FGM layer*

To analyze the effect of the FGM layer, a ceramic plate in TiB contains a r-FGM layer in Ti-TiB with a width h (h=b-a) around a circular lateral notch of radius a (Figure 11).

We observe that an increase of the width h corresponds to a decrease of the Kt value at the rim of the notch and, at the same time, we have an increase of the interface Kt (Figure 12). The Kt at the rim of the notch is noted Ka and the other Kb. The use of a standard FGM layer Ti-TiB

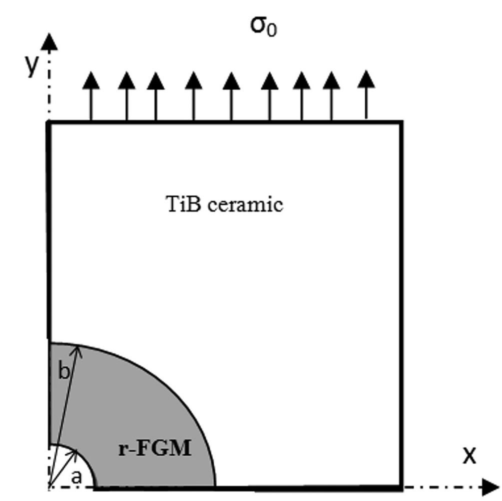


**Figure 11.** A geometric model FGM layer in a ceramic plate.

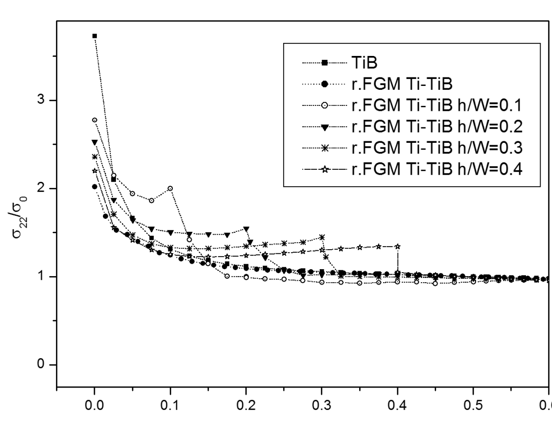


**Figure 12.** Variation of normalized normal stresses vs normalized x for different normalized witdh of FGM layer.

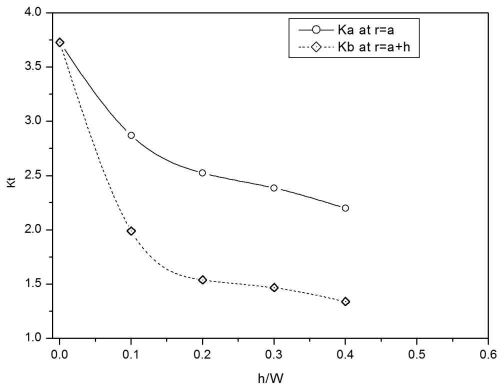


**Figure 13.** Variation of Ka and Kb depending on the normalized width of FGM layer.

**Table 2.** The gain of SCF Ka and Kb for different normalized width of layer.

| Normalized h/W | Gain (%) | |
|---|---|---|
| | ΔKa/Kt | ΔKb/Kt |
| 0.1 | 25.54 | 46.50 |
| 0.2 | 32.26 | 58.60 |
| 0.3 | 36.83 | 60.75 |
| 0.4 | 41 | 64 |

(h/W=0.4) leads to a significant decrease of Ka with a gain of 41% and a gain of 64% for the Kb with respect to a ceramic plate. We observe that the value of Kt = 2.02 of a complete r-FGM is close to that of Ka (Ka = 2.2) of a plate with the same size of the layer.

Figure 13 shows the evolution of Ka and Kb depending on the width of the layer. The values of Ka and Kb tend respectively to 2.2 and 1.32. The gains of Ka and Kb are summarized in Table 2 which are respectively 41% and 64%.

## 5. Conclusion

The use of FG-layer is another way to increase the performance of the notched structures.

The use of a r-FGM or r-FGM layer in a ceramic notched plate allowed to draw the following conclusions:

- The variation of the FGM properties in the direction of notch radius (r-FGM) offers the best favorable stress concentration factor compared to other directions.
- The smallest ratio $E_1$ / $E_2$ is one that gives the best guarantee.
- The use of FGM layer around the notch of a standardized width less than 0.5 gives the best possible performance.

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**Appendix.** User subroutine UMAT x-FGM.

```
c   User subroutine for implementation of a continuous variation
c   of the material elastic properties between integration points.
c
c   ABAQUS 6.11 - user subroutine UMAT for functionally graded materials
c   where E(X)
      SUBROUTINE UMAT (STRESS, STATEV, DDSDDE, SSE, SPD, SCD,
      1 RPL, DDSDDT, DRPLDE, DRPLDT,
      2 STRAN, DSTRAN, TIME, DTIME, TEMP, DTEMP, PREDEF, DPRED, CMNAME,
      3 NDI, NSHR, NTENS, NSTATV, PROPS, NPROPS, COORDS, DROT, PNEWDT,
      4 CELENT, DFGRD0, DFGRD1, NOEL, NPT, LAYER, KSPT, KSTEP, INC)
C
         INCLUDE 'ABA_PARAM.INC'
C
         CHARACTER*80 CMNAME
         DIMENSION STRESS (NTENS), STATEV (NSTATV),
      1 DDSDDE (NTENS, NTENS), DDSDDT (NTENS), DRPLDE (NTENS),
      2 STRAN (NTENS), DSTRAN (NTENS),TIME(2),PREDEF(1),DPRED(1),
      3 PROPS(NPROPS),COORDS(3),DROT(3,3),DFGRD0(3,3),DFGRD1(3,3)
c
           E1=props (1)
             E2=props (2)
                   ANU=props (3)
               X= COORDS (1)
                 Y= COORDS (2)
                  W= 20
c
c   W is Width of plate and a is radus of lateral notch
c   Determine L
      L=W
c   Determine material properties based on global coordinates of gauss points.
c   COORDS(1) is X-coordinates of gauss points.
c   PROPS is defined by users.
c   The function can be also defined by users.
         E=E2*exp (log (E1/E2)/L)*X
c          COMPUTE JACOBIAN
c
           amu=E/2.0d0/(1.0d0+ANU)
           alambda=E*ANU/(1.0d0+ANU)/(1.0d0-2.0d0*ANU)
c
           do i=1, ndi
             do j=1, ndi
               if (i.eq.j) then
                 ddsdde(i,i)=alambda+2.0d0*amu
               else
                 ddsdde(i,j)=alambda
               endif
             enddo
           enddo
           do i=ndi+1, ntens
             ddsdde(i,i)=amu
           enddo
c
c   STRESSES AND STRAINS AT END OF TIME STEP
c
         do i=1, ntens
           do j=1, ntens
             stress(i)=stress(i)+ddsdde(i,j)*dstran(j)
           enddo
         enddo
c
         return
         end
```