

A STUDY ON THE UNIVERSAL ACCESS TO VACCINES IN BRAZIL

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Abstract

In this paper, the universal access to vaccines in Brazil is discussed from an Operations Research (OR) point of view. A classic military model – the Weapon-Target Assignment (WTA) problem – is converted to a humanitarian one, where the objective is to maximize the coverage of the vaccines in the population instead of to destroy military assets. There is an enormous space for OR techniques in the whole healthcare system, as most of the planning is still hand-made, and drawn by professionals with medical background only. In addition, the healthcare data systems open new possibilities to data mining and statistical analysis. There is still a shortage of permanent programs to analyze and use this kind of statistical information for public planning. There are initiatives from the academia and public entities in this direction. In our research, the proposed model was tested with data from an original WTA problem.

Keywords: nonlinear programming; vaccination; public healthcare; mathematical programming.

Resumo

Neste trabalho, a universalização do acesso às vacinas no Brasil é discutida sob a ótica da Pesquisa Operacional (PO). Um modelo militar clássico – o problema de Alocação Arma-Alvo (WTA) – é transformado em um modelo com fins humanitários, cujo objetivo é maximizar a cobertura da distribuição de vacinas na população ao invés de maximizar a destruição de alvos militares. Pode-se afirmar que há um grande espaço para o uso de técnicas de PO no sistema de saúde, pois seu planejamento ainda é feito, em geral, de forma manual por profissionais de formação exclusivamente médica. Além disso, o armazenamento digital de dados do sistema público de saúde abre novas possibilidades para a mineração de dados e análise estatística, ainda que, salvo raras iniciativas acadêmicas ou públicas, haja poucos programas permanentes para fomentar a análise estatística em dados de saúde como subsídio para o planejamento público. Em nossa pesquisa, o modelo proposto foi testado com dados de um problema WTA da literatura.

Palavras-chave: programação não-linear; vacinação; saúde pública; programação matemática.

1. Introduction

Operations Research (OR) is the knowledge field where Applied Mathematics, Economics, Engineering, and Computer Science work in unison to solve real-world problems. Many authors place the beginning of OR in 1936, when the term “Operational Research” appeared for the first time in British military applications (Gass, 2002). Nonetheless, OR’s roots can be tracked to the works of Newton, Fourier, Lagrange, and others luminaries, who built the necessary mathematical foundations of this science (Klerk *et al.*, 2004). During the first decades of the last century, Kantorovich, Koopmans and others researched on the optimal allocation of economic resources, but it was George Dantzig, who worked for the Pentagon during the World War II, who would make the breakthrough that established Operations Research as an invaluable Planning & Management tool: the invention of the Linear Programming Simplex method (Dantzig, 1963). Dantzig’s discovery is an example of a trend that has become quite common in the OR history: the research of military applications. Developing countries, however, have needs that are more urgent than the military ones. In the modern society, it has become more and more common for countries to work out their differences via diplomacy and internal arbitrations, instead of through military actions. Operations research, whose past is deeply attached to military applications, can be employed in many other fields as well, such as the planning of public health policies. Economic resources may be scarce in development countries, where one of the main challenges that the public administration faces is how to distribute them wisely. For example, the access to basic services, like healthcare, education, and food may not be universally available in many of these countries, and the use of Operations Research techniques would surely ease the decision-making process. But as many public administrators may lack the necessary mathematical background to acknowledge this, it is important that the academia, with the aid of OR practitioners in general, divulge the benefits that can be achieved via an optimization-based planning & management approach. Industries like Energy & Electricity, Microelectronics, Telecommunications, Oil & Gas, Water & Wastewater, among others, are living statements of such benefits, being traditional adopters of optimization-based techniques.

In this paper we focus on the Health & Healthcare public administration, where decision-makers usually have excellent background on biological and medical sciences, but little or no formation on mathematics, economics or engineering disciplines. Our study case is the Brazilian National Immunization Program (PNI), coordinated by the country’s Ministry of Health. Our objectives are: (i) to discuss the use of optimization techniques in the improvement of the coverage of the national vaccination program; and (ii) to highlight the flexibility that Mathematical Programming provides, as similar mathematical structures are able to treat subjects so different as the destruction of military targets and the coverage of vaccination policies.

This work is divided as follows: the second section presents the current vaccination policies in Brazil, whereas in the third section a Mathematical Programming model is proposed to optimize the vaccination program. In the fourth section the solving approach is discussed, and in the fifth section a test problem is presented. Our final remarks and proposals for future work are given in the last section.

2. Overview of vaccination policies in Brazil

The Brazilian Constitution states that everyone has the right to Health, which must be ensured by the State, by means of public policies aiming at reducing the risk of diseases and other illnesses, and by the provision of a public, universal and common access to healthcare services, including a variety of health maintenance, disease prevention, and healing services. Vaccination does deserve its high reputation in Brazil. It is the main action responsible for the eradication of smallpox, and the acute reduction of poliomyelitis and measles occurrences in the country. Vaccines evoke a feeling of protection and respect amongst the Brazilian population. Brazil has the world's largest Public Health System, known as SUS ("Sistema Único de Saúde" or Unified Health System), which covers the entire country, but, unfortunately, with different degrees of quality of services (Travassos, 1997). The Brazilian population is very heterogeneous in terms of income, geographical location, and age, posing many challenges for the planning of Health & Healthcare public services. For example, in 1998, 14% of SUS potential users were elders with special and expensive needs (Sawyer *et al.*, 2002). In addition to the public system, there is also a big private network of medical services, concentrated in areas of higher economic importance.

The Brazilian production and distribution of vaccines is mainly public, under the National Immunization Program (PNI). Historically, the development of health policies in the country is deeply related to the development of public institutes and centers of research and production of vaccines and medicines. The most relevant centers are Fundação Instituto Oswaldo Cruz (FIOCRUZ), founded in 1900 in Rio de Janeiro, and Instituto Butantan, founded in 1901 in São Paulo. They were the government's answer to bursts of bubonic plague that occurred in 1889 (Cukierman, 1998). These two institutes are responsible for most of the national production of vaccines and to significant advances, including novel production processes and new types of vaccines, made for Brazilian conditions.

The National Immunization Program (PNI) is a public program designed to produce, distribute and apply vaccines, aiming at the reduction of epidemic diseases in Brazil by providing universal access to a selection of essential vaccines to all people in the country. This program relies on a combination of seasonal vaccination campaigns all over the country, media advertisements and a wide network of perennial vaccination posts, ready to apply the defined selection of vaccines. Most of the vaccines are produced by public institutes (e.g. Oswaldo Cruz and Butantan), but many are imported as well. The worldwide vaccine market was estimated in US\$ 7 billions in 2000 (Greco, 2002). In this same year, Brazil spent US\$ 215,000.00 on the importation of vaccines. Private clinics and physicians are responsible for about 40% of the national market, and almost the totality of the imports, as they offer vaccines that are not included in the PNI, in addition to the ones publicly available (Temporão, 2003).

One of the main challenges faced by the PNI is how to define the best combination of vaccines to be applied in each region of the country. There are vaccines which prevent more than one disease, and there are diseases that can be prevented by more than one vaccine. Moreover, there are different production and transportation costs, demands from private clinics, and distinct probabilities of success on the immunization of different groups. Other instruments can be employed towards the prevention of many diseases as well, such as investments on living conditions, water and sewage treatments, access to healthier diets, and elimination of disease vectors like certain mosquitoes species. Therefore, to find the best allocation of public resources is, in fact, a task of extreme complexity, which can certainly benefit from the use of optimization computational methods.

In the last ten years, SUS operations have been consistently migrating from paper and pen to digital systems in all levels of administration, with the development of a huge Healthcare web-based system (McDonald & Srinivas, 2004). This widespread adoption of computers in SUS made available a significant amount of field and statistical data to help in the decision-making process. Unfortunately, such data are still underused in the planning of public policies, as consequence of the absence of (suitable) models and applications. The vaccination program is certainly one entity that could make a proper utilization of these data.

The local literature is full of epidemiology studies, especially on the establishment of correlations between groups, occurrence of diseases, and the availability of healthcare services (Castro *et al.*, 2002). Epidemiology is the science that studies the development of diseases and possible countermeasures in populations, being a powerful tool to aid the public healthcare planning (Teixeira, 1999). Epidemiology techniques intensively rely on the use of statics and logical inference to create different classes based on the susceptibility to certain diseases, and on the effectiveness of different immunization actions, such as vaccination and improvement of hygienic conditions. The vaccination program can benefit from this knowledge, as it can provide: (i) success probability of vaccines against diseases; and (ii) identification of target groups for immunization.

Unfortunately, the literature is still scarce on optimization-based applications for Health & Healthcare in Brazil. Recent exceptions are Moreira (2003), with a tutorial on the use of Linear Programming as a computer-aided tool to healthcare policies, Gomes *et al.* (2005), with a complex decision support system for prevention and control of the dengue disease, Joly & Pinto (2006) on the use of mathematical programming in the optimal control of the HIV-1 pathogenesis, and Scarpin *et al.* (2007) on the optimal flow of patients in SUS' hospitals in the Brazilian state of Paraná.

3. A Mathematical Programming Model

One of the advantages of the mathematical programming approach is the detection of common patterns in many problems, which can be categorized into distinct classes. A model is an independent mathematical entity, inspired by its original application, but not limited to it, as one same model can be interpreted in different ways. Therefore, whenever modeling a problem, one should check if it belongs to a class of problems already studied in the literature, with known properties like NP-Complexity, convexity and smoothness. A previous knowledge of the problem's properties enlightens the choice of an adequate solving method and advises about pitfalls one might encounter.

The first step in any modeling activity is to understand the problem's nature, by identifying the decision variables, the constraints and the objective to be optimized. In the vaccination program case, one wants to maximize the coverage of the population, by the distribution of vaccines in different quantities, subject to the availability of resources to produce and distribute the vaccines. The vaccination program can be compared to a classic military problem – the Weapon-Target Assignment (WTA) problem (Manne, 1958; Day, 1966). Instead of military facilities, diseases are fought, and instead of weapons, vaccines are employed.

The WTA problem is defined as the problem where one wants to find the best assignment for the weapons available onto military targets. The original application regarded the destruction of enemies' factories and storehouses with different kind of missiles and bombs. Each

weapon has a probability of failure when launched against a target, which is estimated by taking into account defense air-shields, missile flight autonomy, destructive power, and other military issues. The objective is to maximize the destruction of enemies' targets with a limited number of weapons. There are two versions of the WTA problem: the static version, where all targets and weapons are known a priori and all weapons are engaged at the same time, and the dynamic version, where actions occur in rounds (similarly to a game problem).

The WTA original formulation is presented as follows:

$$\text{Maximize } f(x) = \sum_j \mu_j (1 - \prod_i p_{ij}^{x_{ij}}) \quad (1)$$

$$\sum_j x_{ij} \leq c_i \quad (2)$$

$$\sum_i x_{ij} \geq b_j \quad (3)$$

The objective (Eq. 1) is to maximize the damage of enemies' facilities, constrained by the maximum and minimum number of missiles that can be launched to each of the targets (Eq. 2 and 3). In the original formulation, parameter $p_{ij} > 0$ stands for the probability of failure of a missile of type i when launched towards target j , while variable x_{ij} is, analogously, the quantity of missiles of type i which targeted j . Please notice that any probability $p_{ij} = 0$ is removed from the problem, in order to avoid the possible occurrence of a term 0 in the objective function. A parameter μ_j stands for the strategic value of target j , as introduced in the objective function, where c_i is the maximum number of missiles of type i , and b_j is the minimum number of missiles that must be launched to target j . A simple sanity check shows that the model makes sense: the greater the number of weapons allocated to a given target, the lower the probability of failure – except in the case of 100% probability of failure, where the number of weapons is irrelevant to the destruction of the target.

Now, consider the following new WTA formulation:

$$\text{Maximize } f(x) = \sum_j \mu_j (1 - \prod_i p_{ij}^{x_{ij}}) \quad (1)$$

$$\sum_j \mu_j x_{ij} \leq c_i \quad (4)$$

$$\sum_i \mu_j x_{ij} \geq b_j \quad (5)$$

In this formulation, the index j stands for a group of people, which can be defined as a target group of people j , which can be interpreted as the population from certain geographical areas, age groups, social groups, risk groups, etc., susceptible to a disease that could be prevented by vaccination. The parameter μ_j is the group j 's population, while x_{ij} is the per capita dose of vaccine i applied to group j . Please notice that more than one type of vaccine can be applied to the target group, but each group is related to a single disease and population. The original quantity constraints (Eq. 2 and 3) were replaced by new ones (Eq. 4 and 5), whereas the same objective function was kept (Eq. 1). In (Eq. 4) c_i stands for the maximum number of available vaccines of type i , while in (Eq. 5) b_j stands for the minimum number of vaccines that have to be applied to group j , necessary to avoid unreasonable solutions, such as to produce only cheap vaccines or to ignore small populations. Eq. (5) guarantees that even diseases which are affected by a single type of vaccine, possibly expensive, are to be covered in the optimal solution. The probability $(1 - p_{ij})$ is calculated with basis on the effectiveness of vaccine i in population j , which can be either the nominal efficiency as measured in laboratory conditions, or the actual real-world effectiveness, based on historical data, considering the difficulties found in practice: lack of proper refrigeration, defective needles, nonuniform doses, untrained professionals, etc. The same sanity check applied to

the previous formulation can be applied here: the greater the number of doses, the greater the population coverage. Moreover, the model acknowledges the accumulative effect of vaccines on population μ_j as well.

Despite featuring a good adherence to the real-world allocation problem, the previous formulation has a flaw: it does not consider budget constraints, ignoring the fact that vaccination costs vary with vaccine types and target groups. Moreover, a fixed available production c_i for each kind of vaccine is assumed to be known *a priori*. A better formulation is the following, where a total budget B^{max} for the vaccination program is known *a priori*. The optimal solution will not only give the vaccine allocation to each target group, but also the overall production of each type of vaccine.

$$\text{Maximize } f(x) = \sum_j \mu_j (1 - \prod_i p_{ij}^{x_{ij}}) \tag{1}$$

$$\sum_j \mu_j \beta_{ij} x_{ij} \leq c_i \tag{6}$$

$$\sum_i \mu_j x_{ij} \geq b_j \tag{5}$$

$$\sum_i c_i \leq B^{max} \tag{7}$$

The main novelty of this formulation is the following: c_i no longer stands for a fixed production of vaccine i , but it now stands for the amount of budget that is allocated to produce and distribute vaccine i (Eq. 6 and 7). The parameter β_{ij} stands for an average production and distribution cost of one instance of vaccine i to population j , allowing for a better representation of the cost differences one finds in reality. On one hand, different vaccines may have different production costs, and, on the other hand, the same vaccine may have different transportation costs, when distributed to geographically dispersed locations, which is actually the Brazilian case.

There are vaccines which can immunize the population for more than one disease in a single application. These are known as multi-action or combo (combined) vaccines, usually having better costs. In the proposed formulation, these vaccines are modeled as follows: for example, consider a population subject to the occurrence of hepatitis A and B, and a vaccine (Y) for both types of hepatitis, with effectivenesses p_{YA} and p_{YB} , respectively. As variables x_{YA} and x_{YB} are the per capita dose of vaccine Y meant for the combat of hepatitis A and B in the given population, and as Y stands for a single vaccine, x_{YA} and x_{YB} cannot be different. Therefore, we replace the two variables x_{YA} and x_{YB} for a common variable $x_{Y[AB]}$, but keeping different effectivenesses p_{YA} and p_{YB} .

Another consideration that must be taken into account is the definition of the target groups. At the current state of our research, we consider only non-immunized groups, and disregard the possibility of an unaccounted contact of the population with the vaccine's microorganisms, which could have happened in an epidemic occurrence of the disease (during the application of the vaccination program or in the past). In addition, during the problem formulation, one must assure, by construction, that there is no intersection between target groups. Otherwise, the parameter μ_j would not be able to represent the population of each target group j , and the production constraints would be meaningless. As an example, consider one target group as the entire adult population of a given city that may be affected by a certain disease, while another group is composed by the city's elders which may be affected by the same disease. The first group must be redefined as the adult population minus the elderly part, while the second is kept as originally defined.

Finally, we must state that this is still a preliminary model, which can be viewed as an initial step to solve the wider vaccination problem, once other related constraints are imposed by the actual problem. The model can be enhanced to consider more general Health & Healthcare investments, such as media campaigns, medicine distribution, hospitals, staff training, etc. However, this new problem would require correlations between monetary investments and the effects on population health, which is very hard to measure. Moreover, these actions have a combined effect, when one action may interfere in the efficiency of another action, which is also very hard to measure. In the proposed model, this kind of difficulty does not exist, as data on vaccine efficiency are available in the literature and public reports, and the effect of each vaccine can be considered as accumulative in the target population.

4. Solving the problem

One of the reasons why the Weapon-Target Allocation problem has been constantly studied in the literature is the fact that it is nonconvex, nonlinear, and belongs to the class of NP-Complete problems (Lloyd & Witsenhausen, 1986). Moreover, many variants of this problem do exist, with different modeling assumptions – e.g. only one type of weapons, targets that can be bombed by, at most, one weapon, etc. – and different solving approaches – e.g. mixed-integer programming, nonlinear programming, neural networks, genetic algorithms, heuristics, etc. We refer the reader to Maltin (1970), Eckler & Burr (1972), and Murphey (1999) for comprehensive accounts on the subject. A recent report from Ahuja *et al.* (2003) discussed the combined use of heuristics and exact algorithms to solve the problem. Moreover, the authors claim that their branch and bound algorithms are the first implicit enumeration algorithms that can solve moderately size instances of the WTA problem till optimality, and that their heuristics are able to generate almost optimal solutions within a few seconds.

We decided to evaluate the Hyperbolic Penalty (HP) method (Xavier, 2001), which is already employed in different Brazilian public planning and forecasting systems (Xavier *et al.*, 2001), mainly in the Energy & Electricity industry. The HP method is designed to solve nonlinear programming problems with inequalities constraints (Eq. 8), where $f: \mathbb{R}^n \rightarrow \mathbb{R}$, $g_i: \mathbb{R}^n \rightarrow \mathbb{R}$, $i = 1, \dots, m$:

$$\text{Minimize } f(x), \text{ s.t. } g_i(x) \geq 0 \quad (8)$$

As a member of the class of penalty methods, the HP method replaces the original problem (Eq. 8) by an unconstrained one, where the original constraints are penalized in the objective function under a penalty function $P: \mathbb{R} \rightarrow \mathbb{R}$ (Eq. 9):

$$\text{Minimize } f(x) + \sum_i P(g_i(x)) \quad (9)$$

The HP method employs the following convex C^∞ penalty function P (Eq. 10), where $\tau \geq 0$ and $0 \leq \alpha < \pi/2$:

$$P(y, \alpha, \tau) = -(\frac{1}{2} \tan \alpha)y + ((\frac{1}{2} \tan \alpha)^2 y^2 + \tau^2)^{1/2} \quad (10)$$

The graphic representation of $P(y, \alpha, \tau)$, as shown in Fig. 1, is a hyperbole with asymptotes forming angles $(\pi - \alpha)$ and 0 (zero) with the horizontal axis and having τ as the intercept with the axis of the ordinates. Alternatively, the hyperbolic penalty function may be put in a more convenient form (Eq. 11), where $\tau \geq 0$ and $\lambda \geq 0$:

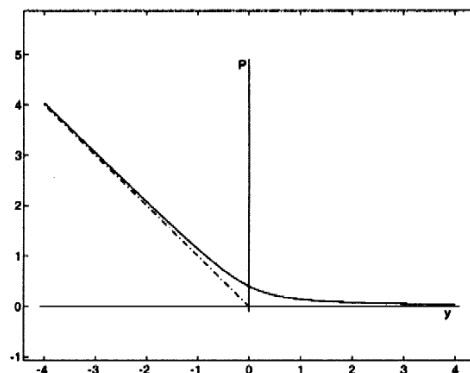


Fig. 1 – Hyperbolic penalty.

$$P(y, \lambda, \tau) = -\lambda y + (\lambda^2 y^2 + \tau^2)^{1/2} \quad (11)$$

In the HP method, the solution of the original NLP (Eq. 8) is obtained by means of solving a sequence of unconstrained subproblems, $k = 1, 2, \dots$, defined by the minimization of the modified objective function (Eq. 12):

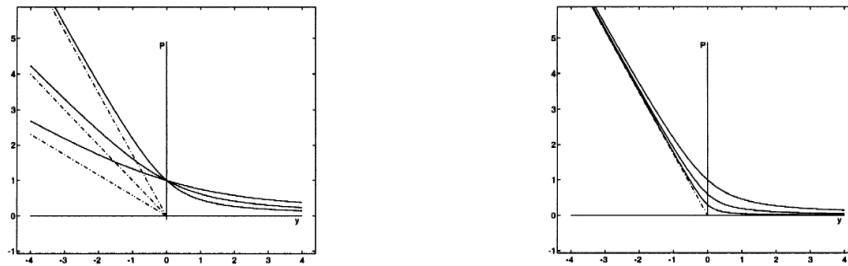
$$F(x, \lambda^k, \tau^k) = f(x) + \sum_i P(g_i(x), \lambda^k, \tau^k) \quad (12)$$

A simplified version of the Hyperbolic Penalty algorithm is presented as follows:

- Step 1: Let $k = 0$. Take initial values x^0 , $\lambda^1 > 0$, and $\tau^1 > 0$.
- Step 2: Let $k := k + 1$. Solve the unconstrained minimization problem (Eq. 12) from the initial point $x = x^{k-1}$, obtaining an intermediate optimal point x^k .
- Step 3: Feasibility Test:
 If x^k is an unfeasible point then go to Step 4;
 Otherwise, go to Step 5.
- Step 4: Increase on parameter λ :
 Make $\lambda^{k+1} := r \lambda^k$, $r > 1$.
 Keep $\tau^{k+1} := \tau^k$.
 Go to Step 2.
- Step 5: Decrease on parameter τ :
 Make $\tau^{k+1} := q \tau^k$, $0 < q < 1$.
 Keep $\lambda^{k+1} := \lambda^k$.
 Go to Step 2.

The sequence of subproblems is obtained by the controlled variation of parameters λ and τ in two different phases of the algorithm. The geometric idea behind the HP method can be described as follows. In Fig. 2a, the parameter λ (or the angle α of the asymptote of the penalty function) increases, causing a significant increase in the penalty outside the feasible region, but, at the same time, reducing the penalty in the feasible region. This process

continues until a feasible point is obtained. From that point on, λ is kept constant and τ is continuously decreased, making the internal penalty more and more irrelevant, while keeping the same degree of prohibitiveness outside the feasible region. Fig. 2b illustrates the second phase of the process.



(2a) First phase: variation of λ^k maintaining τ^k constant. (2b) Second phase: variation of τ^k maintaining λ^k constant.

Fig. 2 – Foundations of the hyperbolic penalty algorithm.

An analysis of the step sequence of the hyperbolic penalty algorithm explicits that it can behave like an internal or an external penalty method, at different moments. As shown in Fig. 3, the hyperbolic penalty function lies in between the internal and the external penalty functions, being able to asymptotically approach either one, as desired. In its first phase, when one manipulates the parameter λ , the method behaves similarly to an external penalty method, such as the quadratic loss function (Fiacco & McCormick, 1966). In its second phase, when one manipulates the parameter τ , the method seems to behave as a barrier method, such as the logarithmic penalty method (Frisch, 1955; Lootsma, 1967). Moreover, when in the limit of $\tau \rightarrow 0$, the HP method asymptotically approaches Zangwill's exact penalty method (Zangwill, 1967).

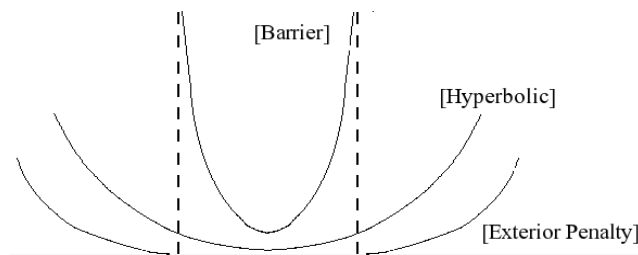


Fig. 3 – Idea of the hyperbolic penalty algorithm.

5. A Test Problem

The model and the computational method were tested with a classic WTA instance from the literature: Himmelblau's problem 23 (Himmelblau, 1972), also described in Bracken & McCormick (1968). The problem is reformulated as a vaccination problem (Table 1), as follows: a given population, spread throughout 20 different administrative regions, is susceptible to epidemic diseases that can be combated by 5 distinct vaccines (V1, V2, V3, V4, V5) with

poor effectivenesses, but equal costs (as all of them are supplied by the PNI). The budget allows a total of 107 vaccines, but with each kind having a different availability, according to current stocks. Moreover, certain regions had already ordered vaccines for their own use (regions 1, 6, 10, 14, 15, 16 and 20). The objective is to maximize the immunization of the overall population.

The HP method was initialized as follows: $x^0 = (0, \dots, 0)T$, $\lambda^1 = 10$, $\tau^1 = 10$, $r = 10^{1/2}$, and $q = 0.1$. Moreover, due to numerical issues on very small values of λ , safeguard measures were taken, including the increase of λ and restart at x^0 . The unconstrained minimization was carried out by a quasi-Newton algorithm employing BFGS updating formula provided by the Harwell routine VA15. Himmelblau's problem 23 is bigger than many real-world vaccination instances, resulting in a nonlinear problem with 100 variables and 112 constraints. This instance was solved in a PC workstation (Intel 486, 150 MHz) in 28.38 CPUs, achieving the same optimal value as found in Murtagh & Saunders (1978). Table 2 presents the optimal distribution of the vaccines amongst the 20 regions.

Table 1 – Himmelblau 23 data as a vaccination problem.

Region	Population (10^4 people)	Vaccines already ordered (10^4 units)	Vaccine Effectiveness				
			V_1	V_2	V_3	V_4	V_5
1	6.0	30	0	0.16	0.04	0	0.08
2	5.0	-	0.05	0.17	0.05	0	0.06
3	5.0	-	0	0.15	0.04	0	0.08
4	7.5	-	0	0.16	0.04	0	0.05
5	4.0	-	0	0.15	0.04	0	0.05
6	6.0	100	0.15	0.19	0.10	0	0.02
7	3.5	-	0.10	0.19	0.08	0	0.02
8	3.0	-	0.15	0.18	0.09	0	0
9	2.5	-	0	0.20	0.08	0	0
10	15.0	40	0	0.14	0.05	0.04	0.10
11	3.0	-	0	0	0.01	0.09	0.05
12	4.5	-	0	0.02	0.02	0.08	0.04
13	12.5	-	0	0	0.01	0.09	0.09
14	20.0	50	0	0.12	0.02	0.08	0.02
15	20.0	70	0	0.13	0.03	0.02	0.01
16	13.0	35	0	0.12	0.02	0.07	0.01
17	10.0	-	0	0.15	0.05	0	0
18	10.0	-	0.05	0.16	0.08	0	0
19	10.0	-	0	0.15	0.07	0	0
20	15.0	10	0	0.15	0.08	0	0
			Stock (10^4 units)				
Total	175.5	335	200	100	300	150	250

The Hyperbolic Penalty method seems to be an adequate choice for the WTA problem, which could be further detailed to be used in the Brazilian National Vaccination Program, as it converged in reasonable computational time to an optimal solution that maximized the immunization of the entire population, covering all regions, even in face of poor effectivenesses and restricted budget. It is also important to notice that in a real-world healthcare application, the vaccination effectivenesses would be much higher than the ones registered in this example, as they came from a military example and not from epidemiological studies.

Table 2 – Optimal distribution of vaccines.

Region	Quantity of vaccines (10^4 units)					Total
	V ₁	V ₂	V ₃	V ₄	V ₅	
1	-	-	-	-	51	51
2	14	1	-	-	45	60
3	-	-	-	-	49	49
4	-	23	-	-	-	23
5	-	21	-	-	-	21
6	100	-	-	-	-	100
7	39	-	-	-	-	39
8	27	-	-	-	-	27
9	20	-	-	-	-	20
10	-	-	-	-	51	51
11	-	-	-	33	-	33
12	-	-	-	41	-	41
13	-	-	-	-	54	54
14	-	-	-	59	-	59
15	-	26	44	-	-	70
16	-	24	-	17	-	41
17	-	4	72	-	-	76
18	-	-	58	-	-	58
19	-	-	64	-	-	64
20	-	-	62	-	-	62
Total	200	99	300	150	250	999

6. Conclusion

In this paper, we proposed an optimization-based approach to help the planning of Brazilian vaccination public policies. We addressed the combat against diseases that still infect the country's population as an optimization problem, where our goal was to maximize the vaccination coverage under budget constraints. The proposed model was derived from a military classic model, which, with simple transformations, is interpreted as a healthcare

policy-making model. Moreover, we presented a suitable optimization method to solve the problem. As Dantzig (1963) once said, “The final test of any theory is its capacity to solve the problems which originated it.” Despite being tested with data from the literature, the results make sense when mapped onto a real-world problem, urging us to the continuation of this research, when the model shall be further enhanced towards real applications within the Brazilian Health Public System (SUS).

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