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ARTICLES

ALTERNATIVES FOR THE COMPOSITION OF INTERACTIVE ENVIRONMENTAL IMPACT FACTORS

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ABSTRACT. Composition of probabilistic preferences is applied here together with life cycle assessment in the evaluation of sewage sludge treatment scenarios. The computations are based on the transformation of preferences for the alternatives elicited applying each criterion into probabilities of being the best. Different forms of probabilistic composition are applied and the possibility of interaction between factors is considered. The results are compared to results previously obtained with different forms of composition modeling the interaction or assuming linearity. Eight scenarios combining different processing techniques and leading to distinct end-uses are compared. These scenarios are ranked combining probabilities of preference according to five environmental impact factors: water use, energy consumption, carbon footprint, human toxicity potential and terrestrial ecotoxicity potential. In the large spectrum of views that the probabilistic approach provides, evidence is found favoring a system that includes recovery of energy to be used in a dryer and in a cement kiln.

Keywords: composition of probabilistic preferences, criteria interaction, sewage management systems.

INTRODUCTION

Treatment and exploitation of sewage sludge is a complex issue. Sewage sludge contains nutrients vital to plant growth that can be applied to agricultural land instead of conventional fertilizers. Some sewage treatment processes also provide the benefit of renewable energy production.

To measure the environmental burdens and financial costs of managing the disposal of sewage sludge in small towns of rural Australia, Peters and Rowley (2009) employed Life Cycle Assessment (LCA) (Suh & Huppes, 2005). Other studies have also demonstrated that sludge treatment

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practices can benefit from the use of LCA methodology to quantify the burdens and benefits of its processes and stages (Yapicioğlu & Demir, 2017; Yoshida et al., 2018).

In LCA, an inventory of impacts jointly affecting successive steps of production processes must be carefully raised. In Peters and Rowley (2009), the life cycle impact assessment (LCIA) was based on evaluation in each step of the life cycle of the impact in five dimensions: water use (WU), total energy consumption (TEC), carbon footprint (CF), human toxicity potential (HTP) and terrestrial ecotoxicity potential (TETP). Similar frameworks have been taken, for instance, by Loiseau et al. (2014), Cartes et al. (2018) and Cremiato et al. (2018).

Loiseau et al. (2014) used the environmental impacts characterized by the hierarchist approach of ReCiPe v1.07 (Goedkoop et al., 2009). ReCiPe calculates 18 midpoint indicators and 3 endpoint indicators. Midpoint indicators include, for example, water use, human toxicity, terrestrial ecotoxicity, global warming, and others. Endpoint indicators show the environmental impact on three higher aggregation levels related to the effect on human health, biodiversity and resource scarcity. Converting midpoints to endpoints simplifies the interpretation of the LCIA results. However, with each aggregation step, uncertainty in the results increases.

Cartes et al. (2018) selected impact categories based on the correspondence between Life Cycle Inventory (LCI) data and potential environmental impacts and previous reports for sludge management (Corominas et al., 2013; Yoshida et al., 2013). The focus was oriented towards categories related to organic matter and nutrients flows, as those are more likely to be affected by the inclusion of pre-treatment in the advanced digestion scenario. The categories selected include climate change, abiotic depletion, acidification and eutrophication (terrestrial, freshwater and marine) impact potentials.

Cremiato et al. (2018) used global warming potential (GWP), which accounts for the emission of greenhouse gases; human toxicity potential (HTP), which addresses a wide range of toxic substances, including, the secondary particulate matter; acidification potential (AP), which accounts for the emissions of NOx, SOx and ammonia; photochemical ozone creation potential (POCP), which accounts for the substances that cause the photochemical ozone production in the troposphere; abiotic resource depletion (ADP) that represent the natural resources consumption such as metals, crude oil and wind energy (Guinée & Lindeijer, 2002); eutrophication Potential (EP) that considerate the conversion factor of phosphorous and nitrogen compounds (waste water discharges and air emissions of nitrogen oxides (NOx) and ammonia (NH3)) into phosphorous equivalents.

Peters and Rowley (2009) selected the categories in their analysis considering the scope of the study. They highlight that environmental burdens are modeled in the LCIA phase by calculating impact category indicators, even though no agreed universal list of impact categories exists, and analysts must choose them. Like most LCA analysts, Peters and Rowley (2009) used CF as a proxy for all climate change effects caused by different greenhouse gases excluding biogenic carbon dioxide (Peters & Lundie, 2001). The use of HTP and TETP models required assumptions about the bioavailability of trace metals.

The five components of Peters and Rowley (2009) cover two important dimensions of the use of resources and two other important dimensions of contamination effects. Finally a most employed measurement of the impact of carbon emissions on climate is included. With a sufficiently high number of dimensions, if distortions from the pressure from modeling relations between the five dimensions are avoided, this set of indicators provides a robust form to access environmental impact.

The sewage treatment may start with thickening in an anaerobic digester (Lohri et al., 2013), from which, besides the biosolids, gas (Wellinger et al., 2013) is a product. A dryer (Dincer & Zamfirescu, 2016) may then be used in a subsequent thickening stage, and the biogas may be used as energy source in the dryer. Other possibilities of treatment, geared for agricultural use, are blending with biodegradable garden waste in windrow composting (Recycled Organic Unit, 2003) and improvement by lime amendment (National Lime association Fact Sheet, 1999). Other end-uses of the biosolids, besides the use as fertilizer, include the direct destination to landfill (Westlake, 1995) or the use as supplementary fuel in a cement kiln (Houillon & Joliet, 2005).

Peters and Rowley (2009) described the LCIA results (raw and relative to each category indicator range) in a table that shows the performance of all scenarios under each impact category. Assuming that there was no universally agreed way to weight the relative importance of different impact categories, and using the min-max approach (Lai et al., 2008), they ranked the relative performance of each scenario within each criterion. Then, considering the average rank of the evaluations by the five indicators, they concluded that the energy recovery obtained with the use of the generated biogas in the dryer and of the biosolids in the cement kiln constituted the best scenario.

Later, in Rowley et al. (2015), the correlation in the matrix of evaluations by the five criteria was explored to take into account the possible interaction between criteria. A different result was then obtained, with the use in agriculture after addition of lime overpassing the alternatives with energy recovery.

Here the application of Combination of Probabilistic Preferences (CPP) (Sant'Anna, 2015) is proposed, to tackle this problem from a probabilistic point of view. The probabilistic treatment of interaction reduced the importance of the water use factor leading to restore the position of the combination of the use of a dryer and a cement kiln as the best treatment.

CPP is used in Souza et al. (2016) to prioritize system alternatives for potential implementation of electronic waste management. In Gavião et al. (2017), CPP is also applied to the problem of supporting multicriteria decision in Life Cycle Assessment. In a broader context, CPP has been used in the most diverse areas of management systems. In (Sant'Anna et al., 2014), a probabilistic framework is used to measure efficiency in services management. In (Sant'Anna, 2012), CPP is used in risk analysis by the evaluation of Failure Modes and Effects Analysis (FMEA) with probabilities of alternatives being the most important according to a set of criteria. In fact, the method has proven to be eclectic in its use in management, from providers contracting (Garcia & Sant'Anna, 2015) to disaster relief operations (Gavião et al., 2020b). This characteristic of the probabilistic approach is especially useful in real life problems, where uncertainty is always present in decision making. However, versatility is a characteristic of many multicriteria methods. This is the case for AHP (Analytical Hierarchy Process) and ANP (Analytical Network Process) (Saaty, 1996), ELECTRE (Elimination et Choix Traduisant la Realité) (Roy, 1968), TOPSIS (Technique for Order Preference with Similarity to Ideal Solution) (Hwang & Yoon, 1981), and many other methods. Reviews of applications of different MCDA methods to environmental issues have been published. Huang et al. (2011) reviewed 312 papers about MCDA applications in the environmental field. Coelho et al. (2017) studied 260 articles related to the application of multi-criteria decision making in solid waste management. Cegan et al. (2017) analyzed approximately 3000 papers concerning multicriteria decision analysis (MCDA) in the environmental field.

The divergence between Peters & Rowley (2009) and Rowley et al. (2015) highlights the importance of taking into account interaction and the need of deepening the investigation on all possible interactions of impact factors. CPP can derive the importance of indicators and of sets of indicators from the evaluations that they provide to a set of alternatives, independent from any previous modeling of relations between them.

This feature of CPP is explored here. Initially, the probabilistic composition of the evaluations is performed additively, assuming absence of interaction. In a second stage, the possibility of interaction is taken into account, with different principles being applied to derive capacities from the probabilities of preference. A large spectrum of views is then provided.

METHODS

The Data Set

The efficiency and environmental impact of sewage treatment and destination depend on the volume operated. Peters and Rowley (2009) based the study on a 'functional unit' (International Organization for Standardization, 2006) which equals the management of normal sludge of 2 dry tonnes generated in a day by an equivalent population (Baumann & Tilman, 2004) of 40,000 people. Peters and Rowley (2009) assumed this to occur in rural Australia, with equipment lifespans of 100 years for civil works and 8.5 years for mechanical devices. This size is not large enough to allow for the biogas produced by digestion being fully utilized, due to the unavailability of a sufficiently small dryer to permit continuous operation. On the other hand, the impact related to transportation costs of fertilizers will be smaller than it would be in the case of larger cities at higher distances from agricultural sites.

In this section, the data set collected by Peters & Rowley (2009) in towns of this size in rural Australia is reviewed. Peters & Rowley (2009) produced a life cycle inventory (LCI) collected for eight scenarios representing different combinations of treatment processes and end-uses observed. The LCI includes consumed energy and material inputs, products and avoided production. The eight scenarios can be briefly identified as follows:

1. The sewage enters a digester that produces gas - which is flared, and biosolids - which are directly used in landfill (Landfill);
2. The sewage enters a digester that produces gas - which is flared, and biosolids - which are directly used in agriculture (DigAg);
3. The sewage enters a digester that produces gas - which is flared, and biosolids - which are combined in windrow composting to be used in agriculture (DigComp);
4. Lime is added to the sewage for use in agriculture (LimeAg);
5. A thermal dryer produces biosolids sent to direct use in agriculture (DryAg);
6. A thermal dryer produces biosolids employed to fuel a cement kiln (Drykiln);
7. The sewage enters a digester that produces gas - which is partially flared and partially used to complement the use of fossil methane in a thermal dryer, where the treatment proceeds to produce biosolids with a final destination to agriculture (DigDryAg);
8. The sewage enters a digester that produces gas - which is partially flared and partially used to complement the use of fossil methane in a thermal dryer, where the treatment proceeds to produce biosolids which are employed to fuel a cement kiln (DigDryKiln).

The LCIA was based on the measurement of indicators of impact on the five dimensions of WU, TEC, CF, TETP, and HTP. Wu is measured in liters, TEC in megaJoules, CF in kg of CO2 equivalents and the toxicity impacts in Kg of 1.4-DCB equivalents.

Water is consumed mainly in the generation of electricity employed to power the dryer. Thus, WU is higher when more sophisticated technologies are applied. The water use of DryAg and DigDryAg is calculated in 2.000 liters approximately. If, besides the dryer, the cement kiln is employed, this value increases to approximately 3.000 liters. The use of water in the production of materials employed, including the quicklime in the LimeAg scenario, is also considered. Even so, LimeAg is the only alternative with negative net water consumption, of -1.366 liters.

TEC, taking into account several aspects, varies considerably across the set of alternatives. It is mainly related to transportation costs. However, exploitation of the energy generated is an important element in the impact evaluation of some scenarios. For this reason, by producing an alternative fuel, DryKiln is a net energy producer, with a total energy consumption of -173mJ. The next best performance with respect to energy use is Landfill, its low-technology process leading to a total of 2806mJ. On the other extreme, the total energy consumption of DryAg and DigDryAg are 28846mJ and 17236mJ, respectively.

CF is higher for scenarios without gas recovery. On the other hand, it is reduced when natural gas is harnessed in scenarios with a dryer. DryKiln presents the best CF performance, of -490kg CO_2-equivalent, followed by DigDryKiln (62kg CO_2-equivalent). There is a positive correlation, of 0.51, between TEC and CF, due to the importance of fossil sources in energy generation in

Australia. Landfill is an exception, with low energy consumption but with the highest carbon footprint, of 2.346kg CO_2-equivalent, due to the uncontrolled emissions of methane in landfill gas.

Finally, a high correlation between TETP and HTP, of 0.94, and of both with TEC and CF, varying from 0.55 for the correlation of HTP with TEC to 0.78 for the correlation of TETP with TEC, is observed. In fact, direct emissions and toxic inorganic chemicals and heavy metals emissions during the combustion of coal for generation of electric energy simultaneously raise carbon footprint and toxicity levels. DryKiln, with -10.7 and -9.5kg of 1.4-DCB equivalents, for TETP and HTP, and DigDryKiln, with -0.12 and -2.52 are the alternatives of negative potential, while the largest amounts are of DryAg, with 11.2 for TETP and 3.11 for HTP, and DigDryAg, with -0.12 and -2.52, respectively, in Peters and Rowley (2009) assessments.

Composition of Probabilistic Preferences

In this section, the aspects of CPP that inform the application to the case of interactive criteria studied are presented. A formal presentation of the main features of CPP is in Appendix A.

The derivation of probabilities of each alternative being the best according to each criterion enables the application of probabilistic composition rules. For instance, a pessimistic and progressive rule combines the evaluations of an alternative using the joint probability of it being the most preferred (for this reason, progressive) according to all the criteria (for this reason, pessimistic). On the other hand, an optimistic and progressive rule combines the evaluations by means of the joint probability of being the most preferred according to at least one of the criteria. A third point of view, pessimistic and conservative, derives the combined score from the probability of not minimizing the preference according to any criterion (this approach is conservative because it takes into account the ability of avoiding the extreme of bad evaluations instead of the ability to approach the extreme of good evaluations and is pessimistic because it asks for avoiding such frontier of worst performances according to all the criteria). Finally, a conservative and optimistic rule combines the evaluations by the joint probability of not minimizing the preferences according to all the criteria.

In the probabilistic composition, the differences between the rankings obtained by the optimistic and the pessimistic approach, after the progressive or the conservative frontier is chosen, are generally small. This may be explained by the proximity between the optimistic and pessimistic probability events.

On the other hand, progressive and conservative rankings present more differences, as proximity to the top and to the bottom may have different effects on human decisions (Vriend et al., 2016; Lount et al., 2017). An association of the progressive with the optimistic point of view and of the conservative with the pessimistic, derived from conservative motivations leading to risk aversion while progressive motivations lead to risk propensity (Gino & Margolis 2011), may reduce to two the number of compositions considered. Associating the optimistic point of view with the progressive and the pessimistic with the conservative has also the advantage of avoiding summing probabilities of very rare events with larger ones.

If the decision maker is able to choose an intermediate point, a compromise score may be used, given by a weighted average between the progressive and the conservative score. No preference by one or other approach would be represented by the use of the arithmetic mean.

Although the CPP method is not exclusive to LCA problems, the idea behind the composition of progressive-conservative and pessimistic-optimistic axes retains some similarities with Cultural Theory and its applications in LCIA. This theory, developed by Thompson et al. (1990), describes five ways of life that are viable combinations of cultural biases and social relations. Three of them have been used in LCA studies: Individualism, Hierarchism and Egalitarianism. Individualists consider the present much more valuable than the future; egalitarians think that a society must adjust its needs to limit the exposure of future generations, and then future is more important; hierarchs state that present and future are equally important. These perspectives have been used to model LCA problems by Hofstetter et al. (2000) and Wolfova et al. (2018). CPP and Cultural Theory share similarity in the way both theories approach the uncertainty present in human decision making. In the present CPPs type of composition, the performance evaluations under the different criteria may be thought subject to conflicts which influence the composition in a form that can be associated to that highlighted by Cultural Theory.

A second type of combination rule employs weighted averages of the individual scores (Fishburn, 1970; Keeney & Raiffa, 1993), which can, in the probabilistic context, be considered as the combination of probabilities of being preferred according to each criterion conditionally on the preference for such criterion. The weights are then the marginal probabilities of each criterion being chosen.

The probabilities of preference among criteria may be derived from the results of the evaluation of a set of alternatives by the criteria, by mechanisms governed by the application of certain principles. Two opposite principles may apply, the principle of preference concentration and the principle of preference dilution (Sant'Anna & Sant'Anna, 2019).

These principles focus on the highest probabilities of choice according to each criterion. The principle of preference concentration leads to seek the maximization of the ability to discriminate the most preferred alternative. This search for maximization of discrimination of the preferred alternative implies that a criterion or a set of criteria is more important for the decision maker the more it is able to point out, from the alternatives among which the choice is processed, one alternative as the preferred one. This corresponds to the belief that high values provide more reliable information about the decision priorities than values assigning lower priorities.

An opposite foundation would relate extreme preferences, instead of the presence of real preferences in the extreme values, to contributions of disturbances whose effect should be disregarded. This would then lead to the application of the opposite principle, of preference dilution, which will give a higher probability to those criteria without a strong preference for an alternative.

Confidence in the practical validity of multicriteria decision analysis favors the principle of preference concentration, but both principles may always be applied and the difference between the results of their application may inform on the sensitivity of the decision to the relative importance assigned to the criteria.

Assuming additivity, the application of the principle of preference concentration leads to assign to each criterion a probability proportional to the maximum of the probabilities of preference that it associates with the evaluated alternatives. If additivity is not assumed, it leads to use the increase in the maximum of the probabilities of preference when one of two criteria is allowed to be applied instead of only one of them to measure the interaction between preferences according to the two criteria.

To take into account interaction in the application of the criteria the multiple preferences are combined through a Choquet integral with respect to a capacity. An essential condition to make possible the use of a capacity in the combination of evaluations by multiple criteria is the commensurability of the criteria. All criteria must employ the same standard in the evaluation of alternatives. This condition is granted by the above highlighted transformation into preference probabilities in the preliminary stage of CPP.

To determine the capacity, we may rely on opinion directly elicited by the decision maker (Grabisch et al, 2008). Directly extracting preferences between sets of criteria of the decision maker is a difficult task, even harder than discovering a probability distribution for the criteria, because the number of sets to be evaluated exponentially increases with the number of criteria. Instead, we may extract the capacity of each set of criteria, indirectly, from an observed distribution of preference among alternatives, in an unsupervised approach. Kojadinovic (2004) and Rowley et al. (2015) were the first to take this route.

The approach here taken differs from that followed by Kojadinovic (2004), which measures the interaction by the information, in the sense of Shannon (1948) or Rényi (1960), contained in the distribution of preferences. The capacity of a set of criteria is estimated by Kojadinovic (2004) by means of the relation between the entropy in the matrix of the assessments of preference according to the criteria in the set and the entropy in the matrix of all the available assessments of preference. This makes the capacities larger as the distribution of the evaluations according to the criteria in the set spreads more uniformly across all the alternatives. Especially, in the case of unitary sets, the capacity of a single criterion increases with the uniformity in the assessments of preference according to the criterion. This is, in a sense, coherent with the application of the principle of preference dilution, even though the probabilistic approach always tries to identify interaction by focusing on the assessments of extreme preference, disregarding other similarities. The principle of preference concentration, even more differently, leads to assign a larger capacity to those criteria with a higher ability to discriminate the best alternative.

The probabilistic approach differs also from the approach of Rowley et al. (2015), which makes the capacity of the sets of criteria grow with the divergence measured in terms of lack of correlation. This lack of correlation is evaluated by a function of the eigenvalues, in a principal component analysis, of the matrix of correlations between the vectors of preference ratings observed. By this way, the different ability of different criteria to individually contribute to the decision is not taken into account when assigning the capacity to unitary sets, and the capacity of sets of more than one criterion decreases with their concordance. Besides, this concordance is evaluated in terms of correlation between whole vectors of preference, while, in opposition, the probabilistic approach centers attention on the extremes of preference. Rowley et al. (2015) apply this correlation approach to the data of Peters and Rowley (2011).The present analysis of the same data set compares the approach based on CPP with that.

A key feature of CPP is the transformation of the numerical vector of evaluations of the different alternatives according to each criterion into a vector of probabilities of each alternative being the most preferred one and a vector of probabilities of being the least preferred one. In the basic CPP framework, these probabilities are determined under simple hypotheses of independence and triangular distributions. The robustness of these assumptions is by now demonstrated by the success of applications in different contexts in (Sant'Anna, 2012; Sant'Anna et al., 2014; Garcia & Sant'Anna, 2015; Gavião et al., 2017, 2020b; Sant'Anna & Barreto, 2020), for instance.

For the data here studied these probabilities, calculated applying R (R-Core Team, 2018; Gavião et al., 2018), are shown in Tables 1 and 2.

Table 1 – Probabilities of being the best according to the Five Criteria.

	WU	TEC	CF	TETP	HTP
Landfill	0.11	0.18	0.03	0.10	0.08
DigAg	0.12	0.15	0.12	0.09	0.07
DigComp	0.11	0.12	0.11	0.08	0.07
LimeAg	0.43	0.11	0.10	0.07	0.07
DryAg	0.06	0.02	0.04	0.05	0.06
DryKiln	0.05	0.27	0.35	0.44	0.47
DigDryAg	0.06	0.04	0.07	0.06	0.06
DigDryKiln	0.05	0.13	0.18	0.12	0.12

Table 2 – Probabilities of being the worst according to the Five Criteria.

	WU	TEC	CF	TETP	HTP
Landfill	0.05	0.06	0.39	0.07	0.10
DigAg	0.05	0.07	0.06	0.08	0.14
DigComp	0.05	0.07	0.07	0.08	0.14
LimeAg	0.03	0.07	0.07	0.13	0.13
DryAg	0.13	0.45	0.22	0.31	0.23
DryKiln	0.29	0.06	0.04	0.03	0.02
DigDryAg	0.13	0.15	0.10	0.24	0.20
DigDryKiln	0.28	0.07	0.05	0.06	0.04

Consider, for instance, the value 0.43 for LimeAg with respect to WU in Table 1. It represents the probability of a system based on this technological approach being preferred when compared to the other seven by the criterion of water use. This probability of preference is much higher than the second highest probability of preference with respect to this criterion, of 0.12 for DigAg. On the other hand, in the last column, the highest probability of maximizing preference, if HTP is applied separately, is 0.47 for DryKiln followed by a distant 0.12 for DigDryKiln. Similar probability distributions are observed in the other columns of Table 1. Thus, Table 1 shows how the probabilistic transformation evidences clear preference for LimeAg only by the WU criterion, and for DryKiln by the other four criteria.

In the probabilities of being the worst, in Table 2, the only high prevalence is of DryAg with respect to TEC, with a probability of 0.45 of being worse than all the others if evaluated by energy consumption isolated. The second highest probability of being worse than all the others by this criterion is 015 for DigDryAg. For the other criteria the probabilities of being the worst are closer to each other.

RESULTS

The evaluations of the eight scenarios resulting from combining with the optimistic progressive and pessimistic conservative rules, and with linear weights derived from the application of the principles of preference concentration and of preference dilution, are shown in Table 3.

Table 3 – Results of Composition without Interaction.

	Progressive	**Conservative**	**Concentration**	**Dilution**
Landfill	0.41	0.46	0.09	0.10
DigAg	0.44	0.66	0.10	0.11
DigComp	0.41	0.65	0.10	0.10
LimeAg	0.60	0.64	0.16	0.15
DryAg	0.22	0.20	0.05	0.05
DryKiln	0.87	0.61	0.32	0.32
DigDryAg	0.25	0.40	0.06	0.06
DigDryKiln	0.47	0.58	0.12	0.12

Table 3 evidences the high global preference for DryKiln. Whether concentration maximization or dilution maximization is applied in the assignment of weights, the score of this alternative, of 0.32, is more than twice the score of the second best alternative, LimeAg. The score of DryKiln is also much higher than any other if the progressive composition is applied.

The conservative approach, which decides on the basis of the probability of avoiding being the worst alternative by at least one of the criteria, is not a natural approach in a search for improvement. It would lead to the choice of DigAg, but with four other alternatives (DigComp, LimeAg, DryKiln and DigDryKiln) with scores close to the highest. A linear composition of the progressive and the conservative scores would lead to the choice of DryKiln even if a very low weight is assigned to the progressive component. At the other end, DryAg is the worst alternative, for whichever composition rule chosen.

These conclusions agree with those obtained by Peters & Rowley (2009) applying average ranks.

The possibility of interaction may turn linearity too restrictive. The combination of the criteria can then use, instead of weighted average, a Choquet integral, with the importance of any set of criteria given by its capacity (Choquet, 1953; Grabish et al., 2008) and the importance of interaction evaluated by Shapley indices (Shapley 1953).

In the evaluation of an alternative by the Choquet integral, the weight of the criterion according to which the alternative has its highest preference assessment is equal to its individual capacity, while the other criteria have their coefficients increased or decreased according to its higher or lower interaction with those assigning higher preference to the alternative. Thus, alternatives with divergent evaluations by criteria with positive interaction have their evaluation by the Choquet integral reduced and those with concordant high evaluations by such criteria have their evaluation increased. On the other hand, the Choquet integral reduces the influence of preferences of small value by criteria with negative interactions with the others.

In CPP, the capacity can be derived from the results of the evaluation of competing alternatives by the criteria in the same form as the probabilistic marginal weights are obtained in the case of assuming no interaction. The principles of preference concentration and of preference dilution are again applied to drive such derivation.

The capacity generated by applying preference concentration is shown in Table 4. The importance of the negative interaction between WU and the other criteria is reflected in the lower values, in the respective columns, of the capacities of the sets of two, three and four criteria that include WU.

Table 4 – Capacity reflecting Preference Concentration for the Five Criteria.

Singleton		**2 criteria**		**3 criteria**		**4 criteria**	
WU	0.49	WU-TEC	0.56	WU-TEC-CF	0.63	WU-TEC-CF-TETP	0.86
TEC	0.31	WU-CF	0.56	WU-TEC-TETP	0.70	WU-TEC-CF-HTP	0.88
CF	0.41	WU-TETP	0.54	WU-TEC-HTP	0.73	WU-TEC-TETP-HTP	0.92
TETP	0.50	WU-HTP	0.58	WU-CF-TETP	0.75	WU-CF-TETP-HTP	0.94
HTP	0.55	TEC-CF	0.61	WU-CF-HTP	0.78	TEC-CF-TETP-HTP	0.99
		TEC-TETP	0.68	WU-TETP-HTP	0.83		
		TEC-HTP	0.71	TEC-CF-TETP	0.85		
		CF-TETP	0.73	TEC-CF-HTP	0.87		
		CF-HTP	0.76	TEC-TETP-HTP	0.90		
		TETP-HTP	0.81	CF-TETP-HTP	0.93		

Other capacities may be employed. To generate conservative scores to be combined with the progressive ones, a capacity may be derived from the probabilities of being the worst alternative. Capacities may be also built applying the principle of preference dilution. The use of all these capacities may be compared to the use of the capacity derived by Rowley et al. (2015), which grows with the inner absence of correlation in the set.

The Shapley value of a criterion averages its marginal contributions to the capacity of the sets it belongs. The Shapley values for these five capacities are different. The highest Shapley value in the capacity derived from the probabilities of maximization of preferences applying the principle of preference concentration is 0.28 for HTP. From probabilities of minimization, such maximum is 0.34 for TEC. The approach of Rowley et al. (2015) leads to the highest Shapley value, of 0.29, belonging to WU. The application of the dilution principle leads to Shapley values around 0.21 for all the criteria.

Even though it is expected that the application of the principle of preference concentration to the probabilities of maximizing preference assigns to each set of criteria a capacity that captures the effective interactions, the computation of the preferences employing the capacities derived from other approaches informs about possible deviations. The prevalence of different criteria in the capacities derived by the different approaches considered brings robustness to the analysis.

The results of the composition by the Choquet integral with respect to the capacities derived from the probabilistic approach are shown in Table 5. The first column presents the results of the application of the progressive point of view with interaction derived from the principle of preference concentration. In the second, dilution substitutes concentration. The two last columns present the results of application of the conservative approach.

Table 5 – Results of Compositions assuming Interaction.

	ProgConc	ProgDil	ConsConc	ConsDil
Landfill	0.12	0.17	0.78	0.63
DigAg	0.11	0.14	0.91	0.86
DigComp	0.10	0.12	0.91	0.86
LimeAg	0.25	0.41	0.90	0.87
DryAg	0.06	0.06	0.63	0.55
DryKiln	0.43	0.47	0.87	0.72
DigDryAg	0.06	0.07	0.81	0.76
DigDryKiln	0.15	0.18	0.86	0.72

Table 5 shows that similar results were obtained whether preference concentration or preference dilution is applied to determine the capacities. The progressive points of view confirm the results obtained without interaction being taken into account, with DryKiln as the best alternative, followed by LimeAg.

The application of the conservative points of view leads again to a large number of options close to the best. This difference confirms the ability of the probabilistic approach to capture the distinction between the two objectives of maximizing and of not minimizing preferences. The linear combinations of the progressive and conservative scores will result in choosing DryKiln even if a very small weight is assigned to the progressive composition. On the other hand, applying the capacity derived from the absence of correlations (Rowley et al., 2015), LimeAg becomes the best alternative.

DISCUSSION

Table 6 summarizes the analysis, presenting the rankings obtained by application of the different composition rules considered. There, the block of the first seven rows starts with the average ranks of Peters & Rowley (2009). They are followed by the probabilistic ranks resulting from combination of the optimistic progressive scores with the pessimistic conservative, with weights 0.8 and 0.2 and conversely. Follow those the ranks derived from linear combination with weights obtained by preference concentration and preference dilution applied to the probabilities of being the best and of being the worst, again with weights 0.8 and 0.2, and conversely.

The lower block, with five rows, presents the results obtained modeling interactions. It starts with the ranking obtained by Rowley et al. (2015). The probabilistic ranks are obtained employing first preference concentration and then preference dilution in the estimation of the capacities and giving weights of 0.8 to the scores obtained combining the probabilities of being the best and 0.2 to the scores obtained combining the probabilities of being the worst and conversely.

Table 6 – Ranks by Different Forms of Composition.

	Landfill	DigAg	DigComp	LimeAg	DryAg	DryKiln	DigDryAg	DigDryKiln
Average rank	4	2 ½	6	5	8	1	7	2 ½
0.8OptProg	6	3	5	2	8	1	7	4
0.2OptProg	6	3	4	2	8	1	7	5
0.8Max Conc	6	4	5	2	8	1	7	3
0.2Max Conc	6	3	4	2	8	1	7	5
0.8Max Dilut	6	4	5	2	8	1	7	3
0.2Max Dilut	6	3	4	2	8	1	7	5
Correlation	6	3	4	1	8	2	7	5
0.8Max Conc	6	4	5	2	8	1	7	3
0.2Max Conc	7	3	4	2	8	1	6	5
0.8Max Dilut	6	3	5	2	8	1	7	4
0.2Max Dilut	7	2	3	1	8	4	5	6

It is noticeable in Table 6, first, the strong agreement in the ranks. Even raising the importance of the proximity to the frontier of worst performance to 0.8 against 0.2 for the proximity to the frontier of best performance rarely produces a change.

This concordance in the probabilistic ranks demonstrates the robustness of the methodology. Taking into account interaction in such a way to considerably vary the Shapley values does not lead to important effects in the final decision. This may be attributed to the homogeneity granted by the initial transformation into probabilities of being the best.

It can also be seen that the different probabilistic points of view generally agree with the choice of DryKiln as the best alternative. Lime Ag is chosen only if preference dilution is applied and the high weight of 0.8 is given to the scores derived from probabilities of not being the worst.

CONCLUSION

The main finding of this work is that probabilistic comparison methods based on the separate evaluation by five impact factors: water use, total energy consumption, carbon footprint, human toxicity potential and terrestrial ecotoxicity provide a robust framework for the evaluation of sewage management systems of small towns in an LCA setup.

The probabilistic evaluation of preferences and the comparison methods based on probabilistic scores employed ensured robustness by avoiding the influence of previous assumptions on the importance of the criteria and on relations between them. The homogenization of the preference assessments granted by the probabilistic transformation was also shown to be important in eliminating the indirect influence of the scales of measurement.

The composition by the Choquet integral with respect to capacities determined by probabilistic unsupervised algorithms highlights those options preferred by the criteria considered, avoiding interfering with the modeling of the problem. Besides, it propitiates a better understanding of the relative importance of each criterion for setting the preferences, both by the capacity allocated to the criteria isolated and by their Shapley values.

The principle of preference concentration, relating interaction to formation of joint preference for the alternatives, leads to Shapley values coherent with the isolate capacity of each criterion. This results in confirming the ranking obtained by the progressive probabilistic composition without interaction, which agree with those obtained in the initial assessment.

On the other hand, the application of the principle of preference dilution leads to equalize the Shapley values for all the criteria. Similar final results are then obtained.

From the point of view of improving sewage sludge systems management, it can be also noticed that CPP supplies new insights into the environmental impact of the possible combinations of treatment processes and end-uses studied.

A key issue in improving the environmental profile of sewage sludge management is the contribution of the system employed to reduce the use of fossile-sourced electrical energy. Different composition approaches taken agree in preferring a scenario that, besides reducing the carbon intensity of the energy sources used, reduces the toxicity of emission by-products.

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APPENDIX A. CPP COMPUTATION RULES

CPP is a methodology that takes into account, in the composition of multiple criteria, the probabilistic character present in the evaluation of preferences. This probabilistic character can result, for example, from imprecision caused by subjective factors, which lead decision makers to attribute different meanings to the same attributes of the alternatives in different circumstances, or by measurement errors that affect the evaluations of such attributes.

A preliminary stage of CPP is the mathematical transformation of exact measurements into random variables followed by the comparison of these random variables in terms of probabilities of presenting the highest values for the individual preferences according to each criterion. To present this step formally, let ($a_{1j}, \dots, a_{nj}$) denote a vector of numerical evaluations of n alternatives, $A_1, \dots, A_n$, by a criterion C_j of a set of criteria C and X_{kj} denote a random variable with the distribution of preference for alternative A_k according to criterion C_j. For any k, a_{kj} will be used as a location parameter for the distribution of X_{kj}. The probability of alternative A_i being the best according to criterion C_j is given by the integral, for x varying on the domain of X_{ij}, of $P[X_{kj}<x$, for all $k \neq i]$ with respect to the density of X_{ij}. Denoting by f_i this density of the distribution of the evaluation of alternative A_i by criterion C_j and by F_{-i} the cumulative distribution function of the joint evaluations by criterion C_j of the n-1 remaining alternatives with which the i-th is compared, this probability of alternative A_i being the most preferred by criterion C_j is given by

$$M_{ij} = \int F_{-i}(x) f_i(x)\, dx \tag{1}$$

Analogously, on the other extreme, the probability of being the least preferred is given by

$$m_{ij} = \int (1 - F_{-i}(x))\, f_i(x)\, dx \tag{2}$$

In the absence of contribution of the decision maker about the form of the statistical distributions, simplicity recommends the triangular distribution. The mode of the distribution is given by the numerical evaluation obtained for the alternative. Besides simplicity, the triangular distribution has the convenience of enabling asymmetrically dealing with the dispersion. Dispersion in the triangular model may be determined by placing the ends at the minimum and maximum effectively or potentially observed in the evaluations resulting from the application of the criterion to all alternatives presently compared. This corresponds to the idea that high measurements are more probably related to positive disturbances, and low measurements to negative disturbances.

There are basically two types of combination rules in CPP. The simplest type employs joint probabilities of events determined by proper points of view. To determine these points of view, first, a choice may be called between a progressive motivation to pursue good outcomes and a conservative motivation to avoid bad outcomes. A second possible choice is between the optimistic motivation to concentrate efforts in one only dimension, risking failure in any other, and the pessimistic motivation to equally tackle all risk dimensions.

If there is statistical dependence between two or more such variables and it can be reliably evaluated, it is natural to take it into account when computing the probability of events involving them.

In general, however, to overcome the practical barriers of the process of estimating dependence parameters, the joint probabilities of preference are calculated under the assumption of independence. An assumption of maximal dependence will be easier to apply but, while independence leads to the calculation of the probabilities of intersections by the product of the probabilities of the intersected events, assuming maximal dependence leads to determine these probabilities of intersections of events by the minimum of their probabilities, so disregarding the other values. Thus, independence is, of these two assumptions, the one that better explores the information gathered.

Thus the best strategy is computation under independence, associated with an effort to guarantee the independent generation of the evaluations of the different alternatives by the different criteria. The calculation under the assumption of maximal dependence may be done additionally, for comparison purposes, to analyze the sensitivity of the results to a possible violation of the independence assumption.

A strong argument in favor of the assumption of independence in the evaluation by multiple criteria comes from the source of randomness, which stems from measurement errors or from subjectivity in the assignment of preferences. These factors generally apply separately to modify each evaluation, and not on blocks. Even if the criteria are based on objective attributes of the alternatives, the presence of these two factors reduces the volume of the possible dependence.

Under the hypothesis of independence, the final scores under these four composite points of view, with $\prod$ denoting the product operator, are:

$$\text{Optimistic and conservative: } OC_i = 1 - \prod_j m_{ij} \tag{3}$$

$$\text{Optimistic and progressive: } OP_i = 1 - \prod_j (1 - M_{ij}) \tag{4}$$

$$\text{Pessimistic and conservative: } PC_i = 1 - \prod_j (1 - m_{ij}) \tag{5}$$

$$\text{Pessimistic and progressive:} PP_i = 1 - \prod_j (1 - M_{ij}) \tag{6}$$

Assuming maximal dependence, the change in these formulae consists in substituting for the product the minimum along i.

The second type of combination rule leads to the classical linear combination of the individual scores, which can, in the probabilistic context, be considered as the combination of probabilities of being preferred according to each criterion conditionally on the preference for such criterion. The weights of the linear combination are then the probabilities of each criterion being chosen.

Rules to assign weights to the criteria, following the principle of preference concentration, proportional to the maximum of the preferences along the set of alternatives, and, following the principle of preference dilution, proportional to the maximum of the complements of such probabilities lead, respectively, to the following weights for criterion j:

$$w_c(j) = \frac{max_i M_{ij}}{\sum_i M_{ij}} \tag{7}$$

and

$$w_d(j) = \frac{max_i\,(1-M_{ij})}{\sum_i\,(1-M_{ij})} \tag{8}$$

To take into account interaction in the application of the criteria we may combine the multiple preferences by a Choquet integral with respect to a capacity.

The principle of preference concentration leads to use the increase in the maximum of the probabilities of preference, when any of the two criteria is allowed to be applied instead of only one of them, to measure the presence of interaction between preferences according to the two criteria.

Extending this rule to two sets of any number of criteria, the comparison will then be between the maximal preference according to each set separately and considering the two sets together. The effect of interaction will then be accessed by measuring the difference between the probability of maximization of preference by at least one element of the union of the two sets combined and when only one of the sets is considered.

This increase in the probability of maximum preference does not precisely access the conflict between the positive interaction that results in the confirmation of the preferred alternative by the two sets and the negative interaction that brings an alternative with divergent evaluations to be the jointly preferred alternative. A correction for this unbalance in the determination of the capacity is done dividing, for every set of criteria, the maximum registered in the preceding step by the maximum for the set of all the criteria. This last operation brings the range of the function to [0,1]. And, by the additivity of the concept of probability, it is nondecreasing.

The following algorithm implements this idea. First, determine, for each i, from 1 to n, and any set of criteria $\{C_{j_1},\ldots,C_{j_s}\}$,

$$PC_i\left(\{C_{j_1},\ldots,C_{j_s}\}\right) = 1 - \prod_{k\in\{1,\ldots,s\}} \left(1-M_{ijk}\right) \tag{9}$$

the probability of the random variable representing the evaluation of alternative A_i presenting the highest evaluation for at least one of the s criteria in $\{C_{j_1},\ldots,C_{j_s}\}$.

Then, compute

$$PC\left(\{C_{j_1},\ldots,C_{j_s}\}\right) = max_{i\in\{1,\ldots,n\}} PC_i\left(\{C_{j_1},\ldots,C_{j_s}\}\right) \tag{10}$$

The capacity of $\{C_{j_1},\ldots,C_{j_s}\}$ will be

$$CC\left(\{C_{j_1},\ldots,C_{j_s}\}\right) = \frac{PC\left(\{C_{j_1},\ldots,C_{j_s}\}\right)}{PC(C)} \tag{11}$$

It is interesting to contrast this with different constructions. For instance, instead of preference concentration, preference dilution might drive the criteria evaluation. Then, instead of CC a capacity CD, assigning to any set $\left\{C_{j_1},\ldots,C_{j_s}\right\}$ a capacity proportional to the maximum of the probabilities of not presenting the highest evaluation by at least one of the criteria in the set, would be built by an algorithm employing the same maximization and standardization steps, with equations (9), (10) and (11) replaced by

$$PD_i = 1 - \prod_{k\in\{1,\ldots,s\}} M_{ijk} \tag{12}$$

$$PD\left(\left\{C_{j_1},\ldots,C_{j_s}\right\}\right) = max_{i\in\{1,\ldots,n\}} PD_i\left(\left\{C_{j_1},\ldots,C_{j_s}\right\}\right) \tag{13}$$

and

$$CD\left(\left\{C_{j_1},\ldots,C_{j_s}\right\}\right) = \frac{PD\left(\left\{C_{j_1},\ldots,C_{j_s}\right\}\right)}{PD(C)} \tag{14}$$

A reverse framework may also be built from a conservative point of view, by using minimization instead of maximization probabilities.