

A BI-OBJECTIVE MULTIPERIOD ONE-DIMENSIONAL CUTTING STOCK PROBLEM

Livia Maria Pierini¹ and Kelly Cristina Poldi^{2*}

Received November 22, 2021 / Accepted April 19, 2022

ABSTRACT. In this paper, we investigate the bi-objective multiperiod one-dimensional cutting stock problem that seeks to minimize the cost of production associated with the total length of cut objects (waste) and the inventory costs related to objects and items. A mathematical model is presented and heuristically solved by a column generation method. Computational tests were performed using the Weighted Sum method, the ε -Constraint method and a variation of the Benson method. The Pearson correlation coefficient was calculated in order to investigate the trade-off between the conflicting objectives of the problem. The results confirmed a strong negative correlation between the objective functions of the problem. All the applied scalar methods were able to find multiple efficient solutions for the problem in a reasonable computational time; however, the ε -Constraint and the modified Benson methods performed better.

Keywords: cutting stock problem, bi-objective optimization, ε -constraint method, weighted sum method, Benson method.

1 INTRODUCTION

In many manufacturing industries, such as paper, textile and furniture, large objects are cut into smaller units to meet a given demand. In the optimization of these processes, there is the Cutting Stock Problem (CSP). The CSP aims to determine how larger objects must be cut into smaller items in order to meet the demanded items and satisfy some optimization criteria, such as minimizing material waste, the number of cut objects or the number of cutting patterns.

The optimization of the cutting process is conditioned on the production of objects and their availability to be cut. Therefore, the CSP can be considered as a fundamental subproblem of the

*Corresponding author

¹ Universidade Estadual de Campinas (UNICAMP), Instituto de Matemática, Estatística e Computação Científica (IMECC), Rua Sergio Buarque de Holanda, 651, 13083-859, Campinas, SP, Brazil – E-mail: liviam.pierini@gmail.com – <http://orcid.org/0000-0002-1469-7596>

² Universidade Estadual de Campinas (UNICAMP), Instituto de Matemática, Estatística e Computação Científica (IMECC), Rua Sergio Buarque de Holanda, 651, 13083-859, Campinas, SP, Brazil – E-mail: poldi@unicamp.br – <http://orcid.org/0000-0002-1649-6843>

Lot Sizing Problem (LSP), which consists of determining the number of objects to be produced and how this production must occur optimizing production costs and meeting demanded objects.

One attribute that can be investigated in both problems is the multi-objective feature. The multi-objective approach delineates the preference relationship in optimization. It makes available alternative solutions that enable a more informed, comprehensible and safe choice by the decision-maker. In the literature, there are many papers that deal with conflicting objectives in the LSP and CSP, such as Rezaei & Davoodi (2011), Romeijn et al. (2014), Mehdizadeh et al. (2016), Kolen & Spieksma (2000), Lopes (2009), Araujo et al. (2014), Aliano Filho et al. (2018). However, there is a lack of research in the literature that explores the Bi-objective Multiperiod Cutting Stock Problem (BMCSPP) that aims to minimize material waste and storage costs.

Among the studies that investigated the bi-objective CSP, there is Kolen & Spieksma (2000), which explored the problem that seeks to minimize the waste in the cutting process and the number of different cutting patterns and proposed an exact solution method applicable for instances with just a few items. Lopes (2009) studied the one-dimensional CSP that aims to minimize the number of cut objects and the number of different cutting patterns by proposing three adaptations of a heuristic method based on the concepts of multi-objective evolutionary algorithms. Araujo et al. (2014) addressed the same problem and proposed a resolution method based on genetic algorithms.

Aliano Filho et al. (2018) applied seven distinct scalarization techniques to the one-dimensional CSP in order to minimize the sum of the frequencies of the cutting patterns and the number of different cutting patterns to be used. Recently, Aliano Filho et al. (2021) proposed a scalarization method to solve bi-objective integer linear optimization problems and studied other three solution methods by proposing extensions and adaptations. Among these techniques, there is the Benson method, a scalarization technique which has been modified to improve its performance.

The bi-objective CSP has two conflicting objectives, and so there is no single solution that optimizes both simultaneously. In this case, the solution of the problem is given by a set of solutions, which are called efficient solutions, in which one objective cannot be improved without harming the other. The image of the efficient solutions set portrays a curve known as Pareto Front, see Branke et al. (2008). In order to find the set of efficient solutions, scalar strategies can be used, in which the multi-objective problem is transformed into a scalar problem that, when optimized, generates an efficient solution. Two widely used scalarization techniques in this process are known as Weighted Sum and ϵ -Constraint methods (Aliano Filho, 2016).

The bi-objective problem considered by Aliano Filho et al. (2018) aims to minimize the frequency of cutting patterns to meet the demand for items and the number of different cutting patterns to be used. Diversely, in this paper, we study the BMCSPP that seeks to minimize the production costs associated with the cut objects and the inventory costs of objects and items. The objective of this research is to investigate the trade-off between the different objectives of the BMCSPP by applying the Weighted Sum, the ϵ -Constraint and the Benson methods. Although, the BMCSPP has already been studied in the literature, the trade-off between the conflicting objec-

tives considered in this paper has not been investigated before. Given the importance of finding in practice alternative solutions for the BMCSP in a reasonable time, it is also analyzed the number of efficient solution found and the computational time spent by each method.

The present paper is organized as follows: Section 2 introduces the studied CSP. Section 3 describes some concepts of the bi-objective problem. The Weighted Sum, the ε -Constraint and the Benson methods are described in Section 4 as well as the Column Generation method. The computational tests and the adopted methodology to solve the problems are discussed in Section 6, and then the performed computational tests are reported in Section 7. Finally, in Section 8 some final considerations are presented.

2 FORMULATION

The CSP consists of optimizing the process of cutting larger objects into a smaller set of items in order to meet the demand and satisfy some optimization criteria such as minimizing the total cost of cut objects, the number of used cutting patterns or the inventory costs. The different ways in which larger objects can be cut are called cutting patterns.

By seeking to minimize waste and improve the cutting process, the studies involving CSP have great value for the industry and have attracted the attention of several researchers. Among these papers addressing the CSP are Gilmore & Gomory (1961), Haessler (1975), Dyckhoff (1990), Vanderbeck (2000), Wang & Wäscher (2002), Umetani et al. (2003), Oliveira & Wäscher (2007), among others.

The CSP can be classified in different ways. For more details, Wäscher et al. (2007), Morabito et al. (2009), Song & Bennell (2014), Gomes et al. (2013) and Delorme et al. (2016) are recommended. In this paper, we study the one-dimensional multiperiod CSP, that is, we consider only one dimension of the object that will be cut and multiple periods in the planning horizon. However, the studied formulation also addresses the cutting of objects with more than one dimension, which is determined according to the way that the cutting patterns are generated. Consider the following notation:

Indices:

i : item type;

t : period;

m : object type;

j : cutting pattern.

Parameters:

N : total number of item types;

T : total number of periods;

M : total number of object types;

N_m : total number of cutting patterns for the object type m ;

c_{mt} : production cost per centimeter of an object type m in period t ;

L_m : length (cm) of object of type m ;

g_t : inventory cost for each object at the end of period t ;

h_{it} : inventory cost for final item type i at the end of period t ;

a_{ijm} : number of items type i cut according to cutting pattern j from the object of length L_m ;

d_{it} : demand of final item type i in period t ;

E_{mt} : number of objects of type m available in period t .

Decision variables:

y_{mt}^j : number of objects type m in period t which are cut according to cutting pattern j ;

s_{it} : number of final items type i held at the end of period t ;

w_{mt} : number of objects type m stored at the end of period t .

Thus, a mathematical model for the BMCSP can be written as:

$$\min \left(f_1 = \sum_{m=1}^M \sum_{t=1}^T \sum_{j=1}^{N_m} c_{mt} L_m y_{mt}^j, f_2 = \sum_{i=1}^N \sum_{t=1}^T h_{it} s_{it} + \sum_{m=1}^M \sum_{t=1}^T g_t w_{mt} \right) \quad (1)$$

$$\text{s.t.:} \quad \sum_{m=1}^M \sum_{j=1}^{N_m} a_{ijm} y_{mt}^j - s_{it} + s_{i,t-1} = d_{it}, \quad i = 1, \dots, N; \quad t = 1, \dots, T \quad (2)$$

$$\sum_{j=1}^{N_m} y_{mt}^j + w_{mt} - w_{m,t-1} = E_{mt}, \quad m = 1, \dots, M; \quad t = 1, \dots, T \quad (3)$$

$$w_{m0} = 0, s_{iT} = 0, s_{i0} = 0, \quad m = 1, \dots, M; \quad i = 1, \dots, N \quad (4)$$

$$y_{mt}^j, w_{mt}, s_{it} \in \mathbb{Z}_+, \quad i = 1, \dots, N; \quad m = 1, \dots, M; \quad t = 1, \dots, T. \quad (5)$$

The BMCSP model (1)-(5) takes into account the production cost of the cut objects (f_1) and the inventory cost of objects and items (f_2). The constraints (2) are the item inventory balancing constraints, i.e., they describe that, in each period and for each item, the total amount of each cut item less the stock of items in the current period plus the number of stored items from the previous period must be equal to the demand of items. The constraints (3) are the object inventory balancing constraints they ensure that the total amount of each cut objects in each period plus the stock of the previous period less the stock of the current period must be equal to the available amount of objects. Without losing generality, the constraints (4) state that initial object inventory

and initial and final item inventory are null. The constraints (5) refer to the non-negativity and integrality of the decision variables.

Observe that the number of available objects, E_{mt} , is given as a parameter in the model. This choice was made in order to preserve the CSP characteristics of the problem since, if E_{mt} were considered as variable, the model would become an integrated LSP and CSP model. Therefore, the model (1)-(5) addresses practical situations in which the production of the standard objects and cutting optimization processes are independently considered. For a multi-objective approach regarding the integrated problem, Campello et al. (2020) is recommended.

Note that the CSP consists of determining the best way to cut the M types of objects available from different cutting patterns. Thus, for the resolution of the BMCSP, it is necessary to define the cutting patterns previously. In view of the difficulties encountered in solving the CSP due to the presence of integrality constraints and a large number of variables associated with the number of cutting patterns, we considered the model (1)-(5) with the relaxed integrality constraints and applied the Column Generation method to solve the model.

3 THE BI-OBJECTIVE PROBLEM

In order to simplify the notation, let us consider the Bi-objective Optimization Problem (BOP) as follows:

$$\min f(x) = (f_1(x), f_2(x)) \quad (6)$$

$$\text{s.t.: } x \in X, \quad (7)$$

where $f_1(x)$ and $f_2(x)$ are the objective functions to be minimized, x is the problem variable and X is the set of feasible solutions. Following, we introduce concepts of multi-objective problems based on Ehrgott (2005).

Definition 1 (Pareto optimal solution) A feasible solution x^* is said to be an efficient solution or Pareto optimal solution of the BOP if there does not exist another solution $x \in X$ such that $f(x) \leq f(x^*)$.

Definition 2 (Ideal vector) The vector $f^- = (f_1^-, f_2^-)$ is the ideal vector of the BOP if its i^{th} component is $f_i^- = \min\{f_i(x) \mid x \in X\}$ for $i = 1, 2$.

Definition 3 (Nadir vector) The vector $f^+ = (f_1^+, f_2^+)$ is the nadir vector of the BOP if its i^{th} component is $f_i^+ = \max\{f_i(x) \mid x \in X\}$ for $i = 1, 2$. When the problem is bi-objective, the nadir solution of $f_1(x)$ is the ideal solution of $f_2(x)$ and vice versa.

4 SOLUTION METHODS

4.1 The Weighted Sum method

The Weighted Sum method scales the set of objective functions of the original multi-objective problem and converts it into a single weighted objective. The method considers a convex combination of each objective function that, when varied, allows the generation of different efficient solutions.

It is important to measure the weights for each objective because the more (less) important that criterion in the problem is, the higher (lower) the value of the weight associated with that objective should be. The assignment of these weights requires an additional task so that the magnitude of each objective function does not affect the generation of efficient solutions. If the difference in the order of magnitude of these objective functions is very large, it is necessary to normalize them. One way to normalize $f_i(x)$ is by determining:

$$\bar{f}_i(x) = \frac{f_i(x) - f_i^-}{f_i^+ - f_i^-},$$

where $\bar{f}_i(x)$ is the i^{th} normalized objective function, f_i^- is the ideal solution of f_i and f_i^+ is the nadir solution. In this way, $\bar{f}_i(x)$ will assume values between 0 and 1.

Considering the problem (6)-(7), in which two objectives must be minimized, with the normalization of the objective functions, the resulting problem of the Weighted Sum method can be described as:

$$\min \quad \alpha_1 \bar{f}_1(x) + \alpha_2 \bar{f}_2(x) \quad (8)$$

$$\text{s.t.:} \quad x \in X, \quad (9)$$

where $\alpha_1, \alpha_2 \geq 0$ and $\alpha_1 + \alpha_2 = 1$. The solution of the scalar problem (8)-(9) is efficient, and the Weighted Sum method is able to find all the efficient solutions for convex problems if α is suitably varied. According to Aliano Filho et al. (2018), there are no clear rules of how to make this variation, nor the uniqueness of this vector weight for each efficient solution found.

4.2 The ε -Constraint method

Proposed by Haimes et al. (1971), this method scales a Multiobjective Optimization Problem (MOP) by taking the objective function with only one objective and restricting the others with upper bounds. As these upper bounds are varied, efficient solutions can be obtained. Consider the problem given by (6)-(7). The constrained problem is given by:

$$\min \quad f_1(x) \quad (10)$$

$$\text{s.t.:} \quad f_2(x) \leq \varepsilon, \quad (11)$$

$$x \in X, \quad (12)$$

where (11) ensures that the second objective function is bounded from above by ε .

With this technique, restricting the value of the constraints, the feasible region of the problem is restricted. Therefore, depending on the value assigned to ε , the problem may be infeasible. A highly used lower and upper bound for ε is the ideal and nadir solution, respectively.

In order to ensure that the solution produced by the ε -Constraint method is efficient, it is necessary to guarantee its uniqueness. In general, it is difficult to know if the subproblem solution is unique. However, multiple solutions can be avoided if the objective function of the restricted problem is modified by:

$$\min f_1(x) + \rho f_2(x),$$

where ρ shall be a small number (Aliano Filho et al., 2018).

4.3 The modified Benson method

Originally proposed by Benson (1978), this scalarization technique can find efficient solutions by setting different feasible solutions and solving the associated scalar problems. Let x^0 be a feasible solution for the problem (6)-(7) and $f^0 = (f_1^0, f_2^0)$ be its image. The scalar problem of Benson method is given by:

$$\max l_1 + l_2 \tag{13}$$

$$\text{s.t.: } f_1^0 - f_1(x) = l_1, \tag{14}$$

$$f_2^0 - f_2(x) = l_2, \tag{15}$$

$$x \in X. \tag{16}$$

Recently, in 2021, Aliano Filho et al. (2021) proposed a modification on the scalar problem which can improve its performance. The author replaces the objective function by maximizing l_1 in order to avoid the generation of equal efficient solutions when varying the feasible solution. Additionally, Aliano Filho (2016) proposed the addition of $\rho \cdot l_2$, where ρ is a real number close to zero, in the objective function to avoid multiple optimal solutions and so prejudice the search for efficient solutions. Therefore, the modified scalar problem is given by:

$$\max l_1 + \rho \cdot l_2 \tag{17}$$

$$\text{s.t.: } (14) - (16). \tag{18}$$

It has been proved by Aliano Filho (2016) that if the modified scalar problem has optimal solution and the set feasible solution satisfies $f_2^- \leq f_2^0 \leq f_2^+$, then the optimal solution is an efficient solution for the MOP.

4.4 The Column Generation method

In 1961, Gilmore & Gomory (1961) proposed a method for solving the relaxed problem that consists of starting its solution with only a subset of cutting patterns. In order to apply the column generation method to determine a solution for the problem (1)-(5), the constraints (5) are relaxed, i.e, the values of the variables are real and non-negative, resulting in a Linear Programming problem. Then, at each iteration, cutting patterns that may potentially improve the current solution are generated, until the optimal solution is obtained. To generate these cutting patterns, at each iteration, a subproblem must be solved.

Observe that a column associated to the cutting pattern j of the constraint matrix of the model (1)-(5) is given by $a_{jm}^T = (a_{1jm}, a_{2jm}, \dots, a_{Njm}, 0, \dots, 1, 0, \dots, 0)$, where each variable a_{ijm} represents the number of items type i cut according to cutting pattern j from the object of length L_m and there is 1 in the position $N + m$ due to the constraints of stock limitation (3).

Let π be the vector of dual variables associated with constraints (2) and (3). Let ℓ_i be the length of item type i , L_m the length of object type m , $m = 1, \dots, M$, and N the quantity of item types. Consider $C_{mt} = c_{mt}L_m$, the production cost of a cut object type m in period t . Thus, the reduced cost associated to the cutting pattern a_{jm} is given by:

$$C_{mt} - \sum_{i=1}^N \pi_i a_{ijm} - \pi_{N+m}.$$

Therefore, for the one-dimensional BMCSP, for each object type m and period t , the subproblem is given by:

$$\min \quad C_{mt} - \sum_{i=1}^N \pi_i a_{ijm} - \pi_{N+m} \quad (19)$$

$$\text{s.t.:} \quad \sum_{i=1}^N \ell_i a_{ijm} \leq L_m, \quad (20)$$

$$a_{ijm} \in \mathbb{Z}_+, \quad i = 1, \dots, N. \quad (21)$$

The objective function (19) seeks for the variable, that is, the cutting pattern, with the minimum reduced cost, which may provide a better solution when entering the base of the problem. Constraints (20) and (21), which characterize a knapsack problem, ensure that the sum of the length of the items that compose the cutting pattern does not exceed the object size to be cut and that the quantities of cut items are non-negative.

Although, the scalar problems generated by the ε -Constraint and modified Benson methods have additional constraints that restrict the feasible region, the generation of new cutting patterns does not affect these additional constraints since cutting patterns have null coefficients in these constraints. Therefore, the subproblem (19)-(21) generates valid columns to the scalar problems associated with each of the three scalarization methods.

Theoretically, null cutting patterns can be advantageous when the scalar problems focus on minimizing the inventory costs, since the remained objects could be cut using null cutting patterns

in order to avoid objects and also items inventory costs. Considering that and the uselessness in practical of cutting patterns with no items, the constraint below (22) was added to all the subproblems of the column generation in order to avoid the generation of these cutting patterns.

$$\sum_{i=1}^N a_{im} \geq 1. \quad (22)$$

Observe that the column generation, in the way that it was exposed, is applied to generate only new cutting patterns a_{jm} and the associated variables y_{jm}^j at each iteration. Therefore, all the other variables and their associated columns are considered in the model since the beginning of the column generation process.

5 METHODOLOGY

Firstly, in order to find the ideal and nadir solutions of each objective function, the model (1)-(5) minimizing only f_1 was solved by applying the Column Generation method and, then, the generated cutting patterns were used in the model resolution minimizing only f_2 . The same set of cutting patterns was used as the set of initial cutting patterns in the scalar problems' resolution. To start the Column Generation method, homogeneous patterns were used for each type of object. The scalar problems associated with each of the three applied methods were solved by the Column Generation method in order to guarantee the optimal solution.

The objective functions were normalized during the application of the Weighted Sum method and, therefore, the objective function of the subproblem in the column generation was modified by:

$$\min \quad \alpha_1 \frac{C_{mt}}{f_1^+ - f_1^-} - \sum_{i=1}^N \pi_i a_{ijm} - \pi_{N+m}. \quad (23)$$

Note that the Column Generation method is applied to the BMCSP with the integrality constraints relaxed. In order to find an integer solution, a number of rounding heuristic techniques proposed in the literature can be used (Hinxman, 1980; Wäscher & Gau, 1996; Belov & Scheithauer, 2002; Poldi & Arenales, 2009; Poldi & Araujo, 2016). However, for multi-objective integer problems, in order to avoid excessive computational effort required by the integer problems' resolution. A suitable strategy is to determine integer solutions just around the region that is attractive to the decision-maker, and it can be done by heuristic procedures (Poldi & Arenales, 2009).

The Weighted Sum, ε -Constraint and the modified Benson methods were applied to the problem in order to solve 50 distinct problems, that is, to find up to 50 different efficient solutions. In the Weighted Sum method, at iteration k , $\alpha_1^k = 0.01 + 0.02(k - 1)$ and $\alpha_2^k = 1 - \alpha_1^k$. In the ε -Constraint method, the second objective function was restricted and $\varepsilon^k = f_2^- + \frac{f_2^+ - f_2^-}{51} k$. In the modified Benson method, the feasible solution at iteration k was considered $f_1^{0(k)} = f_1^+ - \frac{f_1^+ - f_1^-}{51} k$ and $f_2^{0(k)} = f_2^- + \frac{f_2^+ - f_2^-}{51} k$. We considered $\rho = 10^{-4}$ for both the ε -Constraint and the modified Benson methods.

6 COMPUTATIONAL EXPERIMENTS

The models were coded in OPL/CPLEX.12.10 and the computational tests were performed on an Intel Core i7 computer with 3.60 GHz and 16 Gbytes of memory. Six classes were analyzed, with 10 randomly generated instances in each class. The set of instances is available at https://github.com/LiviaPierini/Data_set3. Table 1 presents the characteristics of each class of instances, where N represents the number of item types and T , the number of periods.

Table 1 – Number of type de items, periods and type of objects considered in each class of instances.

Class	Items (N)	Periods (T)	Objects (M)
1	5	8	2
2	10	8	2
3	20	8	2
4	5	12	2
5	10	12	2
6	20	12	2

The lengths of the objects were considered $L_1 = 540$ cm and $L_2 = 460$ cm and the length of each item i was determined by $\ell_i \in [0.1, 0.3] \times \sum_{m=1}^M \frac{L_m}{M}$. The production costs c_{mt} were randomly generated in the interval $\in [0.030 \times L_m, 0.050 \times L_m]$. The inventory costs for objects and items were generated as $g_t \in [0.0000075, 0.0000125]$ (per object unit) and $h_{it} = 0.5 \times g_{it}$ (per item unit) in all periods, respectively. The demands of final items were determined by $d_{it} \in [0, 300]$. If $d_{it} \leq 50$, then $d_{it} = 0$. Finally, the number of available objects were considered $E_{mt} \in [0, 600]$, where $1.1 \times \sum_{i=1}^N \sum_{t=1}^T d_{it} \ell_i \leq \sum_{m=1}^M \sum_{t=1}^T E_{mt} L_m$.

7 COMPUTATIONAL RESULTS

With the obtained results by applying the scalarization methods in the BMCSP, the negative correlation between the two different objectives of the problem was proved. Below, in Fig. 1, the Pareto Fronts of the instance 5 of Class 1 found, by applying the ε -Constraint (a) and Weighted Sum (b) methods, are presented. The modified Benson and the ε -Constraint methods found a similar Pareto front.

In Fig. 1, it is possible to note the trade-off between the different objectives of the BMCSP, that is, the decreasing of one objective function value while the value of the other one increases. In order to analyze the degree of correlation between the objective functions, the Pearson correlation coefficient was calculated for each studied instance. Table 2 presents the values of the Pearson correlation coefficient of the objective functions related to the results found by the ε -Constraint method. In each column, the result of the instances of each class is presented.

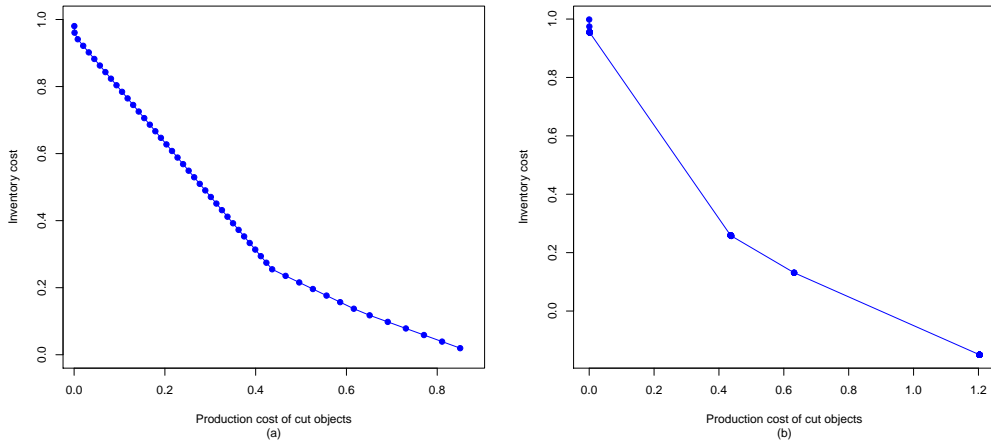


Figure 1 – Pareto Front found by the ε -Constraint method (a) and by the Weighted Sum method (b) in the instance 5 of the Class 1.

Table 2 – Pearson correlation coefficient of the objective functions found by the ε -Constraint method.

Instances	Class					
	1	2	3	4	5	6
1	-0.99957	-0.99880	-0.99233	-0.99965	-0.99593	-0.99745
2	-0.99656	-0.99828	-0.99206	-0.98776	-0.99450	-0.99981
3	-0.99656	-0.99828	-0.99206	-0.98776	-0.99450	-0.99981
4	-0.99630	-0.99845	-0.99739	-0.99999	-0.99860	-0.98629
5	-0.97490	-0.99703	-0.99985	-0.98151	-0.99972	-0.99961
6	-0.99884	-0.99739	-0.99924	-0.99986	-0.99681	-0.99719
7	-0.99792	-0.99961	-0.99983	-1.00000	-0.99663	-0.99941
8	-0.98504	-0.99420	-1.00000	-0.99541	-0.99987	-0.99811
9	-0.99089	-0.99737	-0.99719	-0.99792	-0.99981	-0.99974
10	-0.99831	-0.99880	-0.99912	-0.98580	-0.99753	-0.99945
Average	-0.99349	-0.99782	-0.99691	-0.99357	-0.99739	-0.99769

The closer to -1 is the Pearson correlation coefficient, the stronger the negative correlation between the two objective functions. In Table 2, it is possible to note that the values of the correlation coefficients of the objective functions found by the ε -Constraint method for all the instances studied were less than -0.97 . The correlation coefficients of the objective functions found by the modified Benson method were very similar to the ε -Constraint method, differing only in the third decimal place. It verifies the strong negative correlation between the different objective functions of the BMCSP. Table 3 presents the values of the Pearson correlation coefficient related to the results found by the Weighted Sum method, following the same arrangement as the Table 2.

Table 3 – Pearson correlation coefficient of the results found by the Weighted Sum method.

Instances	Class					
	1	2	3	4	5	6
1	-0.98424	-0.98013	-0.99078	-0.98667	-0.99358	-0.98875
2	-0.99291	-0.98901	-0.96467	-0.99067	-0.99762	-0.99902
3	-0.97251	-0.99540	-0.98907	-0.99175	-0.99436	-0.94412
4	-0.99916	-0.98953	-0.97096	-0.99890	-0.99931	-0.97764
5	-0.96127	-0.98554	-0.99879	-0.98933	-0.99995	-0.98996
6	-0.98191	-0.98475	-0.96764	-0.99859	-0.99542	-0.98148
7	-0.97907	-0.96559	-0.98584	-0.98291	-0.99502	-0.97071
8	-0.96555	-0.97456	-0.97610	-0.98619	-0.96437	-0.95723
9	-0.96744	-0.99378	-0.99707	-0.99006	-0.99998	-0.99842
10	-0.96081	-0.97924	-0.99687	-0.99154	-0.99926	-0.98607
Average	-0.97649	-0.98375	-0.98378	-0.99066	-0.99389	-0.97934

With values less than -0.94 by the Weighted Sum method (Table 3) and -0.98 by the ε -Constraint method (Table 2), the objective functions' correlation coefficients prove the deep negative correlation between the objective functions of the BMCSP. It indicates the impossibility of minimizing both objective functions simultaneously, that is, minimizing the total used material and the inventory costs concomitantly. Therefore, for the BMCSP, multiple efficient solutions should be analyzed by the decision-maker and, then, the most appropriate one based on some criteria should be selected.

Besides the negative correlation between the two objective functions that can be observed in Fig. 1, one can also note that the number of efficient solutions found by the ε -Constraint method is greater than the one obtained by the Weighted Sum method. Tables 4 and 5 show the number of efficient solutions found by the Weighted Sum and the ε -Constraint methods, respectively. Each line refers to each class of instances. In the last column of the table, the average of the efficient solutions found in each class of instances are presented and also the final average, that is, the average number of efficient solutions found in all instances by the methods.

The average number of efficient solutions found by the Weighted Sum method was 11.73. The method found at most 22 efficient solutions for instance 3 of Class 4. In this way, many scalar problems presented the same optimal solution; furthermore, this is a drawback of this method as pointed out by Marler & Arora (2004).

Table 4 – Number of efficient solutions found by the Weighted Sum method.

Class	Instances										Average
	1	2	3	4	5	6	7	8	9	10	
1	4	11	10	11	7	16	14	16	11	16	11.6
2	9	6	18	10	10	9	7	17	14	12	11.2
3	8	12	13	9	9	16	10	18	6	10	11.1
4	11	16	22	11	10	12	12	17	21	16	14.8
5	11	9	10	13	4	6	8	12	5	7	8.5
6	13	10	8	13	13	17	15	13	18	12	13.2
Final average											11.73

Table 5 – Number of efficient solutions found by the ε -Constraint method.

Class	Instances										Average
	1	2	3	4	5	6	7	8	9	10	
1	50	50	50	50	50	50	50	50	50	50	50.0
2	50	50	50	50	50	50	50	50	50	50	50.0
3	50	50	50	50	50	50	50	50	50	50	50.0
4	50	50	50	50	50	50	50	50	50	50	50.0
5	50	50	50	49	50	50	50	50	50	50	49.9
6	50	50	50	50	50	50	50	50	50	50	50.0
Final average											49.98

Note that the ε -Constraint method performed better than the Weighted Sum method regarding the number of efficient solutions found. The method found 50 distinct solutions in all instances, except for the instance 4 of Class 5. The same result was found by the modified Benson method. Therefore, the overall average of efficient solutions found by the ε -Constraint and modified Benson methods was 49.98. Moreover, each one of the scalar problems from both applications, in general, resulted in a distinct efficient solution, which did not occur with the scalar problems of the Weighted Sum method.

The average computational time spent to find the ideal and nadir solutions of each objective function was, on average, 13 seconds, ranging from 2.7 to 32.4 seconds. Tables 6, 7 and 8 present the computational time, in seconds, spent to find the Pareto Front by the Weighted Sum, the ε -Constraint and the modified Benson methods, respectively.

Table 6 – Computational time (seconds) spent to find the Pareto Front by the Weighted Sum method.

Class	Instances										Avg.
	1	2	3	4	5	6	7	8	9	10	
1	36.5	34.6	32.6	40.0	34.4	34.6	33.8	33.3	37.7	36.5	35.4
2	40.8	33.4	43.8	35.6	36.3	36.9	38.2	36.7	35.6	35.6	37.3
3	39.4	41.7	41.1	41.0	36.0	41.1	35.2	43.1	40.2	38.0	39.7
4	56.8	55.8	54.4	53.2	53.8	56.9	56.0	52.1	49.5	56.5	54.5
5	62.8	59.3	55.5	56.9	53.2	58.3	53.6	60.7	52.5	52.9	56.6
6	68.6	63.5	65.8	64.6	69.4	63.6	58.8	72.0	66.9	64.4	65.7
Final average											48.2

Table 7 – Computational time (seconds) spent to find the Pareto Front by the ϵ -Constraint method.

Class	Instances										Avg.
	1	2	3	4	5	6	7	8	9	10	
1	35.2	33.1	32.8	34.0	33.0	34.5	33.0	33.6	34.3	37.3	34.1
2	33.7	32.6	51.1	34.9	36.6	35.7	36.9	36.9	35.4	34.9	36.9
3	33.6	36.3	37.5	35.6	33.2	37.5	34.8	39.9	37.2	34.4	36.0
4	52.5	51.7	53.3	59.2	50.7	52.5	52.0	50.8	46.7	52.9	52.2
5	53.2	68.1	64.3	52.9	51.1	52.7	52.1	51.5	47.4	51.5	54.5
6	56.4	64.9	57.3	59.6	60.6	57.0	55.3	54.4	57.1	53.5	57.6
Final average											45.2

Table 8 – Computational time (seconds) spent to find the Pareto Front by the modified Benson method.

Class	Instances										Avg.
	1	2	3	4	5	6	7	8	9	10	
1	37.3	33.3	32.0	34.1	32.9	34.5	33.0	34.0	33.6	36.7	34.1
2	33.9	33.1	50.5	34.8	35.9	36.1	35.6	39.9	36.2	35.2	37.1
3	34.1	37.5	38.5	36.7	34.3	38.1	34.7	41.0	37.3	34.0	36.6
4	51.6	51.8	54.7	52.4	50.7	52.9	51.9	50.5	46.6	53.3	51.6
5	53.2	68.6	65.4	53.7	51.9	53.8	52.0	51.5	47.6	51.7	54.9
6	58.3	64.3	58.1	61.5	62.4	58.1	56.5	56.0	55.6	53.7	58.4
Final average											45.5

Note that the Weighted Sum method, on average, spent 48.2 seconds to find the efficient solutions, ranging from 33.4 to 65.7 seconds. The longest time to find the Pareto Front was 72 seconds in the instance 8 of Class 6. The ε -Constraint method, on average, spent 45.2 seconds to find efficient solutions, ranging from 34.1 to 57.6 seconds for each class. The longest time spent was 68.1 seconds in the instance 2 of Class 5.

The modified Benson method, on average, spent 45.5 seconds to find efficient solutions, ranging from 34.1 to 58.4 seconds for each class. The ε -Constraint and the modified Benson methods presented a very similar performance, spending around 45 seconds on average to find the Pareto front. The three scalarization techniques presented a reasonable time to find the Pareto front; however, the ε -Constraint and the modified Benson method found more efficient solutions than the Weighted Sum method and, therefore, presented a better performance when applied to the BMCSP.

The Column Generation method behaved differently in the scalar problems' resolution. It is explained by the fact that the ε -Constraint method and the Weighted Sum method result in scalar problems with different objective functions. The average amount of cutting patterns generated in the application of Weighted Sum method was 14.6, while the average for the application of ε -Constraint and modified Benson methods was 7.9. The generated columns in the ε -Constraint and modified Benson methods compromised the generation of efficient solutions in the extreme left side of the Pareto Front, as can be seen in Fig. 2 which shows the Pareto Fronts of the instance 2 of the Class 3.

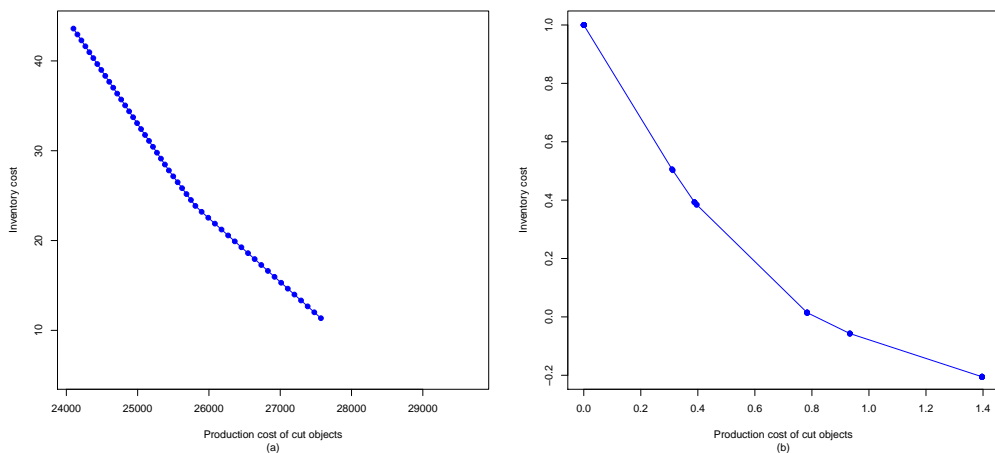


Figure 2 – Pareto Front found by the ε -Constraint method (a) and by the Weighted Sum method (b) for the instance 2 of the Class 3.

The cutting patterns generated during the application of the Weighted Sum method interfered in the normalization of the objective functions, which started to assume values beyond the interval $(0, 1)$. It is due to the fact that the new cutting pattern generations enable to find better values to

f_2 than its ideal value determined at the beginning with a stated cutting pattern set. Moreover, the ideal solution f_2^- became not a lower bound for ε , and then efficient solutions associated with values less than f_2^- were not found by the ε -Constraint method. The same occurred for the modified Benson method, since the feasible solutions were based on the ideal and nadir solutions. Illustrating this, in Fig. 2, it can be seen that the ε -Constraint method did not find efficient solutions for values of f_2 less than 0.

8 CONCLUSIONS

This paper investigates the conflicting objectives of the bi-objective multiperiod cutting stock problem that aims to minimize the production cost associated with cut objects and the inventory costs of objects and items. Computational tests were performed and, for solving the problems, the Column Generation method and three scalarization techniques were applied. The scalarization techniques' performance was also analyzed.

With the computational tests, it was possible to verify the strong negative correlation between the different objectives of the BMCSP indicating the importance of finding alternative solutions for the BMCSP since it is not possible to minimize the cost associated with the total cut material (waste) and the inventory costs simultaneously.

Although all the three scalarization techniques spent a similar amount of computational time to find the Pareto front, the ε -Constraint and the modified Benson methods found many more efficient solutions than the Weighted Sum method. Both techniques found 49.98 distinct efficient solutions on average from 50. Therefore, all the methods were able to find multiple solutions for the BMCSP in a reasonable time; however, the ε -Constraint and the modified Benson methods performed better to the BMCSP. Moreover, the performance of the ε -Constraint and the modified Benson methods were very similar, consisting of satisfactory and alternative options for being applied to the BMCSP.

It was noted that the Column Generation method behaved differently for each scalarization technique, since the scalar problems involved in each method are different. Continuing the study, modifications will be made in the application of the methods, such as in the number of initial columns considered in the Column Generation method, in order to improve the lower bound for ε .

Acknowledgments

This research was funded by the São Paulo Research Foundation - FAPESP (grants 2016/01860-1, 2017/18192-4, 2019/04013-6).

References

ALIANO FILHO A. 2016. *Novas extensões de técnicas de escalarizações no problema de corte unidimensional inteiro multiobjetivo*. Ph.D. thesis. Universidade Estadual de Campinas.

ALIANO FILHO A, MORETTI AC & PATO MV. 2018. A comparative study of exact methods for the bi-objective integer one-dimensional cutting stock problem. *Journal of the Operational Research Society*, **69**(1): 91–107.

ALIANO FILHO A, MORETTI AC, PATO MV & OLIVEIRA WA. 2021. An exact scalarization method with multiple reference points for bi-objective integer linear optimization problems. *Annals of Operations Research*, **296**: 35–69.

ARAUJO SA, POLDI KC & SMITH J. 2014. A genetic algorithm for the one-dimensional cutting stock problem with setups. *Pesquisa Operacional*, **34**(2): 165–187.

BELOV G & SCHEITHAUER G. 2002. A cutting plane algorithm for the one-dimensional cutting stock problem with multiple stock lengths. *European Journal of Operational Research*, **141**(2): 274–294.

BENSON H. 1978. Existence of efficient solutions for vector maximization problems. *Journal of Optimization Theory and Applications*, **26**(4): 569–580.

BRANKE J, DEB K, MIETTINEN K & SLOWIŃSKI R. 2008. *Multiobjective optimization*. Germany: Springer.

CAMPELLO BSC, GHIDINI CTLS, AYRES AOC & OLIVEIRA WA. 2020. A multiobjective integrated model for lot sizing and cutting stock problems. *Journal of the Operational Research Society*, **71**(9): 1466–1478.

DELORME M, IORI M & MARTELLO S. 2016. Bin packing and cutting stock problems: mathematical models and exact algorithms. *European Journal of Operational Research*, **255**(1): 1–20.

DYCKHOFF H. 1990. A typology of cutting and packing problems. *European Journal of Operational Research*, **44**(2): 145–159.

EHRGOTT M. 2005. *Multicriteria optimization*. vol. 491. Berlin: Springer.

GILMORE PC & GOMORY RE. 1961. A linear programming approach to the cutting-stock problem. *Operations Research*, **9**(6): 849–859.

GOMES AM, GONÇALVES JF, ALVAREZ-VALDÉS R & DE CARVALHO V. 2013. Special issue on “Cutting and Packing”. *International Transactions in Operational Research*, **20**(3): 441–442.

HAESSLER RW. 1975. Controlling cutting pattern changes in one-dimensional trim problems. *Operations Research*, **23**(3): 483–493.

HAIMES YY, LASDON LS & WISMER DA. 1971. On a bicriterion formulation of the problems of integrated system identification and system optimization. *IEEE Transactions on Systems, Man, and Cybernetics*, **SMC-1**(3): 296–297.

- HINXMAN A. 1980. The trim-loss and assortment problems: a survey. *European Journal of Operational Research*, **5**(1): 8–18.
- KOLEN AWJ & SPIEKSMAS FCR. 2000. Solving a bi-criterion cutting stock problem with open-ended demand: a case study. *Journal of the Operational Research Society*, **51**(11): 1238–1247.
- LOPES AM. 2009. *Uma abordagem multiobjetivo para o problema de corte de estoque unidimensional*. Master's thesis. Universidade Estadual Paulista "Júlio de Mesquita Filho".
- MARLER RT & ARORA JS. 2004. Survey of multi-objective optimization methods for engineering. *Structural and Multidisciplinary Optimization*, **26**(6): 369–395.
- MEHDIZADEH E, HAJIPOUR V & MOHAMMADIZADEH MR. 2016. A bi-objective multi-item capacitated lot-sizing model: two Pareto-based meta-heuristic algorithms. *International Journal of Management Science and Engineering Management*, **11**(4): 279–293.
- MORABITO R, ARENALES MN & YANASSE HH. 2009. Special issue on cutting, packing and related problems. *International Transactions in Operational Research*, **16**(6): 659–659.
- OLIVEIRA JF & WÄSCHER G. 2007. Cutting and Packing. *European Journal of Operational Research*, **183**(3): 1106–1108.
- POLDI KC & ARAUJO SA. 2016. Mathematical models and a heuristic method for the multiperiod one-dimensional cutting stock problem. *Annals of Operations Research*, **238**(1): 497–520.
- POLDI KC & ARENALES MN. 2009. Heuristics for the one-dimensional cutting stock problem with limited multiple stock lengths. *Computers & Operations Research*, **36**(6): 2074–2081.
- REZAEI J & DAVOODI M. 2011. Multi-objective models for lot-sizing with supplier selection. *International Journal of Production Economics*, **130**(1): 77–86.
- ROMEIJN HE, MORALES DR & VAN DEN HEUVEL W. 2014. Computational complexity of finding Pareto efficient outcomes for biobjective lot-sizing models. *Naval Research Logistics (NRL)*, **61**(5): 386–402.
- SONG X & BENNELL JA. 2014. Column generation and sequential heuristic procedure for solving an irregular shape cutting stock problem. *Journal of the Operational Research Society*, **65**(7): 1037–1052.
- UMETANI S, YAGIURA M & IBARAKI T. 2003. One-dimensional cutting stock problem to minimize the number of different patterns. *European Journal of Operational Research*, **146**(2): 388–402.
- VANDERBECK F. 2000. Exact algorithm for minimising the number of setups in the one-dimensional cutting stock problem. *Operations Research*, **48**(6): 915–926.

WANG PY & WÄSCHER G. 2002. Cutting and packing. *European Journal of Operational Research*, **141**(2): 239–240.

WÄSCHER G & GAU T. 1996. Heuristics for the integer one-dimensional cutting stock problem: a computacional study. *OR Spektrum*, **18**(3): 131–144.

WÄSCHER G, HAUSSNER H & SCHUMANN H. 2007. An improved typology of cutting and packing problems. *European Journal of Operational Research*, **183**(3): 1109–1130.

How to cite

PIERINI LM & POLDI KC. 2022. A bi-objective multiperiod one-dimensional cutting stock problem. *Pesquisa Operacional*, **42**: 258432. doi: 10.1590/0101-7438.2022.042.00258432.