

Pesquisa Operacional (2024) 44: e274910 p.1-14 doi: 10.1590/0101-7438.2023.043.00274910 © 2024 Brazilian Operations Research Society Printed version ISSN 0101-7438 / Online version ISSN 1678-5142 www.scielo.br/pope APPLICATIONS TO THE PUBLIC SECTOR SPECIAL ISSUE ARTICLES

A STATISTICAL APPROACH TO AIRCRAFT CARGO LAUNCHING TO SUPPORT HUMANITARY AID OPERATIONS

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ABSTRACT. Natural disasters often affect communities that become unable to access vital items. Not infrequently, the profile and scale of disasters require the delivery of various types of material and vital items, gaining prominence in these contexts the launching cargo by aircraft. This study presents a mathematical ballistic launch model that incorporates the main random variables that influence the trajectory of the load, making it possible to conduct analyses to improve the accuracy of launchings. In this way, Monte Carlo Simulations were utilized to investigate the trajectory deviations as a function of certain combinations of controlled factors in the model. Based on these findings, a Response Surface method was implemented, enabling the development of flight profiles that minimize load deviations from a desired impact point on the ground.

Keywords: mathematical ballistic launch model, Monte Carlo simulation, response surface method.

1 INTRODUCTION

At certain times of the year, more precisely between December and March, some areas of Brazil, such as the State of Santa Catarina, are hit by torrential rains, causing flooding, landslides, among other tragedies (Magnago et al., 2021). Such natural disasters are catalyzed, in general, by factors such as the increase in disorderly occupation of the land, the degradation of the environment, among others related to socio-environmental vulnerabilities (Francisco, 1996).

In recent years, these types of disasters have worsened, significantly increasing their negative impact on the most vulnerable populations, who in many instances have been isolated due to the

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obstruction of access routes to the affected areas, facing difficulties in gaining access to supplies, medicines, and other items deemed essential (Santos, 2015).

Faced with these challenges, the authorities responsible for rescuing and sheltering victimized populations have used aircraft to deliver supplies as initial aid. In these cases, the cargo launches by Brazilian Air Force aircraft stands out, whose missions include conducting aerial resupply actions in humanitarian aid operations (Brasil, 2020a).

In this context, Brazilian Air Force has the capacity to meet the diverse demands of isolated populations, as it can release materials using three different methods: Bundle Door Dropping, Container Delivery System and Heavy Release. Despite all these methods of executing the mission, this work will address Bundle Door Dropping, which consists of bales weighing up to 226 kilograms that, after being released from the aircraft, are supported by an unguided parachute with only one stage of activation, becoming susceptible to wind action after stabilization in vertical fall (Brasil, 2020b).

The Bundle Door Dropping constitutes a method sufficiently explored in the literature, having its ballistic parameters, physical and deterministic modeling of the load-parachute set, atmospheric models, performance and launch point analyses, accuracy analyses, main sources of errors and dispersion, among other physical and dynamic characteristics addressed in relevant works, such as those by Henry (2011), Boggs (2015), VanderMey (2015) and Patel (2017), referenced in this study.

The present study aims to establish flight profiles to minimize trajectory deviations, in cargo launches, in relation to a desired ground impact point (IP), through analyses conducted with the Response Surface Method (RSM), considering the "Load Shift" as response variable and the Controlled Factors Aircraft Speed, Launch Height and Cargo Weight as explanatory variables. To calibrate this RSM, data was obtained through Monte Carlo Simulations with a mathematical ballistic model of bale release that encompassed the main Non-Controlled Factors responsible for trajectory deviations, presenting as a differential aspect in relation to the works cited the fact of incorporating random variables to the ballistic model, which, characterized by their Probability Density Functions (PDFs), represent the most relevant factors that influence the load deviation.

Moreover, by using 4-year historical records of wind speed to model this Non-Controlled Factor, whose influence on the trajectory stands out before the others, this study presents results more reliable to the physical characteristics found in the region considered. Nevertheless, to demonstrate that the mathematical ballistic model presents results consistent with real launches, 27 launches were simulated using a Wind Speed PDF modeled based on records collected during real experiments conducted by Brazilian Air Force. Thus, it was possible to evaluate the hypothesis of equality between the averages of the load distances obtained in real and simulated launches.

The study consists of seven sections. Next, the mathematical ballistic model of a bundle door launch is presented, along with descriptions of the ballistic parameters of a single-stage unguided parachute and the most significant Non-Controlled Factors, as well as the steps for calculating trajectory deviation. In the sequence, Monte Carlo simulations are executed with the ballistic model for each combination of levels of Controlled Factors, considering the behaviors of the Non-Controlled Factors by their respective PDFs, and tabulating the simulation results. The RSM is then applied, which models the relationship between the response variable "Load Shift" and the explanatory variables using linear regression. After that, Contour and Response Surfaces are introduced, as well as their practical implications and optimization directions. Then, real and simulated launches are compared, and the "two independent samples t-test" is used to determine whether the average distances of real and simulated launches are equal. Finally, the conclusion and potential future directions for the study are presented.

2 THE MATHEMATICAL BALLISTIC LAUNCH MODEL

In this study, the launch profile adopted will be with the aircraft maintaining a fixed course in an opposite direction to the wind speed, which is assumed to be invariant in direction, a feasible profile for door bale launches (Brasil, 2020b). Furthermore, to facilitate the subsequent implementation of the launch model in software for simulation purposes, it will be divided into two calculation stages: (1) Internal and Stabilization Phases, and (2) Stabilized Phase (Patel *et al*., 2017), as highlighted in Figure 1.

Source: Authors.

The first phase involves the handling of cargo inside the aircraft. In this phase, it will be computed an Exit time (ET) of 0.2s, related to the operator's motor reaction to seeing the light indicating free launch and activating the extraction mechanism, since in this study the launch with the cargo

packed directly on the exit ramp was adopted, without the sliding by rails procedure to exit the aircraft (Boggs, 2015).

The next phase corresponds to stabilization, starting when the load leaves the aircraft, with ground airspeed (GS), until its vertical stabilization, with the parachute fully open, passing a time called Time of Fall Constant (TFC) and losing a height called Vertical Distance (VD). Moreover, in this phase, a time called Deceleration Quotient (DQ) elapses until the horizontal stabilization of the load, at which point a horizontal distance on the ground called Forward Throw Distance (FTD) is computed, as shown in Equation (1) below (Henry *et al*., 2011).

$$
FTD = GS * (DQ + ET) \tag{1}
$$

With the parachute fully open, the bale enters the stabilized phase, characterized by the descent in steady state with a descent rate of fall (RoF), without horizontal displacement if the wind speed is zero. It should be noted that the mean values of RoF are different due to different values of ballistic parameters arranged in equation (2), where ρ represents the air density, for simplification considered constant at sea level (1.225 Kg/m3), and Cd represents the drag coefficient of the parachute (Henry *et al*., 2011).

$$
RoF^2 = \frac{2*Weight}{\rho * Parachute\ area * Cd}
$$
 (2)

In this phase, stands out the fact that the trajectory is influenced by two Non-Controlled Factors discussed in this study: the drag coefficient of the parachute (Cd) and the wind speed (for the purposes of this study, the wind direction is considered constant).

The Non-Controlled Factor Cd presents oscillations around an average value, being considered stochastic disturbances. In this way, based on the relationship arranged in Equation (2), the average value of RoF also cannot be considered constant for given values of weight and surface area of the parachute, presenting oscillations around its average value (Henry *et al*., 2011).

Assuming the drag coefficient distributed according to a Normal PDF, $C_d \sim N(\mu_{Cd}; \sigma_{Cd})$, the mean value of RoF will be also distributed according to a Normal PDF, $RoF \sim N(\mu_{RoF}; \sigma_{RoF})$ (Leonard et al., 2017). Therefore, the variations presented by the average value of RoF will affect the time that the cargo-parachute set will be exposed to the wind speed, influencing its horizontal trajectory deviation.

Regarding the second Non-Controlled Factor, the Wind Speed is distributed according to a certain PDF, which can be conceived by the analysis of the historical data of the specific locality. Thus, the horizontal trajectory deviation imposed by this factor can be calculated as provided in Equation (3) below (Boggs, 2015).

$$
Horizontal Trajectory Deviation = Wind speed * ((H - VD)|RoF)
$$
 (3)

After investigating the launch phases and observing how the two Non-Controlled Factors affect the horizontal trajectory deviation, the launch model was implemented in two calculation stages for the execution of the simulations.

The first stage was used to calculate the FTD, considering the turning on of the light indicating free launch in the vertical direction of the IP. After passing the horizontal distance FTD in relation to the IP, the load enters the second stage of the model, equivalent to the stabilized phase, and is then influenced by the Wind Speed in the opposite direction of the FTD displacement.

In this second step, the Launch Height Controlled Factor is now considered in the model, and the horizontal trajectory deviation calculations are performed according to Equation (3), considering the variables Wind Speed and RoF as random variables distributed according to pre-established PDFs.

Thus, as the profiles of the launches are established following a standard, opposite direction to the Wind Speed and without variations in the direction of this factor, the Load Distance in relation to IP (the response variable of interest "Load Shift") is obtained according to Equation (4) (Brasil, 2020b).

Distance to the IP =
$$
FTD - Horizontal Trajectory Deviation
$$
 (4)

Finally, for the calculations via simulation, it is worth noting that the values of the main ballistic parameters used are from a study conducted by Institute of Operational Applications of Brazilian Air Force, in which real launches, with two types of cargo (30 kg and 65 kg), were performed with the RAC-LS parachute, according to data in Table 1 (Brasil, 2021).

Source: Brasil (2021).

3 MONTE CARLO SIMULATIONS

In this stage of the research, Monte Carlo simulations were performed with the ballistic model for each level combination of the Controlled Factors (Aircraft Speed, Launch Height, and Cargo Weight), considering the behavior of the Non-Controlled Factors (Wind Speed and RoF) by their respective PDFs, thereby investigating the load distances in relation to an IP.

Considering that the load is shown to be more susceptible to the action of wind speed when entering the stabilized phase (Patel et al., 2017), thus being the factor with the greatest influence on the trajectory, for a more realistic approach in the simulations, the results from a 2021 paper by Gazola and Ferrari were used.

In that work, the authors analyzed 13,911 Wind Speed records between 2016 and 2019 in the region of Campo Grande - MS, obtained through the Sistema Integrado de Dados Ambientais (SINDA), linked to the Instituto Nacional de Pesquisas Espaciais (INPE), and modeled the data distribution with a Weibull PDF with the parameters shape and scale as shown in Figure 2 (Gazola and Ferrari, 2021).

The suitability tests performed with the PDF Weibull were the Anderson–Darling Test and the Cramer–Von Mises Test. As in both tests the P-Value exceeded $\alpha = 5\%$, it was not possible to reject the Null Hypothesis (*H*0) that the Wind Speed data followed a Weibull PDF (Langat *et al*., 2019).

Figure 2 – Wind Speed Weibull PDF.

Source: Gazola and Ferrari (2021).

As for the Non-Controlled Factor RoF, it was considered as normally distributed, with standard deviation associated to the stochastic perturbations suffered by the Drag Coefficient of the main parachute (C_d) (Leonard *et al.*, 2017).

After the PDFs for the Non-Controlled Factors were designed, launches were simulated for the following combinations of Controlled Factor levels: Aircraft Speed (100 Kt and 120 Kt), Launch Height (300 ft and 500 ft) and Load Weight (30 Kg and 65 Kg). In total, eight profiles, each with three hundred launches, were simulated, and the distances of the cargo in relation to the IP were computed in meters, according to Table 2.

Source: Authors.

Each profile generated a mean value of cargo deviation from the IP, with positive values representing cargo positions ahead of the IP (FTD greater than the horizontal deviation of the trajectory) and negative values representing cargo positions behind the IP (FTD smaller than the horizontal deviation of the trajectory), considering the direction of aircraft displacement. Thus, the focus of the analysis will be on the absolute value of the average deviation, with one of the objectives set to identify a combination of factors that approaches this absolute value to zero.

RESPONSE SURFACE METHOD

After the preliminary simulations, a Factorial Experiment $2³$ was designed, composed of three factors: Aircraft Speed, Launch Height, and Load Weight, with two levels (high "+" and low "-"), and the response variable "Load Shift", whose relationship was designed by linear regression (Montgomery, 2001). We then obtained a second order RSM model with interaction terms, Adjusted R^2 = 0.9915 and statistical significance of the coefficients according to Table 3.

Variables	Estimated Value Coeff.	Int. Coeff. 95%	P-Value
Intercept	$-14,080$	$-15,902$ to $-12,257$	8,23E-09
Speed	10,720	8,897 to 12,542	6,02E-07
Height	$-35,825$	$-37,647$ to $-34,00$	$2.2e-16$
Weight	24,307	22,484 to 26,130	1,06E-12
Speed* Height	0.428	$-1,394$ to 2,250	0.62
Speed* Weight	2,717	0,894 to 4,540	0,0058
Height* Weight	2,617	0,948 to 4,440	0,0075

Table 3 – Coefficients of the RSM Model.

Source: Authors.

As it can be seen in the column related to the P-Value, associated to each factor or interaction between factors, only the Speed*Height interaction term of the model does not present statistical significance, considering a significance level $\alpha = 0.05$. The significance of the other interaction terms confirms that the factor Weight, in interaction with Speed, affects the FTD, and in interaction with Height, through RoF, affects the load exposure time to the effects of Wind Speed, influencing, consequently, the response variable "Load Shift".

Next, graphical examinations and tests were conducted to evaluate the validity of the inferences with the RSM model, assuming that the residuals possess certain characteristics, such as constant variance, uncorrelated pairs of residuals, normally distributed residuals, and an expected value of zero (Mendenhall *et al*., 2003).

Figure 3 depicts a graphical examination of residuals and standardized residuals versus adjusted values that reveals no strong evidence of violation of the homoscedasticity hypothesis (constant variance). In addition, based on the Durbin-Watson correlation test, where the H_0 is uncorrelated residuals and the Alternative Hypothesis (H_a) is the opposite, with a calculated P-Value of approximately 0.2023, it was not possible to reject H_0 .

Figure 3 – Graphical Examination of Residuals.

Source: Authors.

Next, normality tests were conducted by examining the Normal Probability Graph and the Shapiro-Wilk Normality Test, whose *H*⁰ consists of residuals with a Normal distribution. As shown in Figure 4, visual inspection of the "Normal Q-Q" graph and the P-Value, which is approximately 0.499, do not indicate a violation of the residual's normality hypothesis.

Figure 4 – Tests of Normality of Residuals.

Source: Authors.

Once the tests to evaluate the validity of the inferences were conducted, the RSM model could be designed, as set forth in Equation (5). In this way, for its use in the calculation of the Distance from the Load in relation to the IP, the explanatory variables must be replaced by continuous proportional values between -1 and 1, whose extreme values represent the levels used in the design of the Factorial Experiment $2³$.

$$
Distance to IP = -14,08 + 10,72 * Speed - 35,825 * Height -24,307 * Weight
$$

$$
+2,717 * Speed * Weight +2,617 * Height * Weight
$$
(5)

5 RESULTS AND DISCUSSIONS

After designing the RSM model, contour and response surfaces were built to visualize the effect of each factor on the response variable "Load Shift" (Dist. to IP), keeping one of the factors fixed at its mean value for each surface, as shown in Figure 5.

Source: Authors.

The RSM model permits projections of the direction of response optimization as a function of each factor, enabling analyses to be conducted to foresee combinations of factors or how these can be varied to minimize the load distance from the IP, the "Load Shift".

Thus, for flight planning purposes, a launch designed to send as much cargo as possible, within the scope of the model, and with a predicted distance to IP equal to zero, could be conducted approximatively with the parameters listed in Table 4.

Source: Authors.

Other types of analysis can be conducted to characterize the behavior of the response variable as a function of variations in a specific factor. Thus, planning a flight in which the launch will be performed with fixed parameters of Speed and Height of the aircraft, the model could present the behavior of the load distance as a function of changes in Weight, allowing a direction of the optimization of the response variable, i.e., seek a value of Weight that provides the minimization of the distance in absolute values, as shown in Table 5.

Speed (Kt)	Height (ft)	Weight (Kg) IP Distance	
120	400	30	-37
120	400	40	-17
120	400	50	
120	400	60	15
120	400	65	24

Table 5 – Flight Parameters.

Source: Authors.

6 COMPARISON OF ACTUAL LAUNCHING WITH SIMULATED LAUNCHING

To verify whether the mathematical ballistic model presents results consistent with real launches, the "two independent samples t-test" was applied to evaluate the hypothesis of equality between the means of the load distances obtained in the real and simulated launches (Montgomery, 2001).

The real experiment consisted of 27 launches with 30 kg loads at 300 ft height and 110 Kt airspeed, between July 6, 2020, and July 23, 2020, in Campo Grande, MS, where the Wind Speed and Distances were measured.

From these experimental data it was sought the probable PDF associated with the Wind Speed. The graph of Cullen and Frey (Cullen and Frey, 1999) highlighted the Normal distribution. Then, its parameters (mean and standard deviation) were estimated. The Kolmogorov-Smirnov Normality Test was applied. For $\alpha = 0.05$, the P-Value calculated of 0.06948 did not permit the rejection of *H*₀, admitting the random variable $X \sim N(\mu = 12.02; \sigma = 5.23)$ as representative of the Wind Speed in the Real Launches, according to Figure 6 (Mendenhall *et al*., 2003).

Figure 6 – Modeling of the Distribution of Wind Speed in the Real Launching.

Source: Authors.

Once modeled the PDF of the Wind Speed relative to the empirical data, this parameter was used in the simulation of 27 launches, with the mathematical ballistic model, obtaining simulated data of the load's distance to be compared with the real distances through the graphs of Figure 7.

In Figure 7, the Boxplots (a) show that the medians of the real and simulated distances are close to 100 m. The histograms (b) show the mean of the real distances equal to 106.66m and that of the simulated distances equal to 105.58m. Moreover, the distributions of these data can be considered approximately normal, since, for $\alpha = 0.05$, the P-Values of the Shapiro-Wilk Normality Tests (c) do not allow the rejection of *H*⁰ that the Real and Simulated Disparities are normally distributed, meeting the requirement for application of the "two independent samples t-test" (Montgomery, 2001).

In this sense, the "two independent samples t-test" was implemented to evaluate the equality of the real and simulated distance means, adopting as H_0 the difference between the means to be equal to zero. The calculated P-Value of 0.9518 does not allow the rejection of *H*0, offering strong evidence that the mathematical ballistic model, when used in simulations, presents results consistent with real launchings.

Figure 7 – Comparison of Real and Simulated Launching and Test t.

Source: Authors.

7 CONCLUSIONS

The natural disasters that hit the country every year, such as the floods in the State of Santa Catarina that isolate the most vulnerable communities, require the adoption of emergency measures to supply them with vital items. Often, the magnitude of the catastrophe and the number of victims are so great that aerial resupply actions conducted by the Brazilian Air Force become fundamental.

In practical operational terms, the conduct of these actions imposes certain complications of a technical and operational nature, revealing the need to know, in an accurate way, methods and techniques to increase the effectiveness of the launch.

This study presented a method that can maximize the amount of material destined for isolated populations while predicting a minimum distance from the impact point. All that required is to conduct the flight with the aircraft speed and launch height specified by the model.

Moreover, visual analysis of the contour lines of the intermediary graph of Figure 5, summarized in Table 5, allows us to verify that for certain fixed parameters of speed and altitude, assuming an operational or safety need to maintain minimum speed and altitude in a flight, the weight of the load to be established, in order to obtain the predicted minimum distance to the impact point, may not coincide with the maximum launch capacity, thereby reducing the quantity of material towed.

In this way, the RSM model presented has the potential to provide planners of air resupply actions with a method that enables them to conduct analyses in which planning factors (flight parameters) are conjugated to increase the effectiveness of launches, making them with a certain predictability regarding the accuracy of the load impact.

For this predictability to be statistically representative of a particular region, the study incorporated a mathematical ballistic model of launch random variables characterized by their PDFs, using four consecutive years of historical data to model the most influential variable on the trajectory, wind speed.

The fact that wind speed behaves differently in different locations is, however, a distinguishing characteristic of this study that should be highlighted. As the results obtained were based on data for the region of Campo Grande - MS, to conduct studies in other areas, the methodology must be applied to new sets of data.

To improve the analyses of bale launchings, the research can be expanded to include additional Controlled and Uncontrolled Factors with the potential to affect the accuracy of the launching. In addition, Logistic Regression can be used to develop a model for estimating the probability of a load dropping in a particular region, with a binary response variable whose value 1 indicates the region's reach and 0 otherwise.

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