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A SPATIAL PRICE EQUILIBRIUM MODEL IN THE OLIGOPOLISTIC MARKET FOR OIL DERIVATIVES: AN APPLICATION TO THE BRAZILIAN SCENARIO

Fabiano Mezadre Pompermayer
Departamento de Engenharia Industrial / PUC-RJ
Rio de Janeiro – Brazil

Michael Florian
Centre de Recherche sur les Transports / Univ. de Montréal
Montreal – Canada

José Eugenio Leal *
Departamento de Engenharia Industrial / PUC-RJ
Rio de Janeiro – Brazil
jel@rdc.puc-rio.br

Adriana Costa Soares
Departamento de Engenharia Industrial / PUC-RJ
Rio de Janeiro – Brazil
Petróleo Brasileiro SA – Logística – Transporte Marítimo

* *Corresponding author* / autor para quem as correspondências devem ser encaminhadas

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Abstract

This paper presents a spatial price equilibrium model in an oligopoly market for refined oil products. Till 1997 the Brazilian oil market was characterized by the state monopoly of Petrobras, which up to 2001 remained the only firm authorized to import oil derivatives. With several agents operating in the primary oil supply market, the government stopped fixing the prices for Petrobras, which started to determine the prices based on competition with other players. In this new scenario some questions arise regarding the price levels at which refined products will be supplied in different regions across Brazil as well as the capacity of national refineries to compete with imported products. To answer those and other questions, a new oligopoly spatial equilibrium model is herein proposed, taking into account the special characteristics of production of refined oil products. An iterative Gauss-Seidel-like algorithm with sequential adjustments was developed and applied to Brazilian market data. The model, the algorithm and its application are described in this work. Such a model may be used both by regulatory authorities and by companies in the sector.

Keywords: oil industry; spatial price equilibrium model.

Resumo

Este artigo apresenta um modelo de equilíbrio espacial de preços em um mercado oligopolizado de derivados de petróleo. Até o ano de 1997, o mercado brasileiro era caracterizado pelo monopólio estatal da Petrobrás, a qual permaneceu, até 2001, como a única empresa autorizada a importar derivados de petróleo. Com vários agentes operando no mercado, o governo deixou de fixar os preços para a Petrobrás, que passou a determinar os preços baseada na competição com outros agentes. Neste cenário, surgem algumas questões relativas aos níveis de preços a serem oferecidos no mercado e relativas à capacidade das refinarias nacionais de competir com produtos importados. Para responder a estas e outras questões, um novo modelo de equilíbrio espacial de preços para um mercado oligopolizado foi desenvolvido, considerando as características especiais da produção de derivados. Um algoritmo iterativo do tipo Gauss-Seidel, com ajustes seqüenciais, foi desenvolvido e aplicado com dados do mercado brasileiro. O modelo proposto, o algoritmo e a aplicação são discutidos no trabalho. Este modelo pode ser usado por autoridades reguladoras e pelas companhias do setor.

Palavras-chave: indústria petrolífera; modelo de equilíbrio espacial de preços.

1. Introduction

In the last 50 years, the Brazilian oil industry was characterized by the state monopoly of Petrobras, a company created by the Brazilian government for this purpose. Petrobras developed during this period, increasing crude oil production and refining capacity on its way to becoming self-sufficient in 2006.

Since the promulgation of law 9.478/97 in 1997, the government started opening up and deregulating the oil market. Till December 2001, only Petrobras was authorized to import oil derivatives. The government even fixed the prices of derivatives at the sales outlets of Petrobras, although these prices did not reflect the costs of an alternative supply, which would be similar to an open market scenario. But this has changed and other agents may now import directly to the consumption regions without any intermediation by Petrobras.

With several players able to operate in the primary oil supply market, the government stopped fixing the prices for Petrobras, which started to determine the prices based on competition with other players. However, with considerable high international petroleum prices and a lobby on Petrobras for supplying non-expensive fuels, Petrobras has been setting prices below the cheapest import alternative supply. This strategy made all possible direct import by other players unprofitable and no new refined products supplier has entered the Brazilian market till this work was submitted for publication. When the deregulation started in 1997 effectively reaches the refined products market, it is expected that the competition among local and foreign suppliers should result in prices based on cheaper supply alternatives.

There are several ways to supply oil derivatives without the high initial investment on a refinery, basically by purchasing derivatives or solvents from other markets where there is excessive supply and mixing them to form the final derivative. However, control over transportation of these products gives the local refiner a certain advantage over this type of competitor.

Considering the new competitive environment expected after the market was opened up, there are concerns regarding the price levels at which derivatives will be supplied in different regions across the country. Such concerns are evident when one analyzes the likely need to offer subsidies to remote areas, even if they can be supplied by external sources. One must recall that law 9.478/97 determined that the existing subsidies be abolished and grant of new specific subsidies be approved by parliament. Another concern regarding these prices is the capacity of national refineries to compete with imported products. The closure of a refinery, besides resulting in numerous job losses, may also raise the prices of derivatives under the refinery's influence due to a lower degree of competition among the supply sources.

One tool that can be used to estimate these new prices is the spatial price equilibrium model. However, due to the peculiar characteristics of the oil industry and the fact that the Brazilian market is not totally competitive (there is one big company with the capacity to supply a large part of the demand), classical spatial price equilibrium models may be inadequate for this reality. Therefore, a new oligopoly spatial equilibrium model will be proposed in the following sections, taking into account the special characteristics of production of oil derivatives. Such model may be used both by the regulatory authority and by companies in the sector, whose objective is to optimize production considering the probable reactions of competitors.

2. The spatial price equilibrium

The equilibrium between supply and demand among spatially separated markets has been studied for several years. In Cournot's classic work (Fisher, 1898 and Samuelson, 1952), the concept that competitive price is defined by the intersection of the supply and demand curves was established for the first time. Moreover, this work considered two markets spatially apart and transportation costs were considered while determining a competitive price.

The general problem of the spatial price equilibrium assumes that production and consumption exist in various spatially separated locations, which are connected through a transport network. There may be several routes between a production and a consumption site. The problem then is to determine the supply prices, the demand prices and the commercial flows, satisfying the following equilibrium conditions: demand price is equal to supply price plus transportation cost, if there is business between the supply and the demand markets; and if demand price is lower than the supply price plus transportation cost, there is no business between the supply and demand markets (Nagurney, 1993). Spatial price equilibrium models and traffic equilibrium models are closely related. The equivalence between the two problems can be seen in Daza (1989) and Nagurney (1993).

This problem considers that in each supply market there are several firms with similar technology trying to maximize their profits but, due to the competition between them, no one profits in the equilibrium. This concept of zero profit assumes a long-term scenario where the firm's costs include remuneration from investments.

2.1 Mathematical formulation for the spatial equilibrium of prices

To formulate this problem, we consider a set J of supply markets and a set K of demand markets respectively producing and requiring a certain product. We denote by s_j the quantity of products supplied by market j. These supplies may be aggregated in a column vector of supply $\mathbf{s}$. Associated to the supply in market j we have π_j, the supply price of the product in j. Similarly, we aggregate these prices in a line vector of supply prices $\boldsymbol{\pi}$.

On the consumption side, we have d^k, the demand associated to market k and the demand column vector $\mathbf{d}$. The demand price in k is denoted by pr^k and the line vector of the demand prices is $\mathbf{pr}$. The non-negative flow of the product sold between the supply and the demand markets (j,k) is denoted by y_j^k. Similarly, these flows are aggregated in a column vector of dispatches $\mathbf{y}$. Finally, ct_j^k denotes the unit sale cost (including transportation costs) associated with selling the product between (j,k), and $\mathbf{ct}$ is the line vector of the sale cost. Schematically we have:

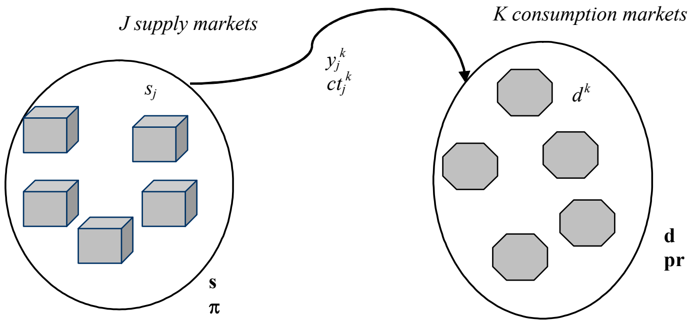

Figure 1 – Scheme of spatial price equilibrium problem.

The following feasibility conditions must be ensured for the entire supply market j and demand market k:

$$s_j = \sum_{k \in K} y_j^k \quad \text{e} \quad d^k = \sum_{j \in J} y_j^k . \tag{1}$$

The market equilibrium conditions presented above assume the following form:

$$\pi_j + ct_j^k \begin{cases} = pr^k, & \text{se } y_j^{k*} > 0 \\ \geq pr^k, & \text{se } y_j^{k*} = 0 \end{cases}, \tag{2}$$

for all pairs of supply and demand markets *(i,j)* ∈ (J,K).

The conditions (2) state that, in equilibrium, if there is business between the *(j,k)* pair, the supply price in j plus the selling costs for the pair *(j,k)* must be equal to the demand price in k; and if the supply price in j plus the selling costs between *(j,k)* exceed the demand price in k, then there will be no dispatch of products between j and k.

The equilibrium conditions defined in (2) may be restated as a variational inequality problem, that is, the levels of production, sale and consumption of a product (**s***,**y***,**d***), satisfying (1), are in equilibrium if and only if they satisfy the following variational inequality problem:

Find (**s***,**y***,**d***), such that:

$$\boldsymbol{\pi}\left(\mathbf{s}^*\right)\cdot\left(\mathbf{s}-\mathbf{s}^*\right)+\mathbf{ct}\left(\mathbf{y}^*\right)\cdot\left(\mathbf{y}-\mathbf{y}^*\right)-\mathbf{pr}\left(\mathbf{d}^*\right)\cdot\left(\mathbf{d}-\mathbf{d}^*\right)\geq 0 , \tag{3}$$

For all (**s**,**y**,**d**) satisfying (1).

2.2 Mathematical formulation of a variational inequality problem

The variational inequality problem is a general formulation covering several mathematical problems such as non-linear equations, optimization problems, complementarity problems and fixed-point problems (Nagurney, 1993).

The variational inequality problem in finite dimensions, $VI(G,L)$, is to determine a vector $\mathbf{x}^* \in L \subset Rn$, such that $G(\mathbf{x}^*)^T.(\mathbf{x}-\mathbf{x}^*) \geq 0, \ \forall \mathbf{x} \in K$, where G is a given continuous function in L for Rn and L is a given closed convex set.

For the case of spatial price equilibrium, it can be shown that the formulation by quantity may be obtained from simple assumptions in the general formulation of variational inequalities. Considering the function G as representing the difference between the production and sales costs and the demand price,

$$G_j^k{*} = \pi_j(\mathbf{s}^*) + ct_j^k(\mathbf{y}^*) - pr^k(\mathbf{d}^*), \tag{4}$$

the variational inequality (3) may be restated as:

$$G_j^{\mathbf{k}}{*}(\mathbf{y}^*)(y_j^{\mathbf{k}} - y_j^{\mathbf{k}}{*}) \geq 0, \qquad \text{for all } j \text{ and } k. \tag{5}$$

The variational inequality (5) may be reduced to equilibrium conditions (2) based on the following considerations (Daza, 1989):

a) If $y_j^k* > 0$ and $G_j^k > 0$, then a flow exists between the supply and demand pair and the "profit" obtained in the transaction is negative. In this case, there may exist some non-negative quantity y_j^k smaller than y_j^k* that makes (5) infeasible.

b) If $y_j^k* > 0$ and $G_j^k < 0$, then a flow exists between the supply and demand pair and the "profit" obtained from the transaction is positive, that is, the producer/agent may obtain advantages in the consumer's market, as the sale price is higher than the sum of purchase price plus transportation cost. In this case, there may exist some non-negative quantity y_j^k greater than y_j^k*, which makes (5) infeasible.

c) Therefore, in order to satisfy (5), there are only two possibilities, which are reduced to the equilibrium conditions (2): if $y_j^k* > 0$, then $G_j^k = 0$; if $y_j^k* = 0$, then $G_j^k \geq 0$. If the demand price in k is equal to the supply price in j plus transportation cost between j and k, then the flow from j to k may be positive in equilibrium and j can supply k. On the other hand, if $y_j^k* = 0$, any $G_j^k* \geq 0$ satisfies (5), given that $y_j^k \geq 0$, that is, if demand price in k is lower than supply price in j plus transportation cost from j to k, there will be no flow from j to k.

3. Mathematical formulation for oligopoly markets

One of the main assumptions about the theory of economical equilibrium and spatial price equilibrium is that the supply of products takes place in a totally competitive market where no firm can obtain actual profit and, given the equilibrium prices, the quantity produced is exactly equal to the quantity demanded. Nevertheless, this concept of perfectly competitive markets is a bit unrealistic for most of markets such as oil and coal. This assumption also ignores the existence of local monopolies, where a firm has certain power over demand in a particular region, obtaining significant income this way.

In the problem of equilibrium in an oligopoly market, consider that there is a set F of firms ($|F|$ finite) that try to maximize their own individual profits. Each firm has knowledge about the demand and tries to maximize its own profits, evaluating the strategy (production level) of other firms.

The classical oligopoly model considers a demand market and a set F of firms supplying a homogenous product in a noncooperative fashion. A Nash equilibrium is established when all the firms set output levels in such a way that no firm alone can increase its own profit by producing more or less products. This problem is usually formulated as a variational inequality, considering the marginal profit of each firm. A more detailed definition of this model can be found in Murphy *et al.* (1982), Harker (1984) and Nagurney (1993).

Harker (1986) and Nagurney (1993) present a spatial oligopoly model where the profit function of each firm considers the transportation costs from the production plants to the demand markets. Harker (1986) compares the spatial oligopoly game with the spatial price equilibrium, in a totally competitive market, and with two possible monopoly situations. All four approaches have a network structure, and can make use of network algorithms to reach the equilibrium. As expected, in the example presented, the oligopoly game profits, prices and flows are between those of the spatial price equilibrium and those of the monopoly model solution.

Okuguchi & Szidarovsky (1999) present a local oligopoly game with multi-product firms. Each firm has a cost function depending on the manufacturing of all products. In a general approach, a given limited input can be used to produce different products, and then the production levels of the products can depend on each other. However, when formulating the problem as a nonlinear complementarity problem, it is assumed that the production levels do not depend on each other and, for each product, they are in the interval [0, *Lp*], where *Lp* is the firm's production capacity for product *p*. Then, the set of all feasible output vectors $\mathbf{X}_f$, for each firm *f*, equals the Cartesian product of the one-dimensional intervals [0, *Lp*].

Equilibration algorithms for oligopoly models usually make use of diagonalization/relaxation algorithms to solve variational inequalities. These algorithms applied to oligopoly problems have an interesting interpretation as a dynamic process of Cournot expectations (Harker, 1984; Kolstad, 1991; Okuguchi & Szidarovsky, 1999). This process assumes that each player (firm) establish its strategy (production level) by trying to maximize its individual profit, considering that the other players will set their own strategies as the same as before. After all the firms make their choice, the game moves to another state and the firms play again, until no firm can improve its profit.

3.1 Classical oligopoly model – single product oligopoly

Consider a single market where a set *F* of producers are involved in the production of a homogeneous commodity. The quantity produced by firm *f* is denoted by s_f, with the production quantities grouped into a column vector $\mathbf{s} \in \mathrm{R}^{|F|}$, and C_f denotes the cost of producing the commodity by firm *f*. Let *pr* denote the demand price associated with the good. Assume that:

$$C_f = C_f(s_f) \text{ and } pr = pr(\sum_{l \in F} s_l) \tag{6}$$

The production cost of the firm *f* is the function of the total amount produced by the firm, and the demand price is the function of the total amount supplied in the market by *m* firms.

The profit of firm *f*, denoted as u_f can be expressed as:

$$u_f(\mathbf{s}) = \underbrace{pr(\sum_{l \in F} s_l) s_f}_{\text{Net revenue}} - \underbrace{C_f(s_f)}_{\text{Cost of production}} \tag{7}$$

Assuming that the competitive mechanism has a non-cooperative behavior, the optimal solution $\mathbf{s}^*$ is a Nash equilibrium if and only if it satisfies the variational inequality (Nagurney, 1993):

$$\sum_{f \in F} -\nabla u_f(\mathbf{s}^*).(s_f - s_f^*) \geq 0$$
$$\sum \left[\frac{\partial C_f(s_f^*)}{\partial s_f} - \frac{\partial pr(\sum_{l \in F} s_l^*)}{\partial s_f} s_f^* - pr(\sum_{l \in F} s_l^*) \right] \times \left[s_f - s_f^* \right] \geq 0 \;\therefore \forall \mathbf{s} \in R_+^{|F|} \tag{8}$$

Equation (8) represents oligopoly equilibrium where all firms achieve zero marginal profits, that is, for each firm, the marginal revenue equals the marginal cost.

4. Spatial oligopoly game with petroleum companies

4.1 General considerations

Some oligopoly models were developed in order to analyze energy markets. Kennedy (1974) analyzed the price levels in the world petroleum market considering supply and demand aspects and the impact of restricted supplies imposed by the OPEC (Organization of Petroleum Export Countries) oil cartel. Murphy & Mudrageda (1998) presented an economic equilibrium model using a decomposition approach to achieve convergence in large-scale problems as part of NEMS, National Energy Modeling Systems. They present a heuristic approach to allow the models in different branches to be run separately. The sub-models are built separately, some of them using linear programming. They point out that that most of the equilibrium models use Gauss-Seidel or Jacobi iterations as part of the solution algorithms. A more detailed description of NEMS and its modules is found in Gabriel *et al.* (2001).

Refined oil products are usually produced in refineries. The profit maximization problem of refining companies is not simple. They must consider the special characteristics of the refining process. Manufacturing of all the products is closely interrelated. How much of each product will be produced depends on crude oil characteristics, complexity of the refinery (i.e., the ability to transform heavy hydrocarbon chains into lighter ones), and the operational mode chosen. Petroleum refining companies usually make use of linear programming to optimize operation of the refineries and to prepare production schedules. In oil refining, several works proposed optimization schemes for the production scheduling of oil types to be processed and the mix of output products, such as the work by Göthe-Lundgren *et al.* (2000). Then, the profit maximization problem of refining companies must consider this cost minimization problem formulated as a linear programming problem. This problem will be briefly presented here and a detailed description about this profit maximization problem can be seen in Pompermayer (2002).

When considering the formulation of the profit maximization problem for petroleum companies, the more complex ones operate several spatially separated refineries and also produce crude oil. Such companies can refine their crude oil in their own refineries, sell crude oil in the global market and buy other kinds of crude oils to refine in their refineries. The formulation of the profit maximization problem for other types of petroleum companies can be considered as particular cases of this one.

The assumptions made in order to formulate the model are the following: each refinery has its own characteristics, producing different product mixes from the same crude oil, with different operational costs. Finished refined products can be swapped among the refineries, and a pipeline network (or other kind of transportation system) links the refineries. Each refinery has a given capacity, R_j, to refine crude oil. The production of some quantity of one "profitable" product such as gasoline implies production of a residual product such as fuel oil. The quantity of each to be produced depends on the characteristics of the refinery, the crude oil and the operational mode of the refinery, which, in turn, depends on the quality of the crude oil. This is formulated by using restrictions on the conversion rates.

4.2 Non-linear profit maximization problem for petroleum companies

This formulation considers that price and demand are not fixed in each demand sub-region and that the demand for each product in each sub-region is given by a function, which depends on the prices of all products.

In this model, the inverse function of demand is considered symmetric, that is, the cross-effects of prices among different products or among different sub-regions are supposed to be symmetric. One particular example of such symmetric price function is when, for a given product in a given sub-region, it depends only on the demand for this product in that sub-region, which means that the cross-effects are irrelevant.

In the case of an oligopoly market, the demand for product *p* in sub-region *k*, d^{pk}, is supplied by a finite number of firms. Therefore, the price function to be considered must take into account the firm *f*'s own production, d_f^{pk}, as well as the optimal supply of the remaining firms in the market, d_{F-f}^{pk*}, where *F* is the set of firms in the market.

The notation used in this model formulation is now presented.

d^{pk} – demand for each product *p*, $p \in P$, in each sub-region *k*, $k \in K$.

$pr^{pk}(d^{pk})$ – price function dependent on total demand d^{pk} for product *p* in the sub-region *k*.

d_f^{pk} – quantity of product *p* sold in the sub-region *k* by firm *f*.

d_{F-f}^{pk*} – quantity of product *p* supplied by other firms ($\neq f$) to sub-region *k*.

$x_{j,i,om(i)}$ – quantity of crude oil *i* processed in the refinery *j* in the operational mode *om*(*i*).

$s_{j,i,om(i)}^{p}$ – quantity of product *p* produced in refinery *j* using crude oil *i* in the operational mode *om*(*i*).

y_j^{pk} – quantity of product *p* produced in refinery *j* and dispatched to demand sub-region *k*.

ct_j^{pk} – cost of selling (including transportation) product *p* of refinery *j* in market *k*.

cp_i – production cost of crude oil of group *C1*, in facility *i*.

ct_{ij} – transportation cost from production facility *i* to refinery *j*.

P_i – production capacity of unit *i*.

pr_{iC2} – CIF acquisition price of oils of group *C2*.

ct_{0j} – crude oil transportation cost from a virtual point on the region's coast under study (serving as a point of imports) to refinery *j*.

pr_{iC1} – FOB price for sale of crude oil of group *C1* to the international market.

x_i^e – quantity of crude oil from production unit *i* sold to other firms or to the international market, that is, not processed in own refineries.

R_j – crude oil processing capacity of refinery *j*.

$n_{om(i)}$ – number of operational modes for crude oil refining.

$r_{j,i,om(i)}^{p}$ – conversion rate of crude oil *i* to product *p*, processed in refinery *j*, using operational mode *om(i)*.

$co_{j,i,om(i)}$ – unit operating cost of refinery *j* processing crude oil *i* in operational mode *om*(*i*).

The objective is to maximize the profit, which schematically may be written as:

Max profit = Sales revenue – cost of sale – crude oil acquisition and production costs – processing cost + revenue from exporting crude oil.

Therefore, the problem may be formulated as:

Maximize the profit function given below

$$\sum_{p=1}^{n_p}\sum_{k=1}^{n_k}\sum_{j=1}^{n_j}\Big(\big(\overbrace{pr^{pk}\left(d_f^{pk}+d_{F-f}^{pk*}\right)}^{\text{Sales revenue}}-ct_j^{pk}\big)\cdot y_j^{pk}\Big)-\sum_{j\in J_f}\left(\underbrace{\sum_{i\in C1}\left(\left(cp_i+ct_{ij}\right)\cdot\sum_{om(i)=1}^{n_{om(i)}}x_{j,i,om(i)}\right)}_{\text{Production costs of crude oil in the firm's own production units}}+\underbrace{\sum_{i\in C2}\left(\left(pr_{iC2}+ct_{0j}\right)\cdot\sum_{om(i)=1}^{n_{om(i)}}x_{j,i,om(i)}\right)}_{\text{Purchase of crude oil in the market}}+\underbrace{\sum_{i\in C1,C2}\sum_{om(i)=1}^{n_{om(i)}}co_{j,i,om(i)}\cdot x_{j,i,om(i)}}_{\text{Cost of processing crude oil in mode } om(i)}\right)+\underbrace{\sum_{i\in C1}\left(pr_{iC1}-\left(cp_i+ct_{i0}\right)\right)\cdot x_i^e}_{\text{Exports of crude oil}} \qquad (9)$$

subject to:

– refinery capacity constraints:

$$\sum_{i\in C1,C2}\sum_{om(i)=1}^{n_{om(i)}}x_{j,i,om(i)}\le R_j\,, \qquad j\in J_f, \qquad (10)$$

– crude oil production capacity constraints:

$$x_i^e+\sum_{j\in J_f}\sum_{om(i)=1}^{n_{om(i)}}x_{j,i,om(i)}\le P_i\,, \qquad i\in C1,C2, \qquad (11)$$

– conversion rates constraints:

$$s_{j,i,om(i)}^{p}=x_{j,i,om(i)}\cdot r_{j,i,om(i)}^{p}\,, \qquad j\in J_f,\ p\in P,\ i,\ om(i), \qquad (12)$$

– production-shipping conservation constraints:

$$\sum_{k\in K}y_j^{pk}=\sum_{i\in C1,C2}\sum_{om(i)=1}^{n_{om(i)}}s_{j,i,om(i)}^{p}\,, \qquad j\in J_f,\ p\in P, \qquad (13)$$

– shipping-demand conservation constraints:

$$\sum_{j\in J_f}y_j^{pk}=d_f^{pk}\,, \qquad p\in P,\ k\in K, \qquad (14)$$

and

$$x\ge 0\,,\quad y\ge 0\,.$$

The first term of the objective function is the revenue obtained from selling refined products in all the markets minus transportation costs. The price of each product p in each sub-region k is a function dependent on the total supply of the product in this sub-region, defined by the supply from the firm under analysis and from all the other firms in market. In a more general formulation, this price function should be dependent not only on the local demand for the product, but also on the demand of all products in all sub-regions. This simplification used in (9) was intended to facilitate the presentation of the oligopoly equilibration algorithm in section 4.4.

The second term is the cost of producing all the refined products expressed as the cost of acquiring crude oil, either from own production or bought in the global market, including transportation of the crude oil to each refinery, plus the operating costs of each refinery, which depend on the crude oil and the operational mode. The difference between the first term and the second term results in the profit obtained from producing and selling refined products. The third term is the profit obtained from exporting crude oil, instead of using it in the firm's own refineries.

The solution of this problem yields the optimal production plan for refineries and crude oil production facilities. The time period considered is short-term; in other words, no increase in production or refining capacity is considered; only the capacities already installed are taken into account. Therefore, the costs of producing and refining crude oil must consider only the variable costs. This means that return on investments, depreciation and amortization are not considered, because they will not affect the firm's profitability in the short term, as the capital has already been spent. Thus, only the costs really involved in the daily production activities are taken into account.

4.3 Formulation of the problem as a variational inequality (classical oligopoly)

Consider a set F of firms, where $|F|$ is finite, supplying to a set K of markets (sub-regions) with a set P of products. Each firm f, $f \in \mathrm{F}$, operates a set of production units J_f, and in each unit j, $j \in J_f$, one product p, $p \in P$, or more than one product may be produced.

In the oil industry, if the competitor is not a trader, seldom can the cost of production be evaluated for each product individually, given the intrinsic characteristics of the oil refining process.

An inverse demand function (price function) is assumed for product p in market k, denoted by $pr^{pk} = pr^{pk}(\mathbf{d})$, depending on the consumption levels of all the products in all the demand markets.

Variables

s_{jf}^{p} – production of product p in unit j of firm f.

$\mathbf{s}_{jf}$ – production vector of unit j of firm f.

$\mathbf{s}_{f}$ – production vector of firm f.

$\mathbf{s}$ – production vector of the industry.

C_{jf} – total production cost of unit j of firm f (cost is dependent on the entire production vector of industry $\mathbf{s}$).

y_{jf}^{pk} – the quantity of a product p sent from a production unit j of a firm f to a demand market k. Similarly to production levels, the flow levels may be aggregated in vectors $\mathbf{y}_f$, a vector of all flows from firm f among all its units, J_f, and all the demand markets K, and in the standard flow vector of industry $\mathbf{y}$.

ct_{jf}^{pk} – transportation cost of a product p from a production unit j, of a firm f, to a demand market k.

d_f^{pk} – quantity of a product p acquired (or consumed) in a demand market k from a firm f. It may be aggregated in vectors $\mathbf{d}_f$, a consumption vector of all the products in all the markets of a firm f, and in $\mathbf{d}$, the consumption vector of all the products in all the demand markets.

Equations of flow conservation like (13) and (14) must be fulfilled, maintaining the relationship between the variables $\mathbf{s}$, $\mathbf{y}$ and $\mathbf{d}$. Considering a firm f in particular, the profit function U_f of this firm may be written as:

$$U_f = \underbrace{\sum_{p\in P}\sum_{k\in K} pr^{pk}(\mathbf{d})\,d_f^{pk}}_{\text{Sales Income}} - \underbrace{\sum_{j\in J_f} C_{jf}(\mathbf{s})}_{\text{Production Cost}} - \underbrace{\sum_{p\in P}\sum_{(j,k)\in(J_f,K)} ct_j^{pk}(\mathbf{y})\,y_j^{pk}}_{\text{Selling Cost}} \tag{15}$$

Production constraints like production capacity and the relationship among the products must also be considered.

A Nash equilibrium exists when no firm in F can unilaterally improve its profit by changing its production strategy. For the equilibrium problem of multi-product spatial oligopoly in the oil industry, this equilibrium solution may be formulated as a variational inequality problem.

The finite dimensional variational inequality problem, $VI(G,L)$, is to determine a vector $\mathbf{x}^* \in L \subset R^n$, such that $G(\mathbf{x}^*)^T(\mathbf{x}-\mathbf{x}^*) \geq 0$, where G is a given function from L to R^n and L is a given closed convex set.

Let $\Omega_{\mathbf{dsy}}$ be the set of constraints for flow conservation, production and non-negativity of the variables $\mathbf{d}$, $\mathbf{s}$ and $\mathbf{y}$. For the oil derivatives market, they would be restrictions like (10), (11), (12), (13) and (14) for each firm in F aggregated to form the set of restrictions of the industry. Therefore, the oligopoly equilibrium problem may be written as:

Find ($\mathbf{d}^*$, $\mathbf{s}^*$, $\mathbf{y}^*$) such that:

$$\sum_{f\in F} \nabla_{\mathbf{y}} U_f(\mathbf{y}).(\mathbf{y}-\mathbf{y}^*) \geq 0\ ,\ \forall(\mathbf{d},\mathbf{s},\mathbf{y}) \in \Omega_{\mathbf{dsy}}\,. \tag{16}$$

or

$$\begin{aligned}&\sum_{f\in F}\sum_{j\in J_f}\sum_{p\in P}\frac{\partial C_{jf}(\mathbf{s}^*)}{\partial s_{jf}^p}\left(s_{jf}^p - s_{jf}^{p*}\right) + \sum_{f\in F}\sum_{p\in P}\sum_{j\in J_f}\sum_{k\in K} ct_j^{pk}(\mathbf{y}^*)\left(y_{jf}^{pk} - y_{jf}^{pk*}\right)\\&-\sum_{f\in F}\sum_{p\in P}\sum_{k\in K} pr^{pk}(\mathbf{d}^*)\left(d_f^{pk} - d_f^{pk*}\right)\\&-\sum_{f\in F}\sum_{j\in J_f}\sum_{p\in P}\sum_{k\in K}\sum_{q\in P}\sum_{l\in K}\left[\frac{\partial pr^{ql}(\mathbf{d}^*)}{\partial d_f^{pk}} - \frac{\partial ct_j^{ql}(\mathbf{y}^*)}{\partial y_{jf}^{pk}}\right] y_{jf}^{ql}\left(y_{jf}^{pk} - y_{jf}^{pk*}\right) \geq 0\ ,\ \forall(\mathbf{d},\mathbf{s},\mathbf{y}) \in \Omega_{\mathbf{dsy}}\,.\end{aligned} \tag{17}$$

The existence of a solution for (16) or (17) is ensured given that the set of restrictions $\Omega_{\mathbf{dsy}}$ is compact and convex and that $\sum_{f \in F} \nabla_{\mathbf{y}} U_f(\mathbf{y})$ is a continuous function (Nagurney, 1993). Conditions of a single solution, however, are not easy to analyze, given the non-differentiability of $\sum_{f \in F} \nabla_{\mathbf{y}} U_f(\mathbf{y})$, especially due to the cost function, in the case of oil companies, and because the problem is spatial and multi-product.

4.4 An algorithm for the oil oligopoly equilibrium

One of the common approaches to solve variational inequalities is using a relaxation algorithm. These algorithms, especially for oligopoly equilibration, solve a sequence of profit maximization problems for each firm. The price functions in each firm's profit maximization problem consider the supply of other firms, assuming that they will follow the same strategy of the previous iteration. The economic interpretation is that each firm assumes that the other firms will not react to its strategy, setting their strategies as the same as before. Okuguchi & Szidarovsky (1999) called this scheme a "Cournot expectations adjustment process". Another adjustment process, called "sequential adjustment process" by Okuguchi & Szidarovsky (1999), assumes that each firm uses the latest information available. Then, instead of using the strategies of other firms obtained in a previous iteration, one can use the strategies obtained at the same iteration for the firms that have already played. These correspond to a Jacobi scheme and a Gauss-Seidel scheme, respectively.

This diagonalization algorithm solves a problem such as (9)-(14) for each firm iteratively. Thus, one firm considers the strategies of the others using the results from a previous iteration. However, considering the characteristics of the price function of each product, updates of this function must be made iteratively, to capture cross-price effects when substitute fuels have different price changes.

A sequential adjustment process, which is a Gauss-Seidel method, seems to provide better convergence results (Okuguchi & Szidarovsky, 1999). It assumes that each firm maximizes its profit by using the latest information available. Let $U_f(t)$ denote the profit function of firm f at iteration t. This algorithm is summarized below:

Step 0 (Initialization): Obtain a feasible strategy for each firm, using historical data on prices, demands and supply or even solving the profit maximization problems (9)-(14) of each firm assuming that the strategies of all other firms are zero supplies. Set $t = 0$.

Step 1 (Iteration): Solve the profit maximization problem (9)-(14) for each firm f assuming

$$d_{F-f}^{pk} = \sum_{g=1}^{f-1} d_g^{pk}(t) + \sum_{g=f+1}^{|F|} d_g^{pk}(t-1). \tag{18}$$

After solving the profit maximization problem (9)-(14) for each firm, update the price function for each p and k with the new prices for $q \neq p$ in sub-region k, capturing the cross-price effects.

Step 2 (Convergence test): If $(\mathbf{d}_f(t) - \mathbf{d}_f(t\text{-}1)) < \varepsilon$, an arbitrary small tolerance, for all firms, then stop. Otherwise, set $t = t + 1$, and go to Step 1.

The price function update in Step 1 could be done in Step 2 after all the firms had played and in this case firms would be less sensitive to changes in the demand function caused by cross-price effects. It can be seen as if the price functions were updated such as in the Jacobi method. Other similar approaches to oligopoly equilibration algorithms can be seen in Kolstad (1991), Marcotte (1987) and Nagurney (1991).

5. Numerical example

The algorithm presented in the previous section was applied to the Brazilian market for refined petroleum products. This application was based on data available for April 2001. At that time, the Brazilian market was not open to competition, and the government established the former refinery's prices for the main products. Price elasticities were estimated based on the time series data from 1977 to 1997. Linear price functions were fitted to data, which include cross-price effects.

Three aggregated products were considered: light distillates, medium distillates and heavy distillates. The main product from light distillates is gasoline; diesel oil from medium distillates, and fuel oil from heavy distillates. Table 1 shows the price and cross-price elasticities for these three products, estimated for the Brazilian market. There are fifteen internal demand sub-regions where the demand functions for these products were calibrated. These sub-regions have been defined to correspond to storage terminals and distribution centers used by distribution companies, where competition is expected in the supply of refined products.

Table 1 – Price elasticities used in the simulation.

Price elasticities	Gasoline	Diesel	Fuel oil
Gasoline	-0.15	0.03	0.00
Diesel	0.05	-0.12	0.04
Fuel oil	0.00	0.03	-0.40

Thus, a linear price function of the form

$$pr^{pk}\left(d^{pk}\right)=\frac{pr_0^{pk}}{\varepsilon^{pp}}\cdot\frac{d^{pk}}{d_0^{pk}}+\frac{pr_0^{pk}\left(\varepsilon^{pp}-1-\Delta pr^{-pk}\right)}{\varepsilon^{pp}} \tag{19}$$

is obtained for each product p and sub-region k, and is dependent only on the total demand for product p in sub-region k, d^{pk}. ε^{pp} is the price elasticity of product p. pr_0^{pk} and d_0^{pk} are the known price and demand for product p in sub-region k, in a given period of time. For this application, these prices and demands are based on data available for April 2001, and are listed in Tables 2 and 3 below, for light distillates only. The term Δpr^{-pk} captures the cross-price effects among different products using the prices obtained in the previous iteration in the algorithm and the cross-price elasticities ε^{pq} ($q \neq p$). It is defined as:

$$\Delta pr^{-pk}=\sum_{q=1,q\neq p}^{n_p}\varepsilon^{pq}\frac{\Delta pr^{qk}}{pr_0^{qk}}\text{, where }\Delta pr^{qk}=pr^{qk}-pr_0^{qk}\,. \tag{20}$$

Three refining companies can supply any of the internal demand sub-regions as well as export to an external sub-region. The costs for transporting to any of the sub-regions are estimated by considering the mode used, and the capacity of each mode and terminal, in the case of multi-modal transportation. Two of these companies operate one refinery each, and buy crude oil in the global market. These are labeled Refiner 2 and Refiner 3 in the Tables below. The other company, the larger one (labeled Refiner 1), operates 10 refineries and several crude oil production facilities. It operates more than 90% of the existing refining capacity in Brazil. Figure 2 shows a map of Brazil with the refineries, terminals and the demand sub-regions considered in this example.

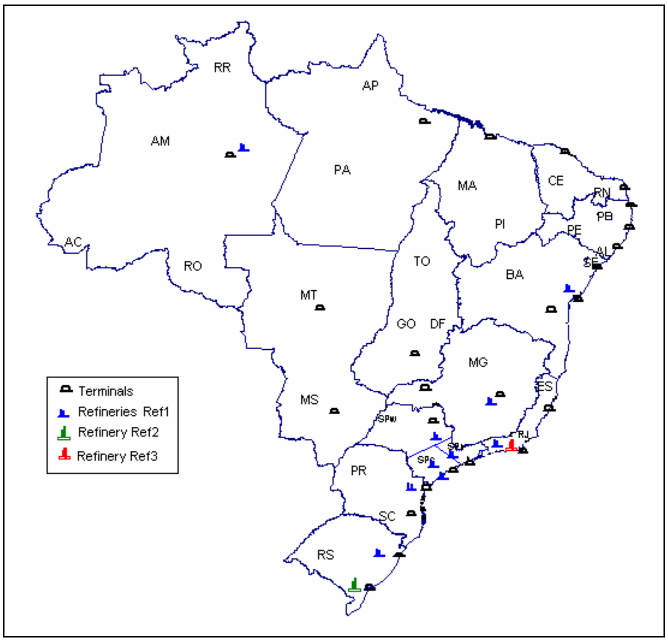

Figure 2 – Brazil, demand sub-regions, refineries and terminals.

Seven types of crude oil are considered: three produced in Brazil and four from other countries that usually trade crude oil with Brazil. Acquisition costs for the "foreign" crude oils are estimated based on prices published in the market and on transportation costs for each refinery. Costs of crude oil production were estimated by considering the type of exploitation (on-shore / off-shore). These production costs consider only the short-term parcels, in order to be comparable to the option of import crude oil. No remuneration for the research and development of the field should be considered, since this investments made in the past can be considered sunk costs.

Each refinery may operate in two modes: "maximum gasoline" and "maximum diesel". Therefore, rates of converting each type of crude oil into each derivative were estimated for each refinery using a given operational mode. This estimation considered the complexity of each refinery and the characteristics of the crude oil, especially density. These data were used in estimating the refining costs as well.

It is assumed that some trading companies may also supply refined products to any of the 15 internal demand regions, acquiring products basically in two foreign points, Argentina and Venezuela. The traders purchase the products at a fixed price. This assumption is based on the fact that the volume acquired by each trader must be small compared to the global market, not influencing the prices in the market (total demand in the acquisition points unilaterally). It is also assumed that transportation costs are constant. Assumption of fixed purchase prices is appropriate especially when representing the newcomers in the petroleum refining market as traders, as it will provide the price at which the first new entrants can supply the demand markets.

Two situations are then considered, with one trader and with two traders, in order to show the impact of one extra firm competing in supplying this market. Table 2 presents the actual consumption (d_0^{1k}) for each sub-region in April 2001 for the light distillates as well as the resulting supplies of the two scenario simulations. Table 3 presents the regulated prices (pr_0^{1k}) for light distillates practiced in April 2001, and the equilibrium prices in the two scenarios. Similar results were obtained for medium and heavy distillates.

Table 2 – Light distillates – Consumption and Supplies [m^3].

d^1		Scenario 1					Scenario 2					
Sub-regions	**(Apr/2001)**	**Ref. 1**	**Ref. 2**	**Ref. 3**	**Trader 1**	**Total**	**Ref. 1**	**Ref. 2**	**Ref. 3**	**Trader 1**	**Trader 2**	**Total**
PA/AP	28,077	10,384	457	566	9,772	21,178	7,813	335	493	7,201	7,201	23,044
AM/RR/RO/AC*	36,631	13,943	279	425	12,746	27,393	10,555	186	399	9,357	9,357	29,853
MA/PI	26,422	9,776	394	497	9,267	19,935	7,339	271	420	6,830	6,830	21,689
CE/RN	53,861	20,040	750	962	18,792	40,544	15,080	549	856	13,832	13,832	44,147
PE/PB/AL	75,001	27,799	1,218	1,543	26,109	56,669	20,929	897	1,353	19,238	19,238	61,655
BA/SE*	82,436	31,678	609	941	28,468	61,696	24,071	482	965	20,861	20,861	67,239
RJ/ES**	183,231	69,968	1,958	3,538	62,530	137,995	52,952	1,820	3,733	45,805	45,805	150,117
MG*	188,306	73,037	1,305	2,942	64,054	141,339	55,537	1,288	3,270	46,855	46,855	153,805
SP W*	144,365	56,038	1,363	1,725	49,158	108,284	42,818	1,292	1,918	35,939	35,939	117,907
SP central**	200,000	77,108	2,530	2,843	67,967	150,447	58,872	2,346	3,022	49,731	49,730	163,701
SP NE*	300,000	114,900	3,949	4,694	102,380	225,923	87,467	3,596	4,885	74,947	74,948	245,843
RS**	154,286	58,675	1,902	1,529	54,473	116,579	44,237	1,471	1,378	40,035	40,035	127,156
PR/SC*	207,805	79,411	1,443	2,093	73,661	156,607	59,884	866	1,897	54,134	54,134	170,915
MS/MT	47,442	17,102	1,941	2,041	15,194	36,278	13,165	1,650	1,823	11,257	11,257	39,151
GO/DF/TO	118,993	45,104	2,360	2,642	39,726	89,833	34,543	2,078	2,567	29,165	29,165	97,517
Sub-total	1,846,856	704,964	22,458	28,980	634,297	1,390,699	535,260	19,128	28,980	465,185	465,186	1,513,739
External	–	1,415,335	0	0	–	1,415,335	1,586,018	0	0	–	–	1,586,018

(*) One refinery in this sub-region. (**) Two refineries in this sub-region.

Table 3 – Light distillates – Prices [$/m^3] - $\mathbf{pr^1}$.

Sub-regions	(Apr/2001)	Scenario 1	Scenario 2
PA/AP	507.70	1,659.86	1,349.98
AM/RR/RO/AC*	492.20	1,634.77	1,331.22
MA/PI	505.50	1,654.04	1,343.13
CE/RN	501.20	1,647.82	1,340.09
PE/PB/AL	507.10	1,650.19	1,340.49
BA/SE*	488.00	1,615.80	1,315.58
RJ/ES**	489.20	1,600.64	1,302.96
MG*	485.30	1,599.21	1,303.70
SP W*	486.20	1,602.39	1,305.58
SP central**	490.60	1,609.15	1,310.93
SP NE*	490.60	1,605.83	1,306.76
RS**	487.80	1,587.88	1,283.56
PR/SC*	485.60	1,588.26	1,284.06
MS/MT	576.20	1,818.92	1,500.13
GO/DF/TO	512.60	1,665.90	1,362.59

The total supplies in the internal sub-regions increase from Scenario 1 to Scenario 2 but are considerably small when compared to the original demand with regulated prices. The prices drop from Scenario 1 to Scenario 2 as a result of a higher level of competition. However, they are still much higher than the regulated prices practiced in April 2001. It can also be seen that the refiners supply a relatively higher quantity of products in the sub-regions close to their refineries, and the traders are more competitive in the sub-regions where there is no refinery and there is easier access to imported products. Results similar to those of light distillates were obtained for medium and heavy distillates. Total supplies increase and prices drop from Scenario 1 to Scenario 2.

These results suggest that the local refiners, especially the bigger one, have an opportunity to achieve high profits by exploiting their advantage in costs and their oligopolistic (almost monopolistic) position. However, in this case other firms would start to enter the market, attracted by the high margins. In the short term the local refiners could impose high prices to the market, based on oligopoly competition, but this could be a dangerous long-term strategy, since other firms would enter the market. It seems that it is very difficult for a firm to enter a new market, but once it has entered, it is even more difficult to leave.

Even with the possibility of trading companies entering the market, these initial results reveal the possibility of the local refiners abusing their market power. Since it takes some time for a new company to enter the market trading refined petroleum products, local refiners may operate using time against new entrants, setting high prices in one moment and reducing them as soon as a new company starts moving to trade products in the local market.

The numerical results obtained are for illustration purposes only. The assumptions made about transportation and production costs, refinery operating modes, and conversion rates were simplified and their values do not correspond to actual values. However, they were estimated with the right order and magnitude. Also, a few taxes that are levied on the sale of certain petroleum products were not considered in the analysis. The numerical example demonstrates the tractability of large problems and the expected decrease in prices when an additional trader takes part in the game and the market becomes more competitive.

6. Conclusions

A model was presented to analyze the problem of multi-product spatial price in an oligopoly, where the supply functions are not explicitly defined. In this model, the supply functions may be defined implicitly from mathematical programming, using, for example, optimization models developed for refineries' production. This model was applied to an oligopoly problem in the oil derivatives market. Some types of firms in this market have complex profit maximization problems where the production of one product is strictly related to the production of others, given the production characteristics of crude oil refining.

An oligopoly model that considers the possibility of newcomers into the market, operating as trading companies, was presented. An application to the Brazilian refined products market has shown that firms should not build strategies considering only their short-term profit maximization problems, since it could attract other firms to the market. These new firms could jeopardize the long-term market share and, possibly, the profits of existing firms. These conclusions were not inferred directly from the model, but analyzing different scenarios. The model itself simulates short-term strategies of firms in oligopoly markets. Long-term strategies considering growth, sustainability and long-term profits may be evaluated with this model.

An analysis of the profit function of each firm in the oligopoly market may lead to a leader-follower oligopoly model, where one of the firms has the power to set the prices while the others simply react to these prices by changing their production plans In this situation, the leading firm tries to maximize its own profit and sets prices by considering the reaction of the other firms in the market and the possibility of new entrants. This is the subject of a forthcoming paper.

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