Pesquisa Operacional (2013) 33(3): 361-382

Printed version ISSN 0101-7438 / Online version ISSN 1678-5142
www.scielo.br/pope

DETERMINATION OF THE CARBON MARKET INCREMENTAL PAYOFF CONSIDERING A STOCHASTIC JUMP-DIFFUSION PROCESS

Fabio Rodrigo Siqueira Batista[1*], Tara Keshar Nanda Baidya[2],
José Paulo Teixeira[3] and Albert Cordeiro Geber Melo[4]

Received December 15, 2011 / Accepted March 15, 2013

ABSTRACT. The objective of this paper is to verify the robustness of the Least Square Monte Carlo and Grant, Vora & Weeks methods when used to determine the incremental payoff of the carbon market for renewable electricity generation projects, considering that the behavior of the price of Certified Emission Reductions, otherwise known as Carbon Credits, may be modeled using a jump-diffusion process. In addition, this paper analyses particular characteristics, such as absence of monotonicity, found in trigger curves obtained through use of the Grant, Vora & Weeks method to valuate these types of project.

Keywords: least square Monte Carlo, grant, Vora & Weeks, jump-diffusion process, carbon market, renewable sources of energy.

1 INTRODUCTION

The enforcement of the Kyoto Protocol and the fines imposed upon European corporations that do not manage to reduce their Greenhouse Gases (GHGs) emissions is making the carbon market a reality in Latin America. According to studies carried out by the Brazilian Government (see Brazil, 2005), Brazil stands out as one of the countries of highest potential in the export of Certified Emission Reductions (CERs), with the capacity to reduce its annual emissions by up to an equivalent of 120.6 million tons of carbon dioxide. It is worth highlighting that a large part of this potential is due to its capacity to produce electrical energy from renewable resources.

As established by the UNFCCC (1998), any project that is developed following the rules established by the Clean Development Mechanism under the Kyoto Protocol, also called CDM

*Corresponding author
[1]Professor Adjunto, Pontifícia Universidade Católica do Rio de Janeiro – PUC/Rio. Pesquisador, Centro de Pesquisas de Energia Elétrica – CEPEL, Brazil. E-mail: fabiorsb@hotmail.com
[2]Professor Associado, Universidade do Grande Rio – UNIGRANRIO/ECT. E-mail: tarabaydia@yahoo.com.br
[3]Professor Associado, Pontifícia Universidade Católica do Rio de Janeiro – PUC/Rio. E-mail: jpt@puc-rio.br
[4]Diretor Geral, Centro de Pesquisas de Energia Elétrica – CEPEL. Professor Associado, Universidade Estadual do Rio de Janeiro – UERJ. E-mail: albert@cepel.br

projects, must have its additionality proven before being registered as such by the United Nations Executive Board. This means that, amongst other things, it must be proven that the project's GHG emissions are lower than those for the baseline scenario, which can be defined as the emission scenario that would be observed if the proposed project were not implemented. Therefore, motivated by the strong conservatism adopted by the Executive Board in recognizing the additionality of proposed projects, different methodologies have been developed to determine a reliable baseline, among which is the ACM0002 methodology – Approved Consolidated Methodology number 2 (see UNFCCC, 2006), which also makes reference to the Tool to calculate the emission factor for an electricity system (see UNFCCC, 2009), that is used in this paper.

According to the scope of the ACM0002 methodology, its objective is to determine the baseline for CDM renewable electricity generation projects, where it is required that they are connected to the electrical grid of the country in which the project will be implemented. In this case it is worth noting that the baseline becomes a direct function of the operation of all of the power plants connected to the grid in which the project will be carried out.

With this in mind, Batista *et al.* (2011a) proposed a methodology capable of determining the incremental payoff of the carbon market for grid-connected renewable electricity generation CDM projects, considering that its baseline behaves as a random variable. In this case, the built-in randomness of the baseline is intended to account for uncertainties associated with the operation of the electricity system in which the CDM project will be connected, in this way including a risk factor in calculating the quantity of CERs that the project will produce and commercialize in the future.

In addition, Batista *et al.* (2011a) considered the uncertainty of the market due to the randomness in CER price when determining the incremental payoff of the carbon market. For this purpose, it was considered that renewable electricity generation projects may have sufficient flexibility to be registered by the United Nations Executive Board, thus allowing the commercialization of CERs generated through their operation. Such flexibility can be understood as an American option, and in this paper it is evaluated as such.

In this analysis the binomial method (see Cox, Ross & Rubistein, 1979) was initially used to evaluate this previously mentioned flexibility. Then the Least Square Monte Carlo (see Longstaff & Schwartz, 2001) and Grant, Vora & Weeks (1996) methods were also used by Batista *et al.* (2011b). Despite the fact that a convergence of results was observed for all of the methods used, the LSM and GVW methods are considered to be more flexible than the binomial method, since they can be used for the evaluation of different types of options or for options with different stochastic factors.

In this context, the main contribution of this paper is to continue the analyses carried out by Batista *et al.* (2011b) verifying the robustness of the Least Square Monte Carlo (LSM) and Grant, Vora & Weeks (GVW) methods when the behavior of the CER price can be modeled using a jump-diffusion process, instead of the traditional geometric Brownian motion used by Batista *et al.* (2011b). As written by Merton (1976), it is known that in the presence of jumps, the

Black-Scholes "hedge" portfolio will not be riskless, and hence, their 'no arbitrage' technique cannot be employed in order to derive an analytical formula for option pricing. Even if one knew the required expected return on the option, the resultant mixed partial differential-difference equation would be difficult to solve. Based on these statements, we decided to resort to the use of numerical methods for option valuation, which justifies the above analysis.

In addition, another contribution of this paper is considered to be the analysis of particular characteristics of the trigger curves obtained through use of the GVW method for CDM projects valuation. Among other characteristics, these curves do not present the generally observed monotonic format found, for example, in the classic works of Siegel, Smith & Paddock (1987) and Dixit & Pindyck (1994).

2 JUMP-DIFFUSION PROCESS

In the international carbon market it can be observed that CER price estimates have a high inherent randomness, owing to existent uncertainties in future supply and demand. For example, between April 24 and May 12 in 2006, the finding that European Union Allowances had been delivered to European corporations in excess caused the drop in the price of these assets from 29.43 to 10.14 €/tCO_2e. More recently the occurrence of jumps on the CER price can also be observed between October 14 and October 27 in 2008, when the price dropped from 20,26 €/tCO_2e to 15,17 €/tCO_2e; between February 2 and February 12 in 2009, when the price dropped from 10,54 €/tCO_2e to 7,6 €/tCO_2e; and between July 26 and August 8 in 2011, when the price dropped from 9,99 €/tCO_2e to 7,75 €/tCO_2e (see http://www.bluenext.fr/statistics/graphs.html).

Taking into account the inadequacy of the standard stochastic diffusion model in representing the reality generally observed in the market, or that the financial and non-financial asset values can present slight random jumps in response to the arrival of new information, alternative stochastic models have been developed. In this paper, we highlight the use of the jump-diffusion model.

The jump-diffusion model was initially proposed by Merton (1976) for the pricing of financial options. In order to adequately model the behavior of financial assets, Merton considered that the irregularity in the price of these assets must be composed of two types of vibrations: normal and abnormal.

According to Merton (1976), normal vibrations can occur due to a temporary imbalance in the supply and demand of the asset, caused by changes in the economic scenario or simply by the arrival of new information capable of causing marginal changes in its price. Merton suggests that these vibrations may be modeled using the geometric Brownian motion.

On the other hand, abnormal vibrations are considered the consequence of new information capable of producing a larger than marginal effect on asset prices. Generally, such information is specific to a particular company or industry. Merton suggests that this type of vibration may be modeled using a Poisson process. It is worth highlighting that in this model the Poisson distribution event is the arrival of new information capable of producing jumps in asset price. These events are considered independent and similarly distributed. Equation 1 describes the jump-diffusion model proposed by Merton:

$$\frac{dS}{S} = (\alpha - \lambda k)dt + \sigma dz + dq\,. \tag{1}$$

In equation 1, S represents the price of the financial asset, α represents the instantaneous expected return of the asset, σ^2 represents the instantaneous variance in the return if the Poisson event does not occur, dz is the increment of the Wiener process, dq is the increment of the Poisson process, λ represents the average number of Poisson event arrivals per unit of time, and k represents the expected percentage variation $(Y - 1)$ of the asset price if the Poisson event occurs, or, $k = E(Y - 1)$, where Y is an impulse function producing a finite jump in S to SY. Note that in this model dz and dq are independent processes.

Supposing that in a small interval of time dt there exists a 100% chance that a maximum of one jump will occur, the previously mentioned stochastic process can be written in the following way:

$$\frac{dS}{S} = (\alpha - \lambda k)dt + \sigma dz + (Y - 1)\,. \tag{2}$$

So, for the opposite situation, where there is a 100% chance that no jump will occur within time interval dt, it must follow that:

$$\frac{dS}{S} = (\alpha - \lambda k)dt + \sigma dz\,. \tag{3}$$

In other words, it can be deduced from equations 2 and 3 that in the absence of jumps the asset price will follow the same dynamics as the model proposed by Black & Scholes (1973), except for the presence of factor λk in the process trend term.

Note that λk must be introduced into equation 1 to correct a potential bias introduced by the use of the Poisson process. As shown in equation 4, this bias owes itself to the fact that the expected value of the Poisson process is not zero:

$$E(dq) = \lambda dt E(Y - 1) + (1 - \lambda dt) \cdot 0 = \lambda dtk\,. \tag{4}$$

Therefore, for the expected return of an asset that follows the jump-diffusion process to be the same as that obtained when the standard diffusion process is used, it is necessary that the process trend term be subtracted from $E(dq)$.

In this paper the jump-diffusion process described in equation 1 is used to model the movement of the CER price. Solving this stochastic differential equation, we obtain the dynamics of the asset price as (see Tsay, 2002):

$$S_t = S_0 \exp\left[\left(\alpha - \lambda k - \frac{\sigma^2}{2}\right)t + \sigma z_t\right] Y(n)\,. \tag{5}$$

In equation 5, z_t represents a random variable with normal distribution, with zero mean and variance t. Still in equation 5, $Y(n) = 1$ if n is equal to zero, and $Y(n) = \prod_{j=1}^{n} Y_j$ if $n \geq 1$, where Y_j are independently and identically distributed, $(Y_j - 1)$ represents the percentage variation in the stock price if the j[th] poisson event occurs, and n is the number of occurrences of the Poisson event distributed within a time interval $[0, t]$. In this paper the price paths obtained from equation 5 will be used in the GVW and LSM methods for the valuation of the considered option.

As described in Section 1, the contributions of this paper are: the proof of the robustness of the GVW and LSM methods in the pricing of the carbon market incremental payoff for some electricity generation projects when the CER price follows a jump-diffusion process, and the analysis of particular characteristics of the trigger curves obtained through the use of the GVW method. In both cases, it is considered that full understanding of the results requires that the reader understand the baseline calculation method used in this paper. This will be discussed in Section 3.

3 BASELINE CALCULATION IN ELECTRICAL INTERCONNECTED SYSTEMS

According to the ACM0002 methodology, the baseline of grid-connected renewable electricity generation projects should be determined using a combination of two types of emission factor: the Operating Margin Emission Factor (EF_{OM}) and the Build Margin Emission Factor (EF_{BM}), as described in equation 6.

$$EF_y = 50\% \cdot EF_{OM,y} + 50\% \cdot EF_{BM,y} \,. \tag{6}$$

In this equation, note that y represents the year for which the baseline is being calculated. Regarding the determination of EF_{OM}, the ACM0002 methodology establishes four different methods that can be applied to grid-connected generation projects: the Simple, Simple adjusted, Dispatch Data Analysis, and the Average Method. Given that the objective of this paper is to verify the robustness of different numerical methods in the valuation of the considered option, and that a comparative analysis of the different methods for baseline emission factor calculations has already been carried out by Batista *et al.* (2011a), in this paper only the Average method will be used. This decision is justified through its simplicity when used in Operating Margin Emission Factor calculations.

According to the methodology adopted by the Average Method, the Operating Margin Emission Factor calculation must be carried out using the following equation:

$$EF_{OM,y} = \frac{\sum_n GEN_{j,y} \cdot COEF_j}{\sum_n GEN_{j.y}} \tag{7}$$

where $GEN_{j,y}$ represents the quantity of electrical energy (in MWh) produced by power plant j in year y, $COEF_j$ represents the carbon dioxide emission coefficient (in tCO_2/MWh) of the primary energy source used by power plant j, and n represents the total number of power plants that belong to the grid where the CDM project is located.

Once the Operating Margin Emission Factor has been determined, the Build Margin Emission Factor must be calculated based on the largest annual generation determined from the following groups of power plants:

- the last five power plants that have been built within the grid where the CDM project is located;
- the most recent additions to the grid capacity of the project, which represent 20% of its total generation.

In both cases, all previously built CDM projects must be excluded from calculation of the Build Margin Emission Factor, which can be determined using equation 7. In this case, parameter n represents the set of power plants as defined by one of the two groups just mentioned. Once EF_{OM} and EF_{BM} have been determined, equation 6 can be used to determine the CDM project baseline emission factor.

It is worth highlighting that the calculated baseline emission factor takes into account that a renewable electricity generation project reduces carbon dioxide emissions by substituting the energy produced by power plants that burn fossil fuels and are connected to the same grid of the project. Therefore, the CDM project baseline becomes a direct function of the operation of all power plants within the system, since the greater the generation due to thermoelectric plants, which in general use fossil fuels extensively, the greater will be the baseline for the CDM project being considered.

Emission reductions achieved by the CDM project can be calculated mathematically in the following way:

$$RE_y = EB_y - EP_Y - F_y \tag{8}$$

where RE represents the reduction in carbon dioxide emissions achieved through operation of the CDM project, EB represents the emissions corresponding to the baseline, EP represents the emissions of the CDM project itself, and F represents any leakages or indirect project emissions. Also in equation 8, note that y represents a period of one year over which project activity is monitored in order to account for the reduction in carbon dioxide emissions.

It is important to say that, for renewable electricity generation projects, such as small hydroelectric plants or wind farms, the ACM0002 methodology determines that both the emissions and the leakages of a CDM project must be considered to be zero. As a consequence, baseline emissions must be determined in the following way:

$$EB_y = EG_y \cdot EF_y \tag{9}$$

where EG_y represents the electricity generation of the CDM project and EF_y represents its baseline emission factor, both determined for a given year y. In this paper it is considered that CDM project generation can be calculated from the product of its installed capacity and the capacity factor of the plant.

Based on the details of the ACM0002 methodology, we concluded that calculating the time evolution of the baseline of such generation projects involves prior recognition of the following variables throughout the period of operation: (1) the thermal and hydraulic generation within the grid; (2) the primary resource of the energy used for each operation within the grid; and, finally, (3) the configuration of the grid's thermal and hydraulic expansion, the latter being of great importance in the calculation of EF_{BM}.

Finally, the estimation of these parameters is important in that it enables the estimation of future baseline scenarios and contributes towards the accuracy of economic-financial viability analyses of CDM projects, the results of which will aid investor decision making.

4 PROPOSED METHODOLOGICAL APPROACH

The objective of this section is to provide a general overview of the solution process adopted in this paper to determine the incremental payoff of the carbon market for renewable electricity generation projects using the GVW and LSM methods. Figure 1 illustrates the proposed methodological approach.

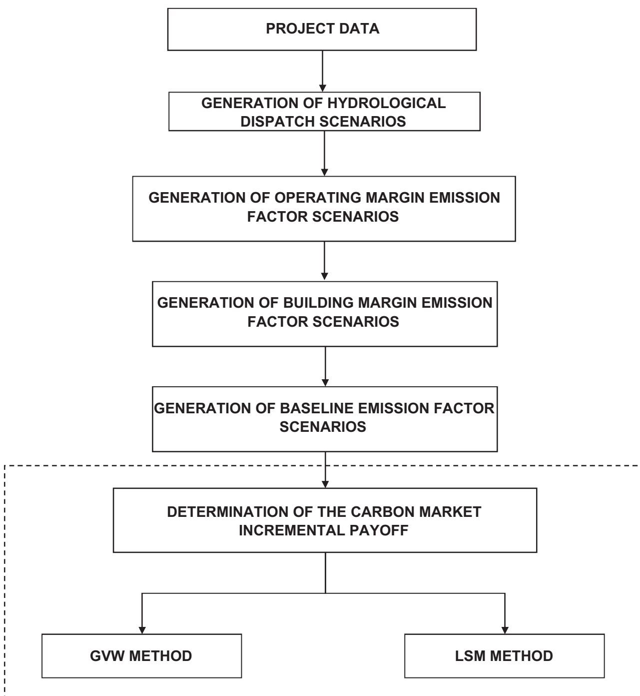

Figure 1 – Proposed methodological approach.

Once the technical and economic characteristics of the CDM project and of the grid are known, the first step is to determine, over time, several hypothetical dispatch scenarios over time for the power plants that belong to the grid configuration.

Assuming that such hydrological scenarios may be determined, the next two steps are to estimate, for each previously defined scenario, the Operating and Build Margin emission factors of the grid. Such factors are determined according to the criteria established within the ACM0002 methodology. After the determination of these emission factors, the next step is to determine scenarios for the CDM project baseline emission factor. To do this equation 6 is used. Once this stage is complete, it is considered that the risk related to the total amount of CERs that will be produced by the project is found to be adequately represented by the scenarios that model the evolution of its baseline.

The final step of the process involves determining the incremental payoff of the carbon market for the CDM project. Once the investor has the right, but not the obligation, to make the additional investment of registering their project with the United Nations Executive Board, it is considered that the project holder has a management option. Such an option allows the investor to apply for and trade CERs during the operational phase of the project. In this paper it is considered that the GVW and LSM methods can be used to valuate this option, and that the CER price can be modeled using a jump-diffusion process.

It is also important to observe that the value of the option is determined for each of the previously estimated baseline scenarios, and that the average of these values represents the final value of the evaluated option.

As explained in Section 3, in order to calculate EF_{OM} and EF_{BM} it is necessary to know which primary energy resources will be used by each power plant within the grid, as well as the configuration of its thermal and hydraulic expansion. In this paper, since we will consider that the CDM project will be developed in the Brazilian Interconnected System, the platform data used by the Brazilian Ministry of Mining and Energy in carrying out its Ten Year Electrical Energy Expansion Plan – PDE (see Brazil, 2006) will be used in our analyses. It is important to say that the PDE analyses are driven by long-term Brazilian electrical energy system guidelines, these being responsible for the identification of main lines of development within electrical energy generation and transmission systems in Brazil.

Note that, in order to obtain thermal and hydraulic energy generation scenarios, which are also required for the calculation of EF_{OM} and EF_{BM} over time, a long-term operation planning model is also needed. In this paper, these information will be provided by the Newave model (see Maceira, 2008), which will use the PDE platform data in order to carry out energy operation planning for the Brazilian Interconnected System. The results are obtained monthly for a period of up to ten years.

It should be noted that the Newave model uses Stochastic Dual Dynamic Programming (see Pereira, 1991) as a solution methodology, as well as being officially used by the System Operator to carry out energy operation planning in Brazil.

Finally, it is important to highlight that the methodology used in this paper comprises some particular features that are not usually found in other papers that provide economic evaluation of renewable electricity generation CDM projetcs. These features are: (i) the random nature of the baseline scenario; and (ii) the random nature of the CER price.

As an example of related works, Roques *et al.* (2006) uses the Real Options Approach to determine the real value of a Nuclear Power Plant when compared to other fossil fuel generation technologies in Europe, thus considering that the gas and the carbon prices are stochastic. In addition to this, Reedman *et al.* (2006) also uses the Real Options Approach to model electricity generation technology adoption in Australia under the carbon price uncertainty, thus considering that the investment decision can be driven by the introduction of a carbon penalty to major emitters.

Concerning to the estimation of baseline scenarios, most works are concentrated on land use projects. For example, Dale *et al.* (2003) projects the effects that land use may have upon land-cover change and carbon storage in the Eastern Panama Canal Watershed. In this case, a spatial modeling approach is used in order to determine regional-scale baseline emissions scenarios. Jong *et al.* (2005) also used a spatial modeling approach for testing and applying a regional baseline for carbon emissions from land-use change in Mexico. None of these works consider the stochastic behavior of the baseline scenario.

5 CASE STUDY

The results and conclusions described in Sections 6 and 7 of this paper are based on the application of the proposed methodology on a wind energy generation project with 135 MW of installed capacity being implemented in the southeast region of Brazil. It is considered that this project is connected to the Brazilian Interconnected System and subject to all terms of the local legislation, such as taxes and industry charges applied in Brazil.

In this case study, transaction costs of US$ 137,500.00 are assumed. These costs represent the additional investment necessary so that all stages of project registration process can be completed with the United Nations Executive Board. It is assumed that the registration process will be complete twelve months after the additional investment has been made, this being the average time taken for the registration of Brazilian projects. Note that only after its registration can the candidate project be classified as a CDM project, thereafter having its future emission reductions converted into CERs.

In the operational phase of the project, annual costs of US$ 9,000.00 are assumed for the verification and certification of CERs, as well as 11% of its total value for issuing and trading. A 43.25% tax payment on revenues gained from the annual sale of CERs was also assumed. It was considered that this revenue should be calculated in the following way:

$$R(y) = P(\iota) \cdot RE_y \cdot (1 - CEC) - CF \tag{10}$$

where $R(y)$ represents liquid revenues for year y, $P(\iota)$ represents the CER price at time (ι) in which the investment option was exercised, RE_y represents the quantity of CERs generated by the project for year y (see Eq. 8), CEC represents the percentage of expenditures involved in CER issuing and trading, and CF represents the annual costs due to CER verification and certification.

Note that the period taken into account for the attainment of CERs is 10 years from the implementation of the project, or in other words, only the CERs obtained during this period will be granted to the project and may then be traded. This assumption is in line with the regulations imposed by the international carbon market.

It should then be clear that in following these adopted assumptions, the decision to register the wind energy project can be made up to twenty-four months after the start of its construction, thereby defining the maturity of the considered investment option. Note that within this period the optimum moment to exercise the option and register the project with the Executive Board is guided by the price level that the CERs might achieve on the international carbon market. To this end, it is considered that the price dynamic follows a jump-diffusion process, with its path being simulated using equation 5. It is assumed that the observed jumps in CER price follow a normal distribution, with an average of 30% and a standard deviation of 15% for the current CER value. In addition, an average occurrence of two jumps per year is assumed for the simulation of CER prices.

All previously outlined assumptions, together with all other considered assumptions, are described in Table 1.

Table 1 – Information needed for the general case study.

Description	Unit	Value
Installed Capacity	MW	135,00
Capacity Factor	%	23,49
Plant Construction Period	months	30
Annual Cost of Capital	%	12,00
Annual Risk-Free Interest Rate	%	8,00
Total Registration Period	months	12
CER Verification/Certification Costs	US$/Year	9.000,00
CER Issuing/Trading Costs	%	11,00
CER Attainment Period	years	10
Exchange Rate	R$/US$	2,20
Global Taxes	%	43,25

Table 2 – Parameters needed for the option pricing under the CER process.

Description	Unit	Value
CDM Total Registration Costs	US$	137.500,00
Initial CER Price	US$/t$CO_2$e	5,00
Annual Volatility of CER Price	%	40,00
Annual Dividend Yield	%	5,00
Option Maturity	months	24
Annual Average Occurance of Jumps	–	2
Average Jump Size	% CER Price	30,00
Jump Size Standard Deviation	% CER Price	15,00

6 RESULTS

In this section we will present the results of the investigation into the robustness of the LSM and GVW methods when used to determine the incremental payoff of the carbon market for grid-connected electricity generation projects. As well as this, specific characteristics of the trigger curves obtained using the GVW method will also be analyzed.

Initially, it is worth noting that the robustness of these methods has already been tested by Batista *et al.* (2011b) for conditions similar to those described in this paper, but considering CER price modeled using a geometric Brownian motion (GBM). In their study the results obtained using the GVW and LSM methods were compared with the option value obtained using the binomial method proposed by Cox, Ross & Rubinstein (1979), the result of which was considered to be a benchmark in the analyses performed. In this paper the convergence of the GVW and LSM methods was tested assuming that the CER price dynamic follows a jump-diffusion process. As described in Section 1, it is known that in the presence of jumps the no-arbitrage assumption is no longer valid, which virtually prevents the pricing of American options through analytical solutions, requiring the use of numerical methods to do so.

Considering that, in the presence of jumps the valuation of options using the binomial method also becomes rather complex, so the results obtained from the GVW method were used as a benchmark in our analyses. Therefore, 100,000 simulated price paths were used in each iteration of this method. Additionally, 96 early exercise dates were used, which is equivalent to dividing the option lifetime (twenty-four months) into weekly time periods. This choice was based on the results obtained by Batista *et al.* (2011b), which in the absence of jumps show that good option value estimates can be obtained using 40,000 simulated paths and 48 early exercise dates.

The measure of accuracy used in these analyses was the percentage Root Mean Square Error (RMSE) of the estimator, whose formula is described in equation 11:

$$CV(\bar{C}) = \frac{RMSE(\bar{C})}{C_{REF}} \times 100 \frac{\sqrt{V(\bar{C}) + [\bar{C} - C_{REF}]^2}}{C_{REF}} \times 100 \tag{11}$$

where CV represents the coefficient of variation or the percentage $RMSE$ of the estimator $\bar{C}$, C_{REF} represents the true value of the option, which will be estimated using the GVW method, and $V(\bar{C})$ represents the variance of estimator $\bar{C}$, which will be determined using the LSM method.

One of the main characteristics of the LSM method is that it supposes that the continuation function of the option can be represented by a linear combination of base functions. Note that the continuation function can be defined as the function that, at any time, provides the value of waiting for new information and decide in the future if it is worth exercising the option. According to Longstaff & Schwartz (2001), various types of functions can be used for this, for example, the Laguerre, Legendre, Chebyshev or Jacobi polynomials.

In this paper the same type of base function was used as that used by Longstaff & Schwartz in their original article:

$$B_1(S) = S^l, \qquad 1 = 1, 2, 3, \Lambda \tag{12}$$

where S represents the price of the underlying asset and l represents the term of the continuation function corresponding to the respective base function. As shown in equation 13, it should be highlighted that Longstaff & Schwartz use the linear combination of two base functions to estimate the continuation value of the option:

$$F_G(w,t) = \sum_{l=0}^{G} a_l B_l(S) = \sum_{l=0}^{2} a_l B_l(S) = a_0 + a_1 \cdot S + a_2 \cdot S^2 \tag{13}$$

Note that the continuation value of the option, when compared to its immediate exercise value, determines, at any time, if it is optimum to exercise the option, given a simulated price path and a given baseline scenario.

As previously described, the methodology proposed in this paper uses various hydrological dispatch scenarios in order to model the uncertainty associated with the CDM project baseline. First, the option is priced for each hydrological scenario, then its final value is obtained from the average of the values obtained in each scenario.

Since the GVW method is intensive in the use of Monte Carlo Simulation, the computational cost of this method is fairly high when compared to the binomial or the LSM method. This problem is further aggravated as the number of baseline scenarios is higher, therefore, for n baseline scenarios, the algorithm proposed by GVW (see Appendix A) should be repeated n times. Because of this, only the first ten hydrological dispatch scenarios generated by the Newave model were used in the convergence analyses.

In the convergence analyses, the variation coefficient related to the least accurate estimative among all baseline scenarios was used. In other words, from all ten baseline scenarios considered, the result used for the convergence analysis is the largest variation coefficient that was found. It was considered in this paper that the estimators with variation coefficients lower than 5% already provide a good approximation of the real option value. The results found are presented in Table 3.

Table 3 – Accuracy of the LSM method.

No. of simulated paths	Number of early exercise dates: 12	24	48	96
5.000	7,89%	9,19%	6,40%	6,25%
10.000	7,10%	6,81%	4,04%	3,61%
20.000	7,12%	4,38%	4,09%	2,74%
40.000	6,26%	4,89%	2,72%	3,40%
100.000	6,34%	3,03%	2,50%	1,59%

Analyzing the results in Table 3 for 100,000 simulated paths and 96 early exercise dates, we concluded that, among all of the considered baseline scenarios, the estimate of lowest accuracy presented a CV value equal to 1.59%, or in other words, the estimates associated with all other scenarios show CV to be lower than this value. In addition, as the number of simulated paths and/or the number of expected option exercise dates increases, the option values calculated by the LSM method tend to converge with the reference value as determined by the GVW method.

From these results, it can be concluded that even when the CER price follows the jump-diffusion process, the LSM and GVW methods can be considered robust in the determination of the carbon market incremental payoff.

Within the particular characteristics of the option evaluated by this study, it is important to note the relationship between the value of the underlying asset (the present value of revenue flows obtained through CER sales), the simulated price for the CER itself, and the considered baseline scenario. For example, considering an investment option (C) under the present value (V) previously mentioned, whose value is a function of CER price (S), it is possible to say that there is linearity between values V and S when S is modeled using some specific stochastic processes. For a given instant, supposing that:

$$V = Q \cdot S \tag{14}$$

where Q represents the number of CERs produced by the project, and S represents the CER price modeled by a jump-diffusion process, it can be deduced that:

$$dS = (\alpha - \lambda k)Sdt + \sigma Sdz + Sdq \tag{15}$$

$$d(Q \cdot S) = (\alpha - \lambda k)QSdt + \sigma QSdz + QSdq \tag{16}$$

$$dV = (\alpha - \lambda k)Vdt + \sigma Vdz + Vdq \tag{17}$$

or, that the value of project V follows the same stochastic process followed by the CER price. Note that this conclusion may have a direct influence on the GVW method solution process. Since the trigger curve of an American option depends only on the dynamics of the value of its underlying asset (V) (see Appendix A), which, in this case, is determined from a random price (S) and from a deterministic Q, it is initially expected that a single trigger curve might be valid for all baseline scenarios.

In addition, note that the trigger curves are generally characterized as monotonic functions (see the work of Siegel, Smith & Paddock, 1987, and Dixit & Pindyck, 1994). For example, it is expected that trigger curves for American options may be strictly decreasing functions of time. According to Hull (1993), this occurs because the closer you get to the expiry date, the lower the opportunity cost for its early exercise, which, as a consequence, reduces its value.

In an attempt to verify the previously stated intuition, trigger curves obtained through use of the GVW method were plotted for two distinct baseline scenarios. The results are shown in Figure 2.

The results in Figure 2 show that the trigger curves do not present a monotonic format, and furthermore, that these curves are hardly equivalent to one another, varying according to the baseline scenario considered. Both results contradict the previously described intuition, and can be explained through analysis of parameter Q described in equation 14.

For the problem in question, Q represents the quantity of CERs produced by the project, which depends as much on the wind energy project generation, considered to be a constant in this

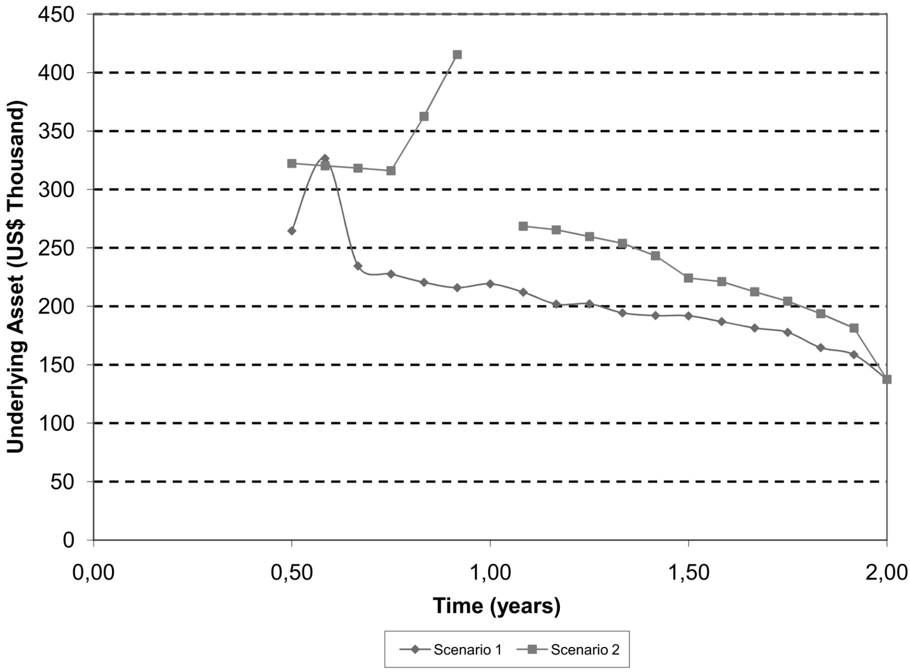

Figure 2 – Comparison of trigger curves of different baseline scenarios.

study, as it does on the baseline emission factor. Since the baseline emission factor is updated on a monthly basis, it can be concluded that Q, despite being able to be considered deterministic at the moment in which the option is valued, is not constant over time. This consideration directly effects the relationship between the probability distributions of V and S, which are then no longer constants.

Note that this finding is independent of the stochastic process considered. For example, considering that S follows an GBM such as that described in equation 3, note that the following result can be obtained:

$$V_{t+\Delta t} = Q_{t+\Delta t} \cdot S_{t+\Delta t} \tag{18}$$

It is known that at $t + \Delta t$ the expected value of a random value that follows an GBM is given by (see Hull, 1993):

$$E\big[S_{t+\Delta t}\big] = S_t \cdot e^{r.\Delta t} \tag{19}$$

Therefore, it follows that:

$$E\big[V_{t+\Delta t}\big] = Q_{t+\Delta t} \cdot S_t \cdot e^{r.\Delta t} = Q_{t+\Delta t} \cdot E\big[S_{t+\Delta t}\big] \tag{20}$$

Using the same rational for the variance of the project value, it follows that:

$$Var\big[V_{t+\Delta t}\big] = Q^2_{t+\Delta t} \cdot S^2_t \cdot e^{2.r\Delta t} \cdot \big[e^{\sigma^2 \Delta t} - 1\big] = Q^2_{t+\Delta t} \cdot Var\big[S_{t+\Delta t}\big] \tag{21}$$

In summary, it can be concluded that if the relationship between the probability distributions for V and S is constant, the trigger curve for the derivative can be characterized as a function that is monotonic and is independent of the baseline scenario. However, in the opposite case nothing can be said about its monotonicity or independence from possible non-financial uncertainty scenarios. The determination of trigger curves with the characteristics described in this paper is original, diverging from what has been seen by Siegel, Smith & Paddock (1987) and Dixit & Pindyck (1994), for example.

Assuming that the baseline emission factor of the project is constant over time, note that the monotonicity of the trigger curves, as much as their independence in relation to the non-financial uncertainty scenario tend to be re-established. These results are illustrated in Figure 3.

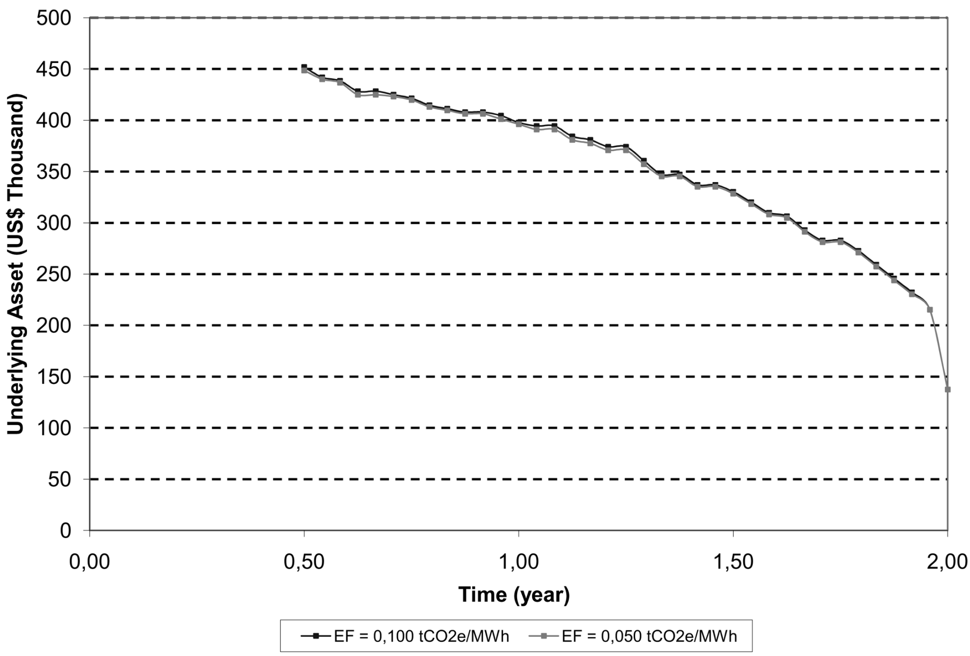

Figure 3 – Comparison of trigger curves for different baseline scenarios with constant emission factors over time.

Back in Figure 2, note that where instant t is equal to one year, it was not possible to find the trigger curve value for baseline scenario 2. This means that regardless of the price that the CER reaches, early exercise of the option would not be optimum at this instant. Such a fact may also be attributed to the variation of the CDM project baseline during the study period.

In Figures 2 and 3 it is also shown that exercise of the option is not optimum in the first six months of its lifetime. This is explained in noting that once the option is exercised, twelve months are necessary for the project to be registered by the Executive Board. In addition, even once the project is registered, CERs will only start to be produced when the project is in operation, or when the construction of the power plant is complete. Since the time period for construction of the plant is eighteen months, it is clear that exercise of the option during the first six months of construction would generate a financial cost due to a larger time interval between the additional investment and the start of CER production.

The results presented until now consider a CER price modeled by a jump-diffusion process whereby the jumps may be either positive or negative, or, in other words, when a random jump occurs, CER price can either rise or fall. Considering only the occurrence of negative jumps, thus reducing the initial value of the CER, it is noted that the trigger curves present different behavior to that previously described. Figure 4 shows the trigger curves determined using different average values (μ_Φ) for random negative jumps.

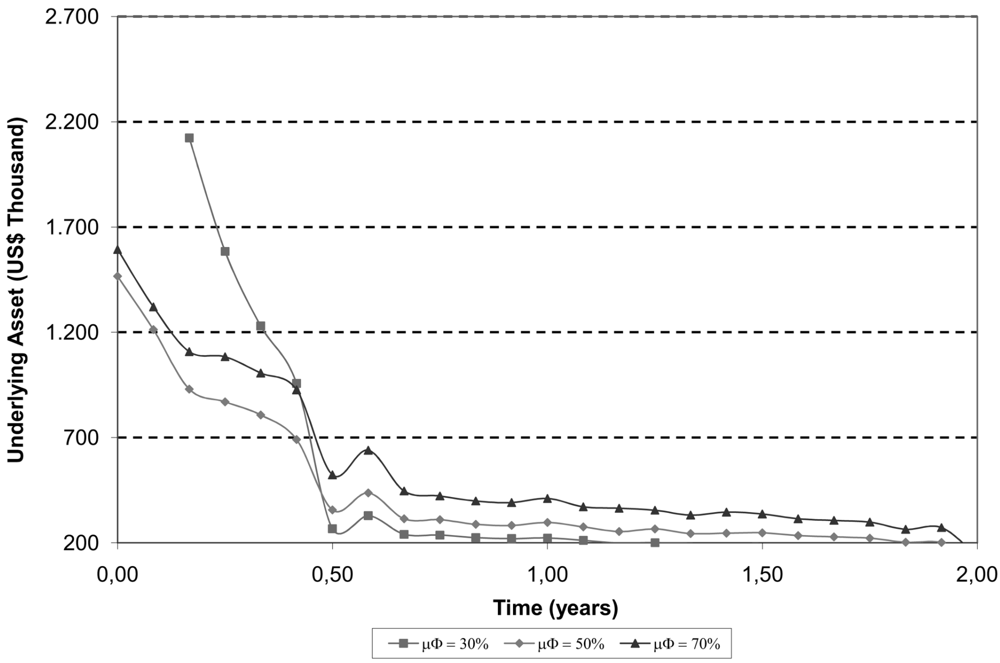

Figure 4 – Trigger curves assuming only negative jumps in the CER price paths.

Contrary to what was observed in Figure 3, the use of a jump-diffusion process for exclusively negative jumps results in the possibility of early exercise of the option being optimum, even during the first six months. In this case, it is noted that for large CER price levels (or large project values), the opportunity cost for non-immediate exercise is high. This is because in the future the CER price would be subject to sudden reductions in value. In this case, it is noted that the immediate exercise value of the option may surpass the benefit of waiting for new information, even if there is a financial cost associated with a larger gap between the registration date of the project and the start of its operation. Upon analyzing the situation where both positive and negative jumps can occur (Fig. 2), note that such an opportunity cost no longer exists, as in the future, sudden price reductions as well as large increases in the value of the underlying asset may be observed.

7 CONCLUSION

This paper verified the robustness of the Grant, Vora & Weeks and Longstaff & Schwartz methods when used to determine the incremental payoff of the carbon market in electricity generation projects, considering that the CER price follows a jump-diffusion process. It was observed that when the number of simulated price paths exceeds 20,000, and when the number of dates for early exercise of the option exceeds 24, in this case being equivalent to monthly time periods, the value of the option as determined using the LSM method converged with the value estimated by the GVW method for all baseline scenarios. This analysis gains relevance, as in the presence of jumps the assumption of no arbitrage is no longer valid and impedes the use of analytical solutions in the valuation of options, making the use of numerical methods necessary in order to do so.

The particularities of the analyses carried out in this paper are due to the special characteristics involved in the determination of the carbon market incremental payoff for grid-connected electricity generation projects. In this case, in light of uncertainties associated with the dispatch of power plants within the system, for example, hydrological uncertainty, different project baseline scenarios may be observed in the future. In each of these scenarios it was noted that the baseline does not remain constant during the study period, but instead varies as the group of power plants utilized within the grid varies.

This means that the relationship between the probability distributions of the CER price and the value of the option's underlying asset may not be constant within its exercise period, eliminating what is known as linearity between the stochastic processes of these two variables. As a consequence, results seen in the literature were not observed. The trigger curves for the underlying asset of the considered management option did not present monotonicity, and hardly presented themselves as continuous, and in addition it was not possible to find a trigger curve that was valid for all baseline scenarios. These findings were considered original contributions of this paper.

Finally, different behavior was observed for the trigger curve of the derivative depending on the nature of the jumps considered in the simulation process of the CER price. When positive and negative variations in value were considered, the trigger curves showed that during the first six months of power plant construction the continuation value of the option was always superior to its immediate exercise value, indicating that such an option should not be exercise during this period. This can be observed for the case study considered, as early exercise does not necessarily imply an increase in revenue due to the sale of CERs. On the other hand, when only negative price variations were considered, the expectation that several CER price drops may be observed in the future reversed this logic.

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APPENDIX

A Grant, Vora & Weeks Method for the valuation of financial options

The basic principle of this method consists in identifying the critical price of the underlying asset for each time prior to its expiry, or in other words, the price at which the investor is indifferent as to whether to exercise the option or not at each point in time. Once these values are known, it can be argued that the American derivative may be valuated in the same way as a European derivative, in other words, through the calculation of the arithmetic average of previously simulated values.

Note that from the critical option prices (S_t^*), two regions may be defined: the early exercise region, where exercising would be the optimum decision, and the continuation region, where the best strategy would be to wait until the next point in time to make a new decision. The indifference curve between these regions is called the Optimum Exercise Frontier, or the derivative Trigger Curve.

Considering an American purchase option that may be exercised at any time $t \in [0, T]$, with an exercise price X, and an asset price at time t represented by S_t, according to the GVW method the value of this option (C_t) can be determined according to equation 22:

$$C_t(S_t, X) = \max\{I_t, F_t\} \tag{22}$$

where

$$I_t = \max\{S_t - X, 0\} \tag{23}$$

and

$$F_t = e^{-r.\Delta t} E_t\big[C_{t+\Delta t}(S_{t+\Delta t}, X)\big]. \tag{24}$$

In equation 22, note that the first term of the maximization operator represents the value of immediate exercise, while the second represents its continuation value. It is worth highlighting that to determine the continuation value using the GVW method, previous knowledge of all of the critical prices between time t and the expiry of the option are required.

Since the critical price represents the price by which the immediate exercise value of the derivative is equal to its continuation value, it is possible to define a boundary condition for S_t^* equating equations 23 and 24:

$$S_t^* - X = e^{-r.\Delta t} E_t\big[C_{t+\Delta t}(S_{t+\Delta t}^*, X)\big]. \tag{25}$$

From this equation, it can be concluded that the value of S^* can easily be determined for the derivative expiry date. Note that at maturity, the continuation value for the derivative is equal to zero, as there would not be any other opportunity for its exercise. Therefore, equation 25 can be rewritten in the following way:

$$S_T^* - X = 0 \tag{26}$$

or, $S_T^* = X$. Since the determination of S_t^* depends on the previous knowledge of all critical prices at times before t, GVW propose that the trigger curvemay be determined recursively, using the Dynamic Programming technique.

The optimization process begins in the instant before expiry of the option, or at $T - \Delta t$. The holder of the purchase option can exercise it immediately or maintain the option "alive" until its maturity. Using equation 22, the value of the option may be determined in the following way:

$$C_{T-\Delta t}\left(S_{T-\Delta t}, X\right) = \max\left\{I_{T-\Delta t}, e^{-r.\Delta t} E_{T-\Delta t}\left[C_T(S_T, X)\right]\right\}.$$

The critical price ($S^*_{T-\Delta t}$) is identified by finding the value of $S_{T-\Delta t}$ which satisfies condition 25. Assuming that it is possible to identify $S^*_{T-\Delta t}$, the optimization goes on to identify the value of $S^*_{T-2\Delta t}$, which depends on the knowledge of values $S^*_{T-\Delta t}$ and S^*_T. Using this logic, the process continues until S_0^* is determined.

According to condition 25, determining the value of S_t^* entails determining the continuation value (F) associated with time t, however, information on future prices is still not known at this time. Grant, Vora & Weeks solved this problem using the Monte Carlo Simulation technique (MCS).

The MCS is initiated at time $T - \Delta t$, adopting $S^*_{T-\Delta t} = S^*_T$ as the initial condition. Once an initial value for $S^*_{T-\Delta t}$ has been arbitrated, S_T values are simulated in order to determine the continuation value of the option. Where condition 25 is not satisfied, the value of $S^*_{T-\Delta t}$ must be incremented and the MCS repeated. This routine must be carried out until condition 25 is met.

The solution process continues recursively for the duration of the option's lifetime. Once the trigger curve of the derivative has been determined, the value of the option can be determined through N Monte Carlo Simulations started at $t_0 (t = 0)$. In this way, an initial price for the underlying asset (S_0) which can be observed in the market is considered. The early exercise happens at the first instant in which the asset price surpasses the trigger curve. The final value of the option is then determined using the average of the values obtained for each simulated path:

$$C_0 = \frac{1}{N} \cdot \sum_{w=1}^{N} e^{-r.\tau} \cdot \left[\max\left(S_\tau^w - X, 0\right)\right] \forall S_\tau^w > S_\tau^* . \tag{27}$$

In this equation, τ represents the first instant at which the simulated price surpasses the trigger curve.

B Least Square Monte Carlo Method (LSM) for the valuation of financial options

As described in Appendix A, determining the immediate exercise value of an option may be considered to be a not so complicated task, however, a good estimate of its continuation value is more difficult to obtain. As previously mentioned, in applying the GVW method this process requires the performance of a large number of Monte Carlo Simulations, which can lead to increased computational costs.

Considering this, Longstaff & Schwartz (2001) proposed a methodology that reduces the computational cost of the simulation methods. Compared to the GVW method, the main difference of the method proposed by Longstaff & Schwartz lies in the calculation of the continuation value. While GVW estimate this value using Monte Carlo Simulations, Longstaff & Schwartz propose that regressions be performed using cross-sectional information about the financial asset. This method is called the Least Square Monte Carlo Method, or LSM.

The first step of the LSM method consists in defining a finite number of dates on which early exercise of the option is possible. Therefore, considering T to be the expiry of the derivative, it is assumed that the lifetime of the option can be divided into D intervals of equal size $\Delta t = T/D$. Once N paths for the price of the underlying asset are simulated, Longstaff & Schwartz consider that the option's continuation price, at a given time t, can be can be initially set using the following equation:

$$F(w,t) = E_Q\left[\sum_{t_j=t+\Delta t}^{T} e^{-r(t_j-t)} \cdot V(w,t_j,t,T) \Big/ \Im_t\right], \tag{28}$$

where t represents any time within a time interval $[0, T]$, w represents one of the simulated paths, Q represents a measure of risk-neutral probabilities and $\Im_t$ represents all of the information available at time t. Also note that in equation 28 $V(w, t_j, t, T)$ represents the cashflow generated by exercise of the option at any time $t_j > t$. Since American options can be exercised only once within each path w, it should be highlighted that there only exists one t_j for which

$$V(w, t_j, t, T) > 0.$$

Despite equation 28 being used to obtain an initial estimate of the option's continuation value at a given time t, Longstaff & Schwartz suppose that the continuation value $F(w, t)$ may be better estimated using cross-sectional regressions of the financial asset price. The algorithm is based on the idea that $F(w, t)$ can be represented using a combination of linear base functions (B_l), whose constants are determined through a Least Squares Regression. This logic is represented by equation 29, where S represents the price of the underlying asset of the option, and al represents the constant associated with each base function B_l.

$$F(w,t) = \sum_{l=0}^{\infty} a_l B_l(S). \tag{29}$$

Equation 29 assumes infinite terms for the calculation of $F(w, t)$, however, for practical reasons this assumption is not computationally viable. In this case, the value of $F(w, t)$ must be estimated using a number $G < \infty$ of base functions:

$$F(w, t) \approx F_G(w, t) = \sum_{l=0}^{G} a_l B_l(S) . \tag{30}$$

From equation 30 the LSM method estimates the value of $F_G(w, t)$ by regressing the continuation values initially calculated using equation 28 in relation to the predefined base functions. At a given time t, such a regression is performed considering only the paths in which the option is found to be "in-the-money", since it is only for these paths that the decision to exercise the option early would be relevant.

Once the continuation value of the option ($F_G(w, t)$) has been estimated for each path w, the decision to exercise it early is made by comparing its immediate exercise value with the estimated continuation value. Just as is seen in the GVW method, the iterative process of the LSM method is recursive. The value of the option (C_{LS}) is approximated by calculating the arithmetic average of the sum of all cashflows $V(w, t_j, t, T)$ where exercise of the option would be optimum, or:

$$C_{LS} = \frac{1}{N} \sum_{w=1}^{N} \sum_{t_j=\Delta t}^{T} e^{-r.t_j} V(w, t_j, 0, T) . \tag{31}$$

It remains clear that the relatively low computational cost associated with the LSM method represents its main advantage when compared with other methods involving MCS in derivative valuation. In addition, such as can be observed for the GVW method, this method may also permit the valuation of different types of options involving different stochastic processes, or even options with different dimensions.