Pesquisa Operacional (2022) 42(nspe1): e263588 p.1-27
doi: 10.1590/0101-7438.2022.042nspe1.00263588

Printed version ISSN 0101-7438 / Online version ISSN 1678-5142
www.scielo.br/pope
ARTICLES

AGE-BASED REPLACEMENT WITH IMPERFECT REPAIR AND RISK AS LOW AS REASONABLY PRACTICABLE

Joyce Araújo[1*], Rodrigo Lopes[2] and Philip Scarf[3]

Received April 30, 2022 / Accepted August 24, 2022

ABSTRACT. Maintenance policies must consider system reliability and the risk of accidents in systems where equipment failures represent a risk. In this context, this work proposes an age replacement policy with Bayesian imperfect repair and considers the "as low as reasonably practicable" (ALARP) principle. The policy determines the age of replacement that minimizes the long-run cost per unit time when the failure rate is ALARP. The model also supposes that failures are either minimally or perfectly repaired, depending on the skill of the maintainer. Numerical applications are performed with and without the disproportion factor in ALARP, both for infinite and one-replacement-cycle horizons. The results show that considering imperfect repair leads to an increase in replacement costs and a decrease in the optimal replacement age when considering the ALARP principle. The model applies to situations where there are conflicts of interest between maintenance management and risk; that is, cases where the aim is to reduce the cost of replacing equipment and minimize the risks. Maintenance policies must consider system reliability and the risk of accidents in systems where equipment failures represent a risk. In this context, this work proposes an age replacement policy with Bayesian imperfect repair and considers the "as low as reasonably practicable" (ALARP) principle. The policy determines the age of replacement that minimizes the long-run cost per unit time when the failure rate is ALARP. The model also supposes that failures are either minimally or perfectly repaired, depending on the skill of the maintainer. Numerical applications are performed with and without the disproportion factor in ALARP, both for infinite and one-replacement-cycle horizons. The results show that considering imperfect repair leads to an increase in replacement costs and a decrease in the optimal replacement age when considering the ALARP principle. The model applies to situations where there are conflicts of interest between maintenance management and risk; that is, cases where the aim is to reduce the cost of replacing equipment and minimize the risks.

Keywords: maintenance, replacement, risk, safety, reliability, ALARP.

*Corresponding author

[1]RANDOM - Research Group on Risk and Decision Analysis in Operations and Maintenance, Management Engineering Department, Federal University of Pernambuco, Rodovia BR-104, Km 59, s/n, Nova Caruaru, 55002-970 Caruaru, PE, Brazil – E-mail: joyce.danielle.araujo@gmail.com – https://orcid.org/0000-0002-8015-0011

[2]RANDOM - Research Group on Risk and Decision Analysis in Operations and Maintenance, Management Engineering Department, Federal University of Pernambuco, Rodovia BR-104, Km 59, s/n, Nova Caruaru, 55002-970 Caruaru, PE, Brazil – E-mail: r.s.lopes@random.org.br – https://orcid.org/0000-0001-5339-3301

[3]RANDOM - Research Group on Risk and Decision Analysis in Operations and Maintenance UFPE, Cardiff Business School, Cardiff University, Cardiff, CF10 3AT, UK – E-mail: ScarfP@cardiff.ac.uk – https://orcid.org/0000-0001-5623-906X

1 INTRODUCTION

Failures of engineered objects can negatively impact cost and availability as well as impact upon safety and the environment (Abbas & Shafiee, 2020). Therefore, maintenance planning needs to be associated with other activities of operators, such as quality control, production planning, and risk management. Also, there is the possibility of inducing failures and accidents during equipment maintenance, leading to explosions, fires, and toxic releases (Vatn & Aven, 2010). In this way, maintenance and risk should be managed simultaneously.

In the search for this integration, some works have introduced security restrictions to maintenance optimization and argued the need to analyze beyond the cost-benefit relationship (Aven & Castro, 2008, 2009; Flage et al., 2012). Other studies implemented the acceptance criteria of the As Low As Reasonably Practicable (ALARP) principle in maintenance optimization to reduce both maintenance costs and the risk of accident (Flage, 2013, 2018).

Nonetheless, most works in the literature assume that maintenance occurs without errors and that items are perfectly restored. Depending on the system studied, replacement policies with perfect repair still present themselves as economical and effective models. However, consideration of imperfect repair brings models closer to reality (Toledo et al., 2015), as it allows the quality of the maintenance action to be affected by factors such as the skills of the maintenance team and the methods, technology, and tools used (Khatab & Aghezzaf, 2016; Sheu et al., 2019). Thus, maintenance actions' quality may harm an engineered object's elements, such as product quality, costs, production time, availability, reliability, and safety (Pham & Wang, 1996; Rivera-GÓmez et al., 2021).

Concerning age-replacement policies based on imperfect repair, it is possible to describe some studies in the literature. Brown and Proschan (1983) suggest an imperfect repair model in which the item is perfectly repaired with probability p or minimally repaired with probability $q = 1 - p$. In the model of Fontenot and Proschan (1984), the item is perfectly repaired (replaced) at age T, and in intermittent failures (corrective maintenance) imperfect maintenance is taken into consideration. The model presented by Lim, Lu and Park (1998) treats the probability of the item being perfectly repaired as a random variable (Bayesian imperfect repair). Lim, Qu and Zuo (2016) considered that the quality of the maintenance action is a random variable governed by a probability distribution, where the probabilities of repairing the system perfectly are different for each employee since they have different repair skills.

The lack of knowledge, incomplete information, and the misunderstanding of data can be real problems for maintenance actions and require adequate attention. In these contexts, the use of Bayesian probabilities becomes opportune as it allows the consideration of knowledge obtained before the occurrence of events and the updating of this knowledge over time, since it allows the insertion of expert knowledge, as well as obtaining reliable results, even though little data is available (Tuan, Yann & Mitra, 2015).

Given the complexity of production systems, the uncertainties related to the quality of maintenance action, the age at which equipment will fail, and the consequences of failures, maintenance managers face the challenge of defining maintenance strategies that jointly consider the costs and risks involved in decisions.

In situations involving risk, ALARP can be used to limit risk following imposed regulation and targeted security practices, considering costs (Jones-Lee & Aven, 2011). The principle states that risks must be reduced, except where there is a gross disproportion between the necessary resources and the benefits of safety measures (Aven & Abrahamsen, 2007). In this way, maintenance activities contribute to mitigating failures and controlling accident risks, which can be prioritized and treated with the support of ALARP.

In this paper, we developed a model based on the age replacement policy with Bayesian imperfect repair, including the ALARP analysis. Given that imperfect repairs can happen and that equipment maintenance is integrated with risk management, we analyze how the uncertainty in the quality of maintenance action impacts on a risk perspective. The proposed model considers that the quality of the maintenance action varies due to the different skills of the maintainers and allows the definition of the optimal replacement age that minimizes maintenance costs while preventing accident risks from exceeding the ALARP region.

The next section presents the proposed model. Numerical studies are performed in Section 3. Section 4 discusses the managerial implications. And Section 5 presents the conclusions.

2 PROPOSED MODEL

This section presents the age replacement model based on Bayesian imperfect repair with ALARP implementation and risk acceptance criteria.

2.1 Model assumptions

The model assumes:

- Time to failure of equipment is a random variable with a known probability density function $f(t)$ with an increasing failure rate;
- The component only supports two states, failed or operational, and system failure is identified immediately;
- The times to perform a replacement and repair are negligible;
- There are k repair teams, each with different repair skills, and the quality of maintenance is a random variable described by a probability distribution;
- The model considers a homogeneous risk;
- The system is renewed at age T or the first perfectly repaired failure, whichever occurs first.

2.2 Age replacement policy

The organization is assumed to have k repair teams, with different repair skills. When team i perform the repair, it is a minimal repair with probability $1-p_i$ and perfect repair (system renewal) with probability p_i, where $i=1,\ldots,k$. Figure 1 illustrates the proposed model with its decisions. As input, there are fixed maintenance costs and parameters related to risk management. The costs incurred for replacement, perfect repair, and imperfect repair are calculated according to the degree of repair, the possible occurrence of the accident, and the approach used for the Value of Preventing a Fatality (VPF).

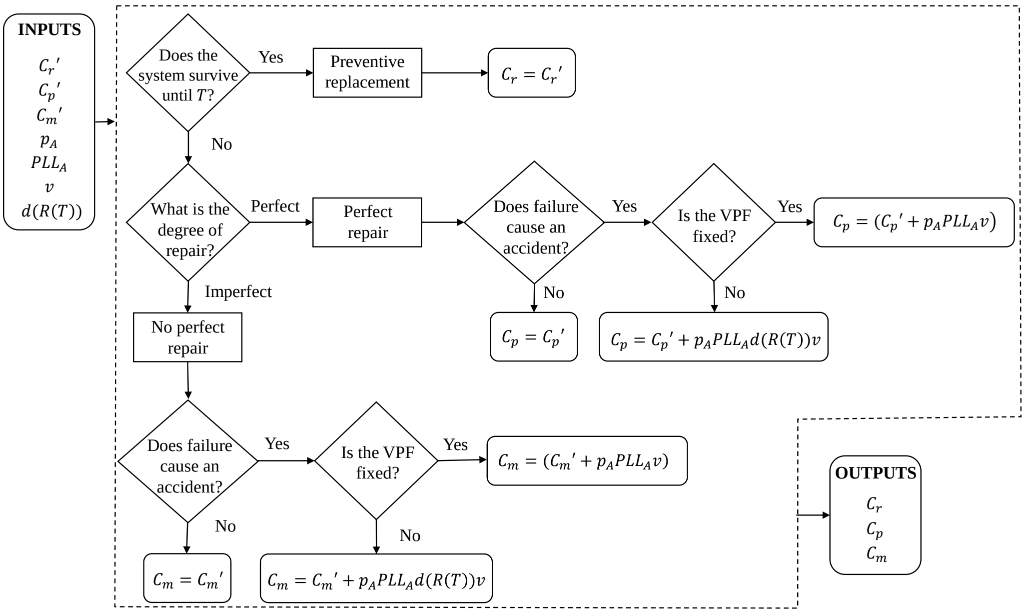


Figure 1 – Proposed model.

The cost of preventive replacement, $C_r = C_r'$, refers to the cost of replacing the system at the programmed age, T.

The expected safety cost, C_s, is considered when the failure results in an accident A. By assuming that the VPF is fixed, C_s is composed by multiplying the probability of occurrence of the accident due to failure (p_A), the expected number of deaths (PLL_A) and the Value of Preventing a Fatality (v).

When considering that the VPF is increasing in the ALARP region, the gross disproportion factor, $d(R(T))$, is inserted into the value of C_s. This factor is expressed as follows.

$$d(R(T)) = \begin{cases} 1, & y \leq r_0 \\ g(R(T)), & r_0 < y < r_1 \\ \infty, & y \geq r_1 \end{cases} \quad (1)$$

Since $R(T)$ refers to the risk index and its value can be an approximation of the upper limit of the PLL when assuming that a person is exposed to the risk related to the maintenance of the system in question, expressed by Equation (2).

$$R(T) = \left[F(T)\left(1+\left\lfloor\frac{u}{T}\right\rfloor\right) - F\left(T-\left(u-\left\lfloor\frac{u}{T}\right\rfloor T\right)\right)\right] p_A PLL_A \tag{2}$$

The perfect repair cost, C_p, is the value of replacing the system perfectly when the system fails before T. If there is no accident due to failure (with probability $1-p_A$), C_p will be equal to the usual perfect repair cost (C'_p). In the situation where the accident A occurs (with probability p_A), C_p will be composed of C'_p and the expected cost of security (C_s).

Equations (3) and (4) present the composition of C_p for the fixed and increasing VPF, respectively.

$$C_p = C'_p + p_A PLL_A v \tag{3}$$

$$C_p = C'_p + p_A PLL_A d(R(T))v \tag{4}$$

The cost of imperfect repair, C_m, is the cost of repairing the system in a less than perfect way. Similarly to the perfect repair cost, if the failure causes an accident, C_m, will be composed of the non-perfect fixed repair cost (C'_m) and C_s. The imperfect repair cost is expressed by Equations (5) and (6) for fixed and increasing VPF, respectively.

$$C_m = C'_m + p_A PLL_A v \tag{5}$$

$$C_m = C'_m + p_A PLL_A d(R(T))v \tag{6}$$

The number of failures at (0, t) is the sum of the number of perfect repairs at (0, t) and the number of non-perfect repairs at (0, t). That is, $N(t) = L(t) + M(t)$. So, $Y_1 = \{t \geq 0 | L(t) = 1\}$ is the waiting time when the first perfect repair occurs and $Z_1 = \{t \geq 0 | M(t) = 1\}$ is the waiting time where the first non-perfect repair occurs. In this way, $M(Y_1)$ refers to the number of non-perfect repairs in $(0, Y_1]$ and, according to Sheu et al. (1999), Y_1 is independent of $\{M(t), t \geq 0\}$.

According to Lim, Lu and Park (1998), $\overline{H}(t) = P(Y_1 \geq t) = \sum_{i=1}^{k} \overline{F}^{p_i}(t)\pi_i$ and $\overline{G}(t) = P(Z_1 \geq t) = \sum_{i=1}^{k} \overline{F}^{(1-p_i)}(t)\pi_i$. Furthermore, $\{L(t), t \geq 0\}$ and $\{M(t), t \geq 0\}$ are Non-Homogeneous Poisson Process (NHPP) with the respective intensity functions (Lim, Qu & Zuo, 2016):

$$r_H(t) = r(t)\frac{A(t,1)}{A(t,0)} \tag{7}$$

$$r_G(t) = r(t)\frac{Z(t,1)}{Z(t,0)} \tag{8}$$

Since $A(t,n) = \sum_{i=1}^{k} p_i^n \overline{F}^{p_i}(t)\pi_i$ and $Z(t,n) = \sum_{i=1}^{k} (1-p_i)^n \overline{F}^{(1-p_i)}(t)\pi_i$. Also, $r_H(t)$ and $r_G(t)$ are the failure rate functions of H and G, respectively. Next, the infinite-horizon and one-replacement-cycle cases will be presented based on Lim, Qu and Zuo (2016) and extended to the situation in which the ALARP risk acceptance criteria are considered.

In sequence, the expected cost per unit of time is formulated for the infinite-horizon planning and the one-replacement-cycle case.

2.2.1 Expected cost per unit of time – Infinite-horizon case

By proposing that $Y_{1,}Y_{2,...}$ are independent and identically distributed random variables (i.i.d.) of $H(y)$, the duration time between two consecutive system renewals is indicated by $Y_i^* = min(Y_i, T)$ for $i = 1, 2, \dots$ as described in Equation (9).

$$Y_i^* = Y_i I_{(0,T)}(Y_i) + T I_{(T,\infty)}(Y_i) \tag{9}$$

The first part of Equation (9) corresponds to the waiting time for the ith perfect repair and the probability of the system failing before T. The second part is composed of the age T and the probability of the system surviving until T. Thus, the expected value of Y_i^* is (Lim, Qu & Zuo, 2016):

$$E(Y_1^*) = \int_0^T t dH(t) + T\overline{H}(T) = \int_0^T \overline{H}(t)dt \tag{10}$$

In turn, the total cost incurred during the renewal interval Y_i^* for $i = 1, 2, \dots, Y_i^*$ is obtained by Equation (11).

$$C_i^* = [C_p + C_m M(Y_i)] I_{(0,T)}(Y_i) + [C_r + C_m M(T)] I_{(T,\infty)}(Y_i) \tag{11}$$

The first part of Equation (11) is composed of the probability of the system failing before T and being repaired perfectly, the cost of the perfect repair (C_p) and the costs of the non-perfect repairs that occurred until performed the first perfect repair, that is, the amount spent on a non-perfect repair (C_m) multiplied by the number of non-perfect repairs performed in the interval.

The second part contains the probability of the system surviving until (C_r), in addition, the costs of non-perfect repairs that occurred until age T are considered. In this way, the expected cost to operate the system when the VPF is fixed is equal to Equation (12).

$$E(C_1^*) = (C'_m + p_A PLL_A v) \int_0^T \overline{H}(t) r_G(t)dt + (C'_p + p_A PLL_A v)H(T) + C'_r \overline{H}(T) \tag{12}$$

Similarly, when the VPF is increasing, the expected cost of operating the system is defined by Equation (13).

$$E(C_1^*) = \left[C'_m + p_A PLL_A d(R(T))v\right] \int_0^T \overline{H}(t) r_G(t)dt + \left[C'_p + p_A PLL_A d(R(T))v\right] H(T) + C_r{}'\overline{H}(T) \tag{13}$$

Note that, $\{(Y_1^*, C_1^*)\}$ comprise a renewal process, in which renewal occurs when there is a replacement or perfect repair. Let $K(t)$ be the expected cost to operate the system during the time interval $[0,t]$, according to Renewal Reward Theorem (Ross, 2010):

$$B(T) = \lim_{t\to\infty} \frac{K(t)}{t} = \frac{E(C_1^*)}{E(Y_1^*)} \tag{14}$$

Given the above, when considering the fixed VPF, the total cost incurred during the renewal interval is obtained by dividing the cost incurred in the renewal interval by the duration of the interval, as shown in Equation (15).

$$B(T) = \frac{(C'_m + p_A PLL_A v) \int_0^T \overline{H}(t) r_G(t)dt + (C'_p + p_A PLL_A v)H(T) + C'_r \overline{H}(T)}{\int_0^T \overline{H}(t)dt} \tag{15}$$

When the total cost incurred during the renewal interval considers the VPF is increasing, Equation (16) is obtained.

$$B(T)=\frac{[C'_m+p_A PLL_A d(R(T))v]\int_0^T \overline{H}(t)r_G(t)dt+[C'_p+p_A PLL_A d(R(T))v]H(T)+C'_r\overline{H}(T)}{\int_0^T \overline{H}(t)dt} \quad (16)$$

Thus, the objective is to find the replacement age that minimizes the expected long-term cost, $B(T)$, and that complies with the risk acceptance criteria, i.e., $R(T)$ is less than r_1.

2.2.2 Expected cost per unit of time – One-replacement-cycle case

When working with only one-replacement-cycle, will analyze the expected cost per unit of time of that cycle. A cycle can be delimited by the occurrence of failures before or after age T, so the total cost per unit of time between two successive replacements when the planned replacement is performed at age T, $W(T)$, is equal to:

$$W(T)=\left(\frac{C_p+C_m M(Y_1)}{Y_1}\right)I_{(0,T)}(Y_i)+\left(\frac{C_r+C_m M(T)}{T}\right)I_{(T,\infty)}(Y_i) \quad (17)$$

Using the independence of Y_1 and $\{M(t),t\geq 0\}$, the expected value of $W(T)$ is (Lim, Qu & Zuo, 2016):

$$E[W(T)]=E\left[I_{(0,T)}(Y_1)\left(\frac{C_p+C_m M(Y_1)}{Y_1}\right)\right]+E\left[I_{(T,\infty)}(Y_i)\left(\frac{C_r+C_m M(T)}{T}\right)\right] \quad (18)$$

$$E[W(T)]=C_p\int_0^T \frac{1}{t}dH(t)+\frac{C_r\overline{H}(T)}{T}+\frac{C_m\int_0^T r_G(t)dt}{T}-C_m\int_0^T\left[\frac{H(t)}{t}r_G(t)-\frac{1}{t}\int_0^t r_G(z)dz\right]dt \quad (19)$$

Equation (20) shows the expected value of $W(T)$ when the VPF is fixed.

$$E[W(T)]=(C'_p+p_A PLL_A d(R(T))v)\int_0^T \frac{1}{t}dH(t)+\frac{C_r\overline{H}(T)}{T}+\frac{(C'_m+p_A PLL_A d(R(T))v)\int_0^T r_G(t)dt}{T}$$
$$-(C'_m+p_A PLL_A d(R(T))v)\int_0^T\left[\frac{H(t)}{t}r_G(t)-\frac{1}{t}\int_0^t r_G(z)dz\right]dt \quad (20)$$

For the case of increasing VPF, the expected value of $W(T)$ is expressed by Equation (21).

$$E[W(T)]=(C'_p+p_A PLL_A v)\int_0^T \frac{1}{t}dH(t)+\frac{C_r\overline{H}(T)}{T}+\frac{(C'_m+p_A PLL_A v)\int_0^T r_G(t)dt}{T}$$
$$-(C'_m+p_A PLL_A v)\int_0^T\left[\frac{H(t)}{t}r_G(t)-\frac{1}{t}\int_0^t r_G(z)dz\right]dt \quad (21)$$

The objective is to find the replacement age that minimizes the expected long-run cost, $E[W(T)]$, and that complies with the risk acceptance criteria, i.e., $R(T)$ is less than r_1.

3 NUMERICAL STUDIES

This section uses numerical examples to illustrate the proposed model. The numerical application comprises the cases with one and two points prior. In addition, the equations for the modeling of four points prior are presented. For the modeling of the cases, the Python (Python Software Foundation, 2020) programming language was used.

Let F be a Weibull distribution with scale parameter $\theta = 1$, shape parameter $\beta > 0$ and let $t \geq 0$. According to the proposal by Lim, Qu e Zuo (2016), we have the following equations.

$$r(t) = \beta t^{\beta-1} \tag{22}$$

$$\overline{H}(T) = \sum_{i=1}^{k} e^{-p_i t^{\beta}} \cdot \pi_i \tag{23}$$

$$h(t) = \sum_{i=1}^{k} \beta p_i t^{\beta-1} e^{-p_i t^{\beta}} \cdot \pi_i \tag{24}$$

$$r_H(t) = \beta t^{\beta-1} \frac{\sum_{i=1}^{k} p_i e^{-p_i t^{\beta}} \cdot \pi_i}{\sum_{i=1}^{k} e^{-p_i t^{\beta}} \cdot \pi_i} \tag{25}$$

$$r_G(t) = \beta t^{\beta-1} \frac{\sum_{i=1}^{k} (1-p_i) e^{-(1-p_i) t^{\beta}} \cdot \pi_i}{\sum_{i=1}^{k} e^{-(1-p_i) t^{\beta}} \cdot \pi_i} \tag{26}$$

3.1 One-point prior

For illustrative purposes, both for the infinite-horizon case and the one-replacement-cycle case, the values are shown in Table 1.

Table 1 – Input values for model illustration.

C'_r	C'_p	C'_m	v	p_A	PLL_A	β	θ	p_1	r_0	r_1	u
1	2	0.25	100	0.1	0.01	2	1	0.9	10^{-5}	10^{-3}	1

The acceptance criteria were defined according to the values suggested by the Society for Risk Analysis (SRA). Weibull's parameters represent a system with a wear pattern (β) equal to 2 and a characteristic life (θ) equal to 1. The proportions outlined for the values of maintenance costs and risk parameters aim to portray the reality of productive systems where is the probability of accident occurrence due to failure.

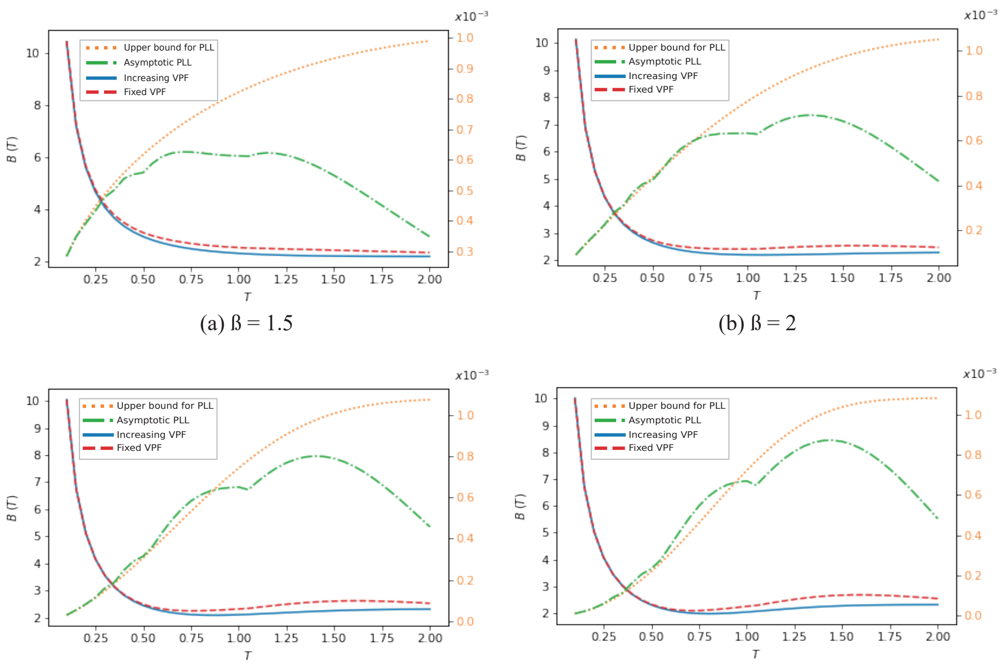
3.1.1 Infinite-horizon case

In the context of one-point prior, it is assumed that $P = p_1$ with probability 1, for the infinite-horizon, Equations (15) and (16) were applied to obtain the expected long-term cost per unit of time. For the case of fixed VPF, the values of C_p and C_m were defined by Equations (3) and (5), whereas Equations (4) and (6) were used to express the case of increasing VPF.

By applying the values from Table 2 and modifying the values of the wear pattern (β), the curves shown in Figure 2 were obtained. The curves represent the expected long-term cost for the fixed VPF (blue line) and the VPF increasing (red line), as well as the asymptotic PLL (orange line) and the upper limit of the PLL (green line). Note that the cost curves appear similar to each other, whereas the risk curves showed greater sensitivity to changing parameters.

Figure 2b shows the curves for $\beta = 2$. When considering the fixed VPF, the optimal replacement age is $T_f^* = 1.05$, with the lowest long-term. Whereas, when using crescent VPF, the ideal replacement age is $T_c^* = 0.95$ and the lowest long-term cost is 2.400.

It can be seen that the case based on the crescent VPF has a lower ideal replacement age, whereas the long-term replacement cost is higher. This situation occurs because the disproportion factor increases over time, causing the risk index and cost to increase. Therefore, replacing the equipment at a younger age becomes more conservative.



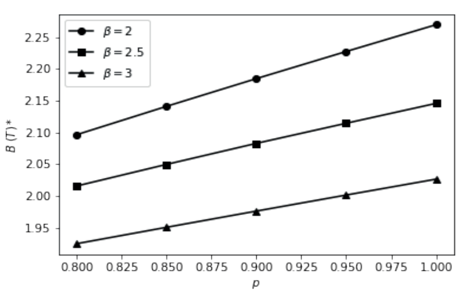
(a) ß = 1.5 (b) ß = 2

(c) ß = 2.5 (d) ß = 3

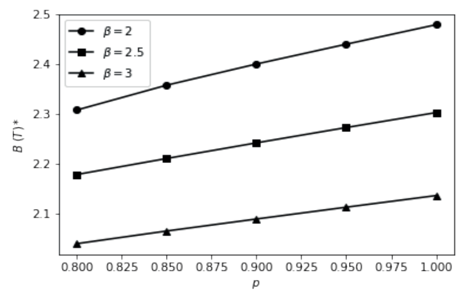
Figure 2 – Long-term expected cost and risk index for (a) β=1.5, (b) β=2, (c) β=2.5 and (d) β=3.

For the values used, it is noted that throughout the analyzed interval, the upper limit of the PLL is included in the acceptance levels (10^{-5} and 10^{-3}), whereas when considering the asymptotic PLL, the acceptance criteria restrict the set of optimal replacement times to the interval $[0, 1.6]$. Thus, both in $T_c^* = 0.95$ and in $T_f^* = 1.05$ the risk values are within the acceptable level and are allowed.

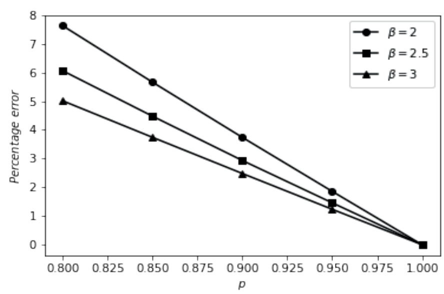
Figure 3a shows the optimal long-term cost for different p values when analyzing the fixed VPF when $\beta = 2$, $\beta = 2.5$, and $\beta = 3$. It appears that, in all three cases, the optimal long-term cost increases when p increases, that is, when the repair tends to be perfect. The greater likelihood of a perfect repair entails more expensive because the cost of such a repair is more expensive than the cost of replacement. In this way, the optimal replacement age decreases as p increases, since it is more economical to replace before failure.



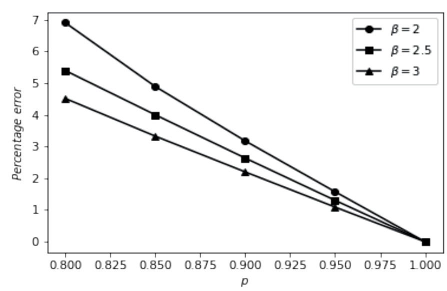
(a) Optimal cost for fixed VPF



(b) Optimal cost for increasing VPF



(c) Percentage error for fixed VPF



(d) Percentage error for increasing VPF

Figure 3 – Long-run optimal cost for different p values (3a and 3b) and percentage error of optimal cost in infinite-horizon (3c and 3d).

Furthermore, it is noted that the long-run cost optimal decreases when the wear pattern (β), of the system increases. Since the non-perfect repair cost is less than the perfect repair cost, it becomes more economical to perform the non-perfect repair as β grows, this causes the long-run optimal cost to decrease.

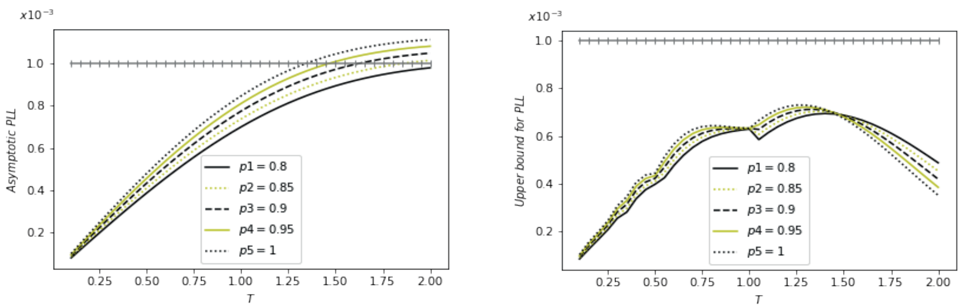
Figure 3b brings information similar to that presented in Figure 3a, this time for the situation where the VPF is increasing. It is possible to observe that the long-run optimal cost also increases when p increases and decreases when β increases.

Figures 3c and 3d show the relative differences between the optimal long-term costs as p varies, that is, the differences between the values found for the optimal cost considering the imperfect repair (with different probabilities) and the values of the optimal cost when having a perfect repair. Note that the closer the probability of perfect repair approaches 1, the smaller the percentage error in cost.

For example, assuming $p = 0.80$ and $\beta = 2$, the relative error is 7.628 and 6.901 when the VPF is fixed and increasing, respectively. On the other hand, when $p = 0.90$ and $\beta = 2$, the relative error is 3.738 and 3.178 when the VPF is fixed and increasing, respectively. The percentage error is also smaller considering the increasing VPF for the other values of p and β.

Figure 4 presents the curves of the asymptotic PLL and upper bound of the PLL for different values of p and $\beta = 2$, indicating that both the asymptotic PLL and the upper limit of the PLL increase when p increases. Note that values above 10^{-3} do not fall within the ALARP region.

In the case of the upper limit of the PLL, the results are in the ALARP region for all p values used. The asymptotic PLL surpassed the ALARP region at ages $T_c = 1.85, T_c = 1.6, T_c = 1.45$ and $T_c = 1.3$ for p equal to $0.85, 0.9, 0.95$ and 1, respectively. Thus, as p increased, the asymptotic PLL increased and the optimal age of replacement decreased.



(a) Asymptotic PLL (b) Upper bound for PLL

Figure 4 – (a) Asymptotic PLL and (b) upper bound of the PLL for different values of p.

A numerical experiment was carried out to investigate the effects of varying the parameters of the model. As can be seen in Table 2, each parameter was changed by $-50, -25, +25$ and $+50$, while the others were kept fixed.

For the three values of p studied, when the C'_m increases, the optimal replacement age (T^*) does not change and the $B(T^*)$ increases.

When the replacement cost (C'_r) increases, T^* and $B(T^*)$ increase.

When the perfect repair cost (C'_p increases, T^* decreases and $B(T^*)$ increases. The better the repair grade, the higher the cost, and as it is cheaper to replace before failure, the optimal replacement age is reduced.

When the expected number of deaths (PLL_A) increases, T^*_c stays or increases, whereas $B(T^*)$ increases. The same behavior is observed for the probability of accident due to failure (p_A) and the Value of Preventing a Fatality (v) when changed separately. This behavior is due to increased risk, so the cost increases and it becomes safer to replace the system at a younger age.

Table 2 – Sensitivity analysis for one-point prior (infinite-horizon).

	Changes (%)	$p=0,8$				$p=0.9$				$p=1$			
		Fixed VPF		**Crescent VPF**		**Fixed VPF**		**Crescent VPF**		**Fixed VPF**		**Crescent VPF**	
		T_f^*	$B(T_f^*)$	T_c^*	$B(T_c^*)$	T_f^*	$B(T_f^*)$	T_c^*	$B(T_c^*)$	T_f^*	$B(T_f^*)$	T_c^*	$B(T_c^*)$
C'_m	**-50**	1.10	2.073	1.05	2.286	1.05	2.174	0.95	2.390	1.05	2.270	0.95	2.479
	-25	1.10	2.085	1.05	2.297	1.05	2.179	0.95	2.395	1.05	2.270	0.95	2.479
	+25	1.10	2.108	1.05	2.320	1.05	2.190	0.95	2.406	1.05	2.270	0.95	2.479
	+50	1.10	2.120	1.05	2.331	1.05	2.196	0.95	2.411	1.05	2.270	0.95	2.479
C'_p	**-50**	2.00	1.205	2.00	1.440	2.00	1.224	2.00	1.416	2.00	1.245	2.00	1.398
	-25	1.55	1.686	2.00	1.930	1.50	1.741	2.00	1.940	1.50	1.795	2.00	1.954
	+25	0.90	2.445	0.80	2.631	0.85	2.560	0.80	2.745	0.85	2.669	0.75	2.852
	+50	0.80	2.751	0.70	2.908	0.75	2.888	0.70	3.045	0.70	3.019	0.65	3.174
C'_r	**-50**	0.60	1.686	0.55	1.784	0.60	1.762	0.55	1.865	0.55	1.838	0.50	1.935
	-25	0.85	1.942	0.70	2.110	0.80	2.028	0.70	2.196	0.80	2.111	0.70	2.280
	+25	1.45	2.177	2.00	2.430	1.40	2.263	2.00	2.472	1.40	2.346	2.00	2.516
	+50	2.00	2.205	2.00	2.440	2.00	2.288	2.00	2.480	1.95	2.368	2.00	2.521
PLL_A	**-50**	1.15	2.049	1.05	2.104	1.10	2.139	1.05	2.195	1.05	2.226	1.00	2.280
	-25	1.10	2.073	1.05	2.193	1.10	2.162	1.00	2.286	1.05	2.248	1.00	2.368
	+25	1.10	2.120	1.05	2.450	1.05	2.207	0.85	2.534	1.00	2.291	0.90	2.613
	+50	1.05	2.143	0.75	2.592	1.05	2.229	0.75	2.681	1.00	2.312	0.80	2.763
p_A	**-50**	1.15	2.049	1.05	2.104	1.10	2.139	1.05	2.195	1.05	2.226	1.00	2.280
	-25	1.10	2.073	1.05	2.193	1.10	2.162	1.00	2.286	1.05	2.248	1.00	2.368
	+25	1.10	2.120	1.05	2.450	1.05	2.207	0.85	2.534	1.00	2.291	0.90	2.613
	+50	1.05	2.143	0.75	2.592	1.05	2.229	0.75	2.681	1.00	2.312	0.80	2.763
ν	**-50**	1.15	2.049	1.05	2.158	1.10	2.139	1.05	2.251	1.05	2.226	1.00	2.334
	-25	1.10	2.073	1.05	2.233	1.10	2.162	1.00	2.328	1.05	2.248	0.95	2.407
	+25	1.10	2.120	1.05	2.383	1.05	2.207	0.90	2.472	1.00	2.291	0.90	2.549
	+50	1.05	2.143	0.85	2.458	1.05	2.229	0.85	2.541	1.00	2.312	0.85	2.619
θ	**-50**	0.55	4.193	0.35	4.986	0.55	4.370	0.30	5.310	0.50	4.540	0.25	5.730
	-25	0.85	2.796	2.00	3.149	0.80	2.913	2.00	3.195	0.75	3.027	2.00	3.255
	+25	1.40	1.677	1.10	1.817	1.35	1.748	1.05	1.888	1.30	1.816	1.05	1.956
	+50	1.65	1.398	1.35	1.509	1.60	1.456	1.35	1.568	1.55	1.513	1.30	1.624
β	**-25**	2.00	2.080	2.00	2.251	2.00	2.201	2.00	2.351	2.00	2.320	2.00	2.449
	+25	0.90	2.016	0.75	2.179	0.85	2.083	0.75	2.242	0.85	2.146	0.75	2.303
	+50	0.80	1.925	0.75	2.040	0.80	1.976	0.70	2.090	0.80	2.027	0.70	2.137

When the scale parameter (θ) increases, T^* and $B(T^*)$ decrease for both the fixed and the variable VPF cases.

Finally, when the shape parameter (β) increases, T^* and $B(T^*)$ decrease. When β decreases, T^* e $B(T^*)$ increase. As the wear pattern increases, the system tends to be repaired more often and the repair performed tends to be non-perfect because its cost is lower than the perfect repair, so there is a decrease in the expected long-term cost.

Furthermore, it was found that in some moments when p increased, the asymptotic PLL increased. It is expected that with increasing p reliability will increase and risk will decrease. However, it is understood that when working with probabilistic models such variations may occur, since the system will not always be perfectly or minimally repaired.

3.1.2 One-replacement-cycle case

By applying the values from Table 1, the curves shown in Figure 5 were obtained. For $\beta = 2$, when considering the fixed VPF, the optimal replacement age is $T_f^* = 0.7$ and the optimal cost is 3.244. When using *crescent* VPF, the optimal replacement age is $T_c^* = 0.6$ and the optimal cost is 3.482.

During the entire analyzed interval, the upper limit of the PLL is included in the acceptance levels (10^{-5} and 10^{-3}), whereas when considering the asymptotic PLL, the acceptance criteria constrain the set of optimal replacement times to the interval $[0, 1.6]$. Thus, both in $T_f^* = 0.7$ and in $T_c^* = 0.6$ the risk values are within the acceptable level and are allowed.

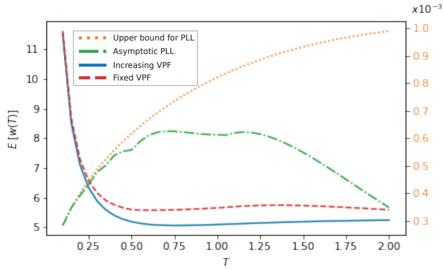
The increase in the risk index results from the increase in β. Similar to the infinite-horizon case, the optimal cost was higher and the optimal age decreased when the VPF was considered to increase. This is due to the increase in the risk index over time.

When working with the one-replacement-cycle case, both in the case of fixed VPF and increasing VPF the optimal cost increases and the replacement age decreases when compared to the infinite time horizon. With the increase in risk, it becomes more appropriate to replace the equipment at a younger age to preserve lives, so there is an increase in cost.

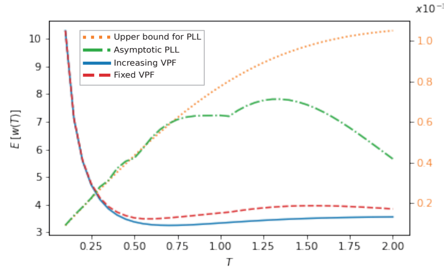
Figures 6a and 6b show the optimal cost for different values of p when $\beta = 2$, $\beta = 2.5$ and $\beta = 3$. By modifying the values of p, it is noted that for both the fixed VPF and the increasing VPF the optimal cost increases as p increases because when the repair tends to be perfect it is more economical to replace the system before failure due to the high cost of the system.

When the wear pattern (β) increases, the optimal cost decreases. This is due to the greater probability of failure and the tendency to perform more non-perfect repairs because they are cheaper. This behavior is similar to the infinite planning horizon.

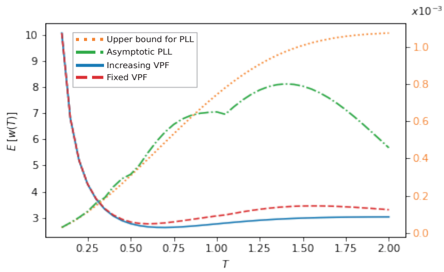
In Figures 6c and 6d, the percentage error of optimal cost decreases as the repair tends to be perfect. When considering $p = 0.80$ and $\beta = 2$, the relative error is 3.084 and 3.311 when



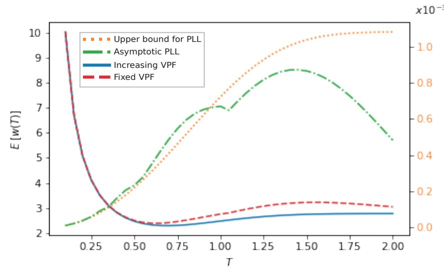
(a) ß = 1.5



(b) ß = 2

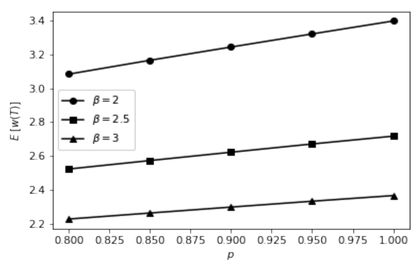


(c) ß = 2.5

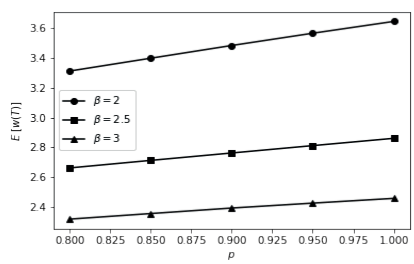


(d) ß = 3

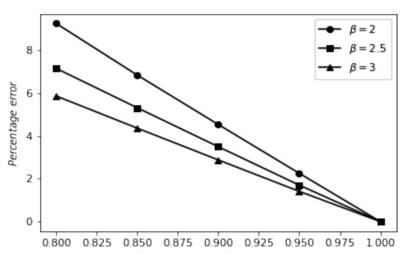
Figure 5 – One-replacement-cycle cost for (a) β=1.5, (b) β=2, (c) β=2.5 and (d) β=3.



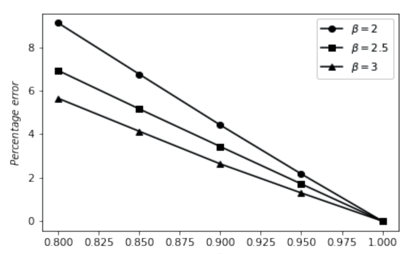
(a) Optimal cost for fixed VPF



(b) Optimal cost for increasing VPF



(c) Percentage error for fixed VPF



(d) Percentage error for increasing VPF

Figure 6 – Optimal one-replacement-cycle cost for different values of p (6a and 6b) and percentage error of the optimal cost for one-replacement-cycle (6c and 6d).

the VPF is fixed and increasing, respectively. Thus, the relative error is smaller when compared to the infinite planning horizon.

In turn, when $p = 0.90$ and $\beta = 2$, the relative error is 3.244 and 3.482 when the VPF is fixed and increasing, respectively. For the other values of p and β the percentage error is also smaller considering the increasing VPF.

The asymptotic PLL and the upper limit of the PLL present the same behavior as in Figure 4 since the same $F(T)$ cumulative distribution function is used . Sensitivity analysis was also performed for the one-replacement-cycle case. Despite the values themselves being different, it was noticed that the results behaved similarly to those presented in Table 2, and in the one-replacement-cycle case, the values of T_c^* were lower, whereas the values of $B(T_c^*)$ were higher.

3.1.3 Synthesis of results for one-point prior

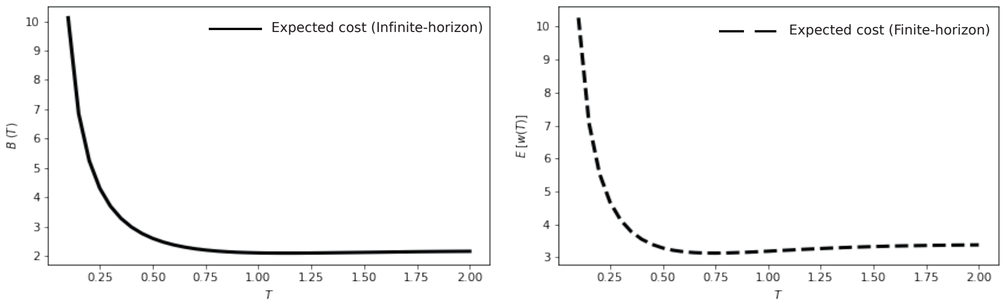
Table 3 presents the summary of the main results obtained for one-point prior. It is possible to notice once again that the optimal age values are higher when the time horizon is infinite and the costs are lower. In the case of both the infinite-horizon and the one-replacement-cycle, the insertion of the ALARP acceptance criteria decreased the value of the optimal age and increased the costs.

Table 3 – Synthesis of results for one-point prior.

		$p = 0.8$		$p = 0.9$		$p = 1$	
		T^*	$B(T^*)$	T^*	$B(T^*)$	T^*	$B(T^*)$
Infinite-Horizon	Without ALARP	1.20	2.001	1.15	2.093	1.10	2.182
	Fixed VPF	1.10	2.097	1.05	2.185	1.05	2.270
	Increasing VPF	1.05	2.308	0.95	2.400	0.95	2.479
One-replacement-cycle	Without ALARP	0.80	2.966	0.75	3.126	0.70	3.278
	Fixed VPF	0.75	3.084	0.70	3.244	0.65	3.398
	Increasing VPF	0.70	3.244	0.60	3.482	0.55	3.643

Figure 7 shows the cost curves when the ALARP risk principles are not introduced in the model, that is, the model is limited to the case proposed by Lim, Qu and Zuo (2016). The curves are similar to the cases in which ALARP was introduced. However, when the risk is not adopted, for the infinite-horizon case, we have $B(T^*) = 2.093$ and $T^* = 1.15$. In turn, for the finite-horizon, $B(T^*) = 3.126$ and $T^* = 0.75$.

Such behavior demonstrates considering risk in the model brings a more conservative result about the replacement age and the costs become higher. In this way, replacements will take place at shorter and more frequent intervals, resulting in the need for greater attention to maintenance personnel.



(a) (b)

Figure 7 – (a) Expected cost and (b) one-replacement-cycle without ALARP.

The different perspectives presented are useful for maintenance managers to adjust maintenance schedules, the choice of the case to be adopted will depend on the planning horizon of interest and the degree of risk acceptance.

3.2 Two points prior

Use of more than one-point prior can be applied to the situation that the organization has $k = 2$ repair teams, its members have different repair skills and are randomly requested to repair a failed system. Thus, the probability of a perfect repair varies randomly due to the skills of the collaborators.

P is considered a random variable with the following distribution when admitting two points prior.

$$P = \begin{cases} p_1 \text{with probability } \pi_1 \\ p_2 \text{with probability } \pi_2 \end{cases} \tag{27}$$

So that $\pi_1 + \pi_2 = 1, 0 \leq p_1 < p_2 \leq 1$. Thus, we have the following equations for the context of two points prior.

$$\overline{H}(t) = \pi_1 e^{-p_1\left(\frac{t}{\theta}\right)^\beta} + \pi_2 e^{-p_2\left(\frac{t}{\theta}\right)^\beta} \tag{28}$$

$$h(t) = \frac{\beta}{\theta} t^{\beta-1} \left(p_1 \pi_1 e^{-p_1\left(\frac{t}{\theta}\right)^\beta} + p_2 \pi_2 e^{-p_2\left(\frac{t}{\theta}\right)^\beta} \right) \tag{29}$$

$$r_H(t) = \frac{\beta}{\theta} t^{\beta-1} \left(\frac{p_1 \pi_1 e^{-p_1\left(\frac{t}{\theta}\right)^\beta} + p_2 \pi_2 e^{-p_2\left(\frac{t}{\theta}\right)^\beta}}{\pi_1 e^{-p_1\left(\frac{t}{\theta}\right)^\beta} + \pi_2 e^{-p_2\left(\frac{t}{\theta}\right)^\beta}} \right) \tag{30}$$

$$r_G(t) = \frac{\beta}{\theta} t^{\beta-1} \left(\frac{(1-p_1)\, \pi_1 e^{-(1-p_1)\left(\frac{t}{\theta}\right)^\beta} + (1-p_2)\, \pi_2 e^{-(1-p_2)\left(\frac{t}{\theta}\right)^\beta}}{\pi_1 e^{-(1-p_1)\left(\frac{t}{\theta}\right)^\beta} + \pi_2 e^{-(1-p_2)\left(\frac{t}{\theta}\right)^\beta}} \right) \tag{31}$$

$$R(T)=\left\{\left[1-\left(\pi_1 e^{-p_1\left(\frac{T}{\theta}\right)^\beta}+\pi_2 e^{-p_2\left(\frac{T}{\theta}\right)^\beta}\right)\right]\left(1+\left\lfloor\frac{u}{T}\right\rfloor\right)-1\right.$$
$$\left.-\left(\pi_1 e^{-p_1\left(\frac{T-\left(u-\left\lfloor\frac{u}{T}\right\rfloor T\right)}{\theta}\right)^\beta}+\pi_2 e^{-p_2\left(\frac{T-\left(u-\left\lfloor\frac{u}{T}\right\rfloor T\right)}{\theta}\right)^\beta}\right)\right\}p_A PLL_A \quad (32)$$

To illustrate the application of the model, the initial values from Table 1 will continue to be used. *However*, p_1 will be equal to 0.8.

3.2.1 Case of the infinite-horizon

Figure 8 shows the expected long-term cost for the fixed VPF and the increasing VPF for the case where $p_1 = 0.8$, $p_2 = 0.9$, $\pi_1 = 0.5$, and $\pi_2 = 0.5$. When the VPF is fixed, the ideal replacement age is $T_f^* = 1,1$, with the lowest long-term cost being 2.139. Whereas, when using *crescent* VPF, the ideal replacement age is $T_c^* = 1.05$ and the lowest long-term cost is 2.355.

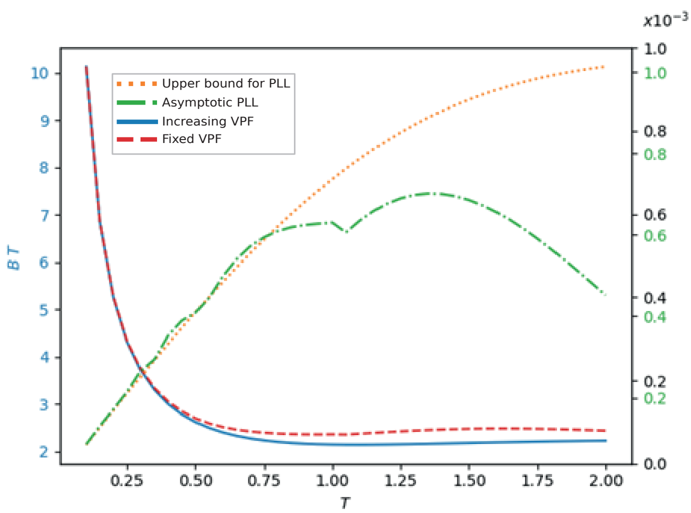
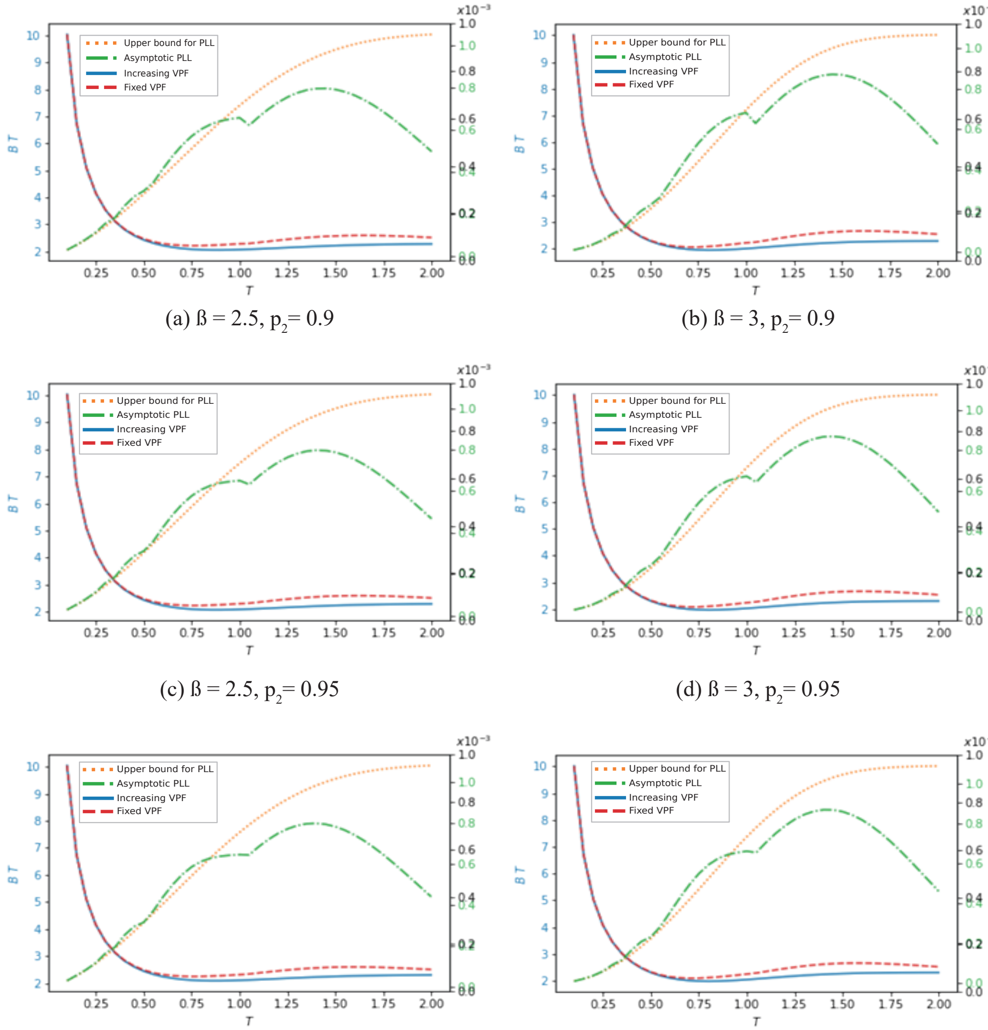


Figure 8 – Long-run expected cost for two points prior for β=2.

Thus, when working with two points prior and reducing one of the p values, it can be seen that the cost has decreased and the replacement age has increased for the fixed VPF and the variable VPF, a situation similar to that portrayed for a point a prior.

The upper bound for PLL is included in the acceptance levels (10^{-5} and 10^{-3}) throughout the range shown, for ages above 1.5 the asymptotic PLL does not fall into the ALARP region.

Figure 9 shows the behavior of the 4 curves for some values of p_2 and β when $\pi_1 = 0.4$ and $\pi_2 = 0.6$. The curves are slightly altered as p varies and undergo a large change as β increases.



(a) ß = 2.5, p_2= 0.9

(b) ß = 3, p_2= 0.9

(c) ß = 2.5, p_2= 0.95

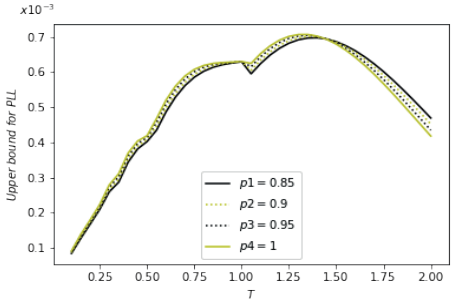
(d) ß = 3, p_2= 0.95

(e) ß = 2.5, p_2= 1

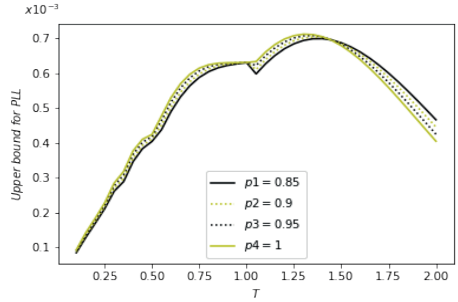
(f) ß = 3, p_2= 1

Figure 9 – Long-run expected cost for two points prior and different values of p_2 and β .

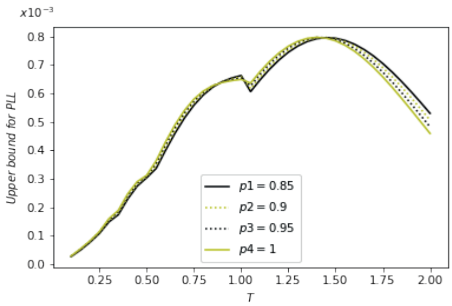
When addressing risk, Figure 10 shows how the upper bound of the PLL behaves when β and p vary. For the values considered in the illustration, in all cases, the upper PLL was found to be within the ALARP region. Similar to a prior point, as it is considered a probabilistic model, even with an increase in p, imperfect repairs can still occur because they follow a probability distribution.



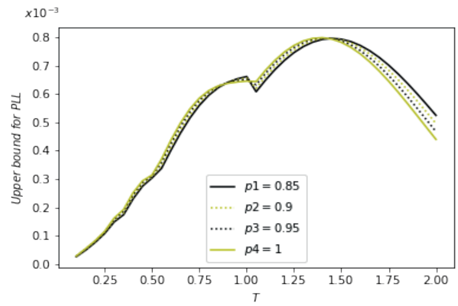
(a) ß = 2, π_1 = 0.5, π_2 = 0.5



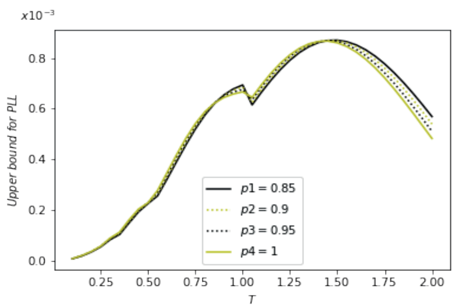
(b) ß = 2, π_1 = 0.4, π_2 = 0.6



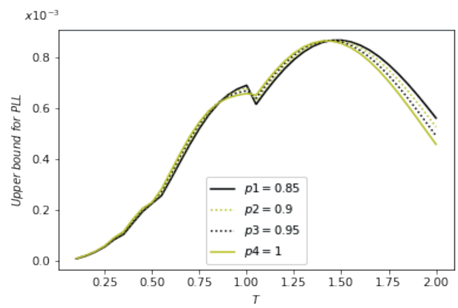
(c) ß = 2.5, π_1 = 0.5, π_2 = 0.5



(d) ß = 2.5, π_1 = 0.4, π_2 = 0.6



(e) ß = 3, π_1 = 0.5, π_2 = 0.5



(f) ß = 3, π_1 = 0.4, π_2 = 0.6

Figure 10 – Upper PLL for two points prior varying β and π.

3.2.2 One-replacement-cycle case

Figure 11 shows the one-replacement-cycle cost for fixed VPF and the crescent VPF for the case where $p_1 = 0.8$, $p_2 = 0.95$, $\pi_1 = 0.5$, and $\pi_2 = 0.5$.

When the VPF is fixed, the ideal replacement age is $T_f^* = 0.70$, with the lowest cost of one-replacement-cycle equal to 2.034. Whereas, when using crescent VPF, the ideal replacement age is $T_c^* = 0.60$ and the lowest cost of one-replacement-cycle is equal to 3.4379.

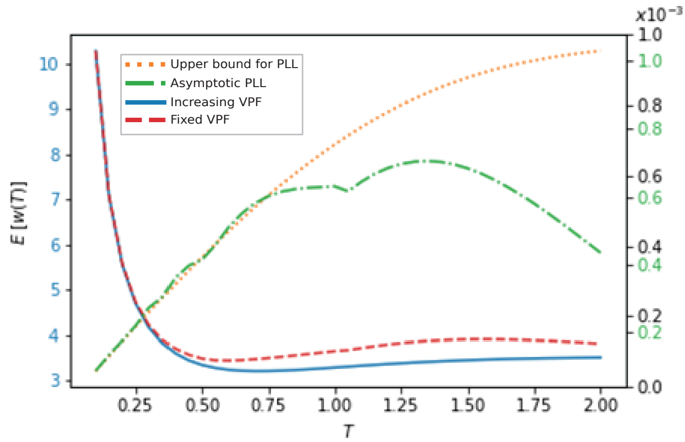


Figure 11 – One-replacement-cycle cost for two points prior for β=2.

When using $\pi_1 = 0.4$ and $\pi_2 = 0.6$, when the VPF is fixed, the ideal replacement age is $T_f^* = 0.70$ and $E[W(T^*)] = 2.227$. In turn, when the VPF is increasing, the ideal replacement age is $T_c^* = 0.60$ and the lowest cost of one-replacement-cycle is equal to $E[W(T^*)] = 3.463$.

For the analyzed values, the optimal age of replacement did not change, and the optimal cost suffered an insignificant variation with the change in π.

3.2.3 Synthesis of results for two points prior

Table 4 has the optimal replacement ages and their related costs. Regarding the optimal age of replacement, it increases when β increases, there is a slight decrease when p changes and there is almost no variation when π is changed. Thus, it is understood that the shape parameter and the repair quality influence the replacement age more than the prior distributions.

However, it is interesting that the prior distributions are considered in the model so that uncertainties are included, and decision-makers have greater support for their choices. In addition, a considerable part of the maintenance actions are still performed by humans and they are prone to errors due to the deficiency in documentation, communication, tools, and methodologies, as well as difficulties that may arise during the maintenance activity (Morag et al., 2018). Thus, due to the high interaction between maintainers and equipment, it is necessary to consider the possibility of human error.

In addition to considering the impact of competencies on team performance, organizations needs to think about the capabilities that teams need, plans for team building, and ways to develop those capabilities of maintainers. It is worth mentioning that in scenarios that involve high risk, maintenance activities are usually carried out with the help of high technology, and with the advent of industry 4.0, the use of digital technologies in maintenance activities has grown (Alvanchi et al., 2021). Thus, this scenario will demand new capabilities and skills that need to be explored, for example, leadership, time management, information analysis, problem-solving, interpersonal skills, collaboration, and knowing how to use virtual and augmented reality technologies (Romero et al., 2016a, b).

Table 4 – Synthesis of results for two points prior (infinite-horizon).

β	π	**VPF**	$p_2 = 0.9$		$p_2 = 0.95$		$p_2 = 1$	
			T^*	$B(T^*)$	T^*	$B(T^*)$	T^*	$B(T^*)$
$\beta = 2$	$\pi_1 = 0.5$ e $\pi_2 = 0.5$	Fixed	1.10	2.139	1.10	2.159	1.10	2.178
		Increasing	1.05	2.356	0.95	2.376	0.95	2.393
	$\pi_1 = 0.4$ e $\pi_2 = 0.6$	Fixed	1.10	2.148	1.10	2.173	1.05	2.196
		Increasing	1.05	2.366	0.95	2.388	0.95	2.409
$\beta = 2.5$	$\pi_1 = 0.5$ e $\pi_2 = 0.5$	Fixed	0.90	2.049	0.90	2.064	0.90	2.080
		Increasing	0.75	2.210	0.75	2.225	0.75	2.239
	$\pi_1 = 0.4$ e $\pi_2 = 0.6$	Fixed	0.9	2.056	0.90	2.075	0.85	2.093
		Increasing	0.75	2.217	0.75	2.234	0.75	2.251
$\beta = 3$	$\pi_1 = 0.5$ e $\pi_2 = 0.5$	Fixed	0.80	1.950	0.80	1.962	0.80	1.974
		Increasing	0.70	2.065	0.70	2.077	0.70	2.088
	$\pi_1 = 0.4$ e $\pi_2 = 0.6$	Fixed	0.80	1.955	0.80	1.970	0.80	1.984
		Increasing	0.70	2.070	0.70	2.084	0.70	2.097

Figure 12 summarizes the behavior of the optimal age and the optimal replacement cost for one and two points prior. In summary, when considering one or more points prior, for both infinite and finite planning horizons, when the risk is introduced, and increased over time, the optimal replacement age decreases and the optimal cost increases. Furthermore, as the probability of perfect repair increases, the optimal age decreases, and the cost increases, this behavior is more noticeable in the case of one-point prior.

3.3 Four points prior

For the case of four points prior, P is considered a random variable with the following distribution.

$$P = \begin{cases} p_1 \text{ with probability } \pi_1 \\ p_2 \text{ with probability } \pi_2 \\ p_3 \text{ with probability } \pi_3 \\ p_4 \text{ with probability } \pi_4 \end{cases} \tag{33}$$

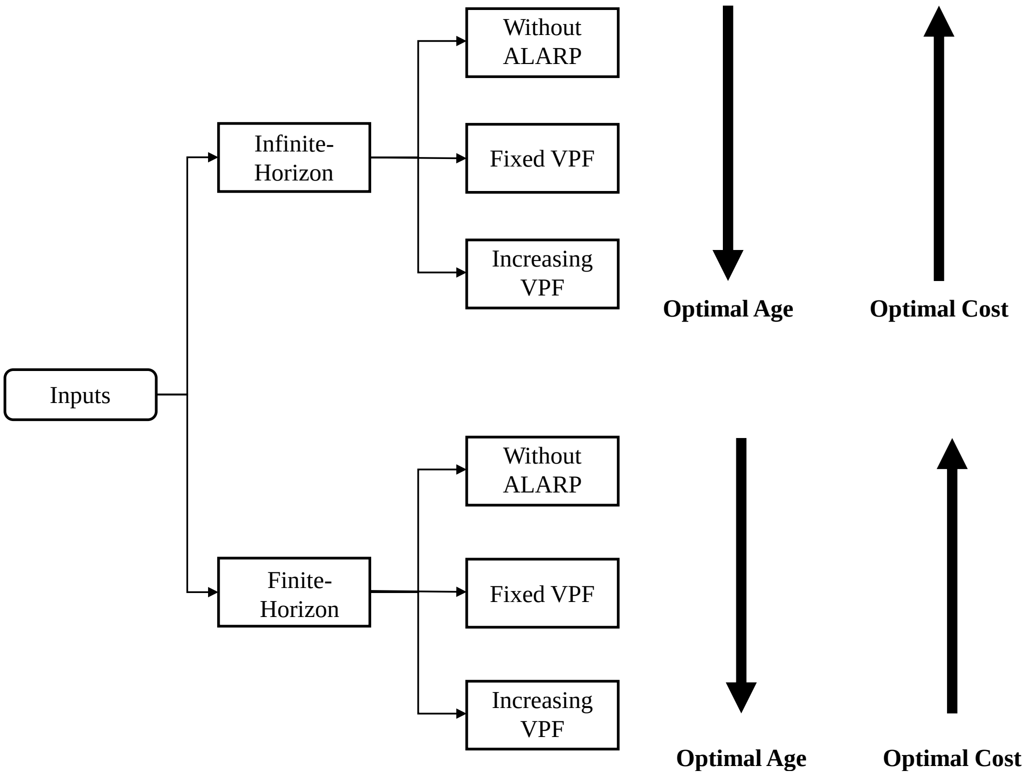


Figure 12 – Synthesis of optimal age performance and optimal cost.

So that $\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$, then, we have the following equations for the context of four points prior.

$$\overline{H}(t) = \pi_1 e^{-p_1\left(\frac{t}{\theta}\right)^\beta} + \pi_2 e^{-p_2\left(\frac{t}{\theta}\right)^\beta} + \pi_3 e^{-p_3\left(\frac{t}{\theta}\right)^\beta} + \pi_4 e^{-p_4\left(\frac{t}{\theta}\right)^\beta} \tag{34}$$

$$h(t) = \frac{\beta}{\theta} t^{\beta-1} \left(p_1 \pi_1 e^{-p_1\left(\frac{t}{\theta}\right)^\beta} + p_2 \pi_2 e^{-p_2\left(\frac{t}{\theta}\right)^\beta} + p_3 \pi_3 e^{-p_3\left(\frac{t}{\theta}\right)^\beta} + p_4 \pi_4 e^{-p_4\left(\frac{t}{\theta}\right)^\beta} \right) \tag{35}$$

$$r_H(t) = \frac{\beta}{\theta} t^{\beta-1} \left(\frac{p_1 \pi_1 e^{-p_1\left(\frac{t}{\theta}\right)^\beta} + p_2 \pi_2 e^{-p_2\left(\frac{t}{\theta}\right)^\beta} + p_3 \pi_3 e^{-p_3\left(\frac{t}{\theta}\right)^\beta} + p_4 \pi_4 e^{-p_4\left(\frac{t}{\theta}\right)^\beta}}{\pi_1 e^{-p_1\left(\frac{t}{\theta}\right)^\beta} + \pi_2 e^{-p_2\left(\frac{t}{\theta}\right)^\beta} + \pi_3 e^{-p_3\left(\frac{t}{\theta}\right)^\beta} + \pi_4 e^{-p_4\left(\frac{t}{\theta}\right)^\beta}} \right) \tag{36}$$

$$r_G(t) = \frac{\beta}{\theta} t^{\beta-1} \left(\frac{(1-p_1)\,\pi_1 e^{-(1-p_1)\left(\frac{t}{\theta}\right)^\beta} + (1-p_2)\,\pi_2 e^{-(1-p_2)\left(\frac{t}{\theta}\right)^\beta} + (1-p_3)\,\pi_3 e^{-(1-p_3)\left(\frac{t}{\theta}\right)^\beta} + (1-p_4)\,\pi_4 e^{-(1-p_4)\left(\frac{t}{\theta}\right)^\beta}}{\pi_1 e^{-(1-p_1)\left(\frac{t}{\theta}\right)^\beta} + \pi_2 e^{-(1-p_2)\left(\frac{t}{\theta}\right)^\beta} + \pi_3 e^{-(1-p_3)\left(\frac{t}{\theta}\right)^\beta} + \pi_4 e^{-(1-p_4)\left(\frac{t}{\theta}\right)^\beta}} \right) \tag{37}$$

$$R(T) = \left\{ \begin{array}{c} \left[1 - \left(\pi_1 e^{-p_1\left(\frac{T}{\theta}\right)^\beta} + \pi_2 e^{-p_2\left(\frac{T}{\theta}\right)^\beta} + \pi_3 e^{-p_3\left(\frac{T}{\theta}\right)^\beta} + \pi_4 e^{-p_4\left(\frac{T}{\theta}\right)^\beta} \right) \right] \left(1 + \left\lfloor \frac{u}{T} \right\rfloor\right) \\ -1 - \left(\pi_1 e^{-p_1\left(\frac{T-(u-\lfloor \frac{u}{T} \rfloor T)}{\theta}\right)^\beta} + \pi_2 e^{-p_2\left(\frac{T-(u-\lfloor \frac{u}{T} \rfloor T)}{\theta}\right)^\beta} + \pi_3 e^{-p_3\left(\frac{T-(u-\lfloor \frac{u}{T} \rfloor T)}{\theta}\right)^\beta} + \pi_4 e^{-p_4\left(\frac{T-(u-\lfloor \frac{u}{T} \rfloor T)}{\theta}\right)^\beta} \right) \end{array} \right\} p_A PLL_A \tag{38}$$

4 MANAGEMENT INSIGHTS

The increased emphasis on sustainable production requires greater commitment from companies to preserve the environment and develop society. By reducing accident risks and negative impacts on nature, organizations gain advantages, namely compliance with environmental legislation, greater competitiveness, better credibility from the surrounding community and customers, and improving their internal processes (Peterson et al., 2021).

On the other hand, failures in production systems can cause great economic, social and environmental losses, in such a way that organizations need to create strategies that contribute to the minimization of accidents and risks must be identified, analyzed and treated.

The maintenance of equipment influences the reliability and, consequently, the safety of the systems. Therefore, effective maintenance actions can cooperate to minimize the risk of accidents.

In managerial terms, the application of the proposed model allows the teams involved with maintenance management to work in association with maintenance costs, equipment reliability, and the risks involved in making decisions about replacement ages. Furthermore, this work draws attention to the fact that the competencies of the members of the maintenance teams can impact the performance of the tasks performed. This study also can alert organizations to the need to reflect on the capabilities required in maintenance teams, strategies for team composition, and plans to leverage these intended capabilities.

The application of the model allows the optimal replacement ages to be obtained, that is, those that minimize costs and allow the risk to keep within the ALARP risk region or the widely acceptable region. The choice of the number of points prior, the planning horizon and the use of the disproportion factor will depend on the number of teams involved, the requirements related to risk exposure and the preferences of maintenance managers, according to the context in question.

The fact of admitting that the repair can be imperfect brings the model closer to reality when compared to models that consider that the system is always perfectly restored since maintenance activities can present errors resulting from deficiencies related to the people involved, the methodologies applied and the technology used.

Both maintenance actions and risk analysis add uncertainties, for example, regarding the quality of the maintenance action, the time when equipment will fail, the consequences of failures, and the absence or inaccuracy of data. Therefore, the use of prior distributions in the proposed model admits that the knowledge of experts is considered so that some of these uncertainties are encompassed.

However, it is important to note that using Bayesian probability does not exclude the need for review and judgment by decision-makers after numerical analyses. For cases where decisions have a high impact on security, it is interesting, for example, to create a "Safety Board" formed by multidisciplinary experts for a broader discussion (Vatn & Aven, 2010).

Given the importance of the decision process being informed about risk and the need for reliable and accurate information, it is believed that the more useful information the teams enjoy, the better the team's knowledge and the greater the prospect of good decisions. Thus, it is expected that this model will contribute to including relevant information in maintenance management decision-making processes and support managers in conflicting situations between risk and maintenance.

5 CONCLUSION

Faced with the need for integration between maintenance management and risk management in organizations, this work aimed to propose an age replacement policy with Bayesian imperfect repair that considers the risk and to analyze how the uncertainty of the quality of the maintenance action impacts ALARP and the total cost of maintenance.

The policy was modeled for the infinite-horizon planning and one-replacement-cycle, so it can be used both in stable situations over time and for cases in which the cycles change. In addition, it encompassed both situations in which the risk index is fixed and cases in which it increases over time, which makes the model adaptable both for circumstances in which decision-makers need to be more conservative about safety and in occasions with greater permissiveness concerning risk restrictions.

Numerical application results showed that cases that relied on increasing VPF had a lower optimal replacement age and higher long-term replacement cost. This was because the disproportion factor became larger over time, causing the risk and cost index to increase, so replacing the equipment at a younger age became more conservative.

When two points prior were considered, it was noticed that the shape parameter and the quality of the repair had a considerable influence on the age of replacement, whereas the prior distributions had a smaller impact. Furthermore, the sensitivity analysis showed that the model behaved as expected when subjected to different variations, showing good performance. Thus, the proposed model is suitable to be used in real cases to support risk management and maintenance management decisions.

The use of adequate maintenance models contributes to assertive decision-making so that accidents can be prevented, culminating in benefits for society, the environment, and the economy. Therefore, it is hoped that applying the proposed model will help companies in their maintenance strategies when there is a conflict between risk management and maintenance.

6 NOTATION

X	Lifetime of a system.
$F(t), f(t), r(t)$	CDF, PDF, and failure rate of X, respectively.
T	Age for planned replacement.
p_A	Probability of an accident (A) given that the system has failed.
PLL_A	Potential Loss of Life given an accident (A), i.e., expected number of fatalities.
v	Value of Preventing a Fatality (VPF).
C_s	Safety cost.
$C'_p, C'_m, C'_r,$	Cost for perfect repair considering only the failure cost, cost for no perfect repair considering only the failure cost and replacement cost, respectively.
C_p, C_m, C_r	Costs for perfect repair considering only the failure and safety cost, no perfect repair considering only the failure and safety cost and replacement cost, respectively.
P	Randon variable representing the probability of a perfect repair
$N(t), L(t), M(t)$	Number of failures, number of perfect repairs, and number of no perfect repair in $(0,t)$, respectively.
Y_1 e Z_1	Waiting times at which the first perfect repair and the first no perfect repair occurs, respectively.
$H(t)$ e $G(t)$	CDFs of Y_1 and Z_1, respectively.
$r_H(t)$ e $r_G(t)$	Failure rates de Y_1 and Z_1, respectively.
Y_1^*	Time duration between two successive renewals of system.
C_1^*	Total cost incurred over the renewal interval Y_1^*.
$R(T)$	Above limit of PLL, approximation of risk index.
$d(R(T))$	Disproportion factor.
$B(T)$	Expected long-run cost per unit time.
W(T)	Total cost per unit time between two successive replacement.

Acknowledgements

The work was supported by funding from the Coordination for the Improvement of Higher Education Personnel in Brazil (Capes).

References

ABBAS M & SHAFIEE M. 2020. An overview of maintenance management strategies for corroded steel structures in extreme marine environments. *Marine Structures*, **71**(Mai 2020): 102718.

ALVANCHI A, TOHIDIFAR A, MOUSAVI M, AZAD R & ROKOOEI S. 2021. A critical study of the existing issues in manufacturing maintenance systems: Can BIM fill the gap? *Computers in Industry*, **131**: 103484.

AVEN T & ABRAHAMSEN E. 2007. On the use of cost-benefit analysis in ALARP processes. *International Journal of Performability Engineering*, **3**(3): 345–353.

AVEN T & CASTRO IT. 2008. A minimal repair replacement model with two types of failure and a safety constraint. *European Journal of Operational Research*, **188**(2): 506–515.

AVEN T & CASTRO IT. 2009. A delay-time model with safety constraint. *Reliability Engineering and System Safety*, **94**(2): 261–267.

BROWN M & PROSCHAN F. 1983. Imperfect Repair. *Journal of Applied Probability*, **20**(4): 851–859.

FLAGE R. 2013. On risk reduction principles in the context of maintenance optimisation modelling. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, **227**(3): 241–250.

FLAGE R. 2018. Maintenance Optimization and Its Relation to Safety. *Handbook of Safety Principles*, pp. 540–564.

FLAGE R, COIT DW, LUXHØJ JT & AVEN T. 2012. Safety constraints applied to an adaptive Bayesian condition-based maintenance optimization model. *Reliability Engineering and System Safety*, **102**: 16–26.

FONTENOT RA & PROSCHAN F. 1984. Some imperfect maintenance models. In: M ABDEL-HAMEED EAE (Ed.), *Reliability Theory and Models*. Orlando: Academic Press.

FOUNDATION PS. 2020. *Python Language Reference*.

JONES-LEE M & AVEN T. 2011. ALARP - What does it really mean? *Reliability Engineering and System Safety*, **96**(8): 877–882.

KHATAB A & AGHEZZAF E. 2016. Selective maintenance optimization when quality of imperfect maintenance actions are stochastic. *Reliability Engineering and System Safety*, **150**: 182–189.

LIM JH, LU KL & PARK DH. 1998. Bayesian imperfect repair model. *Communications in Statistics - Theory and Methods*, **27 (4)**(June 2014): 965–984.

LIM JH, QU J & ZUO MJ. 2016. Age replacement policy based on imperfect repair with random probability. *Reliability Engineering and System Safety*, **149**: 24–33.

MORAG I, CHEMWENO P, PINTELON L & SHEIKHALISHAHI M. 2018. Identifying the causes of human error in maintenance work in developing countries. *International Journal of Industrial Ergonomics*, **68**(July): 222–230.

PETERSON M, MINTON EA, LIU RL & BARTHOLOMEW DE. 2021. Sustainable Marketing and Consumer Support for Sustainable Businsses. *Sustainable Production and Consumption*, **27**: 157–168.

PHAM H & WANG H. 1996. *Imperfect maintenance*. vol. 2217.

RIVERA-GÓMEZ H, GHARBI A, KENNÉ JP, ORTIZ-ZARCO R & CORONA-ARMENTA JR. 2021. Joint production, inspection and maintenance control policies for deteriorating system under quality constraint. *Journal of Manufacturing Systems*, **60**: 585–607.

ROMERO D, BERNUS P, NORAN O, STAHRE J & FAST-BERGLUND A. 2016a. The Operator 4.0: Human Cyber-Physical Systems & Adaptive Automation Towards Human-Automation Symbiosis Work Systems. In: *IFIP International Conference on Advances in Production Management Systems. APMS 2016: Advances in Production Management Systems. Initiatives for a Sustainable World*, vol. 2. pp. 677–686. Springer.

ROMERO D, STAHRE J, WUEST T, NORAN O, BERNUS P, FAST-BERGLUND A & GORECKY D. 2016b. Towards an operator 4.0 typology: A human-centric perspective on the fourth industrial revolution technologies. In: *CIE 2016: 46th International Conferences on Computers and Industrial Engineering*. pp. 29–31. October. Tianjin, China.

ROSS SM. 2010. Renewal Theory and Its Applications. In: *Introduction to Probability Models*. pp. 421–495. Elsevier.

SHEU SH, SHEU UY, TSAI HN & ZHANG ZG. 2019. Optimal replacement policies for a system based on a one-cycle criterion. *Reliability Engineering and System Safety*, **191**(June): 106527.

SHEU SH, YEH RH, LIN YB & JUANG MG. 1999. A Bayesian perspective on age replacement with minimal repair. *Reliability Engineering and System Safety*, **65**(March 1998): 55–64.

TOLEDO MLGD, FREITAS MA, COLOSIMO EA & GILARDONI GL. 2015. ARA and ARI imperfect repair models: Estimation, goodness-of-fit and reliability prediction. *Reliability Engineering and System Safety*, **140**: 107–115.

TUAN ND, YANN D & MITRA F. 2015. Bayesian Analysis of the Brown–Proschan Model. *Stochastics and Quality Control*, **30**(1): 9–20.

VATN J & AVEN T. 2010. An approach to maintenance optimization where safety issues are important. *Reliability Engineering and System Safety*, **95**(1): 58–63.

How to cite

ARAÚJO J, LOPES R & SCARF P. 2022. Age-based replacement with imperfect repair and risk as low as reasonably practicable. *Pesquisa Operacional*, **42** (nspe1): e263588. doi: 10.1590/0101-7438.2022.042nspe1.00263588.