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APPLICATIONS TO THE PUBLIC SECTOR SPECIAL ISSUE ARTICLES

DISTRIBUTION OF INVESTIGATIONS IN THE BRAZILIAN FEDERAL POLICE USING AGENCY THEORY, SHAPLEY'S VALUE AND MCDA MODEL

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ABSTRACT. This paper proposes a decision model for the distribution of two different types of activities in the Brazilian Federal Police, common and special investigations, which usually generate difficulties for the manager when identifying adequate personnel for each category of task. Rational abstractions from Game Theory were used, especially the *Principal-Agent* theory and Shapley's value, building a model that associates both techniques, combined with a multicriteria model to measure the policemen's operational satisfaction. It innovates the model in the combination of concepts that guide the achievement of optimal points for managing teams, without the use of pecuniary benefits. Results were found that manage to balance the distribution of investigations and goals to be met by the police unit. It is necessary to continue the research, improving the development of methodologies for assessing the capabilities and preferences of the Agent with regard to familiarity with special investigations.

Keywords: public administration, game theory, MCDA.

1 INTRODUCTION

One of the main purposes of organizations is the proper distribution of tasks among its employees, in order to maintain a balance between the skills, capabilities and motivation involved, to provide maximum performance. This challenge increases for public managers who do not have full autonomy regarding the payment of financial remuneration, and who need to use incentives directly related to the servant's subjective perceptions.

Mathematical models have been used by organizations, employing various techniques that assist decision-makers (DM), highlighting those that use Game Theory, in areas such as healthcare

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systems prices (Shabbar & Sayama, 2023), energy consumption (Maleki et al., 2023), material quality (Abdzadeh et al., 2023), preventive maintenance (Ding et al., 2023), construction projects (Shehadeh et al., 2022), reducing carbon emissions (Tan et al., 2021), satisfaction services (Lu & Zhang, 2022; X. Wu et al., 2021), group decision (Ziotti & Leoneti, 2020), land allocation (Hasti et al., 2023), tolerance allocation of complex products (L. Li et al., 2021), among others.

However, for cases in which financial compensation cannot be the main factor for increasing productivity and the worker's adherence to the execution of tasks, a possible solution is the appreciation and recognition of the employees' labour interests, so that the possibilities can be explored their best skills by measuring their motivation, then dividing the workload among the team according to these assessments, especially if there are tasks with different or even opposite characteristics, in which each member of the team can express enthusiasm in adhering to one or the other or even indistinctly between them, having as a premise, however, that all activities must be carried out.

Added to this problem is a common characteristic among organizations, whether public or private, which is the need to meet goals established by higher bodies and which must be met by the entire team, also demanding an adequate distribution of goals to be executed, considering the initially existing workload among employees, so that the responsibility for meeting the goal takes into account the workload and is not merely distributed linearly.

There is few research regarding the distribution of tasks in police investigations, something along these lines was proposed in a predominantly deterministic way by James (2014), specifically to investigate crimes involving items of digital evidence, using the algorithm itself with priorities defined by stakeholders, and by Liu et al (2019), who proposed using queueing models that take two levels of the complexity of the investigation into consideration.

Recently, Faveri & Mota (2023) presented a new MCDA outranking sorting method based on the α-cut variation of fuzzy intervals, with application in the distribution of police investigations.

Thus, to contribute to this review, a decision-making model is proposed in which the work processes are optimally distributed, using the *Principal-Agent* theory, but replacing the extraordinary financial incentive with the appreciation of the employee's work interest, which is particularly important in public organizations. The measurement of the employee's aptitude will be carried out through a multicriteria evaluation, which can be adapted to the actual conditions of each police unit in which it is employed, and the result of this evaluation will be used in the computation of the optimization function.

Finally, the game theory technique of Shapley's Value will be used to project goal achievement to be divided and fulfilled by all employees, considering divergent tasks.

This work is organized as follows: Section 2 presents the theoretical framework that will serve as a background for the proposal; in Section 3 the problem is detailed, and the model is built; in Section 4, the model is applied to a situation of distribution of investigations in the Brazilian

Federal Police. Finally, the conclusion of the work follows with the discussion of the results obtained, their limitations and the identification of studies that may be developed in the future.

2 BACKGROUND: GAME THEORY, MULTICRITERIA ANALYSIS, CONCEPTS, AND LITERATURE

Game theory can be defined as the study of mathematical models of conflict and cooperation between rational decision-makers, that is, it is a theory that provides general mathematical techniques for analyzing situations in which two or more individuals make decisions that will influence others (Myerson, 1997).

Therefore, the principle is that individuals, called players, will make a rational choice, that is, they will make the best decision according to their preferences, among all possible actions and without qualitative restrictions, so that their rationality lies in the consistency of their decisions when faced with different sets of available alternatives actions, not in the nature of their likes or dislikes (Osborne, 2002).

Game theory models are abstract representations of real-life situations, so that such abstraction allows them to be used for the study of a wide range of phenomena, through the use of mathematics to formally express ideas, facilitating the definition of concepts accurately, giving consistency to the ideas, to explore the implications and assumptions (Osborne & Rubinstein, 1994).

Nevertheless, it is through practical applications that one understands the theory, as formal arguments about abstract games help but play a minor role, while applications illustrate the model building process, so the variety of applications shows that similar questions arise in different areas and that the same tools can be applied in each environment (Gibbons, 1992).

As a result, examples of game theory applications can be found in the current literature in the most diverse areas, such as traffic management (Ahmad et al., 2023), risk management (X. Li et al., 2023), transportation networks (Ahmad & Al-Fagih, 2023; Chen et al., 2023), supply chain (Y. Zhang & Hezarkhani, 2021), energy (Cheng & Yu, 2019), political, military and economic behaviour (Cho et al., 2023; Colbert et al., 2020; DeAngelo & McCannon, 2020), public security (Imanpour et al., 2019; Jing & Kang, 2022; Yang & Hong, 2023), administration of Justice (Horz & Simpson, 2023), among others.

Game theory comprises several basic models, in addition to others that may yet be deduced, each adaptable from the theoretical version to everyday reality, and it is up to the scholar to identify whether any of them is fruitful in the effort to solve a given problem.

In the case of this work, it was verified that the rationales proposed in the *Principal-Agent relationship* and in the theory developed by Lloyd Shapley (1953) can be useful for the solution of some demands in Public Administration organizations, especially in the Brazilian Federal Police, related to the more convenient distribution of investigative work, as per the following sections.

2.1 Principal-Agent Model or agency theory

An agency relationship is defined as a contract in which one person (*Principal*) engages another person (*Agent*) to perform some service on his/her behalf, with the delegation of some decision-making authority, so that if both parties maximize their utilities, there is good reason to believe that the *Agent* will not always act in the best interests of the *Principal* (Jensen & Meckling, 1976).

The basic structure of the model is the opposition between *Principal* and *Agent*, in which the first delegates to the second the execution of an activity with certain decision-making power, resulting in an asymmetrical relationship of knowledge, and the delegation can be motivated by the possibility of benefit obtained with the division of tasks or any form of limitation of the *Principal*, with the *Agent having* access to information that is not available to the *Principal* (Laffont & Martimort, 2002).

In this way, agency theory is concerned with solving two problems: the first is when there are conflicting desires or goals and it is difficult or costly for the *Principal* to verify what the *Agent* is really doing; the second is the sharing risk, which arises when *Principal* and *Agent* behave differently in the face of a risk. It is up to the theory to determine the most efficient contract to govern the relationship between these parties (Eisenhardt, 1989).

The opportunity cost of the task, the precise technology used and the ability to use this technology are examples of information that may become private knowledge of the *Agent*, the so-called adverse selection, as well as moral hazard the loss of the ability to control the *Agent's* performance, something that cannot be contracted, since there is no way to verify its value (Laffont & Martimort, 2002).

Thus, the focus of agency theory is to determine contracts that efficiently govern the *Principal-Agent* relationship in cases with *adverse selection* or *moral hazard* issues, which is usually obtained through behavior-oriented or result-oriented contracts, through investment in information systems that control opportunism or by trying to align the agent's interests through incentives (Wagner, 2019).

Commonly, two conditions are necessary for the *Agent* to submit to the *Principal's* orders: (a) that the *Agent* doesn't care “much” about which task it chooses or that he/she be compensated in some way for the possibility of the *Principal* chooses an unwanted task for him/her; and (b) that it is advantageous for the *Principal* to offer the *Agent* an additional compensation if he/she cannot predict, with certainty, which task will be the most suitable (Simon, 1951), considering that the satisfaction function (S) for the parties can be described in the Equations 1 and 2.

$$S_p = F_p(x) - q_p w \tag{1}$$

$$S_a = F_a(x) + q_a w \tag{2}$$

where p and a mean *Principal* and *Agent* respectively, for $F(x)$ being the result of performing the task, the incentive (q) and the wage (w), assuming that $S \geq 0$, $F(x) \geq 0$ and $qw > 0$, i.e., zero or positive satisfaction and result, and positive remuneration.

Certainly, accepting to perform the task (x) involves not only pecuniary benefits, but also the utility generated by various non-remunerated aspects, such as the attractiveness of the team, the level of discipline required, personal relationships such as friendship and respect between the team or a good computer to work with (Jensen & Meckling, 1976).

Remunerative negotiations presuppose, on the part of the *Principal*, an expectation of the result of the task, without being sure of achieving the outcome in the desired way, where there lies its disadvantage in terms of information asymmetry, so that in an eventual bad choice regarding to the *Agent*'s ability to carry out the task (*adverse selection*), the *Principal* must describe a set of results to be achieved despite the lack of information, also pointing out a set of incentive compatibility restrictions where there is asymmetric information, combined with a set of participation restrictions, thus creating a contract of possible incentives. Then it must perform a normative analysis and optimize its objective function in the set of viable incentive outcomes (Laffont & Martimort, 2002).

As for the lack of information regarding any actions that the *Agent* may choose to take when performing the task (*moral hazard*), which may result in a change in the value of the business, the conflict is in the actions chosen to carry out the contracted task, so the result linked to the *Agent's* effort is a matter of luck. However, the *Principal* can adjust a contract based on observable performance, to induce, at a reasonable cost, a high effort by the *Agent,* despite the impossibility of directly conditioning the reward to its action, identifying incentives that induce a positive effort, thus satisfying the incentive constraints, resulting in the contract of possible incentives, preferring the one that implements the positive level of effort at the minimum cost (Laffont & Martimort, 2002).

On the subject, Eisenhardt (1989) proposes that for outcome-based contracts the *Agent* tends to behave in accordance with the *Principal's* interests and that, when the *Principal* has the information to verify the *Agent's* behaviour, the *Agent* once again tends to act in accordance with the *Principal's* interests.

In addition, he identified that information systems; uncertainty of results; risk aversion of the *Agent*; task programmability and agency relationship duration are positively related to behaviour-based contracts and negatively related to outcome-based contracts. While *Principal's* risk aversion, the conflict of objectives between one another and the measurability of results are positively related to outcome-based contracts and negatively to behaviour-based contracts (Eisenhardt, 1989).

The current literature is full in research on the application of agency theory, as can be seen in the following areas: remuneration of insurance companies (Turrado García et al., 2023), marketing (Keränen et al., 2023), contracts (Hosseinian et al., 2022; Sales & de Sousa Ramos, 2021; Zhou et al., 2023), water resources (Darbandsari et al., 2020), human resource management (Barrachina & González-Chordá, 2020), police behaviour (DeAngelo et al., 2023; Dharmapala et al., 2016; Gottschalk, 2018), among others.

In this paper, as will be seen later, Equations 1 and 2 will be incorporated into the optimization function, and the understanding of contract types by the DM is important for establishing the relationship between the police unit manager and the investigator.

2.2 Shapley value

Among the fundamentals of game theory is the hypothesis that players can evaluate, on their utility scales, each scenario that arises as a result of a game. It was based on this statement that Lloyd Shapley (1953) included in the class of scenarios the probability of playing the game, i.e., assessing the actual participation of the player, if he participates in the game.

Thus, the Shapley value is defined with reference to other possible games, i.e., it is a function that assigns a single viable payoff profile to each coalition game, where a payoff profile is feasible if the sum of its components is v (N), which is considered the efficiency profile, given that $\{N, v\}$ are a transferable payoff coalition (Osborne & Rubinstein, 1994).

The bargaining procedures that consider non-cooperative and cooperative games between two people were well explored by John Nash (1950, 1951, 1953), coming lates Shapley's formulation for cooperative games of n-persons that considers coalitions, that is, any non-cooperative subset void of a set of players, thus being an axiomatic construction in which the author raises what types of properties can be expected for a satisfactory solution to reach the best expected payoff allocation in a coalition (Myerson, 1997).

Therefore, since player i is a *dummy* in v if Δ_i (S) = v($\{i\}$) for every coalition S that excludes i and players i and j are interchangeable in v if Δ_i (S) = $\Delta_j(S)$ for every coalition S that contains neither i nor j, Shapley defined the following axioms and theorem (Gul, 1989; Myerson, 1997; Osborne & Rubinstein, 1994; Shapley, 1953):

- *Symmetry*: if i and j are interchangeable at v, then $\phi_i(v) = \phi_j(v)$, which states that value is essentially the property of an abstract game.
- *Efficiency*: establishes that the *dummy* player (i) is what will not be in the game and does not contribute to the winning coalition, so at v it will be $\phi_i(v) = v(\{i)$, with $v(\{i\}) = 0$, while the *carrier* coalition N of v is the one that brings together the players who will share the result, without any allocation to the *dummy*, so that $\sum_N \phi_i(v) = v(N)$.
- *Aggregation* or *additivity*: in which for any two games v and w have $\phi(v+w) = \phi(v) + \phi(w)$, that is, when two independent games are combined, their values can be added player by player.

The resulting theorem, whose value function satisfies the axioms stated for games with finite N *carrier* coalitions is given by Equation 3, for the entire universe of players ($i \in U$).

$$\phi_i(v) = \sum_{S \subseteq N} \left[\frac{(s-1)!(n-s)!}{n!} \right] \cdot [v(S) - v(S-i)] \tag{3}$$

Given this, the Shapley value is a important tool to assess structural strength in a coalition game (Myerson, 1997). This is because, the conjunction of its axioms allows us to state that the order of entry of the player in the coalition is not a decisive factor for the division of values, which will be equally divided according to the participation of each player, taking into account all the coalition possibilities.

The value resulting from Shapley's formula conceives a solution whose motivation is normative, which describes a reasonable or fair way to divide gains (or losses) in a cooperation, considering the strategic realities captured by the characteristic form (Mas-Colell et al., 1995).

Since its conception, the Shapley value has been widely used as a fairer method for distributing results, especially in public organizations. It is worth mentioning some recent works on energy management (Bahmani et al., 2021; Cai et al., 2023; Lee & Kim, 2023), group leadership in networks (Belik, 2023), environment (Goli & Mohammadi, 2022; S. Zhao et al., 2023), distribution of profits and funds (Cubukcu, 2020), public transport (Gnecco et al., 2020), allocation of police patrols (C. Wu et al., 2020), among others.

Thus, the Shapley value was used in this research to assist in individualizing the goals to be achieved by each responsible policeman, based on the distributions of special and common investigations determined by the optimization model.

2.3 Multicriteria decision analysis method

Multicriteria decision analysis/aid (MCDA) models are useful for supporting the decision-making process when there are multiple and conflicting decision criteria, for example when recommending the choice or ranking of a set of alternatives. Such models consider the decision-maker's preference structure and involve a value judgment, where these options are incorporated into the decision model to support alternative selection, so that multiple criteria are then analyzed simultaneously (de Almeida et al., 2015).

MCDA methods are widely used for solving real decision problems, such as assessing policies to foster technological innovations (Dias et al., 2018), assessing government performance (Fernández et al., 2022), evaluation of plea bargain proposals (Sant'Anna et al., 2023), strategic decisions on military budget (Santos et al., 2023) and assessing the agility of hospitals in disaster management (Moheimani et al., 2021). There is also several review literature identifying the use and application of various MCDA methods (Almeida et al., 2023; Amor et al., 2023; Basílio et al., 2022; Costa et al., 2022).

Specific applications combining MCDA methods and Game Theory can be found in several areas, such as marine economy using ELECTRE II method (P. Liu et al., 2023), mineral extraction using TOPSIS method (Collins & Kumral, 2022), public security using TOPSIS-GRA method (J. Zhang et al., 2023), selection in higher education using Fuzzy AHP/COPRAS method (Ekinci et al., 2022), public private partnership (PPP) projects using VIKOR method (H. Zhao & Ge, 2023), renewable energy growth using TOPSIS-Grey (Almutairi et al., 2022), healthcare model management using Fuzzy TOPSIS and Fuzzy VIKOR methods (Cuoghi et al., 2022), among others.

In this context, the SMARTS method (Edwards & Barron, 1994) stands out among those with a single synthesis criterion, i.e., that combine all the criteria in a global assessment, and are more easily explained and better understood by DMs, synthesized in Equation 4.

$$v(a) = \sum_{j=1} k_j v_j(a) \tag{4}$$

where *v(a)* is the value function of criterion *j*, for each alternative *a*, considering the scale constant *k*.

In addition to the aggregation of the evaluations of each criterion, the SMARTS method brings with it the evaluation procedure of the scale constants called "swing weights", which takes into account the weights of the decision maker in the process of eliciting the importance between the criteria.

The use of SMARTS and SMARTER methods continues to produce works in the literature, as noted in the areas of biofuel (Shakirov & Kozlov, 2023), consumption preferences (Aloise-Young et al., 2021), public safety planning (Gurgel & Mota, 2013), computation (Parreiras et al., 2006), among others.

In the present problem, the SMARTS method was considered to measure the satisfaction index of the proposed model, as it is suitable for reflecting the DM's and professionals' evaluation in a more transparent manner. Additionally, it brings a compensatory effect among the criteria under evaluation, which is useful in the proposed procedure for optimized task distribution, as will be seen later.

3 MODEL FOR DISTRIBUTING FEDERAL POLICE INVESTIGATIONS

Due to the large number of criminal facts to be elucidated by the Federal Police, around 62,000 police investigations in progress simultaneously (Polícia Federal, 2023), the organization has been using concepts of military special operations to establish teams responsible for investigations considered priorities.

From a military perspective, a special operation requires specific tactics, techniques, procedures, and equipment to be employed in hostile, denied, or politically or diplomatically sensitive environments, characterized by time sensitivity, clandestine or covert nature, low visibility, with foreign forces, need for regional and cultural guidance, with a high degree of risk (USA, 2014) or, simply put, a special operation is conducted by forces specially trained, equipped and supported for a specific target, whose destruction, elimination or rescue is politically or militarily necessary (McRaven, 1996).

In the Brazilian Federal Police, investigations carried out using concepts of special operations, also called special investigations, are those carried out through planned and concentrated actions, which use extraordinary resources and a team that works within a defined period of time in order to neutralize the individual or criminal group. and obtaining evidence (Silva, 2017).

Thus, special investigations, materialized in one or more police investigations, represent a very small amount of the total number of tasks carried out, totalling only 1072 launched in 2021 (Polícia Federal, 2022), i.e., about 1.71% of the total number of ongoing police investigations, but they demand a much greater work effort and present significantly higher results in the fight against crime in the unitary comparison with common inquiries.

Therefore, the distribution of tasks in the organization can be carried out between special and common investigations, thus arising a problem for assigning investigations among the number of policemen responsible for conducting them, because according to the concepts exposed, the special investigation requires dedication, training and development focused on achieving the objective, something that not all policemen are capable of, desire or are destined to face, also because there is no differentiated financial consideration.

In turn, it is frequent for higher management bodies to set goals to reduce the burden of investigations that are already old, generally distributing the objective linearly among the policemen, which may represent an unequal solution in the division of work, generating dissatisfaction and difficulties management problems that are intended to be solved through game theory.

Furthermore, the different types and quantities of crimes investigated, different numbers of investigators between police units and unequal experiences and preparation among policemen also make it difficult to manage the distribution of tasks, so that the distribution of common and special investigations is usually carried out in a non-systematic way, based only on the experience or practical judgment of the head of the unit.

Thus, this paper proposes the creation of a decision model that systematizes the distribution of investigations existing in a police unit, which takes into account local factors regarding both the policemen satisfaction regarding one or another type of investigation, as well as their capacity, experience and preparation, according to evaluation criteria that can be customized to each reality.

For a better distribution of special operations and common investigations among the police chiefs (*delegados*) available in a Brazilian Federal Police station, utility functions will be structured that closely approximate the best representation of the existing agency model in the organization for the distribution of investigative work. This will be done with the support of an MCDA model to identify preferences, and a mechanism will be presented for individualizing management goals based on the Shapley value.

3.1 Maximum police operational effort

In order to carry out its activities, the Brazilian Federal Police has a budget for the payment of wages to the staff, as well as per diem allowances due for the temporary displacement of public servants between districts, these being the costs of the work performed by policemen (Brasil, 1990, 2006).

Principal-Agent model encourages the granting of incentives in addition to the employee's remuneration, however federal policemen only receives fixed remuneration in the form of a subsidy (Brasil, 2023), so that in this paper it is proposed that the stimulus regarding the task to be carried out does not only occur from monetary benefits, but also from other factors to be stimulated.

Thereby, in the utility function of *Principal* (Equation 5), the variables are the budget for paying the employee's remuneration (b), something that is not under his/her control, and the equivalent of the operational effort from the manager's point of view, considered as fulfilment of police duty (d), this under the direct management of the local administrator, so that the Public Administration and society, receives the fruit of police duty (d) at the cost of the budget (b).

$$A = d - b \tag{5}$$

In turn, the policeman, in the role of *Agent* in this modelling, has to receive remuneration in the form of a subsidy (r) for his work, eventual payments of per diem travel (q), at the cost of employing operational personal effort (e), according to Equation 6.

$$B = r + q - e \tag{6}$$

Assuming that the entire financial cost is used in the payment of remuneration and per diem, as well as that the *Agent's* operational effort reverts to the fulfilment of police duty, it is possible to reach Equation 7.

$$d - b = r + q - e \tag{7}$$

The treatment of the police budget or the financial remuneration owed to the employee will not be the object of this research, but only the operational effort, equivalent to police duty (d), with the management of the human workforce under the aspect of the distribution of functions, a context in which such effort can be represented in Equation 8, for each police chief (i).

$$e_i = \left(\frac{\delta_i + \frac{\phi_i}{n_i}\sigma}{2} \right)^2 \tag{8}$$

Therefore, it is conventionally called:

ω_i, the satisfaction index constructed from the MCDA model, with $W = \{\omega_1, \omega_2, \omega_3, \ldots, \omega_i\}$, as explained in the following section, based on information obtained directly from the investigator, reduces the information asymmetry inherent in the *Principal-Agent* model, thus measuring the level of familiarity or aversion of the professional to the type of work (special investigations).

δ_i, with $D = \{\delta_1, \delta_2, \delta_3, \ldots, \delta_i\}$, in a first sense, that is, from common investigations to special ones, corresponds to the inverse of the operational satisfaction rate $(1 - \omega_i)$, multiplied by the quantity of special operations (g_i), with $G = \{g_1, g_2, g_3, \ldots, g_i\}$, to be carried out by the policeman, $\delta_i = (1-\omega_i)g_i$. In a second sense, that is, from special investigations to common ones, it corresponds to the satisfaction rate multiplied by the quantity of special operations, $\delta_i = \omega_i g_i$.

ϕ_i, the rate of police inquiries, which may correspond to a sum of the importance of inquiries, seniority, correctional adequacy or any other form of measuring productivity or an event of nature, with $F = \{\phi_1, \phi_2, \phi_3, \ldots, \phi_i\}$.

n_i, the number of inquiries distributed to the police chief by the *Principal*, with $N = \{n_1, n_2, n_3, \ldots, n_i\}$.

σ, the constant for adjustment between both, the total number of inquiries (N_t) and the total number of operations (G_t) to be distributed, and the sum of ϕ_i/δ_i or $\phi_i/n_i\omega_i$ depending on whether the distribution occurs from investigations to operations or the opposite. This ensures that $\frac{N_t}{\sigma\sum -\frac{\phi_i}{\delta_i}} = 1$ and $\frac{G_t}{\sigma\sum -\frac{\phi_i}{n_i\omega_i}} = 1$.

Based on these parameters, once the investigator's perception of the greater requirements of a special investigation is known (ω_i) and financial attributes are discarded (*b, r* and *q*), the *Principal* promotes the optimization of operational effort (e_i), as described in Equation 9:

$$\underset{\delta_i}{MaxB} = -\left(\frac{\delta_i + \frac{\phi_i}{n_i}\sigma}{2}\right)^2 = -\frac{(\delta_i n_i + \phi_i\sigma)^2}{4n_i^2} = -\left(\frac{\delta_i^2}{4} + \frac{\delta_i\phi_i\sigma}{2n_i} + \frac{\phi_i^2\sigma^2}{4n_i^2}\right) \tag{9}$$

This expression of effort can be optimized with respect to δ_i or n_i without distinction, depending on the chosen direction. When optimizing with respect to δ_i, we obtain (Equation 10), and when optimizing with respect to n_i, we get (Equation 11).

$$f'(\delta_i) = -\frac{\delta_i n_i + \phi_i\sigma}{2n_i} \Rightarrow -\frac{\delta_i n_i + \phi_i\sigma}{2n_i} = 0 \Rightarrow \delta_i = -\frac{\phi_i\sigma}{n_i},\ for\ n_i \neq 0 \tag{10}$$

$$n_i = -\frac{\phi_i\sigma}{\delta_i},\ for\ \delta_i \neq 0 \tag{11}$$

Certainly, the maximization of effort in special investigations implies keeping the number of common investigations at zero and the contrary is also true. But for various reasons of a practical nature, specialization of investigative knowledge and even because a special investigation materializes in one or more common inquiries, the investigator who is dedicated to the special work also conducts common inquiries. As a result, the constant σ needs to be a negative value to maintain equilibrium in maximizing the allocation of work, which results in the graph of the optimization function (Figure 1), for σ = -0.8 and ϕ = 100.

3.2 Multicriteria model for recognition of aptitude in special investigations

The operational satisfaction index (ω) will be constructed here using the SMARTS MCDA method (Edwards & Barron, 1994), using the *swing weights* technique to identify the necessary

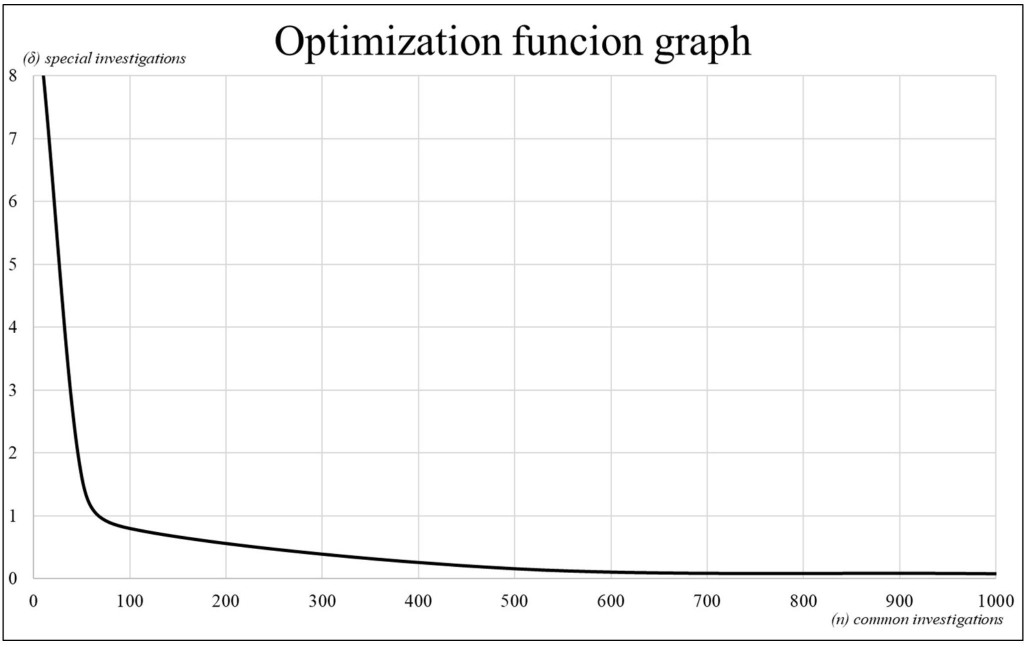

Figure 1 – Optimization function graph.

scale constants (k), based on the criteria defined by the DM, in the case of managing a police unit, responsible for the distribution of activities to be carried out and goals to be met.

As can be observed from the criteria employed in determining the operational satisfaction index (ω), this problem exhibits a nature of compensatory rationality. In essence, the performance of one criterion can be offset by the performance of another. Thus, in the assessment of alternatives, trade-offs between these criteria are taken into account (de Almeida et al., 2015).

Table 1 contains the criteria to be used in this application, already sorted by the importance attributed by the DM, according to the score guided by the *swing weights* technique, normalized in the column *k*.

Table 1 – Allocation of police investigations.

	Criterion	Obj.	Score (SW)	*k*
h1	Motivation	max.	100	0.36
h2	Training	max.	90	0.32
h3	Dedication	max.	60	0.21
h4	Development	min.	30	0.11

It is important to highlight that Federal Police units are not completely homogeneous, with differences between duties, amount of human or material resources, and the experience and training of policemen. Therefore, the trade-off assessments suggested in this standard model must be adapted to each reality, justifying the use of the SMARTS method, which is easy to understand and use by the DM.

To guide the achievement of the satisfaction index, a questionnaire was formulated to be completed by the policemen who will carry out the investigations.

Criterion g1 Motivation:

- What is your level of motivation in conducting special investigations?

- Very motivated (1)

- I am interested (0.75)

- Average (0.5)

- I would not like (0.25)

- No motivation (0)

Criterion g2 Training:

- When did you last train in special investigations?

- Up to 6 months (1)

- Between 6 and 12 months (0.75)

- Between 1 and 3 years (0.5)

- Between 3 and 5 years (0.25)

- More than 5 years ago (0)

Criterion g3 Dedication:

- When did you last conduct a special investigation?

- Currently (1)

- More than 6 months ago (0.75)

- Between 6 and 12 months (0.5)

- Between 1 and 3 years (0.25)

- More than 3 years ago (0)

Criterion g4 Development:

- How long have you been conducting special investigations?

- Never performed (0)

- Between 6 and 12 months (0.25)

- Between 1 and 3 years (0.5)

- Between 3 and 5 years (0.75)

- More than 5 years ago (1)

In this way, obtaining the operational satisfaction index (ω), it is possible to distribute the investigations to be carried out, both special and common, in an optimized way and also establishing a fair division regarding the fulfilment of goals determined by instances superiors, as will be detailed in the following application.

It should be noted that for its use in the optimization function, the ω index is incorporated into the δ parameter, in which $\delta = (1-\omega)g$, for $0 \leq \omega < 1$, necessarily, to avoid division by zero, and it is necessary to carry out an eventual adjustment if the satisfaction index reaches the maximum value.

4 4 APPLICATION OF THE MODEL IN A POLICE STATION

Once the model that maximizes the relationship between police investigations is structured, its application will be demonstrated in a Federal Police station with five police chiefs, $I = \{P_a,\ P_b,\ P_c, P_d, P_e\}$, which currently has a total of 272 common inquiries and 15 special operations.

To assist in tracking the model, Figure 2 presents the flow of procedures that guide the decision-maker until the completion of the investigation distribution. There may be inversion or recursion between steps 4 and 5, as demonstrated further ahead.

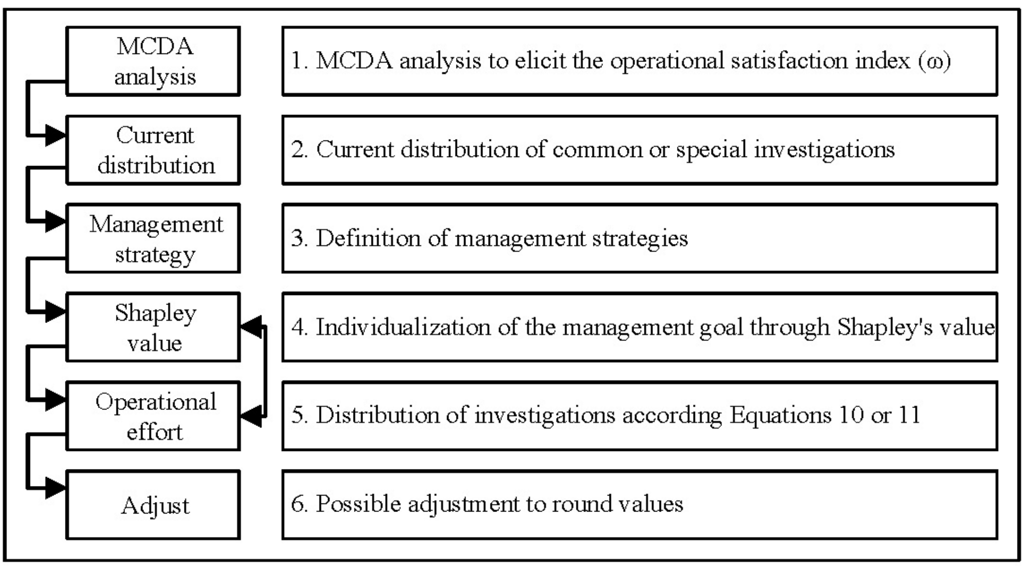

Figure 2 – Flow of procedures for decision model.

In this way, the decision-making process begins with the multicriteria evaluation (Step 1), whose decision matrix and results for the five research police chiefs are shown in Table 2, obtaining the operational satisfaction index (ω).

Table 2 – MCDA model decision matrix.

	h1	h2	h3	h4	ω
k	0.36	0.32	0.21	0.11	-
Pa	0.75	1	1	0.5	0.855
Pb	0.25	0.75	1	0.75	0.623
Pc	0.75	0.75	1	0.25	0.748
Pd	1	0.75	1	1	0.920
Pe	1	1	1	0.25	0.918

Next, the current situation of the police station is evaluated (Step 2), and a decision is made regarding whether the special investigations will be adjusted based on the already distributed common inquiries or vice versa. This decision will influence the order between steps 4 and 5. Subsequently, management strategies are established (Step 3). In the present application, the goal is to reduce the workload of inquiries to 85% of the current amount within a certain period, and to reduce the average age of ongoing inquiries, currently 714 days, to 75% in the same period.

It should be noted that in the application, the age of inquiries was chosen for management using the variable ϕ in the optimization equation, but another attribute of the investigation could have been evaluated, such as importance levels, productivity levels, among others.

Based on this information, two allocation scenarios are identified based on what was established in Step 2. The first scenario focuses on the distribution of special operations, considering the expected quantity of common inquiries and age per policeman at the end of the period. The second scenario focuses on the distribution of current inquiries based on the number of already allocated operations.

In Scenario 1, the Shapley value is initially used (Step 4), assigning each investigator their share of workload according to the organization's defined strategy in the previous step, as shown in Table 3.

Table 3 – Current quantitative inquiries and results with Shapley value.

	Pa	Pb	Pc	Pd	Pe	Total	Average	Strategy
Current Inquiries	50	66	60	74	22	272	54	100%
Current average time (days)	723	732	706	737	671	3569	714	100%
Post-Shapley Inquiries	41	57	51	65	18	231	46	85%
Post-Shapley antiquity	542	549	530	553	503	2677	535	75%

Subsequently (Step 5), the special operations (g) are distributed using Equation 10, with results shown in Table 4, based on the expected inquiries (n) and age (ϕ) at the end of the period (Table 3, post-Shapley). For initial comparison purposes, they were initially distributed without considering the satisfaction rate ($\omega = 0$). Then, the special operations (g') were redistributed again considering the satisfaction rate (ω'). The results in the table (Step 6) are not rounded to allow the manager to better interpret the data and make the final allocation decision.

Table 4 – Allocation of special investigations.

	Pa	Pb	Pc	Pd	Pe	Total
n	41	57	51	65	18	231
ϕ	542	549	530	553	503	2677
ω	0	0	0	0	0	-
σ	-0.213	-0.213	-0.213	-0.213	-0.213	-
g	**2.82**	**2.05**	**2.22**	**1.81**	**6.09**	**15**
ω'	0.855	0.623	0.748	0.920	0.918	**-**
σ'	-0.024	-0.024	-0.024	-0.024	-0.024	**-**
g'	**2.24**	**0.63**	**1.01**	**2.60**	**8.53**	**15**

It is important to compare the two results of operation distribution (g and g') obtained in Table 4, where the first one does not take into account the operational satisfaction index (ω). It is evident that when using the index obtained in the first step, the operations were better distributed and migrated to investigators who demonstrated greater operational aptitude.

In Scenario 2, using Equation 11, starting from the initial allocation of 15 operations (g), the 272 current inquiries (n) are distributed, considering the current overall average age (ϕ) and the satisfaction index (ω). Here, there was a reversal of steps 4 and 5, with results shown in Table 5.

Note that by doing so, the distribution of inquiries (n) may not be balanced (e.g., P_b *vs* P_c). Therefore, a new evaluation was performed disregarding the operational index ($\omega' = 1$), resulting in a more balanced distribution (n').

Based on this, the individual current average ages (ϕ') are identified according to the available inventory of inquiries. Then, the Shapley value is applied to achieve the goal of reducing the workload (n'') and the expected age (ϕ''). It should be noted that with the proposed distribution, the initial average age of each investigator was altered, which initially summed up to 3,311 days but increased to 3,556 days, so that 75% corresponds to 2,667 days.

Taking into account the need for rounding of values, the DM may consider the need for recursion between steps 4 and 5, that is, the cyclical performance of some evaluations, in order to achieve the best balance between the distribution of current and expected common investigations and the special ones, as it became clear in the sequence between the scenarios 1 and 2 demonstrated in this application, consolidated in Table 6.

Table 5 – Allocation of common investigations.

	Pa	Pb	Pc	Pd	Pe	Total
g	2	1	1	3	8	15
ϕ	714	714	714	714	714	3311
ω	0.855	0.623	0.748	0.920	0.918	-
σ	-0.095	-0.095	-0.095	-0.095	-0.095	-
n	**40**	**108**	**90**	**24**	**10**	**272**
ω'	1	1	1	1	1	-
σ'	-0.129	-0.129	-0.129	-0.129	-0.129	-
n'	**46**	**92**	**92**	**31**	**11**	**272**
ϕ'	693	720	719	806	617	3556
n''	**36**	**82**	**82**	**23**	**9**	**231**
ϕ''	520	540	539	605	463	2667

Table 6 – Consolidated results

	Pa	Pb	Pc	Pd	Pe	Total
Current Inquiries	50	66	60	74	22	272
Current average time (days)	723	732	706	737	671	3569
Distributed operations	2	1	1	3	8	15
Corrected current inquiries	46	92	92	31	11	272
Corrected current average time (days)	693	720	719	806	617	3556
Expected inquiries	36	82	82	23	9	231
Expected average time (days)	520	540	539	605	463	2667

5 CONCLUSION

This paper presents two proposals that can be used separately or together. One concerns the allocation of resources and tasks based on established objectives to be achieved, using the Shapley value. Another, in the light of agency theory, is to establish an adequate contract between the manager (*Principal*) and the policeman who will conduct the investigation (*Agent*), based on the identification of their preferences regarding the special work.

Therein resides the innovation of the model to be used in a local police investigative environment, generalizable and adaptable to other units or even to another type of organization, since it mixes rationalizing concepts, which direct to the achievement of optimal points for the management of teams, either in terms of meritocratic valorisation, either as a means of extracting the best qualities of the professional for the benefit of the institution, or even of paying attention to subjective inclinations, failing to consider the *Agent* as a mere task fulfiller, even without using pecuniary benefits as incentives, dealing with likewise with the notion of duties and preferences.

The model contributes to a more customized contract fit as possible regarding the types of work to be balanced, in this case both special and common investigations. This is because the special ones are more suited to contracts for results, since by definition they demand greater investment of resources and dedication from the *Agent*, aimed at a specific objective.

On the other hand, common investigations occur in a much higher volume and necessarily need to share resources to enhance production, requiring fewer concrete results, which are now faced by admitting statistically controlled losses, imposing "correct" actions and behaviours from the *Agent*.

In this sense, the model helps to identify *Agents* with personal characteristics that are more or less suited to one or another type of work, this through a MCDA model that measures the level of operational satisfaction of the policeman, using this elicited information to encourage the employee to work in an activity that holds more aptitude or interest, something that is also advantageous to the *Principal* (Public Administration) by reducing as much as possible the asymmetry of information between the parties involved in this employment relationship.

One limitation of the approach is the need for adaptation of the MCDA model for obtaining the operational satisfaction index to align with the qualifications and training levels of unit leaders (DM) in each group of investigators. Despite this, the chosen model is easy to use and will not be an obstacle to the decision-maker.

Finally, new research papers are shown to be adequate and necessary to better elicit the *Agent*'s preferences with regard to familiarization with working in special operations, including detailing in this type of work any inclinations between specific themes, such as combating the diversion of public resources, crimes against the environment, drug trafficking, terrorism, among others, always with the aim of reducing the asymmetry of information with the *Principal*, seeking to identify the vocation of the police unit itself from its policemen.

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