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ARTICLES

A TYPOLOGY FOR MCDM METHODS BASED ON THE RATIONALITY OF THEIR PAIRWISE COMPARISON PROCEDURES

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ABSTRACT. MCDM methods have been proposed to select, rank, classify, or describe alternatives based on the process of pairwise comparison. While the typologies of MCDM methods present in the literature usually are focused on their amalgamation phase, this paper proposes a typology for classifying MCDM methods based on the rationality of their pairwise comparison procedures. Accordingly, four discriminants were used to provide support in choosing MCDM methods based on the typology proposed. The proposed typology allows the analyst to identify the type of rationality from MCDM methods that best fits the multicriteria problem.

Keywords: multi-criteria decision-making, pairwise comparisons, rationality, elicitation.

1 INTRODUCTION

Since its origins, Operations Research has made use of the concept of optimal solution, which stems for the assumption that a rational choice should be the one that maximizes the expected value or utility. An optimal solution to a problem is considered, therefore, the best outcome among all possible solutions. However, if the problem to be solved encompasses different objectives (which can be associated with each point of view to a different understanding of the problem, as usually occurs in a public choice process) or different criteria (which, in practice, can relate to distinct points of view, such as environmental, financial, strategic, etc.), the concept of seeking an optimal solution loses its effectiveness. Herbert Simon, studying executives' decisions in organizations, clearly and unequivocally explained why the concept of an optimal solution could be eventually insufficient in such context (Simon, 1960).

Multicriteria Decision Making (MCDM) methods employ the concept of satisficing solution. This will be the solution that, being both sufficient and satisfactory, represents the best compromise between multiple objectives (Roy & Bouyssou, 1993). The first phase of a MCDM method

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is the pairwise comparison between the alternatives. At this phase, it is possible to assume different types of rationalities. For example, it can be assumed a universal rationality and incorporate it into the model by a mathematical function that will attribute a value to the trade-off between the alternatives in accordance with the given theory. It also could be assumed a particular rationality, the one of the users itself, which can be obtained from the pairwise comparison that the decision-maker is required to perform. The second phase of MCDM methods involves applying procedures that aim to select, rank, classify, or describe the alternatives based on the pairwise evaluations obtained in the first phase (Slowinski et al., 2012).

The typologies of MCDM methods proposed in the literature are usually focused on the analysis of the procedures performed in their second phase. In a groundbreaking study, Roy (1971) proposed a typology for MCDM methods which divided them into (i) methods that aggregate multiple objective functions into a single function and define a complete order of preferences; (ii) methods that progressively define preferences and exploit the viable set; (iii) methods that define a partial order that is stronger than the product of the complete orders associated with the objective functions; and (iv) methods that reduce uncertainty and incomparability to the greatest degree. Similarly, Jaeger (1982) classified MCDM methods based on how they arrange their final order. While some of these methods has the characteristic of aggregating utilities, others provide outranking relations. Jaeger (1982) was one of the first to mention that the latter would be part of the French school of MCDM methods initiated by Bernard Roy. Other studies have also been developed in the same direction, such as Zavadskas & Turskis (2011). Alternatively, Belton & Stewart (2001) proposed that classification based on the assumption that the acceptance of a satisficing solution should be based on the acceptance of compensations among the criteria. In this scenario, the authors classified the methods into (i) standard value function methods, which are based on different axioms of rationality, and (ii) outranking methods, which create preferences and values as part of the MCDM process. Meinard & Tsoukiàs (2019) proposed a typology for decision analysis methods in general with focus on their different types of rationality. However, none of these typologies focuses on the common aspect shared among the MCDM methods, which is the pairwise comparison phase.

Therefore, the typology proposed herein focuses on the analysis of the different pairwise rationalities applied to the first phase of MCDM methods. First, we aim to clarify which are the possible types of rationalities involved in such pairwise comparisons. This contribution may allow us to extrapolate the typology from within the area of MCDM toward a broader universe. Then it becomes possible to classify the MCDM methods as methods of rationality per convenience, which would distinguish them from multiple goal optimization techniques such as multi-objective linear programming and goal programming. This contribution sets out to minimize what Roy & Vincke (1981) had already identified as a disadvantage of MCDM methods, which is the fact that these techniques are not clearly defined mathematically. Secondly, we focus on differentiating MCDM methods based on how the rationality operates by dividing them into two classes of methods according to the way the rationality is incorporated into the model: (i) those of particular rationality; and (ii) those of universal rationality. For the former, rationality will be integrated into the method by the decision-makers' pairwise comparisons through a specific scale and procedure. In this case, a metric is necessary in order to verify and validate the decision-maker's consistency during this process. For the latter, rationality is assumed to be universal, that is, the pairwise comparisons are proposed based on a specific theory and a mathematical function that contemplates the expected rationality of a universal rational agent. In this case, it would be necessary to include the decision-maker's preferences into the method by means of a prior application of an elicitation technique. Finally, we apply the typology and categorize the most prominent MCDM methods based on the analysis of their pairwise comparison functions[1]. We also demonstrate the complementarity between this typology and existing ones in order to better classify the MCDM methods.

2 THE CONCEPT OF RATIONALITY

Rationality is preeminently a philosophical concept rather than an axiom. For Aristotle, "man is the rational animal" (Oaksford & Chater, 2004) who, according to Thomas Aquinas, uses rational operations to perform voluntary actions since "such are human actions" (Ia IIae, q.6, a.1). In this classical perspective rationality is an objective point of view. Modern and postmodern philosophers first assess whether it is possible to know the object in order to subsequently deal with whether it is real (Kreeft, 2004). Associated with this school of philosophy, there are currently two main schools of thought: (i) the rationalism, as expounded by René Descartes; and (ii) the empiricism, as set forth by David Hume. For rationalists, rationality comes directly from intellectual intuition, while for empiricists, it is limited to sensory experience (Kreeft, 2004). The concept of rationality based on assessments, rather than the classical point of view, has generated intense debate in anthropology and sociology. Nevertheless, this distinction is generally not considered by most of social scientists (Barnes, 1976).

Part of the problem is that rationality is generally seen as a unique concept, a mathematical notion that would be a common domain of statistics, decision theory, game theory, and so forth (Bermúdez, 2009). This generalization is mainly due to the dissemination of the rational choice theory, the mathematical theory which claims that the rational choice maximizes the expected values or utilities, referring to Pascal and Bernoulli (Lengwiler, 2009). Conversely, if we treat agents as though they are self-determining, we neglect the reasons and motivations that may influence a particular individual's actions. Bermúdez (2009) had identified, therefore, that it would be two possible perspectives for framing rationality resulting into two different assumptions, which are either to assume an abstract, although universal, individual called *homo economicus* or to assume a social being essentially located in a scheme of positions and rules called *homo sociologicus*. While the former considers the agent's rationality as universal, the agent's particularities in the latter provide information on how the rationality should be. Based on the evidence that people use heuristics to deal with most decision-making problems, Simon (1960) had named the limitations of the human mind to deal with a universal scheme of rationality as "bounded ratio-

[1] We stress that the criterion for choosing the MCDM methods in this analysis is based on the method's prominence. Therefore, our intention is not to perform a thorough analysis but to provide relevant examples of the proposed typology.

nality”. This perspective could explain why individuals eventually search for satisficing solutions rather than an optimal one (Goodwin & Wright, 2014).

Given the different ways in which rationality can be assumed within mathematical models, it is finally necessary to underscore the evidences in applied psychology that reinforce the thesis that better results relative to absolute scoring methods can be reached through pairwise comparisons procedures (Miller, 1956; Siegel & Siegel, 1972). According to Hair et al. (2006), when all pairs of objects can be compared, pairwise procedures are commonly employed to obtain perceptions of similarities. Saaty (2005) states that pairwise comparisons are biological-based artefacts of the human cognition. Therefore, pairwise comparisons could render as strong as physical scales to apply to objects or phenomena. For example, Vargas (1986) demonstrated that it is possible to derive utility functions from pairwise comparisons. Therefore, the greatest amount of information that a typical observer can provide in a judgment is possible through the use of pairwise comparisons, and this procedure is the core of MCDM methods’ rationality.

3 THE RATIONALITY OF PAIRWISE COMPARISONS IN MCDM METHODS

The rationality in MCDM can be approached from two distinct ways. The first is the one where the rationality is the proper process itself, which is called procedural rationality. From this perspective, it would be enough the use of objective data and a formal process of analysis for overcoming the subjectivity and intuition (Kahneman, 2011). This rationality concerns the extent to which the process leading to the desired solution is sufficiently rational (Eisenführ et al., 2010). Katsikopoulos (2012) refers to this type of rationality as correspondence, which is a criterion external to the individual that is used to verify the effectiveness of the method in translating his/her preferences through its procedures. This feature allows to some of the MCDM methods to deal with the assumption of a *homo sociologicus* decision maker, by providing means to model its particularities and to correct any strong deviation by creating measures for testing the consistency of the judgements made. From the most prominent MCDM methods, the Analytic Hierarchy Process (AHP) is the most notable method of this type.

Developed by the American mathematician Thomas L. Saaty in the 1970s, the AHP method is a MCDM method to support decision-making which allows the modeling of unstructured problems in various areas of knowledge (Saaty, 1975, 1977, 1978). In the first phase of AHP, a pairwise comparison procedure is performed using a numerical scale from 1 to 9 to extract the decision-maker’s preferences for the criteria, wherein 1 corresponds to a neutral preference among the criteria under analysis and 9 signifies an absolute preference for the first criterion over the second. The reciprocity relationship is immediate, which allows the calculations of positive reciprocal matrices. For example, considering a comparison between two criteria A and B, if the decision-maker prefers A than B, it would indicate a preference of A over B in the order of 3 (preferable), 5 (more preferable), 7 (much more preferable), or 9 (absolutely preferable). Subsequently, B would be set inversely, which is given by the unit divided by the value of preference indicated of A over B. If we consider a decision matrix with n criteria, this pairwise comparison is carried out on an amount of $(n^2 - n)/2$ interactions with the user. Subsequently, by the use of the same pairwise comparison structure, the procedure is performed among the m alternatives for each criterion, totaling $m[(n^2-n)/2]$ comparisons required by the user. It is important to note that this procedure proposed by Saaty, including its scale, was not structured on a specific choice or judgment theory (Gass, 2005). Therefore, it is necessary to verify and validate the consistency of the judgments made during the interactions with the decision-maker. Consequently, AHP uses consistency indices to verify the need to redo some pairwise comparisons when there is evidence of transitivity problems in the judgments made.

Procedures similar to AHP can be found in other MCDM methods that propose a framework for integrating decision-making rationality into the method, such as Measuring Attractiveness Through a Categorical-based Evaluation Technique (MACBETH) method. In the specific case of MACBETH, according to Bana e Costa & Vansnick (1994), the procedure includes transforming an initial order into a pre-cardinal judgment on the difference in attractiveness between two alternatives (preferences), based on a six-level semantic scale ranging from weak to extreme attractiveness, in order to finally obtain a cardinal scale that measures the strength of these preferences. To summarize the procedure, the individual is subjected to an interactive process that begins by sorting the elements in decreasing order of attractiveness and entails refining judgments by finding inconsistencies, which are informed by means of a software as the result of four linear programming problems. Therefore, differently from the AHP, the decision maker determines the numerical values of the ordinal scale during the linear programming procedure. An index to test the consistency of judgments is used to review the judgments made until the semantic consistency condition is satisfied. If there is consistency, the algorithm proposed in MACBETH generates a real number for each alternative, which are then plotted in a graphical form for evaluation and agreement by the decision-maker.

The Multiple Attribute Utility (MAUT) method builds its utility function with the decision-maker, which can be conditionally determined based on the hypothesis of independence between the criteria (Keeney & Raiffa, 1976). Given the construction of the utility function for each criterion, which is used to evaluate m alternatives, the procedure begins with obtaining the decision-maker's preferences among all pairs of alternatives and generates an order vector as such, $[x_1,\ldots,x_m]$, where x_1 corresponds to the most preferred alternative and x_m to the least preferred alternative. Next, the most preferred alternative is associated with $u(x_1)=1$, while the least preferred is associated with $u(x_m)=0$. These extremes are used to propose a lottery as such, $[x_1,p;x_m,1-p]$, where one wins x_1 with probability p and x_m with probability $1-p$. For intermediate alternatives, the user is required to evaluate the option between obtaining alternative x_i for sure or playing the lottery, wherein the value of probability p_i is adjusted to measure the decision-maker's indifference to alternative x_i, which constitutes its utility function. The same procedure is performed for the other criteria, which are then grouped in the second phase of the method in the MAUT function and take the form of an additive model.

It should be noted that the first phase of the MCDM methods presented so far have in common the pairwise comparison between alternatives. However, the presence of pairwise comparisons alone is not enough to assume that there is a homogeneity between those MCDM methods. The similarity among the aforementioned MCDM methods is basically the user's interaction required through the pairwise comparisons between the alternatives in order to construct value or utility functions. In other words, these methods incorporate the individual's rationality into the method, with all its scheme of positions and rules.

There is another way in which rationality is approached among the most prominent MCDM methods used to perform this same phase of pairwise comparisons between alternatives. According to Belton & Stewart (2001) this other type of rationality is based on the premises that regard to the internal logic of the individuals. Katsikopoulos (2012) calls it coherence, which is the rationality that is internal to the individual and refers to the logical consistency in which he/she operates the pairwise comparisons. Based on that perspective, some MCDM methods assumes general value or utility functions for modeling the rationality expected from a universal decision-maker. In these methods, the rationality is universal, aligned to the perspective of a *homo economicus*. In other words, it is assumed an expectation that the method can perform a pairwise comparison between the alternatives in an automated way.

Among the methods that assume universal rationality, the family of *ÉLimination et Choix Traduisant la REalité* (ELECTRE) methods is the most well-known. The rationality of ELECTRE methods is characterized by establishing a level of agreement, inspired by Condorcet's ideas, disagreement, and indifference, which are all used in the pairwise comparison process (Roy, 1968). These indicators have values that vary in the range between 0 and 1 and are defined by the decision-maker. For example, in ELECTRE III, the functions $c_i(x,y)$ and $d_i(x,y)$ respectively indicate the levels of agreement and disagreement with the assertion $x \succcurlyeq y$, *x is preferable to* y for criterion i, which, along with the thresholds $q, p,$ and v (indifference, preference, and veto, respectively), construct the outranking relationship between one alternative and another for n criteria under analysis, according to equations (1) and (2).

$$c_j(x,y) = \begin{cases} 1, \text{se} g_j(x) + q_j \geq g_j(y) \\ 0, \text{se} g_j(x) + p_j \leq g_j(y) \\ \frac{p_j + g_j(x) - g_j(y)}{p_j - q_j}, \text{otherwise} \end{cases}, j = 1, \ldots, n \tag{1}$$

$$d_j(x,y) = \begin{cases} 0, \text{se} g_j(x) + p_j \geq g_j(y) \\ 1, \text{se} g_j(x) + v_j \leq g_j(y) \\ \frac{g_j(y) - g_j(x) - p_j}{v_j - p_j}, \text{otherwise} \end{cases}, j = 1, \ldots, n \tag{2}$$

In other words, the pairwise comparison in ELECTRE III is based on the functions $c_i(x,y)$ and $d_i(x,y)$. The former is used in the sum of the weighted values for the criteria that alternative x outranks the respective criteria for alternative y in performance. The latter is based on the maximum difference between the criteria for alternative y that outrank the respective criteria for alternative x in performance. In this sense, although ELECTRE III requires specific information from the user regarding the weights of the criteria and their thresholds of indifference, preference, and veto, it makes pairwise comparisons independently. Then, in the second phase, a procedure is carried out to distill the outranked relationships, which finally allows to select or rank the alternatives by the method.

A similar structure is employed in the Preference Ranking Organization Method for Enrichment Evaluation (PROMÉTHÉE) family of methods, where outranking relationships of alternatives are established based on pairwise comparison functions per criteria, including their respective weights and thresholds, which are defined by the decision-maker. The main difference is that in PROMÉTHÉE it is possible to choose the functions that will serve as the basis for the pairwise comparisons for each of the criteria from a variety of functions. For example, in the context of PROMÉTHÉE II, the user can choose the functions for criteria with linear preferences and indifference, according to the respective equations (3) and (4).

$$P_j(x,y) = \begin{cases} |0, & \text{se } g_j(x) - g_j(y) \leq 0 \\ |\frac{g_j(x)-g_j(y)}{p_j}, & \text{se } 0 \leq g_j(x) - g_j(y) \leq p_j \\ |1, & \text{otherwise} \end{cases} \tag{3}$$

$$P_j(x,y) = \begin{cases} |0, & \text{se} g_j(x) - g_j(y) \leq q_k \\ |\frac{g_j(x)-g_j(y)-q_k}{p_k-q_k}, & \text{se} q_k \leq g_j(x) - g_j(y) \leq p_k \\ |1 & \text{otherwise} \end{cases} \tag{4}$$

Another example of a method that assumes a universal rationality is the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. It was developed based on the rationality that the best alternative should be geometrically closer to the positive ideal alternative and geometrically further from the negative ideal alternative, following Coombs's theory of data (Coombs, 1964). The positive and negative ideal alternatives are essentially the solutions that constitute the best and worst scores, respectively, of each criterion for each alternative. Then, the pairwise comparison process based on Euclidean distances occurs between the m alternatives, and these two exogenous alternatives that were created (Hwang & Yoon, 1981), according to equations (5) and (6).

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}, i = 1, \ldots, m \tag{5}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}, i = 1, \ldots, m \tag{6}$$

It is noteworthy that, unlike the methods of the ELECTRE and PROMETHEE families, in TOPSIS the pairwise comparisons are not carried out between all the alternatives but only between each one and the positive and negative ideal alternatives. Therefore, the type of generalization assumed in TOPSIS is more restricted than the previous methods, since here it is assumed that a universal agent would only care to make pairwise comparisons with regards to these ideal alternatives. Moreover, the decision-maker's particular rationality is not represented in these pairwise comparisons in TOPSIS, as they are approached generally through mathematical functions D_i^+, the Euclidean distance between alternative i and the positive ideal alternative, and D_i^-, the Euclidean distance between alternative i and the negative ideal alternative. To execute the pairwise comparisons correctly, TOPSIS first requires (i) standardizing the scales for the criteria and (ii) weighting each criterion's by using a weight vector, which is determined by the application of an elicitation method. In the second phase of the method, the relative proximity of each alternative are calculated and become the basis for ranking the *m* alternatives under evaluation.

The *TOmada de Decisão Interativa e Multicritério* (TODIM) method (Gomes & Lima, 1991; 1992) is yet another method that assumes a generalized rationality, which was inspired in Kahneman and Tversky's prospect theory (Kahneman & Tversky, 1979). The TODIM method makes use of the notion of a global measure that is calculated by applying the rationality of the theory of prospects, wherein gains and losses have a relative difference. Therefore, the method is based on a description of how people effectively make choice when facing risk, which is substantiated by empirical evidence. To apply the rationality of the prospect theory, the TODIM method constructs a multi-attribute value function, also called the additive difference function (Phi function), based on the projection of the differences between the values of any two alternatives (perceived in relation to each criterion) on a referential criterion, according to the equation (7).

$$\varphi_j(x_i,x_k) = \begin{cases} \sqrt{\frac{w_{jr}\left(x_{ij}-x_{kj}\right)}{\sum_{c=1}^{n} w_{cr}}} if \left(x_{ij}-x_{kj}\right) > 0 \\ 0 if \left(x_{ij}-x_{kj}\right) = 0 \\ \frac{-1}{\theta}\sqrt{\frac{\left(\sum_{c=1}^{n} w_{cr}\right)\left(x_{kj}-x_{ij}\right)}{w_{jr}}} if \left(x_{ij}-x_{kj}\right) < 0 \end{cases} \tag{7}$$

The TODIM method is similar to the methods of the ELECTRE and PROMÉTHÉE families as it proposes a general pairwise comparison structure among all the alternatives. However, it requires only the set of a weight vector, which can be obtained prior to applying the method, and no thresholds. Finally, the concept of the additive difference function adopted in the TODIM method, which is based on the analytical processing of the multidimensionality of a value function in Tversky (1969), is implemented in the aggregation phase. Different forms of the TODIM's Phi function have been investigated in the literature for better adherence of the method to the prospect theory, which includes the use of exponential and logarithmic form as presented by Leoneti & Gomes (2021).

Summarizing, MCDM methods allow for the integration of either an particular or a universal rationality within the method's procedures. While some MCDM methods assume a universal rationality by using a mathematical function in their pairwise comparisons phase, others incorporate the individual's rationality through interactive procedures for the same task of pairwise comparisons.

4 A TYPOLOGY OF MCDM METHODS BASED ON THE RATIONALITY OF THEIR PAIRWISE COMPARISONS

Given the variety of ways of incorporating rationality into the procedures of MCDM methods, this paper proposes a new typology based on the analysis of the rationality that supports their pairwise comparisons. From the analysis of pairwise comparisons within the prominent MCDM methods it can be defined four discriminants, namely: (i) the decision maker's effort, (ii) the dependence on judgment and decision theories, (iii) the dependence on exogenous measuring, and (iv) the necessity of verification of accuracy. We shall further explain each of these four discriminants.

The first discriminant, the decision-maker's effort, is mainly related to the number of interactions or settings required by the method to perform the pairwise comparison. There is a significant difference between the MCDM methods with regards to this question. For example, in the AHP, MACBETH, and MAUT methods, classified as particular rationality methods, the number of interactions required from the user for pairwise comparisons is significantly higher than in universal rationality methods. This is because the elicitation procedures in the former are part of their structure, while a prior elicitation procedure is required in the latter. Even if it is argued that this prior elicitation procedure should factor into measuring the decision-maker's effort in using universal rationality methods, the user can at least choose a relatively simple procedure, such as direct ranking or ordering methods, to establish their preferences for the criteria. Still, it is necessary to make a distinction between the universal rationality methods as some of these methods depend on determining thresholds, such as ELECTRE III and PROMÉTHÉE II, while others do not, such as TODIM and TOPSIS. Particularly, within PROMÉTHÉE II the disadvantages of determining thresholds may be eventually disregarded by the decision-maker if quasi or pseudocriteria are not used. Therefore, there is a continuum between two extremes of higher and lower effort by the decision-maker, which can help one to distinguish between the two types of methods with regard to the rationality of pairwise comparisons. Particular rationality methods usually require more effort from the user than universal rationality methods.

The discriminant dependence on judgment and decision theories indicates the essence of the MCDM method's, including the theoretical basis for its pairwise comparison procedures. It is a widely known fact that the structure of pairwise comparisons needs to be properly grounded in universal rationality methods in order to allow for a satisfactory generalization of the agent rationality for conducting the comparisons in an automated way. Nevertheless, these foundations are not always directly associated with judgment and decision theories, which would assume that there is a continuum between two extremes of higher and lower theoretical foundation in judgment and decision theories. For example, in TOPSIS, the adoption of the notion of distance of each alternative to the ideal alternatives in a geometric sense is based on Coombs' theory of data (Coombs, 1964), which brings important elements of rationality, such as the fact that individuals' utility always declines monotonically when an alternative apart from the ideal positive alternative. In its turn, ELECTRE III is based on Condorcet's fundaments. On the other hand, the TODIM method is based on a widely used and recognized theory of decision and judgment, which is the theory of prospects, which states that: (i) the judgment is relative to a reference point; (ii) a principle of decreasing sensitivity must be considered; and (iii) there is an aversion to loss greater than the proneness to gains. In general, while particular rationality methods do not depend on the support of specific decision and judgment theories, since the individual's rationality is itself incorporated into the method by the process of pairwise comparisons, theoretical justification is fundamental in universal rationality methods. One particular case is the one of MAUT, which has predominant features of an particular rationality method while it is extremely dependent of strong axioms from utility theory.

Dependence on exogenous measurements indicates how much the method depends on the prior measurement of objectives through natural, constructed, or proxy criteria. Aside from determining the weights for the criteria, one of the main advantages of particular rationality methods, such as AHP, is that this measurement is obtained through the process of relatively evaluating each alternative based on the user's judgment in the pairwise comparison process. In this sense, these methods are less dependent on direct measurement instruments, allowing the valuation of qualitative phenomena, such as comfort or beauty, that can be satisfactorily obtained from their pairwise comparison process. Conversely, for universal rationality methods, it will be necessary for the criteria to be preliminarily measured in an absolute fashion, which, in many circumstances, will make its application unfeasible for the situations where there are no ways to measure the phenomenon. Thus, this is an important discriminant that can help the analyst to choose which MCDM method to use in an application, which will depend on whether the phenomenon can be measured exogenously, as required by universal rationality methods, or obtained endogenously based on the user's judgments in the pairwise comparison process, as in the particular rationality methods.

Finally, there is a trade-off between the MCDM methods with regards to satisfactorily translating users' actual preferences and the effort that it requires to provide the parameters necessary to these operations within the method. While the decision-making effort is already included in the first discriminant, here we emphasize the advantages of each specific rationality in decision analysis. Despite the greater effort required by methods based on particular rationality, they are more precise with respect to the individual's thinking process, which Katsikopoulos (2012) refers to as correspondence. Accordingly, it can be assumed that particular rationality methods are more adherent to the concept of *homo sociologicus* while universal rationality methods are more adherent to the concept of *homo economicus*. Given the greater specificity of the former, in a continuum between higher and lower necessity of verification of accuracy, it is expected that particular rationality methods would have greater necessity of verifying correspondence while universal rationality methods would have less. However, among particular rationality methods there are still differences, since, for instance, AHP tolerates up to 10% of inconsistency, while MACBETH does not accept inconsistency. In its turn, the construction of utility functions within MAUT requires the compliance with axioms such as the one of transitivity. Finally, it should be noted that the idea of a universal rationality method achieving high coherence through the structures of pairwise comparisons is possible by means of a grounded theory of decision and judgement, thus creating a range of these methods based on their respective theory capacity. Recent examples of high prediction performance of universal rationality methods can be seen in the literature (Leoneti & Gomes, 2021; Leoneti, 2016).

Finally, in order to jointly operationalize the four discriminants for classifying MCDM methods, Figure 1 presents the continuum for each of the discriminant. Consequently, the typology is defined based on the majority adherence to one of the two extremes. While the method is going to be classified as Particular Rationality Method (PRM) if the dominance occurs on the left, it is going to be classified as Universal Rationality Method (URM) if the dominance occurs on the right. Therefore, based on the simultaneous application of the four discriminants, it is possible to classify MCDM methods according to the typology proposed herein.

PARTICULAR RATIONALITY METHODS (PRM)		UNIVERSAL RATIONALITY METHODS (URM)
Higher	**Decision-Maker's effort**	Lower
Lower	**Dependence on Judgement and Decision Theory**	Higher
Lower	**Dependence on Exogenous Measurements**	Higher
Higher	**Necessity of Verification of Accuracy**	Lower

Figure 1 – Discriminants of the typology for classifying MCDM methods based on the analysis of the rationality of their pairwise comparisons.

For instance, AHP could be considered a type of particular rationality method, since: (i) it requires high effort of the decision maker, due to the $m[(n^2 - n)/2]$ pairwise comparisons needed; (ii) it has low dependency on judgement and decision theory, due to the fact that AHP uses its own scale and procedures for performing pairwise comparisons; (iii) it has low dependency on exogenous measurements, due to the fact that all necessary data is generated within the AHP's through the process of pairwise comparisons; and (iv) it has a consistency ratio index that indicates whether or not the pairwise comparisons are satisfactory. Table 1 presents the typology of the other methods.

Table 1 – Classification of the MCDM methods based on the four discriminants with relation to the pairwise rationalities.

MCDM Method	**Decision-maker's effort**	**Dependence on Judgement and Decision Theory**	**Dependence on Exogenous Measurement**	**Necessity of Verification of Accuracy**	**Typology***
AHP	Very high	Very low	Very low	High	PRM
MACBETH	Very high	Very low	Low	Very high	PRM
MAUT	Very high	Very high	Very low	High	PRM
ELECTRE III	High	High	High	Low	URM
PROMÉTHÉE II	Eventually Low	Low	High	Low	URM
TOPSIS	Very Low	High	Very high	Very low	URM
TODIM	Very Low	Very High	Very high	Very low	URM

* PRM = Particular Rationality Methods; URM = Universal Rationality Methods.

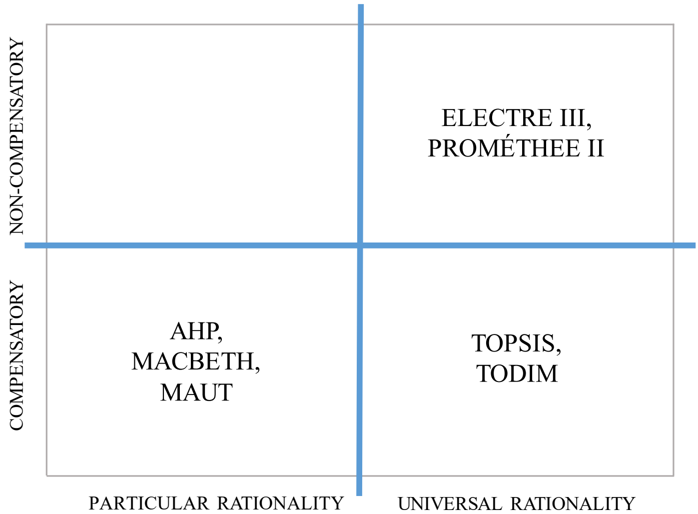

Figure 2 – Typology of MCDM methods based on the rationality of the pairwise comparison phase and the type of procedure in the operationalization phase.

The typology can also be used along with another existing typology, such as the one that refers to the second phase of MCDM methods. A typology which dates back to Roy (1971) and then later to Jaeger (1982), distinguishes these methods into two main types: (i) compensatory and (ii) non-compensatory methods. Based on this typology, MCDM methods are classified by the procedure applied in the operationalization phase of the mathematical objects obtained in the pairwise comparison phase. For the former, aggregation is computed as an index extracted from all the pairwise comparisons performed. One immediate example of compensatory method is the weighted sum. For the latter, instead of producing an overall index, the premise is that the comparison by criterion is necessary to obtain its outranking performance. Consequently, along with the typology presented herein, we can classify MCDM methods, as suggested by Figure 2.

By considering the pairwise rationality analysis, it is expected that the efficiency on the classification of MCDM methods can be increased. Firstly, it is possible to note that particular rationality methods provide a good descriptive support to the decision-making process, which is based on the own individual's rationality that is very close to the *homo sociologicus* assumptions. This perspective is similar to the view of MCDM method as constructivists tools. On the other hand, universal rationality methods can provide a normative support, related to the assumptions of the *homo economicus*, since it is assumed that a general objectivity is incorporated within the method in order to find the best decision. Note that this is an axiomatic/analytic perspective of MCDM methods, which is close related to the ones of the quantitative analysis in decision sciences.

5 FINAL CONSIDERATIONS

This paper presents a new typology for MCDM methods based on the analysis of the rationality adopted in their pairwise comparison phase. The typology proposed herein is built on the assumption that the pairwise comparison is a fundamental characteristic of MCDM methods. In other words, with the focus on the pairwise comparison procedures, we have shown the possibility to establish discriminating criteria between MCDM methods and distinguish them comparatively based on an analysis of the different types of rationalities. The four discriminants proposed for joint application in the classification of MCDM methods based on the pairwise rationality are: (i) the decision-maker's effort, (ii) the dependence on judgment and decision theories, (iii) the dependence on exogenous measurements, and (iv) the necessity of verification of accuracy. The typology makes possible to classify MCDM methods into those that assume either a (i) particular rationality, which are composed by procedures that require and, in some way, incorporate the decision-maker's rationality (they require the measure of the judgements consistency), or (ii) universal rationality, which are composed by functions that assume a theoretical rationality (while they do not require a consistency index, they do require a prior elicitation method). The typology can be used in combination with other typologies that refer to the phase of amalgamation of the mathematical objects generated in the pairwise comparison phase.

In this study, it was classified some of the most prominent MCDM methods by analyzing their pairwise comparison procedures. It was also used a well-known typology to classify MCDM methods, which divides MCDM methods into compensatory and non-compensatory methods. As a result, it was observed the following: (i) TOPSIS and TODIM are universal rationality methods that use an aggregating procedure to propose their final solution; (ii) ELECTRE III and PROMÉTHÉE II are universal rationality methods that use an outranking procedure to propose their final solution; and (iii) MAUT, AHP, and MACBETH are particular rationality methods that use an aggregation procedure to propose the final solution. To the best of our knowledge, we found no examples of particular rationality methods that make use of a non-compensatory procedure to propose the final solution, which offers room for a new field of research in MCDM methods.

Finally, given the absence of a homogeneous procedure for performing pairwise comparisons within MCDM methods, the proposed typology has the potential to help one to choose the MCDM method that more fits the problem. This potential is also concerned to other types of methods that use similar procedures of pairwise comparisons, including, eventually, the adaptations of MCDM methods for group decision-making applications.

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