

## BI-LEVEL MODEL: NEW APPROACH TO DYNAMIC VEHICLE ALLOCATION IN SUPPLY CHAIN

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**ABSTRACT.** There are different decision levels with distinct decision makers in a decentralized Supply Chain, for example, shippers and carriers. Nevertheless, most studies are conducted considering only one decision-making level. The carrier is the decision-making agent in Dynamic Vehicle Allocation (DVA) problem and allocates vehicles to maximize its profit, usually delaying shipping. It is necessary to respect the partnership principle. This paper presents the DVA problem using bi-level programming. The shipper's objective is to minimize shipping delays, while the carrier's objective is to maximize profits. The exact algorithm is used to solve the Bi-level DVA problem. The results show potential applications in logistics, decreasing both transportation delays and costs when synchronizing carrier's and shippers' decisions.

**Keywords:** supply chain management, transportation, mixed integer linear programming, bi-level optimization, dynamic vehicle allocation, multi-period.

### 1 INTRODUCTION

Supply Chain Management (SCM) has been widely used to globally integrate and optimize the logistic functions, extending the management concept beyond the organization boundaries. There is increasing literature on quantitative models that guide the SCM decision-making procedures. Review of SCM models were presented in Mula *et al.* (2010), Arumugham & Parameswaran (2017), and Liao & Widowati (2021).

Most of the presented models consider only one decision maker agent. In reality, each organization makes decisions toward its objectives and these decisions impact the entire chain, influencing each other decisions. Multi-level programming is a suitable mathematical model to address

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this type of problem (Lachhwani & Dwivedi, 2018). The bi-level problem is a special case with only two decision levels. It considers a hierarchical structure similar to the Stackelberg Problem, involving interactive agents at two distinct levels: leaders and followers (Colson, Marcotte & Savard, 2005). Bi-level optimization provides a more realistic model of the distribution and manufacturing processes of decentralized SC (Calvete, Galé and Iranzo, 2014). Besides these processes, this approach has been addressed for modeling upstream and downstream elements of SC, for example: raw material supplier and plant (Yue & You, 2017); manufacturer and retailer (Ma, Wang and Zhu, 2014; Reisi & Fahimnia, 2018; Tantiwattanakul & Dumrongsiri, 2019); suppliers and retailers (Hsueh, 2015); manufacturer and distributor (Yang *et al.*, 2015; Amirtaheri *et al.*, 2017; Nourifar *et al.*, 2017) and production and transportation (Guo *et al.*, 2016; Jalil *et al.*, 2019).

Transport of raw materials, intermediate products, and finished goods are essential in SC. Improvement of transport networks could enhance the efficiency of supply chain networks (Yamada *et al.*, 2011). Transport allocation from their respective origins to their destinations is a problem in logistic called Dynamic Vehicle Allocation (DVA). The classical DVA model has been addressed with only one decision making level. In this case, the carrier is the decision-making agent and allocates its vehicles to maximize profit. This type of approach opposes the SCM principle of partnership between clients and suppliers since decision influences the overall performance of the whole chain. In reality, there are two decision-making agents: carrier and shipper, each having its objectives, and their decisions impact one another. This paper proposes a new model to approach this type of problem by applying bi-level linear programming in DVA. In this model, the shipper strives to minimize delays in shipment, while the carrier makes the vehicle allocation decisions to maximize profits. It consists of a trade-off between shipping delays and the shipping agent's profits. The following text starts with a literature review in section 2. Its first part gives an overview of Dynamic Vehicle Allocation, showing the need for a new mathematical approach with bi-level modeling. The second part takes a look into research concerned with Bi-level Programming theory, algorithms, and applications. Section 3 shows the new Bi-level DVA Model. Examples of applications are shown in Section 4. The section Conclusion describes the contribution of this paper and gives suggestions for new works.

## 2 LITERATURE REVIEW

The literature review about Dynamic Vehicle Allocation and Bi-level Programming are presented in these sections below.

### 2.1 Dynamic Vehicle Allocation

Since Dantzig and Ramser (1959) introduced Vehicle Routing Problem as a generalization of the Traveling-Salesman Problem, mathematical modelling has been widely used in fleet planning. These models approaching freight planning, fleet sizing, loaded vehicle positioning, fleet allocation, vehicle inventory management, fleet expansion, fleet replacement, empty vehicle reposition-

ing, and vehicle routing, were reviewed in SteadieSeifi *et al.* (2014) and Baykasoğlu *et al.* (2019). Empty vehicle repositioning is a frequent problem in long-haul freight. This problem has been related to other problems like empty containers (Kuźmicz & Pesch, 2018; Khakbaz & Bhattacharjya, 2018); fleet sizing (Song & Earl, 2008; Dong & Song, 2009), and vehicle routing (Huth & Mattfeld, 2009). Empty wagon allocation and train scheduling problems are combined into a single mathematical formulation in Upadhyay and Bolia (2014). Vasco and Morabito (2016a) study fleet management in freight transportation in Brazil. The authors use metaheuristic techniques to solve the Brazilian freight carrier problem in realistic-size instances. Cruz *et al.* (2020) and Cruz *et al.* (2022) propose an exact method, using Branch and Price techniques, to reach optimality in reasonable running times for large-scale instances. The decision related to empty vehicle repositioning and vehicle inventory management is addressed by the Dynamic Vehicle Allocation (DVA) Problem. Cargoes remaining unattended during a certain period are lost to the system, which results in losses to the shipping company. Repositioning vehicles to address a forecast demand may result in empty vehicles traveling between regions on time (network nodes). Another factor that must be considered is the imbalance between origins and destinations in the regions being served. This imbalance can in turn cause disequilibrium between the number of vehicles in a region and its potential for cargoes. Therefore, it is necessary to recommend repositioning the areas to avoid this issue.

Let us consider the following integer programming model shown by Powell and Carvalho (1998). In this formulation, one assumes that the time is split into discrete sets  $t = (0, 1, \dots, T)$  where  $T$  is the planning horizon. It is considered that one vehicle can hold only one cargo.

Here  $\sigma$  is the set of regions  $i$  in the network, and  $\tau_{ij}$  is the travel time from region  $i$  to region  $j$ ;

Network Variables:

$N$  is the set of nodes  $(i, t)$ ,  $i \in \sigma$ ,  $t \in T$ , in the dynamic network.

$\Lambda$  is the set of cargoes  $l$  available in the planning horizon.

$\Lambda_{ijt}$  is the set of loads  $l \in \Lambda$  with origin  $i$ , destination  $j$  and feasible time  $t$ .

$R_{it}$  is either the entry ( $R_{it} > 0$ ) or exit ( $R_{it} < 0$ ) flow of vehicles at point  $i$  and time  $t$ .

It is usually assumed that  $R_{it} = 0$  for  $t > 0$ .

$$\max_{x,y} F(x,y) = \sum_{t=0}^T \sum_{i \in \sigma} \sum_{j \in \sigma} \left( \sum_{l \in \Lambda_{ijt}} r_{lt} x_{lt} - c_{ij} y_{ijt} \right) \tag{1}$$

s.t :

$$\sum_{t \in T} x_{lt} + z_l = 1 \quad \forall l \in \Lambda \tag{2}$$

$$\sum_{l \in \Lambda_{ijt}} x_{lt} + y_{ijt} - w_{ijt} = 0 \quad \forall i, j \in \sigma, \forall t \leq T \tag{3}$$

$$\sum_{j \in \sigma} w_{ijt} - \sum_{j \in \sigma} w_{j,i,t-\tau_{ij}} = R_{it} \quad \forall (i, t) \in \mathbb{N} \tag{4}$$

$$y_{ijt}, w_{ijt} \geq 0 \quad (5)$$

$$x_{lt} = \{0, 1\} \quad (6)$$

Parameters:

$r_{lt}$  is the net revenue due to choosing time  $t$  to ship cargo load  $l$ .

$c_{ij}$  is the cost of positioning a vehicle along arc  $(i, j, t)$ .

Decision variables:

$x_{lt} = 1$  if cargo  $l$  is shipped at time  $t$ .

$z_l = 1$  if cargo  $l$  was never shipped (within the planning horizon).

$y_{ijt}$  is the number of positioned vehicles using arc  $(i, j, t)$ . If  $i = j$ ,  $y_{iit}$  represents the number of vehicles idling at region  $i$  from instant  $t$  to instant  $t+1$ .

$w_{ijt}$  is the total vehicle flow in the arc  $(i, j, t)$ .

In this model, the carrier tries to maximize the profit from transported cargo by maximizing revenues and minimizing costs by reducing empty trip costs. The cost of not moving a vehicle is not considered in this model. This situation is represented by objective function (1). The constraint (2) represents the fact that only one vehicle is allocated to each cargo. In addition, constraint (2) checks if the carrier actually ships the cargo during the planning horizon. Constraints (3) and (4) represent the flow conservation conditions in each node.

Several DVA approaches are found in the literature. Crainic (2003) published a review of the existing models. The main differences among them are the objective function formulation, the decision variables, the applicability of each model, and the solution methods. All models consider only one decision level, where the carrier decides the optimal vehicle allocation and the shipping period. Since the objective function used in the DVA model does not include a penalty for shipping delays, the carrier decides to pick up the load whenever it is most convenient to maximize revenues and reduce trip costs. Therefore, several delays that are undesirable to the shipper may occur. In a just-in-time system, the delay may result in additional storage, and production stoppage costs, setup costs, and administrative costs as well as production re-planning costs.

In SC, there are different decision levels with different decision makers such as shippers and carriers. Shippers generate the freight transportation demand and Carriers perform the transport for the shippers (Crainic, Perboli & Rosano, 2017). Nevertheless, the problems are mostly modeled with only one decision making level, despite this old concern. LeBlanc and Boyce (1986) addressed a bi-level model where carriers determine shipment rates and transit times while shippers choose the best mode of shipment. In maritime freight network, the relationship between shippers and carriers was investigated by Lu (2003) and Lee *et al.* (2013), being a bi-level model developed by Boile, Lee and Theofanis (2013) considering hierarchical interactions, where the carriers (leaders) in the upper level, and the shippers (followers) in lower level. In the highway

freight network, the bi-level model was used in Apivatanagul and Regan (2010) considering in the upper level, the transportation agency seeking to reduce the highway congestion and in the lower level, the shipper selecting the transportation services, and the carrier model routes vehicles based on this demand. In hazardous material distribution, Kheirkhah, Navidi and Bidgoli (2016) proposed two meta-heuristics for the bi-level model with regulatory agency in the upper level and distributor in the lower.

This kind of approach, bi-level programming presented in the next section, can be suitable for DVA Problem in SC.

### 3 BI-LEVEL PROGRAMMING

The bi-level programming involves two hierarchical levels in the decision-making process. In this hierarchy, the second level agent, termed follower, depends on the first level agent, termed leader. This type of model is adequate for addressing problems with two decentralized decision levels. Wen and Hsu (1991) define this type of problem as described below.

First, one assumes that there are two hierarchical levels in the decision-making process: upper and lower. Let  $(x, y) \in \mathfrak{R}^n$  be a vector of decision variables split between both decision makers. The upper-level decision maker controls the vector  $x \in \mathfrak{R}^{n_1}$ , while the lower level controls the vector  $y \in \mathfrak{R}^{n_2}$ , where  $n_1 + n_2 = n$ . Next, assuming that  $F, f, \mathfrak{R}^{n_1} \times \mathfrak{R}^{n_2} \Rightarrow \mathfrak{R}^1$  are linear, the bi-level linear problem can be put as:

$$P1: \max_{x, y} F(x, y) = a^T x + b^T y$$

where  $y$  is a solution of:

$$P2: \max_y f(x, y) = c^T x + d^T y,$$

subject to  $Ax + By \leq r$ ,

where:  $a, c, x \in \mathfrak{R}^{n_1}$ ;  $b, d, y \in \mathfrak{R}^{n_2}$ ;  $r \in \mathfrak{R}^m$ ;  $A$  is an  $m \times n_1$  matrix;  $B$  is an  $m \times n_2$  matrix.

Let  $S = \{(x, y) / Ax + By \leq r\}$  be the set of feasible solutions of the problem.

For any given  $x$ , let  $Y(x)$  be the set of the optimal solutions of problem P2.

Ben-Ayed and Blair (1990) demonstrated that bi-level programming is NP-hard, even in problems involving linear functions. There are problems with decision variables continues, integer or mixed-integer in leader, follower or both levels. The bi-level problem is non convex, even when leader's and follower's objective functions are convex as in the linear case (Ben-Ayed, 1993). The bi-level programming theory and solution algorithms are presented by Kalashnikov *et al.* (2015).

Different approaches have been proposed to solve bi-level programming problems. Two streams of research have been taken: exact algorithms and heuristic methods. The first of them can use different methods: Extreme point algorithms (Candler and Townsley 1982; Bialas and Karwan, 1984; Chen, Hansen and Jaumard, 1991); Complementarity pivot algorithms (Bard and Falk, 1982); Branch and bound (Bard and Moore, 1992; Hansen, Jaumard and Savard, 1992; Xu and Wang, 2014); Descent methods (Judice and Faustino, 1992; Vicente, Savard and Júdice, 1994)

and Branch and Cut (Audet, Savard and Zegal, 2007; DeNegre and Ralphs, 2009; Fischetti *et al.*, 2017; Dempe and Kue, 2017). Yue *et al.* (2019) propose an algorithm for global optimization using the reformulation and decomposition method. The second stream uses metaheuristics like Genetic Algorithms (Marinakis, Migdalas and Pardalos, 2007; Deb and Sinha, 2009); Ant colony (Calvete, Galé and Oliveros, 2011) and Tabu Search (Rajesh *et al.*, 2003; Balakrishnan *et al.*, 2013). Talbi (2013) provides background on metaheuristics to solve bi-level problems. Said *et al.* (2021) present a co-evolutionary algorithm solving combinatorial bi-level optimization. Another kind of approach is using supervised learning techniques. Bagloee *et al.*, 2018 apply a hybrid method of machine learning and optimization to solve real-life applications of bi-level problems.

Bi-level programming has been addressed to model several practical problems in different areas, such as Economics; Engineering; Management; Pricing; Energy; Telecommunication; Gas, etc. A list of applications and a taxonomy literature review on bi-level programming were presented in Lachhwani and Dwivedi (2018). In transport, LeBlanc and Boyce (1986) proposed a bi-level model for the network design problem, concluding that the approach can be readily extended to a large class of transportation planning problems. Since then, the bi-level formulation has been addressed by other authors in different modal of transportation problems. In urban passenger transportation, operational decisions on the competitive environment are made considering the upper-level management authority and lower-level described the three operators: bus, taxi, and subway (Hu, Wang, and Sun, 2012). In airlines, operative decisions on fares and frequencies of services have been addressed (Zito, Salvo and La Franca, 2011). In railways, operational decisions of running trains to optimal fare prices have been taken (Kumar, Gupta, and Mehra, 2018). On the highway, the network pricing problem has been addressed (Labbé, Marcotte and Savard, 1998; Brotcorne and Mont Houy, 2001) and transportation network (Alizadeh, Marcotte and Savard, 2013). In the next section, a bi-level model to approach Dynamic Vehicle Allocation Problem was developed.

#### 4 BI-LEVEL DVA MODEL

Given the increase in transportation costs and logistics delays, it is necessary to use planning models that consider all organizations involved in the SCM. Logistics require a systemic approach with synchronization between carrier and shipper decisions. In reality, there are two decision-making agents: carrier and shipper, each one with different objectives and their decisions impact each other. The bi-level programming is suitable for this kind of problem.

In the classic DVA model, cargoes unattended in their due period are lost to the system, resulting in a loss to the shipping company. In SC, shippers and carrier partnerships, unmet cargoes in time are postponed to another period causing shipment delays. To account for the number of shipment delays, constraint (7) was added as follows:

$z_{lt} = 1$  if cargo  $l$  is not shipped in time  $t$ .

$$x_{lt} + z_{lt} = 1 + z_{lt-1}, \forall l \in \Lambda, t = 1, 2, \dots, T \quad (7)$$

As such, every time the carrier does not ship a cargo within period  $t$ ,  $z_{t-1}$  is equal to 1. During the next period, the carrier will have to pick up the load left behind in addition to the new load. This does not change the problem results; it only emphasizes the number of delays during the planning horizon. The travel time between terminals was considered even 1. In addition, constraint (8) was added to guarantee trips do not over fleet capacity.

$$\sum_{i \in \sigma} \sum_{j \in \sigma} w_{ijt} \leq F \quad \forall i, j \in \sigma, t = 0 \tag{8}$$

The bi-level DVA model proposed in this paper considers that the shipper is the leader (first level), and the carrier is the follower (second level). In this case, the shipper controls shipping periods, these become the leader’s decision variables. The carrier controls vehicle allocations; these become the follower decision variables.

The analytical formulation of this problem according to a bi-level programming model is:

$$\min_z f(z) = \sum_{t=0}^T \sum_{i \in \sigma} \sum_{j \in \sigma} \sum_{l \in \Lambda_{ijt}} z_{lt} \tag{9}$$

$$\max_{x,y} F(x,y) = \sum_{t=0}^T \sum_{i \in \sigma} \sum_{j \in \sigma} \left( \sum_{l \in \Lambda_{ijt}} r_{lt} x_{lt} - c_{ij} y_{ijt} \right) \tag{10}$$

s.t :

$$\sum_{t \in T} x_{lt} + z_{lt} = 1 \quad \forall l \in \Lambda \tag{11}$$

$$x_{lt} + z_{lt} = 1 + z_{l,t-1}, \quad \forall l \in \Lambda, t = 1, 2, \dots, T \tag{12}$$

$$\sum_{l \in \Lambda_{ijt}} x_{lt} + y_{ijt} - w_{ijt} = 0 \quad \forall i, j \in \sigma, \quad \forall t \leq T \tag{13}$$

$$\sum_{j \in \sigma} w_{ijt} - \sum_{j \in \sigma} w_{j,i,t-\tau_{ij}} = R_{it} \quad \forall (i,t) \in N \tag{14}$$

$$\sum_{i \in \sigma} \sum_{j \in \sigma} w_{ijt} \leq F \quad \forall i, j \in \sigma, t = 0 \tag{15}$$

$$y_{ijt}, w_{ijt} \geq 0 \tag{16}$$

$$x_{lt}, z_{lt} = \{0, 1\} \tag{17}$$

The mixed integer bi-level linear model above was implemented in Mathematica (<http://www.wri.com>), using the branch and bound method proposed by Xu and Wang (2014) to a global optimal solution:  $(x^*, y^*, \zeta^*)$ .

**Algorithm 1.**

Let  $N$  integer variable to record the number of active nodes in the branch-and-bound tree and  $h$  binary variable to indicate whether ( $h=1$ ) or not ( $h=0$ ) the MILP (39)–(44) has been solved.

**Step 0** (Initialization): Create the root node, Initialize  $x^* = \emptyset$ ,  $y^* = \emptyset$ ,  $\zeta^* = -\infty$ ,  $N = 1$ . Here,  $N$  is the number of remaining nodes.

**Step 1** (Node management): For all  $j \in \{1, \dots, N\}$  such that  $z^k \leq \zeta^*$  or  $l_j^k > u_j^k$ ,  $\exists j$ , remove node  $j$ . Update  $N$  as the number of remaining nodes optimal.

If  $N=0$  then: if  $x \neq \emptyset$  return optimal solution; else  $\zeta^* = -\infty$ , return infeasible,  $\zeta^* = -\infty$ , return unbounded.

If  $N \neq 0$  select a node  $k$  from  $[1, \dots, N]$ ; set  $(\hat{l} = l^k, \hat{u} = u^k, \hat{w} = w^k)$ , remove node  $k$ ; reorder the remaining nodes from 1 to  $N - 1$ , reduce  $N$  by 1; and go to Step 2.

**Step 2** (Relaxation): Solve the high point problem:  $H(\hat{l}, \hat{u}, \hat{w})$ :

If infeasible go to Step 1. If unbounded, go to Step 3. 1,

else, let  $(x^H, y^H)$  denote an optimal solution to  $H(\hat{l}, \hat{u}, \hat{w})$ , if  $C^T x^H + d_1^T y^H \leq \zeta^*$  go to Step 2, else go to Step 3.

**Step 3** (Lower level): Solve  $\mathcal{L}(x^H)$

If is unbounded go to Step 4, then return: BMILP is infeasible, else, let  $y^L$  denote an optimal solution to  $\mathcal{L}(x^H)$ : if  $d_2^T y^H = d_2^T y^L$  then 3 update  $(x^* = x^H, y^* = y^H, \zeta^* = c^T x^H + d_1^T y^H)$ ; if  $H(\hat{l}, \hat{u}, \hat{w})$  is unbounded go to Step 4: else go to Step 1.

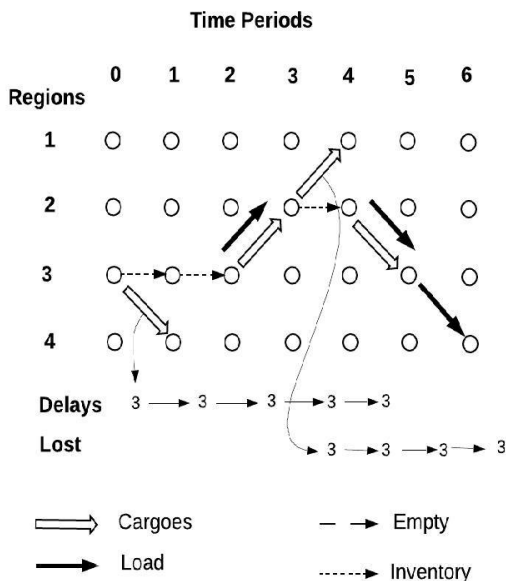
If  $A_1 x^H + B_1 y^L \leq b$  and  $c^T x^H + d_1^T y^L > \zeta^*$  then update  $(x^* = x^H, y^* = y^L, \zeta^* = c^T x^H + d_1^T y^L)$  go to Step 4.

**Step 4**: Create  $(m_2 + 1)$  new nodes, increase  $N$  by  $(m_2 + 1)$ , and go to Step 1. For  $k = 1; \dots; m_2$ , node  $(N+k)$  is characterized by  $(l^{N+K}, u^{N+K}, w^{N+K}, z^{N+K})$ , which is defined in Xu and Wang (2014).

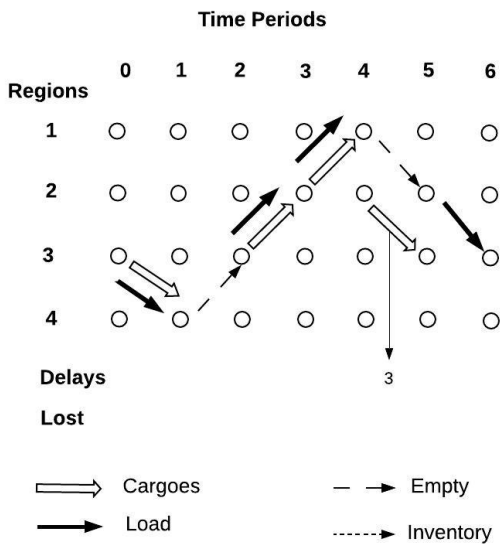
**5 EXAMPLES**

Let us consider 4 different shipping locations, 1, 2, 3 and 4, a planning horizon consisting of 6 periods, the travel time between locations, and a time span. Let us consider an example of cargo shipment per period: three cargoes from region 3 to 4, time 0; three cargoes from region 3 to 2, time 2; three cargoes from region 2 to 1, time 3; and three cargoes from region 2 to 3, time 4; three-vehicle fleet, the trip cost equal to 1, and the net revenue per load shipment equal to 2. Figure 1 and Figure 2 present the results of the Classic DVA model and the new Bi-level DVA Model.





**Figure 1** – Allocation by Classic DVA Model.



**Figure 2** – Allocation by Bi-level DVA Model.

Figure 1, Classic DVA model, shows that the carrier tries to maximize its profit, equal 9, avoiding all empty trips and delaying 24 shipments, and lost 3 shipments. Figure 2, Bi-level DVA Model, shows that the shipper makes 6 empty trips, decreases its profit to 6, but all loads are shipped and there were just 3 delays.

The goal of this example is to demonstrate the difference between the Classic DVA and the Bi-level DVA solutions. It can be observed that with a small reduction in carrier’s profits, a significant decrease in delays is obtained. Other eight hypothetical examples, Table 1, with random data, were analyzed.

**Table 1 – Examples.**

Ex.	Region	Cargoes (Period)			
		1	2	3	4
2	1	0	3(3), 3(4),1(5),1(6)	2(2), 2(3),3(4), 3(5)	2(3),2(5)
	2	3(1),1(4),2(5)	0	3(1), 2(2), 3(5)	3(2),2(4),1(5),2(6)
	3	1(2),2(4),1(5),2(6)	2 (1),3(2),3(3), 2(5)	0	1(3), 2 (4), 3(6)
	4	2(2), 2(3),3(6)	1(3), 3(4),2(6)	3(1), 2(2), 1 (4)	0
3	1	0	3(2),2(3),1(4),2(5)	2(2), 2(3),3(4),1(6)	3(5),2(6)
	2	3(1),2(3),3(5)	0	2(1),2(2)	1(4),1(6)
	3	2(1),2(2),1(3),2(6)	1(2),3(3), 1(5),2(6)	0	1(3),1(4),2(5)
	4	1(3),1(5),2(6)	1(3),1(5),2(6)	1(2),3(3), 1(5),2(6)	0
4	1	0	1(3),3(5)	3(1), 2(5),1(6)	1(2),2(3),3(4),1(5)
	2	1(3),3(4),1(5)	0	0	2(1),2(4)
	3	1 (2),1(3), 1(5),2(6)	2(2),1(3),3(4),1(5)	3(4),1(5)	1(3), 1(5),2(6)
	4	1(1),2(3), 1(6)	3(1),1(3),1(6)	3(1),3(2),2(3),3(5)	0
5	1	0	2(2), 3(4)	3(1),2(6)	3(1),1(3)
	2	3(3),3(4),1(5)	0	1(4),1(5)	1(2),2(4),1(5),2(6)
	3	1 (2),1(3), 1(6)	1(3), 3(5),3(6)	0	1(1), 3(4)
	4	3(1),1(3),2(4),2(6)	1(2),3(3),1(4),1(5)	3(1),3(3),2(4),3(6)	0
6	1	0	3(1), 2(2), 2(3),2(6)	3(1),3(2),1(4),3(5)	2(1),2(6)
	2	3(1), 2(2), 3(5),3(6)	0	1(1),1(2), 1(4),2(5)	2(1),3(4)
	3	1 (2),3(3),3(5),1(6)	2(1), 1(3),3(4),2(6)	0	1(2),2(4), 1(5),2(6)
	4	2(1), 2(3),3(4),1(5)	3(1),1(3),2(4),1(5),3(6)	3(1),1(4),2(5),2(6)	0
7	1	0	1(3),3(4), 2(6)	1(3),3(4),1(5)	1(1),1(2),3(5)
	2	3(2),1(3),3(5),1(6)	0	1(1), 1(4), 1(6)	3(2),1(3),2(5),2(6)
	3	1(1),1(2),1(3)	1(1),1(2),1(3),3(4),1(5)	0	1 (2),1(3), 3(4)
	4	1(2),2(4),3(5),2(6)	1(2),2(5),3(6)	1(3),3(5),1(6)	0
8	1	0	1 (1),3(2),3(5)	3(1),3(2),3(3),2(5)	2(1),3(2),2(3),2(6)
	2	3(3), 3(4),1(5),1(6)	0	1 (2),3(4),3(5)	3(1),2(3),3(4),3(6)
	3	1(1),2(4),1(5)	2(1),1(3),1(4),1(6)	0	2 (2),1 (4), 2 (5)
	4	2(2),3(3),1(4)	1(2),2(3),2(5),3(6)	3(2),1(3),3(4),2(6)	0
9	1	0	1(1),2(3),1(4),1(5),1(6)	3(1),3(2),3(3),2(5)	2(1),1(2),2(3),3(4),1(6)
	2	3(1),3(2),2(3),3(6)	0	1(2),1(3),3(4),1(5), 2(6)	3(2), 2(6)
	3	1 (2),1(3),3(4),1(5)	3 (2),1 (4), 2 (6)	0	2 (3),1 (4), 3 (5)
	4	3(1), 2(2), 3(4),3(5)	2(1),3(2),1(3),1(5)	3(2),1(3),3(4),2(6)	0

Costs: 10 for region 1 to 2, 2 to 3 and 3 to 4; 20 for region 1 to 3, and 2 to 4; 30 for region 1 to 4.  
 Net Revenue: 20 for region 1 to 2, 2 to 3 and 3 to 4; 40 for region 1 to 3 and 2 to 4; 60 for region 1 to 4.

Shipments between locations were randomly obtained as follows: loads follow a discrete distribution with the following probabilities: no cargo =1/2; one, two and three cargoes =1/6.

Costs: 10 for region 1 to 2, 2 to 3 and 3 to 4; 20 for region 1 to 3 and 2 to 4; 30 for region 1 to 4.  
 Net Revenue: 20 for region 1 to 2, 2 to 3 and 3 to 4; 40 for region 1 to 3 and 2 to 4; 60 for region 1 to 4.

Shipments between locations were randomly obtained as follows: loads follow a discrete distribution with the following probabilities: no cargo =1/2; one, two and three cargoes =1/6.

Fleet sizes varying between 10 and 15 vehicles were analyzed in each example with both the DVA model and the Bi-level DVA model.

Table 2 shows the profits, delays, and lost shipments for the examples. The columns with profit, delay and lost shipments show changes between the DVA and the Bi-level DVA models. The Bi-level DVA succeeded in allocating vehicles such that, with a small reduction in carrier’s profits, a significant decrease in delays and, in some examples, a decrease in lost shipments too.

Another DVA example has been present in Ghiani (2004) and in Vasco and Morabito (-a Murty is a motor carrier operating in the Andhraachuki region (India). On July 11, four TL transportation requests were made: from Chittoor to Khammam on July 11, from Srikakulam to Ichapur on July 11, from Ananthapur to Chittoor on July 13 (two loads). On July 11, one vehicle was available in Chittoor and one in Khammam. A further vehicle was currently transporting a previously scheduled shipment and would be available in Chittoor on July 12. Transportation times between terminals, cost and revenue are shown in Table 3.

Let  $T = \{11 \text{ July}, 12 \text{ July}, 13 \text{ July}\} = \{1,2,3\}$  and  $N = \{\text{Ananthapur, Chittoor, Ichapur, Khammam, Srikakulam}\} = \{1, 2, 3, 4, 5\}$ . The optimal ADV solution (Fig. 3):  $X^*_{241} = 1, X^*_{123} = 1, Y^*_{441} = Y^*_{442} = 1, Y^*_{443} = 2$  and  $Y^*_{212} = 1$ , while the remaining variables are zero.  $Z^* = p_{24} + p_{12} - c_{21} = 3.6+1.8-1=4.4$ . It is worth noting that the requests from Srikakulam to Ichapur on July 11 and from Ananthapur to Chittoor on July 13 are not satisfied.

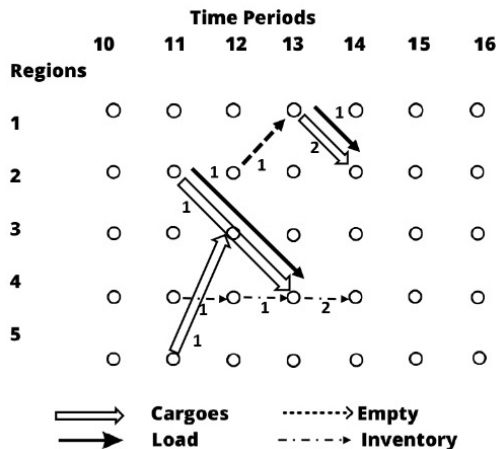


Figure 3 – Murty problem: allocation by DVA Model.

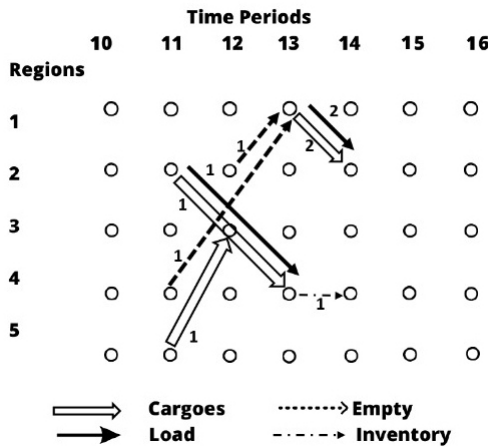
**Table 2 – Results.**

Example	Profit				Delay			Lost		
	Fleet	ADV	Bi-level	%	ADV	Bi-level	%	ADV	Bi-level	%
2	15	1360	1340	-1%	32	25	-22%	6	3	-50%
	14	1340	1330	-1%	34	31	-9%	5	5	0
	13	1300	1290	-1%	41	38	-7%	9	9	0
	12	1270	1270	0	48	47	-2%	12	12	0
	11	1220	1220	0	63	63	0	17	17	0
	10	1150	1150	0	84	84	0	23	23	0
3	15	960	920	-4%	23	4	-83%	1	1	0
	14	950	920	-3%	26	7	-73%	2	2	0
	13	940	910	-3%	24	10	-58%	3	3	0
	12	930	900	-3%	25	14	-44%	4	4	0
	11	910	840	-8%	26	14	-46%	6	6	0
	10	870	850	-2%	37	24	-35%	10	9	-10%
4	15	990	940	-5%	24	4	-83%	0	0	0
	14	990	930	-6%	23	4	-83%	0	0	0
	13	990	910	-8%	22	5	-77%	0	0	0
	12	980	860	-12%	23	8	-65%	0	0	0
	11	970	940	-3%	27	16	-41%	2	1	-50%
	10	940	940	0	39	29	-26%	5	4	-20%
5	15	1070	990	-7%	28	8	-71%	0	1	0
	14	1060	1050	-1%	28	17	-39%	1	2	100%
	13	1050	980	-7%	29	13	-55%	2	3	50%
	12	1040	1020	-2%	27	20	-26%	3	3	0
	11	1010	1010	0	26	24	-8%	5	5	0
	10	980	960	-2%	35	29	-17%	9	8	-11%
6	15	1070	1060	-1%	35	18	-49%	8	6	-25%
	14	1040	1010	-3%	41	20	-51%	11	8	-27%
	13	1010	980	-3%	42	22	-48%	12	9	-25%
	12	980	950	-3%	45	28	-38%	15	12	-20%
	11	950	920	-3%	54	34	-37%	18	15	-17%
	10	900	870	-3%	51	42	-18%	20	19	-5%
7	15	1070	1060	-1%	35	18	-49%	8	6	-25%
	14	1040	1010	-3%	41	20	-51%	11	8	-27%
	13	1010	980	-3%	42	22	-48%	13	9	-31%
	12	980	950	-3%	45	28	-38%	15	12	-20%
	11	950	920	-3%	54	34	-37%	18	15	-17%
	10	900	880	-2%	51	42	-18%	20	19	-5%
8	15	1230	1150	-7%	47	25	-47%	6	5	-17%
	14	1220	1160	-5%	51	33	-35%	8	7	-13%
	13	1200	1150	-4%	51	40	-22%	10	10	0
	12	1160	1130	-3%	59	47	-20%	14	13	-7%
	11	1110	1100	-1%	65	55	-15%	18	16	-11%
	10	1060	1060	0	77	70	-9%	22	21	-5%
9	15	1490	1480	-1%	36	29	-19%	4	4	0
	14	1470	1470	0	40	39	-3%	7	7	0
	13	1420	1420	0	55	54	-2%	12	12	0
	12	1370	1370	0	70	70	0%	17	17	0
	11	1310	1310	0	91	91	0%	23	23	0
	10	1250	1250	0	112	112	0%	29	29	0

**Table 3** – Murty problem: Cost - Revenue (Travel times in days between terminals are same to costs).

	Ananthapur	Chittoor	Ichapur	Khammam	Srikakulam
Ananthapur	0	1 – 1.8	2 – 3.6	2 – 3.6	2 – 3.6
Chittoor	1 – 1.8	0	2 – 3.6	2 – 3.6	2 – 3.6
Ichapur	2 – 3.6	2 – 3.6	0	2	1 – 1.8
Khammam	2 – 3.6	2 – 3.6	2 – 3.6	0	2 – 3.6
Srikakulam	2 – 3.6	2 – 3.6	1 – 1.8	2 – 3.6	0

The optimal Bi-level DVA solution (Fig. 4):  $X^*_{241} = 1, X^*_{123} = 2, Y^*_{411} = 1, Y^*_{443} = 1$  and  $Y^*_{212} = 1$ , while the remaining variables are zero.  $Z^* = p_{24} + 2p_{12} - c_{21} - c_{41} = 3.6 + 2 \times 1.8 - 1 - 2 = 4.2$ , while the remaining variables are zero. One request from Srikakulam to Ichapur on July 11 is not satisfied. Another optimal solution:  $X^*_{241} = 1, X^*_{123} = 1, X^*_{533} = 1, Y^*_{451} = 1, Y^*_{443} = 1$  and  $Y^*_{212} = 1$ , while the remaining variables are zero.  $Z^* = p_{24} + p_{12} + p_{53} - c_{21} - c_{45} = 3.6 + 1.8 + 1.8 - 1 - 2 = 4.2$ , One request from Ananthapur to Chittoor on July 13 is not satisfied.



**Figure 4** – Murty problem: allocation by Bi-level DVA Model.

The Bi-level DVA model showed a slightly lower profit, but a higher load was achieved. In the ADV model, the vehicle remained in inventory in Khammam, as the transport cost would be 2 and the value to be received would be 1.8, which would result in loss to the carrier. In the DVA Bi-level model the vehicle was to meet the load, even causing a loss, because the model considers the partnership in the logistics chain and searches the objectives of both players. In this case, the financial loss is justified, because the cargo that was waiting to be transported could not be delayed. In several situations it is common the existence of an urgency and serious implications for the customer, such as, for example, a production stop, which would certainly harm the chain as a whole. From the carrier’s point of view, it may not be reasonable, on the other hand, from the shipper’s perspective, it would be.

Other five examples are shown with the same Murty problem’s parameters to travel time, cost, revenue. Loads were generated randomly. Fleet sizes varying between 15 and 20 vehicles were analyzed in each example, with both the DVA and the Bi-level DVA models (Table 4).

**Table 4 – Examples.**

Ex.	Region	Cargoes (Period)				
		1	2	3	4	5
11	1	0	1(2), 2(3), 3(4),1(5)	2(1),1(2),2(3),2(4),3(5)	2(4)	3(2), 1(3), 1(4), 3(5)
	2	2(1), 1(2), 1(3), 1(4)	0	(1)	2(1)	1(2), 2(5)
	3			0	2(5)	1(1), 1(3),
	4	2(1), 2(2),	1(3),	1(1), 3(3),	0	1(1), 2(2), 1(3), 1(5)
	5	1(1), 2(3), 3(4)	1(1), 1(2), 1(4), 2(5)	1(1), 1(2), 1(4), 1(5)	2(3)	0
12	1	0	3(1)	1(1), 2(2), 2(4)	3(1), 3(2), 3(3)	3(1), 3(3)
	2	2(3), 2(4)	0	2(4)	1(1), 3(3), 2(4), 2(5)	1(1), 3(3), 1(5)
	3	3(5)	1(3)	0	3(2)	3(2), 3(2), 3(5)
	4	2(5)	1(1), 1(2), 2(4)	3(1), 1(3), 3(4)	0	3(1), 1(5)
	5	1(1), 3(3), 2(4), 2(5)	3(2), 3(4), 3(5)	3(4)	1(3)	0
13	1	0	3(1), 3(2), 1(4)	3(2), 2(3)	1(1), 2(2), 1(4)	3(2), 3(3), 3(5)
	2	3(2)	0	3(4), 3(5)	1(1), 2(3)	3(2), 2(4), 2(4), 2(5)
	3	2(1), 1(3), 2(4), 1(5)	2(1), 3(3), 2(4), 1(5)	0	3(3)	2(2)
	4	3(3), 2(4), 2(5)	2(1), 1(3), 1(4), 2(5)	2(5)	0	1(1), 2(2), 2(4)
	5	3(2), 3(5)	1(2), 1(4)	2(3), 2(5)	2(1), 3(2), 1(4), 2(5)	0
14	1	0	2(2), 3(3), 1(5)	3(2), 1(5)	3(1), 3(4), 3(5)	1(2), 1(3), 1(4)
	2	1(2), 3(3), 3(5)	0	2(1), 3(4), 2(5)	2(3), 3(5)	1(2), 1(3), 1(5)
	3	1(3), 1(4)	1(2), 2(3), 2(4)	0	3(4), 1(5)	2(2), 3(5)
	4	2(1),3(2),2(3),3(4),3(5)	3(2), 3(3), 1(4)	2(1)	0	1(1), 1(4), 3(5)
	5	2(1), 3(2), 1(5)	3(2), 3(4)	2(2), 1(3), 2(5)	1(1), 2(1), 1(4), 3(5)	0
15	1	0	3(1), 3(2), 1(4)	3(2), 2(3)	1(1), 2(2), 1(4)	3(2), 3(3), 3(5)
	2	3(2), 1(3)		3(4), 3(5)	1(1), 2(3)	3(2), 2(4), 2(5)
	3	2(1), 1(3), 2(4), 1(5)	2(1), 3(3), 2(4), 1(5)	0	3(3)	2(2)
	4	3(3), 2(5)	2(1), 1(4), 2(5)	2(5)	0	1(1), 2(2), 2(4)
	5	3(2), 3(5)	1(2), 2(3), 1(4)	2(5)	2(1), 3(2), 1(4), 2(5)	0

The DVA model has one decision level and assumes that the carrier has flexible shipping dates. This situation is the most favorable for the carrier since s/he controls two decisions: shipping period and vehicle programming. Because this model does not assign penalties for shipping delays, it will maximize the carrier’s profit by ignoring shipping delay problems. In the Bi-level DVA model, the shipping date is also flexible, but this decision is made by the shipper. This model assumes that on the first decision level, the shipper strives to minimize shipping delays, and the carrier, on the second level, maximizes the transportation profit by controlling the vehicles programming. The Bi-level DVA model analyzes from the perspective of both players.

**Table 5 – Results.**

Example	Profit				Delay			Lost		
	Fleet	ADV	ADVBL	%	ADV	ADVBL	%	ADV	ADVBL	%
11	20	195.6	180.4	-7.77%	68	41	-39.71%	17	15	-11.76%
	19	189.4	176.8	-6.65%	74	46	-37.84%	19	16	-15.79%
	18	182.8	173.2	-5.25%	71	51	-28.17%	20	17	-15.00%
	17	176	167.8	-4.66%	72	57	-20.83%	20	19	-5.00%
	16	168.6	161.2	-4.39%	76	63	-17.11%	21	20	-4.76%
	15	160	153.4	-4.13%	79	69	-12.66%	24	23	-4.17%
12	20	233.4	230.2	-1.37%	136	115	-15.44%	36	31	-13.89%
	19	224.2	219.4	-2.14%	140	123	-12.14%	38	34	-10.53%
	18	220.8	214.8	-2.72%	143	132	-7.69%	37	35	-5.41%
	17	210.4	205.8	-2.19%	149	141	-5.37%	40	38	-5.00%
	16	202	195	-3.47%	155	150	-3.23%	41	41	0.00%
	15	191.2	184.2	-3.66%	168	159	-5.36%	45	44	-2.22%
13	20	240	232.2	-3.25%	109	100	-8.26%	27	29	7.41%
	19	230.8	222.4	-3.64%	113	107	-5.31%	29	32	10.34%
	18	221.2	211.6	-4.34%	118	114	-3.39%	32	35	9.38%
	17	211.6	200.8	-5.10%	125	121	-3.20%	36	38	5.56%
	16	200.8	188.4	-6.18%	134	129	-3.73%	39	42	7.69%
	15	190	177.6	-6.53%	146	138	-5.48%	43	45	4.65%
14	20	222	215.4	-2.97%	133	116	-12.78%	42	42	0.00%
	19	212.8	195.2	-8.27%	139	122	-12.23%	45	49	8.89%
	18	203.6	194.2	-4.62%	141	129	-8.51%	46	49	6.52%
	17	194.4	186	-4.32%	145	136	-6.21%	48	51	6.25%
	16	185.2	176.2	-4.86%	153	143	-6.54%	51	54	5.88%
	15	175.6	166.4	-5.24%	161	150	-6.83%	56	57	1.79%
15	20	240	232.2	-3.25%	109	100	-8.26%	27	29	7.41%
	19	230.8	222.4	-3.64%	113	107	-5.31%	29	32	10.34%
	18	221.2	211.6	-4.34%	118	114	-3.39%	32	35	9.38%
	17	211.6	200.8	-5.10%	125	121	-3.20%	36	38	5.56%
	16	200.8	188.4	-6.18%	134	129	-3.73%	39	42	7.69%
	15	190	177.6	-6.53%	146	138	-5.48%	43	45	4.65%

## 6 CONCLUSION AND FUTURE RESEARCH

The challenge of providing a high level of service at low cost in logistics has been much studied. Many models have been developed considering only one level of decision, for example, the carrier. However, there are several decision-making agents involved in supply chain. In the case of this work, the carrier and the shipper have been considered. The carrier aims to maximize the profit of the transported cargo and the shipper seeks to minimize cargo dispatch delays. An optimal solution to this problem should consider both points of view. This involves a negotiation between these two players that can be solved using the bi-level approach presented in this work. The two-level DVA model proposed in this work synchronizes the decisions of carriers and shippers. This model can help decide vehicle allocations in a shipper/transporter partnership in

the supply chain, making a compensatory analysis between maximizing profits and minimizing delays.

This paper uses an exact solution algorithm. The objective is to be able to compare the results of the classic DVA with the new Bi-level DVA model using a global optimum solution. The study uses a small instance because bi-level problems are known to be NP-hard. For future work, we suggest including several computational instances and computing time.

The real-world problems may involve large data; therefore, a global optimization algorithm may not be adequate.

Thus, for future studies we propose the use of metaheuristics or approximate dynamic programming for large-scale problems.

The Bi-level DVA problem studied in this paper assumes that the demands are distributed through time and can be forecast with some certainty in a given time horizon. For future research, we suggest to extend the proposed approach to scope uncertainties in the problem parameters, for example, uncertain demands.

### **Disclosure statement**

No potential conflict of interest was reported by the authors.

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### **How to cite**

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