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ARTICLES

A STOCHASTIC OPTIMIZATION MODEL FOR THE IRREGULAR KNAPSACK PROBLEM WITH UNCERTAINTY IN THE PLATE DEFECTS

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ABSTRACT. The present research deals with the two-dimensional knapsack problem by considering the cutting of irregular items from a rectangular plate with defects. While the defects are only known at the time of cutting (in the future), we need first to select which items to produce from cutting the plate. The final items cannot have any defects and the goal is to maximize the profit from cutting the plate and producing the items. We propose a two-stage stochastic optimization model that makes use of a discrete set of scenarios with the realization of the plate defects. The first-stage decisions involve selecting items for cutting and possible production. The second-stage decisions consider the positioning of items in the plate given the scenarios with defects, and then the cancellation and non-production of some selected items, if any. We also extend this model to include a measure of risk, aiming at robust solutions. We perform computational tests on instances adapted from the literature that consider three types of defects, eight scenarios, and four cases for determining each scenario's probability. The tests evaluate the impact of uncertainties on the problem by calculating the expected value of perfect information and the value of the stochastic solution. The results indicate a percentage reduction in the profit of up to 27.7%, on average, when considering a fully risk-averse decision-maker.

Keywords: two-dimensional knapsack problem, irregular shaped items, plate defects, two-stage stochastic optimization.

1 INTRODUCTION

In companies concerned with the cutting of leather, cloth, and metal, among others, a plate (i.e. a large piece of leather, cloth, or metal) is available to be cut to produce (irregularly shaped) items. These items may be used for the production of clothing, bags, shoes, sporting goods, machines, etc. (Mundim et al., 2017; Peralta et al., 2018; Souza Queiroz & Andretta, 2020). In some

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cases, the plate presents defects in some parts, either from extraction, handling, or transportation, making it impossible to obtain items on these parts (Baldacci et al., 2014).

In this paper, we consider the cutting of irregular items from a rectangular plate that may have defects. We assume that the plate is purchased in advance, due to preparation and transportation issues on the part of the supplier. The plate has dimensions and material characteristics known in advance. However, defects in the plate are only identified after delivery and when it enters the cutting stage. At this point, previously selected items may have their production canceled, incurring a cancellation cost. This situation may appear, e.g., in the leather and metal-mechanic industries. The industry confirms with its customers that their items are going to be produced, expecting to obtain a certain profit from this. The industry orders the plate specifying its dimensions and material characteristics based on a set of customers' items to produce, knowing that the plate may contain parts with defects (e.g., arising from extraction, handling, and/or transportation) that are not precisely known. When the plate arrives at the industry, it undergoes a control check. If the plate has no defect, it is certain that all items selected initially will be produced. On the other hand, in the presence of defects, the industry needs to reconfirm which items will be produced, so that some selected items will be canceled and, in this case, there will be a cost (contractual fine) for not producing these items (e.g., not be able to deliver them within the time agreed with the customers). Clearly, the industry wants to minimize the impact that cancellation costs bring on the profit from cutting the plate.

The objective is to obtain a production plan that gives the maximum net profit, which depends on the profit obtained with the production of items minus the cancellation cost of selected items. The production plan considers the selected items to be obtained from the plate and the realization of possible defects through scenarios, so it may be necessary to cancel items when knowing the defects. We treat the defects as uncertain data. We call this problem a Two-dimensional Irregular Knapsack Problem with Defects Uncertainty (2IKP-DU).

The problem of optimizing the cutting of irregularly shaped items has been studied in the literature on cutting and packing problems, with a relatively smaller number of contributions compared to problems that consider regular items, e.g., rectangles (Wäscher et al., 2007; Cherri et al., 2016). Common constraints in these problems concern ensuring that items do not overlap each other and items are entirely inside the plate. Due to the geometry of irregular items, it is necessary to use additional tools to deal with these constraints. Some tools include discretizing items into arrays of points (raster method), calculating the relative position between items using distance functions (phi-functions), constructing polygons between pairs of items indicating if they are overlapping (no-fit polygon), and checking the intersection between segments and the inclusion of points (direct trigonometry method) (Bennell & Oliveira, 2008).

Regarding the knapsack problem with irregular items, Scheithauer & Terno (1993) developed mixed-integer linear optimization models to deal with convex and non-convex polygons. Martins & Tsuzuki (2010) proposed simulated annealing when the problem has rectangular and irregular plates. The feasible regions to positioning items are obtained with the no-fit polygon. Valle et al. (2012) developed heuristics based on the greedy randomized adaptive search procedure to solve the problem variants in which there is only one copy of each item and when there are unlimited copies of each item. The authors calculated the no-fit polygon to better positioning items in the plate. Mundim & Queiroz (2012) used simulated annealing as the local search in the algorithm of Valle et al. (2012).

In Dalalah et al. (2014), the problem considers rectangular and irregular plates. These authors proposed a constructive heuristic in which items are positioned according to the occupancy of the plate. Baldacci et al. (2014) solved the problem with an irregular plate that arises in the leather industry. The plate is discretized into a matrix, and the authors presented an integer linear optimization model, a Lagrangean relaxation, and heuristics obtained from the relaxation they proposed. Mundim et al. (2018) developed a general heuristic for different irregular cutting problems such as knapsack and cutting stock. The authors proposed many positioning rules, some of them based on bottom-left, and were able to improve much of the solutions presented by the previous literature, such as Valle et al. (2012). Souza Queiroz & Andretta (2020) proposed two heuristics, the first based on a biased random key genetic algorithm. The other is a variable neighborhood search. The heuristics consider the solution coded as a vector of numbers. The positioning of items observes the no-fit polygons and three rules inspired on the bottom-left. Román (2020) considered the positioning of a subset of items of maximum value and respecting the maximum weight that can be cut from the plate. The author assumed that items could be rotated and the plate could be irregular and with defects. The author developed several heuristics for the problem, including an evolutionary algorithm. The non-overlap of items is guaranteed with a Python library.

Some studies have considered defects and quality zones in plates in irregular cutting problems. In Chung et al. (1990) there is a hybrid heuristic that allocates items to regions of the plate in accordance with the position of the defects. Heistermann & Lengauer (1995) handled a real case found in the leather industry, where the plate is irregular and contains both quality and defect zones. Their heuristic considers a set of candidate items and tries to position them without generating overlap if it is possible; otherwise, it places only the item that results in the best occupancy of the plate. Han & Na (1994) developed a two-stage approach, which initially positions items by observing the plate and its defects. The plate occupancy is improved with a heuristic based on simulated annealing. For problems involving irregular plates, Tay & Lee (2002) developed a genetic algorithm, while Yuping et al. (2005) considered quality zones and used a penalty strategy to find feasible positions. Crispin et al. (2005) developed genetic algorithms for a problem in the footwear industry. The algorithms use the no-fit polygon to determine positions that maximize the plate utilization or that let items touching each other. In Lee et al. (2008), the problem considers multiple irregular and defective plates. Items are sorted by value and initially positioned in such an order. To improve the solution, items can be rotated and/or translated from their initial position.

Alves et al. (2012a) and Alves et al. (2012b) solved the problem of cutting leather parts for the automotive industry. In Alves et al. (2012b), they searched for the maximization of the occupancy of a single plate. A variable neighborhood search heuristic is developed and considers the search over the sequence of items. The authors developed four operators to move items in search of a sequence that results in the better occupancy of the plate. Mundim & Andretta (2014) considered the problem with multiple irregular plates, aiming to minimize the number of plates needed to cut the demanded items. The authors presented two heuristics based on the bottom-left. The non-overlapping between items and defects is guaranteed by the no-fit polygon. In Pinto et al. (2016) there is an improvement over the results of Alves et al. (2012b), with the use of a constructive heuristic. This heuristic simulates the positioning of items on points in the plate, selecting that item and point which generate the best occupancy of the plate. To improve the solution, the authors proposed a local search, where already positioned items are removed from their positions and items not yet positioned are tested on those positions.

Reijntjes (2016) considered the plate with defects and quality zones, proposing constructive heuristics based on the bottom-left. They were considered inside a branch-and-bound algorithm. The non-overlapping between items is guaranteed by a collision-free algorithm based on the concepts of the no-fit polygon. Chen et al. (2020) handled the knapsack problem where items are rectangular and the plate is irregular and defective, with application to slate cutting. The objective is to maximize the occupancy of the plate considering that items are obtained from guillotined cuts. Their heuristic considers the plate subdivided by horizontal levels and items are positioned on the sub-plates. Sequences of items are generated and controlled by a genetic algorithm.

There are few contributions in the literature for the treatment of uncertainties in irregular cutting problems. Mundim (2017) considered uncertainties in the demand of items for the problem with multiple plates, proposing a two-stage stochastic model. The model penalizes the lack or excess of items produced more than needed, respecting a sampled set of scenarios. In Souza Queiroz & Andretta (2022) there is a two-stage stochastic model for the strip packing problem with uncertainties associated with the demand of items. The results obtained with the model indicated that disregarding the uncertainties would lead to higher total cost solutions. We summarize in Table 1 the cited literature, which is related to two-dimensional irregular cutting problems, focusing on knapsack type problems, problems whose plate has quality zones or defects, and problems with uncertainties. Each row contains information of the given contribution, including the authors and the solution method used to solve the problem.

Differently from the authors in Table 1, we solve the knapsack problem with irregular items by assuming that defects in the plate are uncertain, i.e., which defects and their position in the plate are not known in advance. We propose a two-stage stochastic optimization model with recourse: the first stage considers decisions about which items to select to get from the plate; the second stage contains the scenarios with the realization of the defects in the plate and then some items may be canceled because there is no longer a feasible positioning in the plate. To obtain a feasible production plan in each scenario, we use the no-fit raster method, which is a combination of the raster method and the no-fit polygon.

The initially proposed stochastic model is risk-neutral, i.e., it considers the optimization of the expected value of the net profit to take the best decisions. In this case, the net profit is optimized on average, and for particular realizations of the uncertain data, the profit could be much smaller than

Table 1 – Literature related to the investigated problem.

Group	Authors	Solution method	Group	Authors	Solution method
	Scheithauer & Terno (1993)	mixed-integer linear optimization models		Chung et al. (1990)	hybrid heuristic
	Martins & Tsuzuki (2010)	simulated annealing		Heistermann & Lengauer (1995)	constructive heuristic
	Valle et al. (2012)	greedy randomized adaptive search procedure		Han & Na (1994)	two-stage heuristic
	Mundim & Queiroz (2012)	greedy randomized adaptive search procedure with simulated annealing		Tay & Lee (2002)	genetic algorithm
knapsack type problems	Dalalah et al. (2014)	constructive heuristics		Yuping et al. (2005)	simulated annealing
	Baldacci et al. (2014)	mixed-integer linear optimization models	quality zones or defects	Crispin et al. (2005)	genetic algorithm
	Mundim et al. (2018)	general heuristic		Lee et al. (2008)	constructive heuristic
	Souza Queiroz & Andretta (2020)	biased random key genetic algorithm and variable neighborhood search		Alves et al. (2012a)	constructive heuristic
	Román (2020)	evolutionary algorithm		Alves et al. (2012b)	variable neighborhood search
				Mundim & Andretta (2014)	hybrid heuristic
	Mundim (2017)	two-stage stochastic model		Pinto et al. (2016)	constructive heuristic
uncertainties	Souza Queiroz & Andretta (2022)	two-stage stochastic model		Reijntjes (2016)	branch-and-bound algorithm
				Chen et al. (2020)	level based heuristic

their average values (Shapiro et al., 2013). Aiming to control the variability, which may occur by the defect realizations, we consider a risk measure to find a compromise between maximizing the average profit and trying to control the variability of second-stage decisions (Ahmed & Sahinidis, 1998), resulting in a risk-averse model. We perform computational experiments with instances adapted from the literature on irregular cutting problems. For each instance, scenarios are created observing the occurrence of each defect. Four probability cases for each scenario are studied. The computational experiments assess the expected value of perfect information, the value of the stochastic solution, and the risk-averse model, concluding on the need to consider uncertainties to obtain profitable solutions to the problem.

This paper is organized as follows. The problem definition is given in Section 2. Section 3 presents the proposed model, discusses the analyses to evaluate the impact of uncertainties on the problem, and introduces the risk-averse model. Section 4 contains the computational study performed on the models, which are solved by a branch-and-cut algorithm. Instances adapted from the literature are tested. Concluding remarks and directions for continuing this research are given in Section 5.

2 PROBLEM DEFINITION

The two-dimensional knapsack problem (with irregular items) is NP-hard (Garey & Johnson, 1979). We define the problem over the Cartesian plane. The plate is positioned on the first quadrant and has the origin at $(0,0)$. The x-axis is associated with the length dimension L, while the y-axis is associated with the height/width dimension H. These dimensions have fixed and known values. There is a set I of items available for selection. Each item $i \in I$ has area a_i and a rectangular envelope, which is the smallest rectangle, in terms of sides and without rotation, that circumscribes the item. An item cannot be rotated and is positioned by its reference vertex, which is the vertex of the smallest y-coordinate and, in the case of a tie, of the smallest x-coordinate. Furthermore, each item i has profit v_i, given its selection and production, and the cancellation cost c_i, given its selection but non-production.

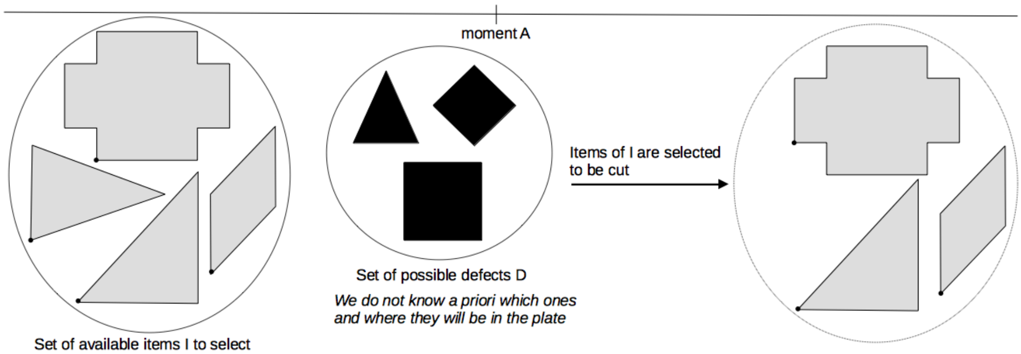
The 2IKP-DU considers a set D of possible defects that the plate may have. Each defect $d \in D$ is modeled as a regular/irregular item of known dimensions, area, and reference vertex. On the other hand, the defects and their position in the plate are not known and, thus, we assume them as uncertain data. The goal is to determine a feasible production plan that results in the maximum net profit when cutting the plate. A production plan is feasible when the items to be produced can be positioned without overlapping, entirely inside the plate, and contain no defective parts.

Selection of items that can be part of the production plan occurs at an early moment A, when the plate is ordered. At this time, items from the set I are selected to be produced from cutting the rectangular plate of known dimensions. Since the plate may have defects that are only revealed at moment B (when items are produced), some selected items may be canceled, incurring a cancellation cost due to non-production. The production plan is only confirmed at the second moment when defects are realized, and thus we can confirm which of the selected items can be feasibly positioned, while others may be canceled. In this way, we look for a feasible production plan that results in the maximum net profit. Figure 1 illustrates an example of the 2IKP-DU and the decisions that need to be made at each moment.

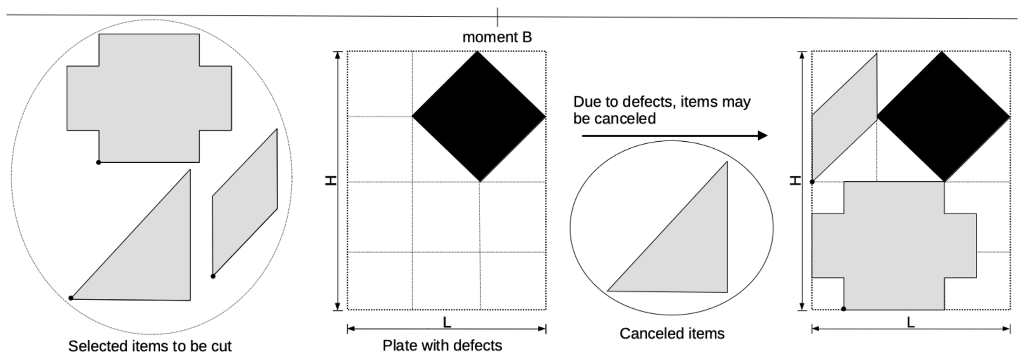
Regarding the positioning of items, we employ the no-fit raster to ensure that items do not overlap and the inner-fit raster to ensure that items are totally inside the plate (Toledo et al., 2013). The no-fit raster NFR_{ij} consists of computing the no-fit polygon for each pair of items i and j, one item is fixed (i) and the other is orbiting (j). The orbital item is translated around the fixed item, always touching the fixed item but without overlapping it, forming a polygon whose interior indicates the items are overlapping. This polygon is then discretized into a binary matrix, whose cells with 1 indicate that items are overlapping; otherwise, cells have the value 0, as we can see in Figure 2(a). On the other hand, the inner-fit raster IFR_i consists of the polygon obtained by translating each item i, internally, always touching one of the edges of the plate. This polygon is then discretized into a binary matrix. The cells with values 0 indicate that the item can be positioned without extrapolating the dimensions of the plate; otherwise, cells have the value 1, as we can see in Figure 2(b).

3 STOCHASTIC OPTIMIZATION MODEL WITH RECOURSE

We develop a two-stage stochastic optimization model for the 2IKP-DU. The number and location of the defects in the plate are considered random variables with discrete realizations according to a known probability distribution. Let $\Omega = \{1, 2, \ldots, S\}$ be the set of possible states, i.e., scenarios ($s \in \Omega$) sampled with the realization of the defects in the plate.

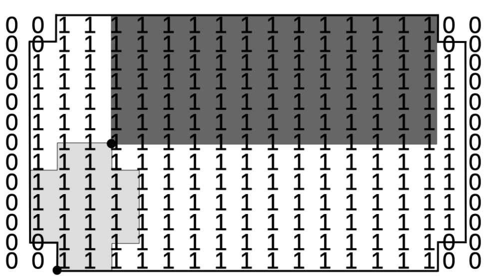


(a) Selection of items at moment *A*.

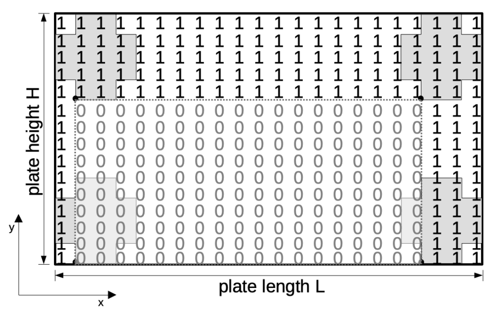


(b) Confirmation of the production plan at moment *B*.

Figure 1 – Decisions to take in the 2IKP-DU.



(a) No-fit raster of a rectangle (fixed item) and a cross-shaped item (orbital item).



(b) Inner-fit raster of a cross-shaped item.

Figure 2 – Example of the no-fit raster and inner-fit raster matrices.

In the proposed model, the first stage of the problem is related to decisions here-and-now involving the selection of items to be produced from cutting the plate. In the second stage, wait-and-see decisions are made considering the positioning of the selected items. Due to the realization of the random variables in this stage, i.e., the defects are revealed in the plate, there are also decisions involving the possible cancellation of items. The production plan must be feasible and have the maximum net profit, which involves the profit from the selected and produced items minus the expected cost of the canceled (i.e. selected but not produced) items.

We assume the plate is discretized over a mesh of points, and thus an item is positioned by allocating its reference vertex to one point of this mesh. We use the no-fit raster and inner-fit raster to obtain the feasible points for positioning items (Toledo et al., 2013). The parameters and decision variables of the model are:

- $I = \{1, 2, \ldots, n\}$: the set of irregular items available for selection;
- v_i: profit associated with the selection of item i in the first stage;
- c_i: the cost associated with canceling item i in the second stage;
- M: sufficiently large number.
- π^s: the probability of occurrence of the scenario $s \in \Omega$;
- D^s: the set of defects in the plate, given the scenario $s \in \Omega$;
- $NFR_{ij}^{(p,q)}$: the set of points (u, v) from the no-fit raster between items i and j (i.e., NFR_{ij}). Item i has its reference vertex positioned at (p, q), while item j is orbiting around i. The points (u, v) are those where item j having its reference vertex positioned at them will cause an overlap with i. In other words, this set contains all points that j positioned on any of them will overlap i.
- IFR_i^s: the set of points of the inner-fit raster of item i (i.e., IFR_i). These points allow i to be positioned without overlapping a defect of scenario $s \in \Omega$ and entirely inside the plate.
- y_i: a first-stage binary variable that receives the value 1 if item $i \in I$ is selected for production; otherwise, it receives the value 0;
- x_{ipq}^s: a second-stage binary variable that receives the value 1 if item i has its reference vertex positioned at point (p, q) of the mesh of points associated to the plate; otherwise, it receives the value 0; for the scenario $s \in \Omega$;
- z_i^s: a second-stage binary variable that receives the value 1 if item i is not canceled; otherwise, it receives the value 0; for the scenario $s \in \Omega$;

We propose a two-stage, risk-neutral integer linear optimization model for the 2IKP-DU. The objective is to optimize the expected value of the net profit, considering the selection of items to produce (first stage) and the placement and possible cancellation of items according to plate defects realization (second stage). The objective function (1) of the first stage considers the profit by selecting items and the expected cost by cancelling items given by the recourse function $Q(y, \xi)$. Let $\xi = [\xi_s]$, with $\xi_s = \{x_{ipq}^s, z_i^s\}$, be the random vector of the scenario s. Constraints

(2) imposes that the selected items respect the total area of the rectangular plate. Constraints (3) define the domain of the first-stage decision variables.

$$\text{Maximize} \sum_{i \in I} v_i y_i + Q(y, \xi) \tag{1}$$

$$\sum_{i \in I} a_i y_i \leq LH, \tag{2}$$

$$y_i \in \{0,1\}, \qquad \forall\, i \in I. \tag{3}$$

The second-stage model has objective function (4), which looks for the minimization of the expected cost of canceling items considering the probability of occurrence π^s of each scenario $s \in \Omega$. Since canceled items cannot have their profit accounted for in the solution, we assume that $c_i \geq v_i$, for each item $i \in I$.

$$Q(y, \xi) = \text{Minimize} \sum_{s \in \Omega} \pi_s \left(\sum_{i \in I} c_i (y_i - z_i^s) \right) \tag{4}$$

$$z_i^s \leq y_i, \qquad \forall\, s \in \Omega,\ i \in I; \tag{5}$$

$$\sum_{j \in I} \sum_{(u,v) \in NFR_{ij}^{(p,q)}} x_{juv}^s \leq (1 - x_{ipq}^s) M, \qquad \forall\, s \in \Omega,\ i \in I,\ (p,q) \in IFR_i^s; \tag{6}$$

$$\sum_{(p,q) \in IFR_i^s} x_{ipq}^s = z_i^s, \qquad \forall\, s \in \Omega,\ i \in I; \tag{7}$$

$$z_i^s \in \{0,1\}, \qquad \forall\, s \in \Omega,\ i \in I; \tag{8}$$

$$x_{ipq}^s \in \{0,1\}, \qquad \forall\, s \in \Omega,\ i \in I,\ (p,q) \in IFR_i^s. \tag{9}$$

Constraints (5) of the second-stage model assure that an item not selected in the first stage must be assigned with the same state as canceled items, given each scenario $s \in \Omega$. Constraints (6) ensure, for each scenario $s \in \Omega$, that if item i is positioned at (p,q) in the plate, then item j cannot be positioned at any of the points (u,v) of the set $NFR_{ij}^{(p,q)}$. Constraints (7) ensure that only selected and non-canceled items i are positioned in the plate, for each scenario $s \in \Omega$. Constraints (8) and (9) define that the second-stage variables are binary.

The solution of the two-stage optimization model for the 2IKP-DU can be obtained by solving a deterministic equivalent model (by any integer linear optimization solver). Such a model has the objective function (1), with the recourse function $Q(y, \xi)$ given by the function (4), and constraints (2), (3), and (5)-(9). It is also common to refer to this as Recourse Problem (RP), or stochastic problem, obtaining the RP solution from solving the deterministic equivalent model (Birge & Louveaux, 1997). This model has $|I| + |\Omega| \times |I| \times |IFR_i^s| + |\Omega| \times |I|$ variables, and $1 + 2 \times |\Omega| \times |I| + |\Omega| \times |I| \times |IFR_i^s|$ constraints, where $|IFR_i^s|$ is limited by $O(L \times H)$.

3.1 Solution analysis of the stochastic problem

A solution to a deterministic problem contains a single production plan, while in the solution of the stochastic problem there are $|\Omega|$ production plans. In this way, we apply two well-known metrics to evaluate the solutions of the stochastic model for the 2IKP-DU, allowing us to conclude about the importance of considering (or not) the uncertainties in the problem (Birge & Louveaux, 1997; Bakker et al., 2020). In other words, we calculate the Expected Value of Perfect Information (EVPI) and the Value of Stochastic Solution (VSS) to measure the outcome of the stochastic program in comparison to the outcome of an approximative, deterministic problem.

The Expected Value of Perfect Information (EVPI) indicates the maximum value a decision-maker would pay to have complete and accurate information about the future. In other words, the value that a decision-maker is not able to gain due to the imperfect information. For the 2IKP-DU, this information is related to the exact knowledge of the future number of defects and their locations in the plate, making it possible to obtain a production plan with the maximum possible profit.

To compute the EVPI of an instance of the problem: (i) we obtain the solution of the stochastic problem (i.e., Recourse Problem - RP) by solving the model (1)-(9) with all scenarios in Ω, resulting in the RP solution; (ii) we obtain the solution WS_s^* of the wait-and-see problem for each scenario $s \in \Omega$, i.e., we solve the model (1)-(9) considering only scenario s, so the variables associated to the other scenarios are disregarded (this means that we need to solve $|\Omega|$ times the model, each time for a different scenario s to obtain the respective WS_s^* solution); (iii) we compute the expected value WS of the solutions of the wait-and-see problems, i.e., $WS = \sum_{s\in\Omega} \pi_s WS_s^*$; (iv) we compute EVPI as (WS - RP). A small value of EVPI indicates that the uncertainties of the problem are not that impactful on the solution, and it is not worth solving a stochastic problem. Thus, it is more timely/easier to use an approximate solution, e.g., to consider the solutions of the wait-and-see problems.

While obtaining the EVPI may require solving several deterministic problems, another alternative for evaluating the solutions of the stochastic model is to calculate the Value of the Stochastic Solution (VSS). It indicates the cost of ignoring uncertainty by using the solution of an expected value problem. In other words, the advantage of solving a stochastic model over a deterministic model where all stochastic parameters are replaced by their expected value. For 2IKP-DU, this would correspond to making decisions based on a reference scenario whose random variables values are known, i.e., assuming a given realization of the defects and their locations in the plate.

To obtain the VSS of an instance of the problem: (i) we obtain the RP solution of the model (1)-(9) with all scenarios in Ω; (ii) we obtain the solution of an Expected Value problem (EV), i.e., we assume a reference scenario, which among the available scenarios for the instance corresponds to the worst one in terms of defects realization. The solution of the EV is obtained by solving the model (1)-(9) for this reference scenario, resulting in the values $\bar{\mathbf{y}}$ for variables y_i; (iii) we calculate the Expectation of the Expected Value Problem (EVV), i.e., the solution of EVV is obtained from solving the model (1)-(9) with all scenarios in Ω, but with the y_i (first-stage) variables fixed at the values in $\bar{\mathbf{y}}$. (iv) we calculate VSS as (RP - EVV). A small value of VSS indicates that the gain of the solution of the stochastic problem (RP) over the solution of an expected value problem (EV) is small. Thus, it is more convenient/easier to consider an approximate solution, e.g., to use the solution of EVV.

3.2 Risk aversion

In a risk-neutral model, we assume the solution has a long-term performance, so the scenarios represent general trends for the random variables. This means that a dispersion of the random variables that significantly impacts the obtained solution is not desirable. When there is variability in the uncertain parameters, in turn causing dispersion of the random variables, the expected value of the cost, which represents the second-stage objective function, can have high variability. In situations of high risk (high variability of the parameters), the expected cost of the second stage should be controlled/limited. One way to control the risk is to consider a risk-averse model so that the expected cost of each scenario is close to the optimal expected cost when considering all scenarios.

One risk measure to limit the variability of the second-stage variables, consequently obtaining solutions that are less sensitive to scenario variability (i.e., risk-averse) is to minimize the variance of the total expected value, but this introduces nonlinearities into the model. On the other hand, Ahmed & Sahinidis (1998) proposed a risk measure, called Upper Partial Mean (UPM), which can be used as a "variance measure" defined over linear functions of the form:

$$\sum_{s\in\Omega} \pi_s \Delta_s \tag{10}$$

where Δ_s is the difference between the expected value of the cost of scenario s and the total expected value of the cost of all scenarios.

According to Ahmed & Sahinidis (1998), the measure (10) is added to the two-stage stochastic optimization model as new constraints. In this case, we reduce the variability of the second-stage solution (i.e., of the expected value of the cancellation cost) by imposing a maximum tolerance $\Delta_{\max}$. Therefore, the risk-averse model for the 2IKP-DU is defined by the objective function (11) and constraints (2), (3), (5)-(9), (12)-(14):

$$\text{Maximize} \sum_{i\in I} v_i y_i - \sum_{s\in\Omega} \pi_s \left(\sum_{i\in I} c_i (y_i - z_i^s) \right) \tag{11}$$

$$(2), (3), (5), (6), (7), (8), (9),$$

$$\Delta_s \geq \sum_{i\in I} c_i (y_i - z_i^s) - \left(\sum_{\bar{s}\in\Omega} \pi_{\bar{s}} \left(\sum_{i\in I} c_i (y_i - z_i^{\bar{s}}) \right) \right), \qquad \forall s \in \Omega; \tag{12}$$

$$\sum_{s\in\Omega} \pi_s \Delta_s \leq \alpha \Delta_{\max}, \tag{13}$$

$$\Delta_s \geq 0, \qquad \forall s \in \Omega. \tag{14}$$

In constraint (13), when $\alpha\Delta_{\max}$ is small, the resulting solution is expected to be less sensitive to scenario variations and thus the decision-maker is more conservative (or risk-averse). Small values of this parameter can turn the model infeasible, while large values can result in a risk-neutral solution. The idea is to construct a solution curve by varying the parameter $\alpha \in [0,1]$. We assume that $\Delta_{\max} = \sum_{s\in\Omega} \pi_s \Delta_s$ is obtained from the RP model (1)-(9) with all scenarios.

4 COMPUTATIONAL RESULTS

The risk-neutral (1)-(9) and risk-averse (11)-(14) models were implemented in the C++ language, inside the branch-and-cut algorithm framework of the Gurobi Optimizer, version 9.1, using the default settings. The computer used in the experiments has Linux Ubuntu 16.04.7 LTS as the operating system, a 3.5 GHz Intel Xeon E3-1245v5 processor with 8 threads, and 32 GB of RAM. We set a time limit of 7200 seconds per instance.

4.1 Instances

The instances used in the experiments were adapted from the literature of the two-dimensional irregular knapsack problem, following the methodology of Souza Queiroz & Andretta (2020). Some of these instances can be found in the EURO Special Interest Group on Cutting and Packing (ESICUP[1]) and were made available by Souza Queiroz & Andretta (2020). The main data of each instance are presented in Table 2, as the name, the total quantity of items available for selection, the dimensions of the plate, the maximum area (in percentage) that it is possible to occupy if defects are not present, i.e., $\text{Max}_{area} = 100 \times \frac{\sum_{i\in I} a_i}{LH}$, and the best known solution (i.e., occupied area) (BKS) calculated by the heuristics of Souza Queiroz & Andretta (2020). We adopt the scale of 1 point for every 1 unit of distance to obtain the no-fit raster and inner-fit raster binary matrices. Regarding the first and second stage objective functions of the models, we consider $v_i = a_i$, i.e., the profit corresponds to the item area, and $c_i = 1.5v_i$, for each item $i \in I$.

Observing Table 2, even if the total sum of the area of the irregular items is smaller than the total area of the plate without defects, the irregular items bring additional complexity, such as the issue of not having a perfect fit between them, differently from the positioning of rectangular items. Applying the heuristics of Souza Queiroz & Andretta (2020) on these instances, we could observe that the occupied area ranges from 52.88% (instance shirts1-2) to 95.40% (instance dagli1), and the BKS is not equal to the Max_{area} for 35 out of 40 instances.

The number of scenarios impacts on the models' size and, thus, we assume the generation of the scenarios considers a tree of possibilities (Ma et al., 2010). We consider there are three different defect types (a triangle, a square, and a rhombus). Each scenario considers the presence or absence of each of the defects, resulting in 8 scenarios per instance, which are the possibilities between having no defect at all in the plate and having all defects in the plate. The position of each defect in the plate was defined randomly using a uniform distribution. Besides that, each instance is associated with four probability cases for the occurrence of the scenarios, which are:

[1] https://www.euro-online.org/websites/esicup/data-sets

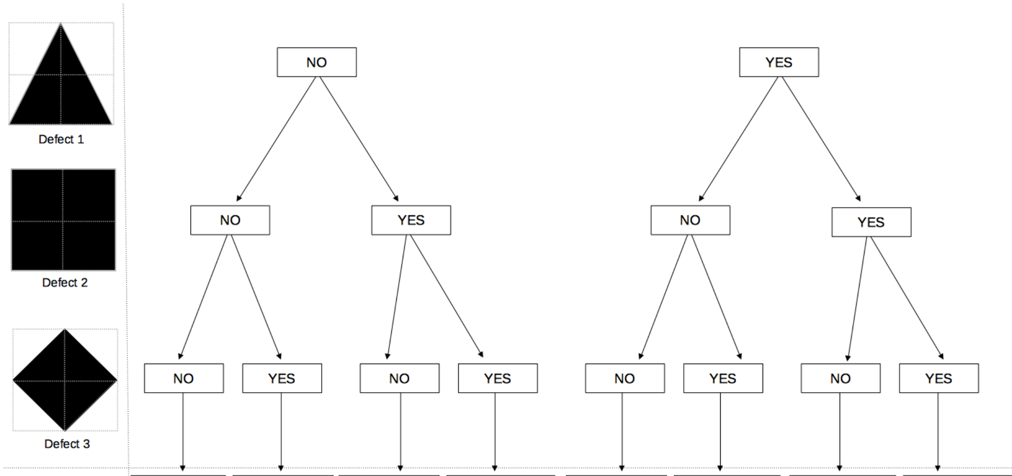
Table 2 – Instances used in the computational tests.

Instance	#Items	H	L	Max_{area}	BKS	Instance	#Items	H	L	Max_{area}	BKS
blasz2	16	15	16	94.17	79.17	rco2	14	15	13	96.92	91.79
blazewicz1	7	15	6	90.00	77.78	rco3	21	15	19	99.47	92.46
blazewicz2	14	15	11	98.18	84.85	rco4	28	15	26	96.92	91.79
blazewicz3	21	15	17	95.29	86.67	rco5	35	15	32	98.44	91.77
blazewicz4	28	15	22	98.18	87.27	shapes2	8	40	14	57.14	57.14
blazewicz5	35	15	27	100.00	88.02	shapes4	16	40	16	100.00	74.38
dagli1	10	60	23	219.89	95.40	shapes5	20	40	20	100.00	73.00
fu	12	38	29	98.28	92.56	shapes7	28	40	28	100.00	77.50
fu5	5	38	14	82.33	73.12	shapes9	34	40	33	98.18	77.58
fu6	6	38	17	98.14	78.64	shapes15	43	40	40	99.75	76.50
fu7	7	38	19	97.51	87.81	shirts1-2	13	40	13	52.88	52.88
fu8	8	38	20	98.55	90.26	shirts2-4	26	40	14	98.21	81.61
fu9	9	38	23	96.91	89.70	shirts3-6	39	40	21	98.21	86.79
fu10	10	38	26	97.87	90.79	shirts4-8	52	40	28	98.21	88.62
poly1a	15	40	13	78.85	76.73	shirts5-10	65	40	35	98.21	87.75
poly1b	15	40	13	95.10	80.48	three	3	7	4	82.14	60.71
poly1c	15	40	13	60.67	60.67	threep2	6	7	7	93.88	69.39
poly1d	15	40	11	74.32	74.32	threep2w9	6	9	6	85.19	70.37
poly1e	15	40	10	72.25	72.25	threep3	9	7	10	98.57	70.00
rco1	7	15	7	90.00	76.67	threep3w9	9	9	8	95.83	76.39

pessimistic with the probability of 75% associated with having d in the board and the other 25% for not having such a defect in the board; *moderate*, with the probability of 40% associated with having d in the board and the other 60% for not having such a defect; *equiprobable*, with the probability of 50% percent associated with having the defect d in the board and the other 50% percent for not having such a defect in the board; and *optimistic*, with the probability of 25% associated with having d in the board and the other 75% for not having such a defect. For example, for the optimistic case, the scenario where there is no defect in the board has an occurrence probability equal to $75\% \times 75\% \times 75\% = 42.19\%$, given the 75% probability associated with the event of *not* having the defect d in the board. On the other hand, in the scenario whose plate contains all defects, its probability of occurrence is given by $25\% \times 25\% \times 25\% = 1.56\%$, with the probability of 25% associated with the event *yes*, that is, of having the defect d in the plate, for $d = 1,2,3$. Figure 3 presents the scenario tree with the probability of occurrence of each scenario.

4.2 Results of the risk-neutral model

Tables 3 to 6 contain the results obtained with the risk-neutral model (1)-(9) over the instances of Table 2, considering the 8 scenarios and four cases (pessimistic, moderate, equiprobable, and optimistic) presented in Figure 3. In general, it was possible to obtain the optimal solution for 42.5% of the instances within the time limit of 7200 seconds, with an average gap of 5.0% and an



Scenario	#1. NO-NO-NO	#2. NO-NO-YES	#3. NO-YES-NO	#4. NO-YES-YES	#5. YES-NO-NO	#6. YES-NO-YES	#7. YES-YES-NO	#8. YES-YES-YES
Probabilities								
Pessimistic:	1.56%	4.69%	4.69%	14.06%	4.69%	14.06%	14.06%	42.19%
Moderate:	6.40%	9.60%	9.60%	14.40%	9.60%	14.40%	14.40%	21.60%
Equiprobable:	12.50%	12.50%	12.50%	12.50%	12.50%	12.50%	12.50%	12.50%
Optimistic:	42.19%	14.06%	14.06%	4.69%	14.06%	4.69%	4.69%	1.56%

Figure 3 – Scenario tree for the instances used in the computational tests.

average computational time of 4284.1 seconds. The instances not optimally solved tend to have a larger number of items for selection or a larger plate. This may also indicate that the model could find it harder to solve instances with more scenarios.

Observing the results for the pessimistic case, in Table 3, the average gap and average time are 4.7% and 3919.1 seconds, respectively. The number of optimal solutions occurred for 47.5% of the instances, with the largest gap of 20.6% for instance shirts4-8. There was no item cancellation for 67.5% of the instances, including poly1c, shapes2, and shirts1-2, which also had all items selected. The highest number of cancellations occurred for scenario #8, while instances blazewicz1 and threep3 had the largest number of canceled items. Scenario #8 has all defects and, thus, it would be expected to have more cancellations on it. Besides that, the location of the defects in the scenarios of instances blazewicz1 and threep3 makes hard the positioning of the selected items, resulting in more canceled items.

For the moderate case, in Table 4, the gap and average time are 5.0% and 4185.9 seconds, respectively. In this case, the number of optimal solutions occurred for 45.0% of the instances, while the instance with the largest gap is shapes15, with a value of 24.3%. There was no item cancellation for 62.5% of the instances, including poly1c, shapes2, and shirts1-2, which also had all items selected. The highest number of cancellations occurred in scenarios #4, #6, #7, and #8, while instances rco1 and threep2w9 had the largest number of item cancellations. Compared to the pessimistic case, we observe for the moderate case less instances solved to optimality, larger gaps, larger computational times, and more canceled items per instance/scenario. On the other

Table 3 – Results of the pessimistic case.

Instance	Total of items	#Items selected	Objective	Gap (%)	Time (s)	#Items canceled in scenario							
						#1	#2	#3	#4	#5	#6	#7	#8
blasz2	16	11	162.0	0.0	27.7	0	0	0	0	0	0	0	0
blazewicz1	7	5	42.6	0.0	0.7	0	0	0	1	0	0	1	1
blazewicz2	14	11	107.4	0.0	136.0	0	0	0	0	0	0	1	1
blazewicz3	21	17	196.4	0.8	7200.0	0	0	0	0	0	0	0	1
blazewicz4	28	23	262.0	1.7	7200.0	0	0	0	0	0	0	0	0
blazewicz5	35	28	328.2	4.4	7200.0	0	0	0	0	0	0	0	0
dagli1	30	14	1110.9	16.0	7200.0	0	0	0	0	0	0	0	1
fu	12	8	887.0	6.2	7200.0	0	0	0	0	0	0	0	0
fu5	5	4	296.7	0.0	1.5	0	0	0	0	0	0	1	1
fu6	6	4	434.0	0.0	6.9	0	0	0	0	0	0	0	0
fu7	7	4	441.0	0.0	212.4	0	0	0	0	0	0	0	0
fu8	8	6	515.7	0.0	612.1	0	0	0	0	0	0	0	1
fu9	9	8	651.0	0.0	796.8	0	0	0	0	0	0	0	0
fu10	10	8	775.0	5.3	7200.0	0	0	0	0	0	0	0	1
poly1a	15	10	304.0	19.4	7200.0	0	0	0	0	0	0	0	0
poly1b	15	10	321.5	18.0	7200.0	0	0	0	0	0	0	0	0
poly1c	15	15	315.5	0.0	355.1	0	0	0	0	0	0	0	0
poly1d	15	12	268.5	17.8	7200.0	0	0	0	0	0	0	0	0
poly1e	15	13	237.5	4.3	7200.0	0	0	0	0	0	0	0	0
rco1	7	5	50.9	0.0	3.8	0	0	0	1	0	0	0	1
rco2	14	12	157.0	0.0	19.2	0	0	0	0	0	0	0	0
rco3	21	18	234.0	0.1	7200.0	0	0	0	0	0	0	0	0
rco4	28	25	337.2	3.4	7200.0	0	0	0	0	0	0	0	0
rco5	35	31	414.3	6.3	7200.0	0	0	0	0	0	0	0	0
shapes2	8	8	315.8	0.0	20.8	0	0	0	0	0	0	0	0
shapes4	16	11	376.2	0.0	3362.6	0	0	0	0	0	0	0	0
shapes5	20	12	496.0	3.4	7200.0	0	0	0	0	0	0	0	0
shapes7	28	18	782.8	6.5	7200.0	0	0	0	0	0	1	0	0
shapes9	34	23	913.2	10.3	7200.0	0	0	0	0	0	0	0	0
shapes15	43	29	1062.3	20.3	7200.0	0	0	0	0	0	0	0	0
shirts1-2	13	13	275.0	0.0	6.0	0	0	0	0	0	0	0	0
shirts2-4	26	21	437.5	0.1	7200.0	0	0	0	0	0	0	0	0
shirts3-6	39	37	716.0	1.2	7200.0	0	0	0	0	0	0	0	0
shirts4-8	52	49	885.0	20.6	7200.0	0	0	0	0	0	0	0	0
shirts5-10	65	62	1138.5	20.2	7200.0	0	0	0	0	0	0	0	0
three	3	0	0.0	0.0	0.0	0	0	0	0	0	0	0	0
threep2	6	3	13.2	0.0	0.1	0	0	0	0	0	1	0	1
threep2w9	6	3	14.7	0.0	0.2	0	0	0	0	1	0	0	1
threep3	9	5	30.9	0.0	0.2	0	0	0	0	0	1	0	2
threep3w9	9	5	34.7	0.0	0.2	0	0	0	0	0	0	1	1
Average	19	15	408.5	4.7	3919.1	-	-	-	-	-	-	-	-
Std. Dev.	14	13	323.3	7.1	3533.0	-	-	-	-	-	-	-	-

Table 4 – Results of the moderate case.

Instance	Total of items	#Items selected	Objective	Gap (%)	Time (s)	#Items canceled in scenario							
						#1	#2	#3	#4	#5	#6	#7	#8
blasz2	16	12	164.4	0.0	35.3	0	0	0	0	0	1	1	0
blazewicz1	7	5	46.9	0.0	1.5	0	0	0	1	0	0	1	1
blazewicz2	14	11	113.3	0.0	220.7	0	0	0	0	0	1	0	0
blazewicz3	21	18	198.9	1.6	7200.0	0	0	0	0	0	0	0	1
blazewicz4	28	23	263.2	3.4	7200.0	0	0	0	0	0	0	0	0
blazewicz5	35	28	320.5	7.4	7200.0	0	0	0	0	0	0	0	0
dagli1	30	14	1094.3	18.4	7200.0	1	0	0	0	0	1	1	0
fu	12	9	887.0	8.2	7200.0	0	0	0	0	0	0	0	0
fu5	5	4	314.4	0.0	2.2	0	0	0	0	0	1	1	0
fu6	6	4	434.0	0.0	6.5	0	0	0	0	0	0	0	0
fu7	7	4	441.0	0.0	272.6	0	0	0	0	0	0	0	0
fu8	8	6	537.3	0.0	3650.5	0	0	0	0	0	0	0	1
fu9	9	8	651.0	0.0	4336.4	0	0	0	0	0	0	0	0
fu10	10	8	790.1	5.6	7200.0	0	0	0	0	0	0	0	1
poly1a	15	11	310.8	18.7	7200.0	0	0	0	1	0	0	1	1
poly1b	15	10	313.0	20.6	7200.0	0	0	0	0	0	0	0	0
poly1c	15	15	315.5	0.0	328.7	0	0	0	0	0	0	0	0
poly1d	15	13	261.5	22.5	7200.0	0	0	0	0	0	0	0	0
poly1e	15	12	241.5	4.8	7200.0	0	0	0	0	0	0	0	0
rco1	7	6	57.0	0.0	10.8	0	0	0	1	0	1	1	2
rco2	14	12	157.0	0.0	126.8	0	0	0	0	0	0	0	0
rco3	21	19	236.5	1.3	7200.0	0	0	0	0	0	0	0	0
rco4	28	24	333.0	3.8	7200.0	0	0	0	0	0	0	0	0
rco5	35	30	411.3	7.8	7200.0	0	0	0	0	0	0	0	0
shapes2	8	8	315.7	0.0	36.3	0	0	0	0	0	0	0	0
shapes4	16	11	375.9	0.3	7200.0	0	0	0	0	0	0	0	0
shapes5	20	13	510.6	3.0	7200.0	0	0	0	0	0	0	0	0
shapes7	28	18	780.0	7.2	7200.0	0	0	0	0	0	0	0	0
shapes9	34	23	903.3	12.1	7200.0	0	0	0	0	0	0	0	0
shapes15	43	27	1032.0	24.3	7200.0	0	0	0	0	0	0	0	0
shirts1-2	13	13	275.0	0.0	6.1	0	0	0	0	0	0	0	0
shirts2-4	26	22	439.2	0.2	7200.0	0	0	0	0	0	0	0	0
shirts3-6	39	37	714.3	1.7	7200.0	0	0	0	0	0	0	0	0
shirts4-8	52	48	948.9	4.9	7200.0	0	0	0	0	0	0	0	0
shirts5-10	65	62	1117.0	23.1	7200.0	0	0	0	0	0	0	0	0
three	3	1	2.2	0.0	0.0	0	0	0	1	0	0	1	1
threep2	6	3	17.8	0.0	0.1	0	0	0	0	0	1	0	1
threep2w9	6	4	20.8	0.0	0.1	0	0	0	1	1	0	1	2
threep3	9	5	35.2	0.0	0.2	0	0	0	0	0	1	0	2
threep3w9	9	5	37.1	0.0	0.4	0	0	0	0	0	0	1	1
Average	19	15	410.5	5.0	4185.9	-	-	-	-	-	-	-	-
Std. Dev.	14	13	321.2	7.6	3478.9	-	-	-	-	-	-	-	-

Table 5 – Results of the equiprobable case.

Instance	Total of items	#Items selected	Objective	Gap (%)	Time (s)	#Items canceled in scenario							
						#1	#2	#3	#4	#5	#6	#7	#8
blasz2	16	12	167.6	0.0	68.9	0	0	0	0	0	1	1	0
blazewicz1	7	5	49.1	0.0	2.1	0	0	0	1	0	0	1	1
blazewicz2	14	11	117.0	0.0	510.1	0	0	0	0	0	1	0	0
blazewicz3	21	18	201.6	1.6	7200.0	0	0	0	0	0	0	0	1
blazewicz4	28	23	263.6	4.2	7200.0	0	0	0	0	0	0	0	0
blazewicz5	35	28	328.5	5.2	7200.0	0	0	0	0	0	0	0	0
dagli1	30	14	1115.3	19.1	7200.0	0	0	0	1	0	1	0	0
fu	12	10	902.4	7.7	7200.0	0	1	1	1	0	1	1	1
fu5	5	4	328.7	0.0	1.9	0	0	0	0	0	1	1	0
fu6	6	4	434.0	0.0	7.4	0	0	0	0	0	0	0	0
fu7	7	5	443.9	0.0	343.0	0	0	0	0	1	0	0	1
fu8	8	6	549.1	1.4	7200.0	0	0	0	0	0	0	0	2
fu9	9	8	651.0	2.0	7200.0	0	0	0	0	0	0	0	0
fu10	10	8	796.8	5.5	7200.0	0	0	0	0	0	0	0	1
poly1a	15	11	317.9	16.7	7200.0	0	0	0	1	0	0	0	1
poly1b	15	10	319.5	18.5	7200.0	0	0	0	0	0	0	0	0
poly1c	15	15	315.5	0.0	350.5	0	0	0	0	0	0	0	0
poly1d	15	12	265.5	22.0	7200.0	0	0	0	0	0	0	0	0
poly1e	15	12	241.5	6.2	7200.0	0	0	0	0	0	0	0	0
rco1	7	6	62.1	0.0	7.5	0	0	0	1	0	1	1	2
rco2	14	13	157.6	0.0	2094.7	0	0	0	0	0	0	1	1
rco3	21	19	239.2	1.4	7200.0	0	0	0	0	0	0	0	0
rco4	28	25	337.5	2.7	7200.0	0	0	0	0	0	0	0	0
rco5	35	31	415.6	4.0	7200.0	0	0	0	0	0	0	0	0
shapes2	8	8	316.3	0.0	13.1	0	0	0	0	0	0	0	0
shapes4	16	11	377.5	1.4	7200.0	0	0	0	0	0	0	0	0
shapes5	20	12	514.5	4.3	7200.0	0	0	0	0	0	0	0	0
shapes7	28	19	789.3	6.6	7200.0	0	0	0	0	0	0	0	0
shapes9	34	23	904.0	12.8	7200.0	0	0	0	0	0	0	0	0
shapes15	43	29	1057.0	21.8	7200.0	0	0	0	0	0	0	0	0
shirts1-2	13	13	275.0	0.0	6.1	0	0	0	0	0	0	0	0
shirts2-4	26	22	440.4	0.2	7200.0	0	0	0	0	0	0	0	0
shirts3-6	39	37	716.0	1.7	7200.0	0	0	0	0	0	0	0	0
shirts4-8	52	49	863.5	25.7	7200.0	0	0	0	0	0	0	0	0
shirts5-10	65	61	1126.3	22.1	7200.0	0	0	0	0	0	0	0	0
three	3	1	3.9	0.0	0.0	0	0	0	1	0	0	1	1
threep2	6	4	20.2	0.0	0.1	0	1	1	1	0	2	1	2
threep2w9	6	4	23.9	0.0	0.1	0	0	0	1	1	0	1	2
threep3	9	5	37.3	0.0	0.4	0	0	0	0	0	1	0	2
threep3w9	9	5	38.4	0.0	0.4	0	0	0	0	0	0	1	1
Average	19	15	413.1	5.4	4405.2	-	-	-	-	-	-	-	-
Std. Dev.	14	13	320.9	7.8	3482.0	-	-	-	-	-	-	-	-

hand, in the moderate case, the solutions, in general, have better objective function values and more selected items.

Table 6 – Results of the optimistic case.

Instance	Total of items	#Items selected	Objective	Gap (%)	Time (s)	#Items canceled in scenario							
						#1	#2	#3	#4	#5	#6	#7	#8
blasz2	16	12	177.2	0.0	45.8	0	0	0	0	0	0	0	0
blazewicz1	7	5	53.1	0.0	5.4	0	0	0	1	0	1	1	1
blazewicz2	14	11	123.6	0.2	7200.0	0	0	0	0	0	1	0	0
blazewicz3	21	18	206.0	2.0	7200.0	0	0	0	0	0	0	0	0
blazewicz4	28	23	264.4	5.7	7200.0	0	0	0	0	0	0	0	0
blazewicz5	35	29	332.8	5.1	7200.0	0	0	0	0	0	0	0	0
dagli1	30	17	1090.6	23.5	7200.0	0	0	0	0	1	0	1	0
fu	12	10	923.9	7.3	7200.0	0	1	1	1	0	1	1	1
fu5	5	4	360.1	0.0	5.7	0	0	0	1	0	1	1	0
fu6	6	4	435.0	0.0	16.5	0	1	1	1	1	1	1	1
fu7	7	4	458.3	0.0	499.0	0	0	0	0	0	0	0	1
fu8	8	6	570.9	3.2	7200.0	0	0	0	0	0	0	0	2
fu9	9	7	680.1	0.0	4018.5	0	1	1	1	1	1	1	1
fu10	10	8	822.1	3.9	7200.0	0	0	0	0	1	1	1	1
poly1a	15	12	323.2	19.9	7200.0	1	0	0	1	1	1	1	2
poly1b	15	11	347.8	13.3	7200.0	0	1	1	0	0	1	1	1
poly1c	15	15	315.5	0.0	369.5	0	0	0	0	0	0	0	0
poly1d	15	14	280.6	16.1	7200.0	0	1	1	0	1	0	0	0
poly1e	15	13	252.3	5.1	7200.0	0	0	0	1	1	1	1	1
rco1	7	6	71.3	0.0	8.3	0	0	0	1	0	1	1	2
rco2	14	12	163.7	0.0	7200.0	0	0	0	0	0	0	0	0
rco3	21	19	245.6	1.5	7200.0	0	0	0	0	0	0	0	1
rco4	28	26	339.2	3.4	7200.0	0	0	0	0	0	0	0	0
rco5	35	31	416.1	9.0	7200.0	0	0	0	0	0	0	0	0
shapes2	8	8	318.6	0.0	69.0	0	0	0	0	0	0	0	0
shapes4	16	11	384.1	0.6	7200.0	0	0	0	0	0	0	0	0
shapes5	20	12	526.4	3.6	7200.0	0	0	0	0	0	0	0	2
shapes7	28	19	801.8	6.7	7200.0	0	0	0	0	0	0	0	0
shapes9	34	23	900.1	15.4	7200.0	0	0	0	0	0	0	0	0
shapes15	43	29	1065.8	21.8	7200.0	0	0	0	0	0	0	0	0
shirts1-2	13	13	275.0	0.0	6.0	0	0	0	0	0	0	0	0
shirts2-4	26	22	443.3	0.2	7200.0	0	0	0	0	0	0	0	0
shirts3-6	39	37	716.0	2.3	7200.0	0	0	0	0	0	0	0	0
shirts4-8	52	49	933.2	7.9	7200.0	0	0	0	0	0	0	0	0
shirts5-10	65	62	1117.0	23.1	7200.0	0	0	0	0	0	0	0	0
three	3	2	9.6	0.0	0.0	0	0	0	2	1	1	2	2
threep2	6	4	27.2	0.0	0.1	0	0	0	1	0	1	0	1
threep2w9	6	4	28.9	0.0	0.2	0	0	0	1	1	0	1	2
threep3	9	6	42.7	0.0	0.3	0	1	1	0	1	1	1	2
threep3w9	9	6	42.3	0.0	0.4	0	0	0	0	1	0	1	1
Average	19	16	422.1	5.0	4626.1	-	-	-	-	-	-	-	-
Std. Dev.	14	13	322.3	7.2	3421.3	-	-	-	-	-	-	-	-

In Table 5, for the equiprobable case, we observe that the optimal solution was obtained for 40.0% of the instances, with an average gap of 5.4% and an average time of 4405.2 seconds. The instance that presented the largest gap is shirts4-8, with a value of 25.7%. In instances poly1c, shapes2, and shirts1-2, it was possible to select all items and no cancellation occurred for any scenario. Overall, there was no cancellation of items in any scenario for 55.0% of the instances. On the other hand, in the instances with canceled items, the cancellation occurred the most for scenarios #4, #6, #7, and #8, which are scenarios having more defects. The instance with the largest cancellation of items is threep2. Compared to the pessimistic case, we observe for the equiprobable case fewer instances solved to optimality, larger gaps, larger computational times, more canceled items per instance/scenario. On the other hand, in the equiprobable case, the solutions, in general, have better objective function values and more selected items, justified by the equal probability of occurrence of scenarios.

Observing Table 6, for the optimistic case, the optimal solution was obtained for 40.0% of the instances, with an average gap of 5.0% and an average time of 4626.1 seconds. The instance with the largest gap is dagli1, with 23.5%. In instances poly1c, shapes2, and shirts1-2, it was possible to select all items and there was no item cancellation. Besides, there was no item cancellation for 45.0% of the instances. On the other hand, in the instances with canceled items, the cancellation occurred the most for the scenarios from #4 to #8. The instance with the highest item cancellation is three. When comparing the values of the optimistic case with the pessimistic case, we notice for the optimistic case that fewer instances are solved to optimality, the computational times are larger, and there are more canceled items per instance/scenario. On the other hand, in the optimistic case, the solutions, in general, have better objective function values and more selected items, justified by the lower probability of occurrence of the scenarios with more defects.

The EVPI and VSS analyses consider 14 instances for which it was possible to obtain the optimal solution, in all cases, within the imposed time limit, as reported in Tables 3 to 6. The results for the EVPI are given in Tables 7 and 8, for the four cases: moderate, equiprobable, optimistic, and pessimistic. All reported solutions, including those for the wait-and-see problems, are optimal. The calculation of the relative EVPI (%) is given by $100 \times (\text{EVPI}/\text{WS})$.

The results of Tables 7 and 8 show relative EVPI values greater than 5% for instances blazewicz1, three, threep2, threep2w9, threep3 and threep3w9 (in all cases), fu5 (except in the optimistic case), fu6 (only in the optimistic case), and rco1 (except in the pessimistic case). The least impacted instances in terms of EVPI are those whose defects have little influence on the scenarios, such as poly1c, shapes2, and shirts1-2, resulting in no difference between solving the wait-and-see problems or the two-stage stochastic model. On the other hand, the highest relative EVPI values occurred for instances three, threep2, and threep2w9, with an overall average value of 56.3%, 18.0%, and 15.0%, respectively. In general, the values of EVPI are higher for the cases whose scenarios with more defects are more likely to occur. We observed this especially for the pessimistic, moderate, and equiprobable cases. In other words, the relative average value of the EVPI is 14.2% for the pessimistic case (with the standard deviation of 25.4%), 10.9% (with the

Table 7 – EVPI for instances with an optimal solution in all cases - part I.

	Pessimistic	Moderate	Equiprobable	Optimistic	Pessimistic	Moderate	Equiprobable	Optimistic
Scenario	WS_s^* for blasz2				WS_s^* for blazewicz1			
1	2.8	11.7	22.8	76.8	1.0	4.1	8.0	27.0
2	8.5	17.5	22.8	25.6	2.6	5.4	7.0	7.9
3	8.5	17.5	22.8	25.6	2.6	5.4	7.0	7.9
4	22.8	23.3	20.3	7.6	6.8	6.9	6.0	2.3
5	8.5	17.5	22.8	25.6	2.8	5.7	7.4	8.4
6	23.4	24.0	20.8	7.8	7.7	7.9	6.8	2.6
7	23.4	24.0	20.8	7.8	6.8	7.0	6.1	2.3
8	68.3	35.0	20.3	2.5	17.1	8.8	5.1	0.6
WS	166.4	170.3	173.1	179.3	47.4	51.1	53.4	58.8
RP	162.0	164.4	167.6	177.2	42.6	46.9	49.1	53.1
EVPI	4.4	6.0	5.6	2.1	4.7	4.2	4.3	5.7
EVPI (%)	2.6	3.5	3.2	1.2	10.0	8.2	8.0	9.7
	WS_s^* for fu5				WS_s^* for fu6			
1	6.1	24.9	48.6	164.1	7.6	31.0	60.6	204.6
2	18.2	37.3	48.6	54.7	20.3	41.7	54.3	61.0
3	15.8	32.5	42.3	47.5	22.7	46.6	60.6	68.2
4	52.7	54.0	46.9	17.6	61.0	62.5	54.3	20.3
5	18.2	37.3	48.6	54.7	20.3	41.7	54.3	61.0
6	47.5	48.7	42.3	15.8	61.0	62.5	54.3	20.3
7	45.8	46.9	40.8	15.3	61.0	62.5	54.3	20.3
8	121.9	62.4	36.1	4.5	183.1	93.7	54.3	6.8
WS	326.4	344.1	354.1	374.3	437.2	442.2	446.7	462.7
RP	296.7	314.4	328.7	360.1	434.0	434.0	434.0	435.0
EVPI	29.7	29.7	25.4	14.2	3.2	8.2	12.7	27.7
EVPI (%)	9.1	8.6	7.2	3.8	0.7	1.8	2.9	6.0
	WS_s^* for fu7				WS_s^* for poly1c			
1	7.6	31.0	60.6	204.6	4.9	20.2	39.4	133.1
2	22.7	46.6	60.6	68.2	14.8	30.3	39.4	44.4
3	20.7	42.3	55.1	62.0	14.8	30.3	39.4	44.4
4	68.2	69.8	60.6	22.7	44.4	45.4	39.4	14.8
5	22.1	45.2	58.9	66.2	14.8	30.3	39.4	44.4
6	68.2	69.8	60.6	22.7	44.4	45.4	39.4	14.8
7	68.2	69.8	60.6	22.7	44.4	45.4	39.4	14.8
8	186.1	95.3	55.1	6.9	133.1	68.2	39.4	4.9
WS	463.7	469.9	472.2	476.1	315.5	315.5	315.5	315.5
RP	441.0	441.0	443.9	458.3	315.5	315.5	315.5	315.5
EVPI	22.7	28.9	28.3	17.8	0.0	0.0	0.0	0.0
EVPI (%)	4.9	6.2	6.0	3.7	0.0	0.0	0.0	0.0

standard deviation of 15.4%) for the moderate case, 9.2% (with the standard deviation of 11.8%) for the equiprobable case, and 5.5% for the optimistic case (with the standard deviation of 5.6%).

Overall, the EVPI values in Tables 7 and 8 indicate that future knowledge of plate defects would be advantageous for making accurate decisions about which items to select and produce. Thus, solving the two-stage stochastic model is important to the problem, since the EVPI values are relatively high for the majority of the instances and also in the overall average for the cases. The lower EVPI values observed for the optimistic case are justified by the low probability of occurrence of the scenarios with more defects.

Table 8 – EVPI for instances with an optimal solution in all cases - part II.

	Pessimistic	Moderate	Equiprobable	Optimistic	Pessimistic	Moderate	Equiprobable	Optimistic
Scenario	WS_s^* for rco1				WS_s^* for shapes2			
1	1.2	5.0	9.8	33.1	5.0	20.5	40.0	135.0
2	3.5	7.2	9.3	10.5	15.0	30.7	40.0	45.0
3	3.5	7.2	9.3	10.5	15.0	30.7	40.0	45.0
4	8.2	8.4	7.3	2.7	45.0	46.1	40.0	15.0
5	3.7	7.5	9.8	11.0	15.0	30.7	40.0	45.0
6	9.1	9.3	8.1	3.0	42.2	43.2	37.5	14.1
7	10.2	10.4	9.1	3.4	45.0	46.1	40.0	15.0
8	20.0	10.3	5.9	0.7	135.0	69.1	40.0	5.0
WS	59.4	65.3	68.6	75.0	317.2	317.1	317.5	319.1
RP	59.0	57.0	62.1	71.3	315.8	315.7	316.3	318.6
EVPI	0.4	8.3	6.5	3.7	1.4	1.4	1.2	0.5
EVPI (%)	0.7	12.7	9.5	5.0	0.4	0.4	0.4	0.1
	WS_s^* for shirts1-2				WS_s^* for three			
1	4.3	17.6	34.4	116.0	0.3	1.1	2.1	7.2
2	12.9	26.4	34.4	38.7	0.7	1.4	1.9	2.1
3	12.9	26.4	34.4	38.7	0.4	0.9	1.1	1.3
4	38.7	39.6	34.4	12.9	0.0	0.0	0.0	0.0
5	12.9	26.4	34.4	38.7	0.4	0.9	1.1	1.3
6	38.7	39.6	34.4	12.9	1.3	1.3	1.1	0.4
7	38.7	39.6	34.4	12.9	0.0	0.0	0.0	0.0
8	116.0	59.4	34.4	4.3	0.0	0.0	0.0	0.0
WS	275.0	275.0	275.0	275.0	3.1	5.6	7.4	12.2
RP	275.0	275.0	275.0	275.0	0.0	2.2	3.9	9.6
EVPI	0.0	0.0	0.0	0.0	3.1	3.4	3.4	2.7
EVPI (%)	0.0	0.0	0.0	0.0	96.8	60.4	46.5	21.7
	WS_s^* for threep2				WS_s^* for threep2w9			
1	0.5	2.2	4.3	14.3	0.5	2.2	4.3	14.3
2	1.2	2.5	3.3	3.7	1.5	3.1	4.0	4.5
3	1.2	2.5	3.3	3.7	1.5	3.1	4.0	4.5
4	3.4	3.5	3.0	1.1	3.7	3.7	3.3	1.2
5	1.6	3.3	4.3	4.8	1.1	2.3	3.0	3.4
6	2.5	2.6	2.3	0.8	4.5	4.6	4.0	1.5
7	3.7	3.7	3.3	1.2	3.7	3.7	3.3	1.2
8	3.8	1.9	1.1	0.1	3.8	1.9	1.1	0.1
WS	17.9	22.2	24.6	29.8	20.3	24.7	26.9	30.8
RP	13.2	17.8	20.2	27.2	14.7	20.8	23.9	28.9
EVPI	4.7	4.3	4.4	2.5	5.6	3.9	2.9	1.9
EVPI (%)	26.1	19.5	18.0	8.5	27.6	15.6	10.9	6.0
	WS_s^* for threep3				WS_s^* for threep3w9			
1	0.8	3.1	6.1	20.7	0.8	3.1	6.1	20.7
2	2.0	4.1	5.4	6.1	1.9	3.9	5.1	5.8
3	2.1	4.3	5.6	6.3	2.0	4.1	5.4	6.1
4	5.8	5.9	5.1	1.9	5.8	5.9	5.1	1.9
5	2.0	4.1	5.4	6.1	2.1	4.3	5.6	6.3
6	4.9	5.0	4.4	1.6	5.8	5.9	5.1	1.9
7	6.3	6.5	5.6	2.1	4.9	5.0	4.4	1.6
8	11.4	5.8	3.4	0.4	14.3	7.3	4.3	0.5
WS	35.3	39.0	41.0	45.2	37.6	39.7	41.1	44.8
RP	30.9	35.2	37.3	42.7	34.7	37.1	38.4	42.3
EVPI	4.5	3.8	3.8	2.5	3.0	2.6	2.7	2.5
EVPI (%)	12.6	9.8	9.1	5.5	7.8	6.5	6.6	5.6

The results for the VSS are presented in Table 9 considering the four cases under study (pessimistic, moderate, equiprobable, and optimistic). All solutions reported in the table are optimal. In the expected value problem, the reference scenario consists of scenario #8 in Figure 3, which is the one with the largest number of defects and would represent the worst situation. In general, the VSS values show that solving the two-stage stochastic model for the problem is more advantageous than solving an expected value problem, especially for the optimistic, equiprobable, and moderate cases. In other words, the cost of ignoring the uncertainties in the problem is high and this is reflected in the low objective value of the expected value problem.

In the results of Table 9, the instances with higher VSS values that consequently have higher relative differences (i.e., percentage increase) of the RP solutions over the EVV ones are blazewicz1, fu5, rco1, threep2, threep2w9, threep3, and threep3w9, for all cases. For example, in all cases, in instances rco1 and threep2, the relative differences are greater than 30%, while in instance threep2w9, the values exceed 60%. On the other hand, there is no difference for some instances, such as poly1c, shapes2, and shirts1-2, because they are instances whose effect of defects has little influence on the item cancellation (i.e., there are few defects or they are in positions that allow the selected items to be repositioned inside the plate to avoid cancellation).

Analyzing the values of the EVV between the cases, in Table 9, we notice that the optimistic case presents the highest average relative difference, of 45.0%, followed by the equiprobable case, with an average value of 30.1%, then the moderate case, with an average value of 23.2%, and finally the pessimistic case, with an average value of 12.1%. The lower values for the pessimistic case are justified by the EVV considering the solution from solving the expected value problem with the worst-case scenario (i.e., scenario #8, which has more defects). Therefore, the tendency is to select fewer items and thus cancel fewer items in the scenarios. This results in a smaller difference between the RP and EVV solutions in the pessimistic case compared to the other ones.

We summarize in Table 10 the results of the risk-neutral model. Each line of this table has a case; the results for all instances in Tables 3 to 6: the percentage number of optimal solutions, the average gap in percentage, the average computational time in seconds, and the percentage of instances with no item cancellation; the average relative EVPI for the instances in Tables 7 and 8; and the average relative difference of the RP solutions over the EVV ones for the instances in Table 9, for the VSS results. These results evidence the importance of solving a stochastic optimization model to obtain profitable solutions and, at the same time, the considerable computational effort that is required to obtain optimal solutions.

4.3 Results of the risk-averse model

We start solving the risk-averse model (11)-(14) with the parameter $\alpha = 1$, for $\Delta_{\max}$ obtained from the solution of the risk-neutral model. The results presented in Table 11 consider the same 14 instances used in the experiments for the EVPI and VSS analyses, over the four cases under study. The table contains the objective value of the optimal solution of the risk-averse model for

Table 9 – VSS for the 14 instances with optimal solutions in all cases.

	RP	EVV	VSS	Diff. (%)	RP	EVV	VSS	Diff. (%)
Instance	Pessimist				Moderate			
blasz2	162.0	162.0	0.0	0.0	164.4	162.0	2.4	1.47
blazewicz1	42.6	40.5	2.1	5.2	46.9	40.5	6.4	15.7
fu5	296.7	289.0	7.7	2.7	314.4	289.0	25.4	8.8
fu6	434.0	434.0	0.0	0.0	434.0	434.0	0.0	0.0
fu7	441.0	441.0	0.0	0.0	441.0	441.0	0.0	0.0
poly1c	315.5	315.5	0.0	0.0	315.5	315.5	0.0	0.0
rco1	59.0	43.8	15.2	34.7	57.0	43.7	13.3	30.4
shapes2	315.8	315.8	0.0	0.0	315.7	315.7	0.0	0.0
shirts1-2	275.0	275.0	0.0	0.0	275.0	275.0	0.0	0.0
three	0.0	0.0	0.0	0.0	2.2	0.0	2.2	0.0
threep2	13.2	9.0	4.2	47.1	17.8	9.0	8.8	98.2
threep2w9	14.7	9.0	5.7	63.1	20.8	9.0	11.8	131.1
threep3	30.9	27.0	3.9	14.3	35.2	27.0	8.2	30.3
threep3w9	34.7	34.0	0.7	2.0	37.1	34.0	3.1	9.1
Average	-	-	-	12.1	-	-	-	23.2
Std. Dev.	-	-	-	20.8	-	-	-	40.7
	Equiprobable				Optimistic			
blasz2	167.6	162.0	5.6	3.4	177.2	162.0	15.2	9.4
blazewicz1	49.1	40.5	8.6	21.3	53.1	40.5	12.6	31.1
fu5	328.7	289.0	39.7	13.7	360.1	289.0	71.1	24.6
fu6	434.0	434.0	0.0	0.0	435.0	434.0	1.0	0.2
fu7	443.9	441.0	2.9	0.7	458.3	441.0	17.3	3.9
poly1c	315.5	315.5	0.0	0.0	315.5	315.5	0.0	0.0
rco1	62.1	44.2	17.9	40.4	71.3	46.3	25.0	54.1
shapes2	316.3	316.3	0.0	0.0	318.6	318.6	0.0	0.0
shirts1-2	275.0	275.0	0.0	0.0	275.0	275.0	0.0	0.0
three	3.9	0.0	3.9	0.0	9.6	0.0	9.6	0.0
threep2	20.2	9.0	11.2	124.3	27.2	9.0	18.2	202.6
threep2w9	23.9	9.0	14.9	166.0	28.9	9.0	19.9	221.4
threep3	37.3	27.0	10.3	38.0	42.7	27.0	15.7	58.2
threep3w9	38.4	34.0	4.4	12.9	42.3	34.0	8.3	24.5
Average	-	-	-	30.1	-	-	-	45.0
Std. Dev.	-	-	-	51.3	-	-	-	73.5

Table 10 – Summary of the results obtained with the risk-neutral model.

Case	Results for all instances				EVPI	VSS
	Opt. sol. (%)	Gap (%)	Time (s)	No canceling (%)	results (%)	results (%)
Pessimistic	47.5	4.7	3919.1	67.5	16.2	12.1
Moderate	45.0	5.0	4185.9	62.5	18.4	23.2
Equiprobable	40.0	5.4	4405.2	55.0	12.7	30.1
Optimistic	40.0	5.0	4626.1	45.0	6.2	45.0

$\alpha \in \{1; 0.75; 0.5; 0.25; 0.0\}$. In addition, we present the relative difference (i.e., percent decrease) of the objective value for a given α compared to the value obtained for $\alpha = 1$, which corresponds to the risk-neutral model solution (i.e., the RP solution presented in the previous tables).

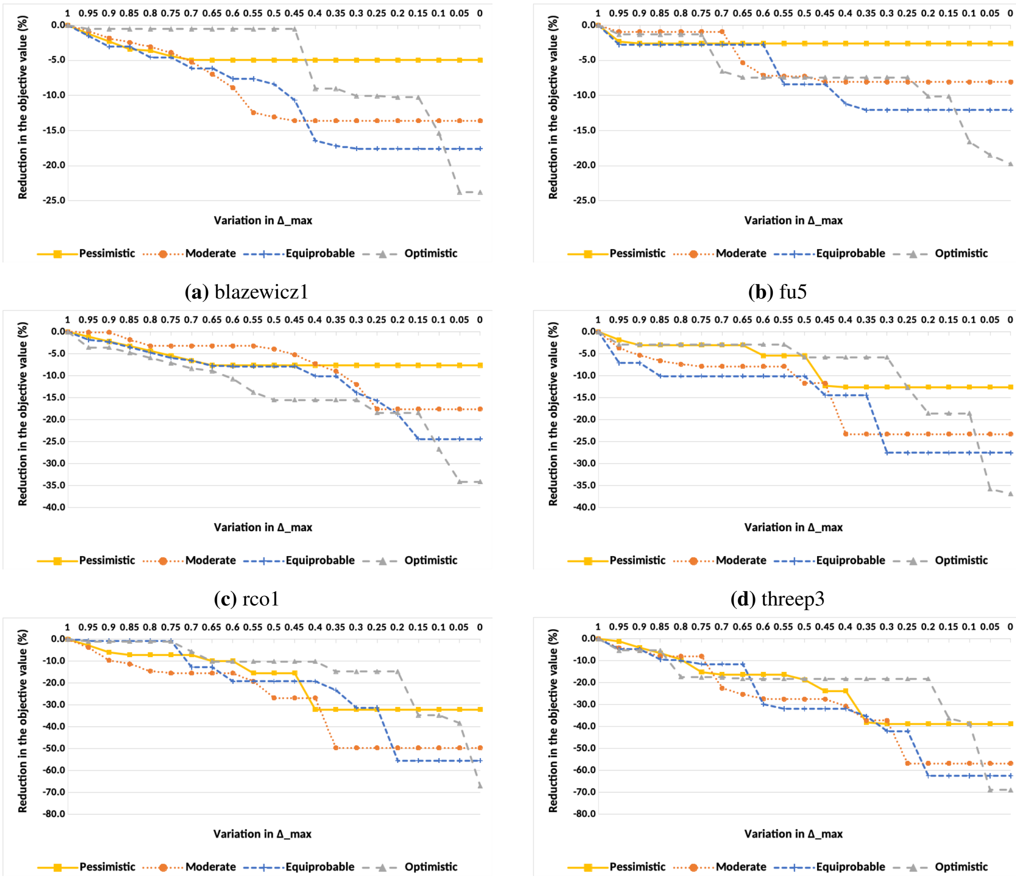
The results in Table 11 show that the solution gets worse (i.e., the profit from cutting the plate decreases) as α is reduced (i.e., as the aversion to risk increases). Observing all cases, the instances most affected by the α reduction are blazewicz1, fu5, rco1, three, threep2, threep2w9, and threep3. For example, for instances threep2 and threep3, the reduction in the objective value was over 7% (for $\alpha = 0.75$) and 23% (for $\alpha = 0.25$) in the moderate case. On the other hand, instances such as fu6, poly1c, and shirts1-2 had little or no worsening in the objective value while increasing the risk aversion, justified by the little or no influence of defects on the selection and positioning of items in the plate.

Observing Table 11, in general, the case with the largest reduction in the objective value is the optimistic one, with an average relative difference of 27.7% when $\alpha = 0$ (i.e., assuming a fully risk-averse problem), followed by the equiprobable case whose average relative difference is 22.8%, and the moderate case, with an average relative difference of 20.3%. The pessimistic case presents the smallest reductions in the objective value as the risk aversion increases (i.e., as the value of α is reduced), with an average relative difference of 7.5% when $\alpha = 0$. We notice that in the pessimistic case there is a higher probability of occurrence of the scenarios with more defects, resulting in solutions with the selection of fewer items to avoid cancellations. This is also observed in the results of the risk-neutral model. On the other hand, when considering a 25% or 50% reduction in $\Delta_{\max}$, i.e., $\alpha = 0.75$ or $\alpha = 0.5$, the largest reductions occur for the equiprobable and moderate cases, followed by the optimistic case.

Figure 4 illustrates the percentage reduction in the objective value of instances blazewicz1, fu5, rco1, threep2, threep2w9, and threep3 for all cases (pessimistic, moderate, equiprobable, and optimistic). The construction of the objective value curve for each case considers solving the risk-averse model to α varying in intervals from 5% until reaching $\alpha = 0$. We observe in this figure that the curves decrease as the values of α decrease. As the risk-averse problem controls the variability that exists among scenarios, the largest impact occurs in the optimistic case, which considers a low probability of occurrence for the scenarios with more defects. On the other hand, the pessimistic case is the one with the smallest reduction in the objective function, ranging from -2.6% (fu5) to -38.7% (threep2w9) when $\alpha = 0$. In instances blazewicz1 and fu5, the optimistic case presents a reduction of -23.7% and -19.7%, for $\alpha = 0$. This same situation, in instances rco1

Table 11 – Solutions of the risk-averse model for the 14 instances with optimal solutions.

Instance	$1.0 \times \Delta_{max}$ Objective	$0.75 \times \Delta_{max}$ Objective	$0.75 \times \Delta_{max}$ Diff. (%)	$0.5 \times \Delta_{max}$ Objective	$0.5 \times \Delta_{max}$ Diff. (%)	$0.25 \times \Delta_{max}$ Objective	$0.25 \times \Delta_{max}$ Diff. (%)	$0.0 \times \Delta_{max}$ Objective	$0.0 \times \Delta_{max}$ Diff. (%)
	Pessimistic								
blasz2	162.0	162.0	0.0	162.0	0.0	162.0	0.0	162.0	0.0
blazewicz1	42.6	40.7	-4.4	40.5	-5.0	40.5	-5.0	40.5	-5.0
fu5	296.7	289.0	-2.6	289.0	-2.6	289.0	-2.6	289.0	-2.6
fu6	434.0	434.0	0.0	434.0	0.0	434.0	0.0	434.0	0.0
fu7	441.0	441.0	0.0	441.0	0.0	441.0	0.0	441.0	0.0
poly1c	315.5	315.5	0.0	315.5	0.0	315.5	0.0	315.5	0.0
rco1	50.9	48.1	-5.4	47.0	-7.6	47.0	-7.6	47.0	-7.6
shapes2	315.8	300.0	-5.0	300.0	-5.0	300.0	-5.0	300.0	-5.0
shirts1-2	275.0	275.0	0.0	275.0	0.0	275.0	0.0	275.0	0.0
three	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
threep2	13.2	12.3	-7.1	11.2	-15.4	9.0	-32.0	9.0	-32.0
threep2w9	14.7	12.5	-15.2	11.9	-18.7	9.0	-38.7	9.0	-38.7
threep3	30.9	29.9	-3.0	29.2	-5.4	27.0	-12.5	27.0	-12.5
threep3w9	34.7	34.0	-1.9	34.0	-1.9	34.0	-1.9	34.0	-1.9
Average	-	-	-3.2	-	-4.4	-	-7.5	-	-7.5
Std. Dev.	-	-	4.2	-	6.0	-	12.4	-	12.4
	Moderate								
blasz2	164.4	162.0	-1.4	162.0	-1.4	162.0	-1.4	162.0	-1.4
blazewicz1	46.9	45.1	-3.9	40.8	-13.1	40.5	-13.6	40.5	-13.6
fu5	314.4	311.5	-0.9	291.6	-7.3	289.0	-8.1	289.0	-8.1
fu6	434.0	434.0	0.0	434.0	0.0	434.0	0.0	434.0	0.0
fu7	441.0	441.0	0.0	441.0	0.0	441.0	0.0	441.0	0.0
poly1c	315.5	315.5	0.0	315.5	0.0	315.5	0.0	315.5	0.0
rco1	57.0	55.2	-3.2	54.8	-3.9	47.0	-17.5	47.0	-17.5
shapes2	315.7	300.0	-5.0	300.0	-5.0	300.0	-5.0	300.0	-5.0
shirts1-2	275.0	275.0	0.0	275.0	0.0	275.0	0.0	275.0	0.0
three	2.2	1.5	-33.6	0.0	-100.0	0.0	-100.0	0.0	-100.0
threep2	17.8	15.1	-15.5	13.1	-26.8	9.0	-49.6	9.0	-49.6
threep2w9	20.8	19.1	-8.0	15.1	-27.5	9.0	-56.7	9.0	-56.7
threep3	35.2	32.4	-7.8	31.1	-11.7	27.0	-23.2	27.0	-23.2
threep3w9	37.1	34.0	-8.4	34.0	-8.4	34.0	-8.4	34.0	-8.4
Average	-	-	-6.3	-	-14.6	-	-20.3	-	-20.3
Std. Dev.	-	-	9.1	-	26.2	-	29.3	-	29.3
	Equiprobable								
blasz2	167.6	163.4	-2.5	162.0	-3.3	162.0	-3.3	162.0	-3.3
blazewicz1	49.1	46.9	-4.6	45.0	-8.4	40.5	-17.5	40.5	-17.5
fu5	328.7	319.6	-2.8	301.1	-8.4	289.0	-12.1	289.0	-12.1
fu6	434.0	434.0	0.0	434.0	0.0	434.0	0.0	434.0	0.0
fu7	443.9	441.0	-0.6	441.0	-0.6	441.0	-0.6	441.0	-0.6
poly1c	315.5	315.5	0.0	315.5	0.0	315.5	0.0	315.5	0.0
rco1	62.1	58.4	-5.9	57.3	-7.8	52.4	-15.7	47.0	-24.3
shapes2	316.3	300.0	-5.1	300.0	-5.1	300.0	-5.1	300.0	-5.1
shirts1-2	275.0	275.0	0.0	275.0	0.0	275.0	0.0	275.0	0.0
three	3.9	2.6	-33.5	0.0	-100.0	0.0	-100.0	0.0	-100.0
threep2	20.2	20.1	-0.6	16.3	-19.2	13.9	-31.3	9.0	-55.4
threep2w9	23.9	21.2	-11.5	16.3	-31.9	13.9	-42.0	9.0	-62.4
threep3	37.3	33.5	-10.1	33.5	-10.1	27.0	-27.5	27.0	-27.5
threep3w9	38.4	34.0	-11.4	34.0	-11.4	34.0	-11.4	34.0	-11.4
Average	-	-	-6.3	-	-14.7	-	-19.0	-	-22.8
Std. Dev.	-	-	8.9	-	26.0	-	26.7	-	29.9
	Optimistic								
blasz2	177.2	174.5	-1.5	165.5	-6.6	162.0	-8.6	162.0	-8.6
blazewicz1	53.1	52.8	-0.5	52.8	-0.5	47.8	-10.1	40.5	-23.7
fu5	360.1	355.4	-1.3	333.4	-7.4	333.4	-7.4	289.0	-19.7
fu6	435.0	434.0	-0.2	434.0	-0.2	434.0	-0.2	434.0	-0.2
fu7	458.3	441.0	-3.8	441.0	-3.8	441.0	-3.8	441.0	-3.8
poly1c	315.5	315.5	0.0	315.5	0.0	315.5	0.0	315.5	0.0
rco1	71.3	66.2	-7.1	60.3	-15.5	58.2	-18.4	47.0	-34.1
shapes2	318.6	300.0	-5.8	300.0	-5.8	300.0	-5.8	300.0	-5.8
shirts1-2	275.0	275.0	0.0	275.0	0.0	275.0	0.0	275.0	0.0
three	9.6	7.5	-21.6	7.5	-21.6	0.0	-100.0	0.0	-100.0
threep2	27.2	27.0	-1.0	24.5	-10.1	23.2	-14.7	9.0	-66.9
threep2w9	28.9	23.9	-17.5	23.7	-18.3	23.7	-18.3	9.0	-68.9
threep3	42.7	41.5	-2.9	40.3	-5.8	37.3	-12.7	27.0	-36.8
threep3w9	42.3	40.4	-4.6	40.4	-4.6	40.4	-4.6	34.0	-19.7
Average	-	-	-4.8	-	-7.2	-	-14.6	-	-27.7
Std. Dev.	-	-	6.7	-	6.9	-	25.4	-	30.9



(a) blazewicz1 **(b)** fu5

(c) rco1 **(d)** threep3

(e) threep2 **(f)** threep2w9

Figure 4 – Percentage reduction of the objective value of instances from Table 11 considering α reducing from 5% to 5%.

and threep3, results in a reduction of -34.1% and -36.8%, while in the threep2 and threep2w9, the reduction is even larger, of -66.9% and -68.9%, respectively. The equiprobable case presents the second largest reductions in the objective value, especially when α approaches zero. We notice also that the reductions in the objective function in the equiprobable case are compatible with those in the optimistic case when α varies in the interval $[0.15;\ 0.55]$.

5 CONCLUDING REMARKS

We present a two-stage stochastic optimization model for the two-dimensional irregular knapsack problem with uncertainties associated with the plate defects. The first stage of the model decides which items to select, while the second stage handles the decisions to define a feasible production plan as the plate defects are realized. While this model is risk-neutral, we have extended it to consider a risk aversion measure that limits the variability of the second-stage decisions. The models are solved with a branch-and-cut algorithm, using the no-fit raster and inner-fit raster methods to ensure a feasible positioning of items in the plate.

Computational tests with the risk-neutral model show that the cases where the probability of occurrence of the scenarios with fewer defects is higher tend to have better objective values (i.e., result in higher profit when cutting the plate), as in the optimistic case. The solutions in the pessimistic case consider selecting fewer items to avoid cancellation costs since the scenarios with more defects have higher chances of occurring. Besides that, the analyses of the expected value of perfect information and the stochastic solution value show that the uncertainties about defects directly impact the profit when cutting the plate. They indicate it is advantageous to solve the stochastic optimization model instead of considering an approximate solution by solving wait-and-see problems or an expected value problem. On the other hand, when evaluating the solutions of the risk-averse model, we observe a significant deterioration of the objective value when having a conservative decision-maker, in particular, for the optimistic case, since this case considers a low probability of occurrence for the scenarios with more defects. We note it is possible to control the reduction of the objective value by adjusting the risk aversion parameter.

As a way of continuing this research, future directions may consider having more items, more defects, different dimensions for the plate, and/or more scenarios. Concerning the number of items and dimensions of the plate, we could investigate how the decisions related to selecting and positioning of items impact on the final solution. Regarding the number of defects and scenarios, we could evaluate the models' scalability and the trade-off between solution quality and computational effort. These may demand more computational effort to have an optimal solution, creating opportunities for the proposal of heuristics and their integration with the proposed models. Another line of research seeking to accelerate the resolution of the models is related to the development of valid inequalities related to the positioning of items and the proposal of methods based on Benders decomposition (Almeida & Conceição, 2021).

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References

AHMED S & SAHINIDIS NV. 1998. Robust Process Planning under Uncertainty. *Industrial & Engineering Chemistry Research*, **37**(5): 1883–1892. Available at: https://doi.org/10.1021/ie970694t.

ALMEIDA JFF & CONCEIÇÃO SV. 2021. A decomposition approach for the two-stage stochastic supply network planning in light of the rolling horizon practice. *Pesquisa Operacional*, **41**(e234451): 1–23. Available at: https://doi.org/10.1590/0101-7438.2021.041s1.00234451.

ALVES CMM, BRÁS PAF, CARVALHO JMV & PINTO TMP. 2012a. New constructive algorithms for leather nesting in the automotive industry. *Computers & Operations Research*, **39**(7): 1487–1505. Available at: https://doi.org/10.1016/j.cor.2011.08.021.

ALVES CMM, BRÁS PAF, CARVALHO JMV & PINTO TMP. 2012b. A variable neighborhood search algorithm for the leather nesting problem. *Mathematical Problems in Engineering*, **183**: 28. Available at: https://doi.org/10.1155/2012/254346.

BAKKER H, DUNKE F & NICKEL S. 2020. A structuring review on multi-stage optimization under uncertainty: Aligning concepts from theory and practice. *Omega*, **96**: 102080. Available at: https://doi.org/10.1016/j.omega.2019.06.006.

BALDACCI R, BOSCHETTI MA, GANOVELLI M & MANIEZZO V. 2014. Algorithms for nesting with defects. *Discrete Applied Mathematics*, **163, Part 1**: 17–33. Available at: https://doi.org/10.1016/j.dam.2012.03.026.

BENNELL J & OLIVEIRA J. 2008. The geometry of nesting problems: A tutorial. *European Journal of Operational Research*, **184**(2): 397–415. Available at: https://doi.org/10.1016/j.ejor.2006.11.038.

BIRGE JR & LOUVEAUX F. 1997. *Introduction to stochastic programming*. New York: Springer.

CHEN K, ZHUANG J, ZHONG S & ZHENG S. 2020. Optimization Method for Guillotine Packing of Rectangular Items within an Irregular and Defective Slate. *Mathematics*, **8**(11). Available at: https://www.mdpi.com/2227-7390/8/11/1914.

CHERRI LH, CARRAVILLA MA & TOLEDO FMB. 2016. A model-based heuristic for the irregular strip packing problem. *Pesquisa Operacional*, **36**(3): 447–468. Available at: https://doi.org/10.1590/0101-7438.2016.036.03.0447.

CHUNG J, SCOTT III DR & HILLMAN DJ. 1990. Intelligent nesting system on 2-D highly irregular resources. In: TRIVEDI MM (Ed.), *Applications of Artificial Intelligence VIII*, vol. 1293. pp. 472–483. International Society for Optics and Photonics. Orlando, FL, United States: SPIE. Available at: https://doi.org/10.1117/12.21096.

CRISPIN A, CLAY P, TAYLOR G, BAYES T & REEDMAN D. 2005. Genetic algorithm coding methods for leather nesting. *Applied Intelligence*, **23**: 9–20. Available at: https://doi.org/10.1007/s10489-005-2368-2.

DALALAH D, KHRAIS S & BATAINEH K. 2014. Waste minimization in irregular stock cutting. *Journal of Manufacturing Systems*, **33**(1): 27–40. Available at: https://doi.org/10.1016/j.jmsy.2013.11.003.

GAREY MR & JOHNSON DS. 1979. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. San Francisco: Freeman.

HAN GC & NA SJ. 1994. A New Multi-Stage Layout Approach for Optimal Nesting of 2-Dimensional Patterns with Boundary Constraints and Internal Defects. *Transactions of the Korean Society of Mechanical Engineers*, **18**(12): 3236–3245. Available at: https://doi.org/10.22634/KSME.1994.18.12.3236.

HEISTERMANN J & LENGAUER T. 1995. The nesting problem in the leather manufacturing industry. *Annals of Operations Research*, **57**: 147–173. Available at: https://doi.org/10.1007/BF02099695.

LEE WC, MA H & CHENG BW. 2008. A heuristic for nesting problems of irregular shapes. *Computer-Aided Design*, **40**(5): 625–633. Available at: https://doi.org/10.1016/j.cad.2008.02.008.

MA Z, KWON RH & LEE CG. 2010. A stochastic programming winner determination model for truckload procurement under shipment uncertainty. *Transportation Research Part E: Logistics and Transportation Review*, **46**: 49–60. Available at: https://doi.org/10.1016/j.tre.2009.02.002.

MARTINS TC & TSUZUKI MSG. 2010. Simulated annealing applied to the irregular rotational placement of shapes over containers with fixed dimensions. *Expert Systems with Applications*, **37**: 1955–1972. Available at: https://doi.org/10.1016/j.eswa.2009.06.081.

MUNDIM LR. 2017. *Mathematical models and heuristic methods for nesting problems*. Doutorado em ciências da computação e matemática computacional. Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo. São Carlos, SP, Brasil.

MUNDIM LR & ANDRETTA M. 2014. Problema de corte de itens irregulares na fabricação de luvas de couro. In: *XLVI Simpósio Brasileiro de Pesquisa Operacional*. pp. 1–12. Salvador-BA, Brasil.

MUNDIM LR, ANDRETTA M, CARRAVILLA MA & OLIVEIRA JF. 2018. A general heuristic for two-dimensional nesting problems with limited-size containers. *International Journal of Production Research*, **56**(1-2): 709–732. Available at: https://doi.org/10.1080/00207543.2017.1394598.

MUNDIM LR, ANDRETTA M & QUEIROZ TA. 2017. A biased random key genetic algorithm for open dimension nesting problems using no-fit raster. *Expert Systems with Applications*, **81**(15): 358–371. Available at: https://doi.org/10.1016/j.eswa.2017.03.059.

MUNDIM LR & QUEIROZ TA. 2012. A hybrid heuristic for the 0-1 knapsack problem with items of irregular shape. In: *XXXVIII Conferência Latino-americana em Informática (CLEI)*. pp. 1–6. Medellín, Colombia. Available at: https://doi.org/10.1109/CLEI.2012.6427170.

PERALTA J, ANDRETTA M & OLIVEIRA JF. 2018. Solving irregular strip packing problems with free rotations using separation lines. *Pesquisa Operacional*, **38**(2): 195–214. Available at: https://doi.org/10.1590/0101-7438.2018.038.02.0195.

PINTO TMP, ALVES CMM, CARVALHO JMV & BRÁS PAF. 2016. Heuristic methods for the leather nesting problem in the automotive industry. *International Journal of Business Excellence*, **9**(3): 332–347. Available at: https://dx.doi.org/10.1504/IJBEX.2016.075594.

REIJNTJES IE. 2016. *Solving a nesting problem using the collision free region*. Master in mathematics and computer science. Eindhoven University of Technology. Eindhoven, Netherlands.

ROMÁN AE. 2020. *An Evolutionary Algorithm for Solving the Two-Dimensional Irregular Shape Packing Problem Combined with the Knapsack Problem*. Master in artificial intelligence. Polytechnic University of Catalonia. Barcelona, Spain.

SCHEITHAUER G & TERNO J. 1993. Modeling of packing problems. *Optimization*, **28**(1): 63–84. Available at: https://doi.org/10.1080/02331939308843904.

SHAPIRO A, TEKAYA W, DA COSTA JP & SOARES MP. 2013. Risk neutral and risk averse Stochastic Dual Dynamic Programming method. *European Journal of Operational Research*, **224**(2): 375–391. Available at: https://doi.org/10.1016/j.ejor.2012.08.022.

SOUZA QUEIROZ LR & ANDRETTA M. 2020. Two effective methods for the irregular knapsack problem. *Applied Soft Computing*, **95**: 106485. Available at: https://doi.org/10.1016/j.asoc.2020.106485.

SOUZA QUEIROZ LR & ANDRETTA M. 2022. A branch-and-cut algorithm for the irregular strip packing problem with uncertain demands. *International Transactions in Operational Research*, **29**(6): 3486–3513. Available at: https://doi.org/10.1111/itor.13122.

TAY FEH & LEE CFC. 2002. Pattern nesting on irregular-shaped stock using Genetic Algorithms. *Engineering Applications of Artificial Intelligence*, **15**: 551–558. Available at: https://doi.org/10.1016/S0952-1976(03)00009-5.

TOLEDO FMB, CARRAVILLA MA, RIBEIRO C, OLIVEIRA JF & GOMES AM. 2013. The Dotted-Board Model: a new MIP model for nesting irregular shapes. *International Journal of Production Economics*, **145**: 478–487. Available at: https://doi.org/10.1016/j.ijpe.2013.04.009.

VALLE AMD, QUEIROZ TA, MIYAZAWA FK & XAVIER EC. 2012. Heuristics for two-dimensional knapsack and cutting stock problems with items of irregular shape. *Expert Systems with Applications*, **39**(16): 12589–12598. Available at: https://doi.org/10.1016/j.eswa.2012.05.025.

WÄSCHER G, HAUBNER H & SCHUMANN H. 2007. An improved typology of cutting and packing problems. *European Journal of Operational Research*, **183**(3): 1109–1130. Available at: https://doi.org/10.1016/j.ejor.2005.12.047.

YUPING Z, SHOUWEI J & CHUNLI Z. 2005. A very fast simulated re-annealing algorithm for the leather nesting problem. *International Journal of Advanced Manufacturing Technology*, **25**(11): 1113–1118. Available at: https://doi.org/10.1007/s00170-003-1966-6.

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