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THE MULTIPLE CHOICE PROBLEM WITH INTERACTIONS BETWEEN CRITERIA

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ABSTRACT. An important problem in Multi-Criteria Decision Analysis arises when one must select at least two alternatives at the same time. This can be denoted as a multiple choice problem. In other words, instead of evaluating each of the alternatives separately, they must be combined into groups of n alternatives, where $n = 2$. When the multiple choice problem must be solved under multiple criteria, the result is a multi-criteria, multiple choice problem. In this paper, it is shown through examples how this problem can be tackled on a bipolar scale. The Choquet integral is used in this paper to take care of interactions between criteria. A numerical application example is conducted using data from SEBRAE-RJ, a non-profit private organization that has the mission of promoting competitiveness, sustainable development and entrepreneurship in the state of Rio de Janeiro, Brazil. The paper closes with suggestions for future research.

Keywords: Multi-Criteria Decision Analysis, bipolar Choquet integral, multiple choice problem, formation of portfolios, fuzzy logic.

1 INTRODUCTION

The project portfolio management process involves different stages of decision making. At the end of those stages, projects that add value to organizations are selected and prioritized. Companies that work with multiple projects require a vision or integrated form of management that encompasses all of the projects from their portfolios. The methods that are used to form a portfolio of projects tend to emphasize the importance of uncertainty as a process variable. The term uncertainty usually refers to both the resources and the results that must be achieved (He & Zhou, 2011; Yu et al., 2012). This statement is especially true in a multiple interacting criteria context.

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The issue of dependency between criteria has been approached by different authors in Multi-Criteria Decision Analysis (Čančer, 2010; Dalalah et al., 2012; Öztürk, 2006; Roy, 2009). However, that issue has been virtually untouched within the context of portfolio selection under multiple criteria.

Selecting a portfolio of projects is a problem that has been approached using Multi-Criteria Decision Analysis (Vetschera & Almeida, 2012; Anagnostopoulos & Mamanis, 2010; Carazo et al., 2010). However, in many cases when portfolios must be selected under multiple criteria, the interactions among the criteria are not considered as much as they should be, and additive aggregation procedures are used. A mathematical model that has been quite useful for modeling the interactions between criteria is the Choquet integral (Choquet, 1953). The objective of this paper is to show how the bipolar Choquet integral can be used for determining two or three combinations of choices for projects to be performed at the same time under multiple criteria, given that there are interactions between criteria. Ordering the two or three project combinations is indeed a multiple choice problem. A numerical application example is conducted using the data from Gomes et al. (2009).

2 THE CHOQUET INTEGRAL AS A MULTICRITERIA RANKING MODEL IN THE UNIPOLAR SCALE

The Choquet integral makes use of fuzzy measures. Those measures are very important for problems that require reliability (in a sense that an element belongs to a set) and plausibility which is dual to reliability. The fuzzy measures also assign different degrees of importance to preferences and verify whether the criteria are met.

Following Grabisch & Labreuche (2010), consider the set $X = X_1 \times X_2 \ldots \times X_n$ of feasible alternatives. The decision maker has preferences with respect to X that are expressed by a binary relation of the type $\succsim$.

Now consider the function that is given by $x \geq y \rightarrow F(\mu_1(x_1), \ldots, \mu_n(x_n)) \geq F(\mu_1(y_1), \ldots, \mu_n(y_n))$, where F is the Choquet integral and $\mu_i : X_i \rightarrow S, i = 1, \ldots, n$ are aggregation functions. $S \subset R^+$ is a scale that represents the decision maker's preferences.

There are two types of scales: the first scale, the limited unipolar scale, applies when $S = [0; 1]$, where zero means the absence of a property and 1 means the total certainty about the existence of such a property. In modeling, one can affirm the existence in X_i of two elements that have the notations U_i and P_i, where U_i is an element of X_i that represents the complete dissatisfaction of the decision maker and P_i represents his complete satisfaction, then $\mu_i(U_i) = 0$ and $\mu_i(P_i) = 1$. The second scale, the unlimited unipolar scale, applies when $S = R^+$. This scale serves to represent the priorities and relative importance. For convenience, we use the notation $\mu_i(S_i) = 1$.

According to Sugeno (1974), the function $u : 2^N \rightarrow R$ is a capacity if $u(\phi) = 0$. A capacity μ that satisfies $\mu(A) \leq \mu(B), A \subseteq B$ is a fuzzy measure. This fuzzy capacity is normalized if $\mu(N) = 1$, where N is the set of natural numbers. The fuzzy capacity is additive if, for all disjoint sets $A, B \subseteq N$, one has $\mu(A \bigcup B) = \mu(A) + \mu(B)$. It is symmetrical if, for all subsets A, B, we have $|A| = |B| \Rightarrow \mu(A) = \mu(B)$.

The formal definition of the discrete Choquet integral in a unipolar scale can be defined as follows: Let $f : N \to R^+$ to be the Choquet integral f in relation to a capacity μ given by:

$$C_\mu(f) = \sum \big(f_{\sigma(i)} - f_{\sigma(i-1)}\big)\mu\big(\{\sigma_1, \ldots, \sigma_n\}\big)$$

where σ is a permutation in N such that $f_{\sigma_1} \leq \ldots \leq f_{\sigma_n}$ and $f_{\sigma_0} = 0$.

To construct an example, assume that the scores of 4 students in 3 subjects are as shown in Table 1.

Table 1 – Students' evaluations in 3 subjects.

Criterion	Alternative			
	Student A	Student B	Student C	Student D
Subject 1	4	3	1	2
Subject 2	6	5	6	5
Subject 3	3	5	3	2

The dean of the school wants to give a full scholarship to a student by sticking to the following rule: every chosen student must be good in subjects 1, 2 and 3 (exactly in this order, that is subject 1 is more important than subject 2 and subject 2 is more important than subject 3) (i.e. *Subject* 1 $\succ$ *Subject* 2 $\succ$ *Subject* 3).

The ordering of these 4 students can be determined by using the Choquet integral as shown in Table 2. The steps below are then followed.

Step 1 – Determining the fuzzy measures

A fuzzy measure indicates the degree of evidence that an element belongs to a set. It was used a 2-additive model and Shapley-Schubik index to determine the fuzzy measures.

For example, considering three subjects in this order *Subject* 1 $\succ$ *Subject* 2 $\succ$ *Subject* 3. The fuzzy measures used in this example were:

$$\mu(\{1,2,3\}) = 1, \quad \mu(\phi) = 0, \quad \mu(\{1\}) = 0.35, \quad \mu(\{2\}) = 0,$$
$$\mu(\{3\}) = 0, \quad \mu(\{1,2\}) = 0.34, \quad \mu(\{1,3\}) = 0, \quad \mu(\{2,3\}) = 0.33.$$

Step 2: Calculating the Choquet integral

These calculations are performed by summing the values along each column. This sum gives the values of the Choquet integral. The ranking of the alternatives that are provided by the Choquet integral is then obtained by ordering these alternatives from the highest to the lowest values. The results are presented in Table 2.

The resulting order is: *Student A* $\succ$ *Student C* $\succ$ *Student D* $\succ$ *Student B*. This arrangement means that Student A is preferable to the other students.

Table 2 – Ranking obtained by using the Choquet integral in the unipolar scale.

Criterion	Alternatives			
	Student A	Student B	Student C	Student D
Subject 1	$4\mu(\{1\})$ $= 4 \cdot 0.35$ $= 1.4$	$3\mu(\{1\})$ $= 3 \cdot 0.35$ $= 1.05$	$\mu(\{1\})$ $= 0.35$	$2\mu(\{1\})$ $= (2) \cdot 0.35$ $= 0.7$
Subject 2	$2\mu(\{1, 2\})$ $= (6 - 4) \cdot 0.34$ $= 0.68$	$2\mu(\{1, 2\})$ $= (5 - 3) \cdot 0.34$ $= 0.68$	$5\mu(\{1, 2\})$ $= (6 - 1) \cdot 0.34$ $= 1.7$	$3\mu(\{1, 2\})$ $= (5 - 2) \cdot 0.34$ $= 1.02$
Subject 3	$3\mu(\{2, 3\})$ $= 3 \cdot 0.33$ $= 0.99$	$(5 - 5)\mu(\{2, 3\})$ $= 0$	$(6 - 3)\mu(\{2, 3\})$ $= (3) \cdot 0.33$ $= 0.99$	$7\mu(\{2, 3\})$ $= (5 - 2) \cdot 0.33$ $= 0.99$
Choquet integral	$1.4 + 0.68 + 0.99$ $= 3.07$	$1.05 + 0.68 + 0$ $= 1.73$	$0.35 + 1.7 + 0.99$ $= 3.04$	$0.7 + 1.02 + 0.99$ $= 2.71$
Ordering	1	4	2	3

3 THE CHOQUET INTEGRAL IN THE BIPOLAR SCALE

Using the same notation as in the previous section, for the bipolar Choquet we have the following (Grabisch & Labreuche, 2005):

Let $f : N \to R^n$, $A \subseteq R^n$ be the Choquet integral of f with respect to the capacity μ given by

$$C_\mu(f) = \sum_{i=1}^{n} \big[f(\sigma(i)) - f(\sigma(i-1)) \big] \mu(A_{\sigma_i}),$$

where σ is a permutation in N such that $f_{\sigma(1)} \leq \ldots \leq f_{\sigma(n)}$ and $f_{\sigma(0)} = 0$.

According to Greco & Figueira (2003), given a finite set $J = \{1, 2, \ldots, n\}$, a fuzzy measure μ is a function of the form: $\mu : 2^J \to [0, 1]$ such that $\mu(\phi) = 0$, $\mu(J) = 1$ (boundary conditions) and $\mu(C) = \mu(D)$ if $D \subseteq C, \forall C, D \subseteq J$ (monotonicity condition).

Let $P(J)$ be a set of pairs of subsets of

$$J : P(J) = \big\{(C, D), C, D \subseteq J, C \bigcap D = \phi\big\}.$$

A bi-capacity μ in J is a function $\mu : P(J) \to [0, 1]X[0, 1]$ such that $\mu(C, \phi) = (c, 0)$ and $\mu(\phi, D) = (0, d), c, d \in [0, 1]$; $\mu(J, \phi) = (1, 0)$ and $\mu(\phi, J) = (0, 1)$ (boundary conditions). For each $(C, D), (E, F) \in P(J)$ such that $E \subseteq C, D \subseteq F$, we have $\mu(C, D) = (c, d)$ and $\mu(E, F) = (e, f), e, f \in [0, 1]$ with $c \geq e$ and $d \geq f$ (monotonicity condition). We use the following notation: $\mu^+(C, D) = c$, $\mu^-(C, D) = d$. A bi-capacity $\hat{\mu}$ on the set J is a function $\hat{\mu} : P(J) \to [-1, 1]$ such that $\hat{\mu}(\phi, \phi) = 0$; $\hat{\mu}(J, \phi) = 1$ and $\hat{\mu}(\phi, J) = -1$ (boundary conditions). If $E \subseteq C, D \subseteq F$, then $\hat{\mu}(C, D) \geq \hat{\mu}(E, F)$ (monotonicity condition). From each bi-polar capacity μ in J, we can obtain a bi-capacity $\hat{\mu}$ in $J : \hat{\mu}(C, D) = \mu^+(C, D) - \mu^-(C, D), \forall C, D \in P(J)$ (Greco & Figueira, 2003).

For each $x \in R^n : x^+ = \max\{x, 0\}$ is the positive part of x; for each $x \in R : x- = \max\{-x, 0\}$ is the negative part of x; for each $x \in R : x^+ = (x_1^+, x_2^+, \ldots, x_n^+)$ is the positive part of

$x(x_1, x_2, \ldots, x_n) \in R^n$; and $x^- = (x_1^-, x_2^-, \ldots, x_n^-)$ is the negative part of $x(x_1, x_2, \ldots, x_n) \in R^n$. Given $x \in R^n$, we consider a permutation $(\cdot)$ of the elements of J such that $|x_{(1)}| \leq |x_{(2)}| \leq \cdots \leq |x_{(j)}| \leq |x_{(n)}|$. For each element $j \in J$, we have two subsets, $C(j) = \{i \in J : x_i \geq |x_{(j)}|\}$ and $D(j) = \{i \in J : -x_i \geq |x_{(j)}|\}$. Considering a bi-capacity μ in J and a vector $x \in R^n$, we can define the positive part of the bipolar Choquet integral as follows:

$$Ch^+(x, \mu) = \sum_{j \in J^>} \left(|x_{(j)}| - |x_{(j-1)}|\mu^+(C_{(j)}, D_{(j)})\right),$$

where $J \geq \{j \in J/|x_j| > 0\}$. In the same way, we can write the negative part of the bipolar Choquet integral as follows: $Ch^-(x, \mu) = \sum_{j \in J^>} \left(|x_{(j)}| - |x_{(j-1)}|\mu^-(C_{(j)}, D_{(j)})\right)$. Therefore, the bipolar Choquet integral is $Ch^B(x, \mu) = C^+(x, \mu) + Ch^-(x, \mu)$.

To illustrate the use of the bipolar Choquet integral, we now consider an example of the evaluation of apartments for rent based on three alternatives: near downtown, near a subway station and low cost, which are given in Table 3. In this example, we have used a Likert scale, with which the opinions of experts varying from 1 (worst value) to 5 (best value). To select the best apartment, the client expresses his preferences as follows: (i) for an apartment near downtown, a low price is more important than being near the subway; therefore, apartment #1 is better than apartment #2; and (ii) for an apartment far from downtown, being near the subway station is more important than a low price; therefore, apartment #3 is better than apartment #4.

Table 3 – Decision matrix for a bipolar example.

Criterion	Alternative			
	Apartment # 1	Apartment # 2	Apartment # 3	Apartment # 4
Near downtown	5	5	2	2
Near subway station	4	5	5	4
Near subway station	3	4	4	3

Step 1 – Determining the fuzzy measures

Consider the following ordering of criteria: (i) low price $\succ$ near subway station for an apartment near downtown; and (ii) near subway station $\succ$ low price for an apartment far from downtown. This arrangement allows us to establish a relation between the fuzzy measures using a 2-additive model and the Shapley-Chubik index [Grabisch & Labreuche (2010)]. Those measures are presented below:

$$\mu(\{1,2,3\}) = 1, \quad \mu(\phi) = 0, \quad \mu(\{1\}) = 0.39, \quad \mu(\{2\}) = 0,$$
$$\mu(\{3\}) = 0, \quad \mu(\{1,2\}) = 0.33, \quad \mu(\{1,3\}) = 0, \quad \mu(\{2,3\}) = 0.31$$

Step 2 – Calculating the Choquet integral

In Table 4, we present the rank ordering obtained by using the bipolar Choquet integral.

Computations are performed by determining the Min and Max values along each column. The MaxMin operator gives the values of the Choquet integral. The ranking of the alternatives that are provided by the Choquet integral is then obtained by ordering these alternatives from the highest to the lowest values. The results are presented in Table 4.

Table 4 – Rank ordering for a bipolar example.

Criterion	Alternatives			
	Apartment #1	Apartment #2	Apartment #3	Apartment #4
Near downtown	$5\mu(\{1\})$ $= 5 \cdot 0.39$ $= 1.95$	$5\mu(\{1\})$ $= 5 \cdot 0.39$ $= 1.95$	$3\mu(\{1\})$ $= 3 \cdot 0.39$ $= 0.78$	$3\mu(\{1\})$ $= 3 \cdot 0.39$ $= 0.78$
Near subway station	$\mu(\{1, 2\})$ $= (5 - 4) \cdot 0.33$ $= 0.33$	$\mu(\{1, 2\})$ $= (5 - 5) \cdot 0.33$ $= 0$	$\mu(\{1, 2\})$ $= (5 - 2) \cdot 0.33$ $= 0.99$	$\mu(\{1, 2\})$ $= (4 - 2) \cdot 0.33$ $= 0.66$
Low price	$(4 - 3)\mu(\{2, 3\})$ $= 1 \cdot 0.31$ $= 0.31$	$(5 - 4)\mu(\{2, 3\})$ $= 1 \cdot 0.31$ $= 0.31$	$(5 - 4)\mu(\{2, 3\})$ $= 1 \cdot 0.31$ $= 0.31$	$(4 - 3)\mu(\{2, 3\})$ $= 1 \cdot 0.31$ $= 0.31$
Min operator	$1.95 \times 0.33 \times 0.31$ $= 0.2$	$1.95 \times 0 \times 0.31$ $= 0$	$0.78 \times 0.99 \times 0.31$ $= 0.24$	$0.78 \times 0.66 \times 0.31$ $= 0.16$
Max operator	$1.95 + 0.33 + 0.31$ $= 2.59$	$1.95 + 0 + 0.31$ $= 2.26$	$0.78 + 0.99 + 0.31$ $= 2.08$	$0.78 + 0.66 + 0.31$ $= 1.75$
Choquet integral	2.59	2.26	2.08	1.75
Rank Ordering	1	2	3	4

By using the bipolar Choquet integral the logic and desired solution is obtained. This solution is the following: Apartment #1 $\succ$ Apartment #2 and Apartment #3 $\succ$ Apartment #4.

4 THE SEBRAE-RJ CASE STUDY

A numerical application example is conducted using data from Gomes et al. (2009). SEBRAE-RJ is a non-profit private organization that has the mission of promoting competitiveness and sustainable development and encouraging entrepreneurship in the state of Rio de Janeiro, Brazil. In conjunction with the Strategies and Guidelines area of that organization, nine criteria were defined to evaluate different development projects. These criteria were C_1 = cost of project; C_2 = generated revenue/total cost of project; C_3 = degree of synergy in the use of SEBRAE-RJ's products in the project; C_4 = capacity to contribute to the sustainable development of the region; C_5 = capacity to interact with other sectors of the economy; C_6 = capacity to generate employment and income; C_7 = degree of adherence of the partnerships in the management as well as governance of the project; C_8 = chance of success; and C_9 = degree of visibility that the project would bring to SEBRAE-RJ. The decision matrix is presented in Table 5. In Table 5, 'Cr' stands for 'Criterion'.

Table 5 – Decision matrix for the SEBRAE-RJ Case Study.

Cr	Projects										
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}
C_1	775.79	393.06	343.93	1572.24	327.55	884.38	2354.2	884.39	929.54	982.67	3684.94
C_2	0.08	0.06	0.63	0.10	0.10	0.19	0.09	0.09	0.12	0.31	0.06
C_3	4	6	4	6	6	8	10	4	10	6	4
C_4	10	10	10	8	10	6	10	10	10	10	10
C_5	10	8	10	10	10	10	10	8	10	10	10
C_6	8	10	10	8	10	6	10	10	10	8	10
C_7	10	8	10	10	8	10	10	8	10	8	10
C_8	10	10	10	10	10	10	10	10	10	10	10
C_9	10	10	10	10	10	10	10	10	10	10	10

By using the mean and standard deviation of each line of the decision matrix a Gaussian membership function can be utilized in order to minimize the spreading of the data (Oliveira et al., 2007). This approach is illustrated in Figure 1 for line 1 of the decision matrix.

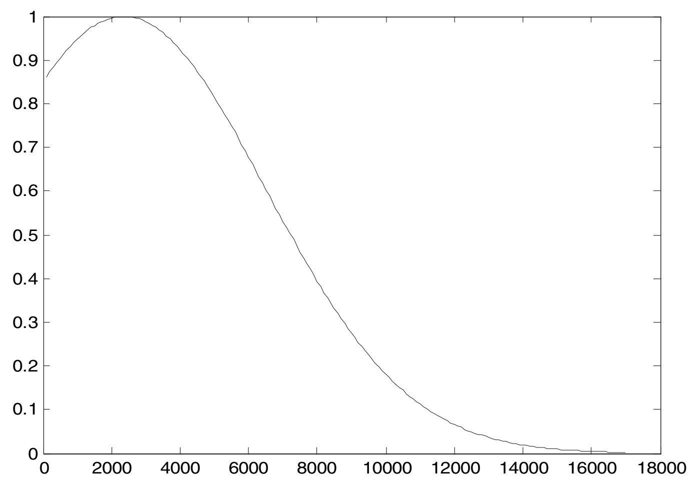

Figure 1 – Membership function adjusted to the data of line 1 of decision matrix.

The membership values for line 1 of the decision matrix are presented in Table 6.

This task is accomplished for the whole decision matrix, as shown in Table 7. In this last table m_i is the membership value for line i of the decision matrix ($i = 1, \ldots, 7$).

The Choquet integral was calculated for all two portfolio combinations and for all possible two criteria combinations. The MinMax operator was used for each two of them. Similarly, all three portfolio combinations for all possible three criteria combinations were taken. Since criteria 8 and 9 are irrelevant for the analysis as they lead to the same figures they were removed from Table 8.

By applying these values, we obtain a new decision matrix, which is presented in Table 8.

Table 6 – Membership values for of the decision matrix.

Project	Membership values
P_1	0.93
P_2	0.89
P_3	0.89
P_4	0.98
P_5	0.89
P_6	0.93
P_7	0.01
P_8	0.94
P_9	0.94
P_{10}	0.95
P_{11}	0.97

Table 7 – Membership values for all data of decision matrix.

Project	m_1	m_2	m_3	m_4	m_5	m_6	m_7
P_1	0.93	0.85	0.60	0.90	0.90	0.55	0.91
P_2	0.89	0.79	0.99	0.90	0.10	0.66	0.45
P_3	0.89	0.02	0.60	0.90	0.90	0.66	0.77
P_4	0.98	0.91	0.99	0.50	0.90	0.55	0.77
P_5	0.89	0.91	0.99	0.90	0.90	0.66	0.45
P_6	0.93	0.99	0.70	0.02	0.90	0.01	0.77
P_7	0.91	0.88	0.21	0.90	0.90	0.66	0.77
P_8	0.94	0.91	0.60	0.90	0.10	0.66	0.45
P_9	0.94	0.95	0.21	0.90	0.90	0.66	0.77
P_{10}	0.95	0.68	0.99	0.90	0.90	0.55	0.45
P_{11}	0.97	0.79	0.60	0.90	0.90	0.66	0.77

Table 8 – Fuzzified decision matrix for the SEBRAE-RJ case study.

Criterion	Projects										
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}
C_1	0.93	0.89	0.89	0.98	0.89	0.93	0.91	0.94	0.94	0.95	0.97
C_2	0.85	0.79	0.02	0.91	0.91	0.99	0.88	0.91	0.95	0.68	0.79
C_3	0.60	0.99	0.60	0.99	0.99	0.70	0.21	0.60	0.21	0.99	0.60
C_4	0.90	0.90	0.90	0.50	0.90	0.00	0.90	0.90	0.90	0.90	0.90
C_5	0.90	0.10	0.90	0.90	0.90	0.90	0.90	0.10	0.90	0.90	0.90
C_6	0.55	0.66	0.66	0.55	0.66	0.01	0.66	0.66	0.66	0.55	0.66
C_7	0.77	0.45	0.77	0.77	0.45	0.77	0.77	0.45	0.77	0.45	0.77

Step 1 – Determining the fuzzy measures

As fuzzy measures indicate the degree of evidence that an element belongs to a set, considering a 2-aditive model and the Shapley-Schubik index, the fuzzy measures used in this paper were:

$$\begin{array}{llll}
\mu(\{1,2,3,4,5,6,7,8,9\}) = 1, & \mu(\phi) = 0, & \mu(\{1\}) = 0.56, & \mu(\{2\}) = 0, \\
\mu(\{3\}) = 0, & \mu(\{4\}) = 0, & \mu(\{5\}) = 0, & \mu(\{6\}) = 0, \\
\mu(\{7\}) = 0, & \mu(\{8\}) = 0, & \mu(\{9\}) = 0, & \mu(\{1,2\}) = 0.54, \\
\mu(\{1,3\}) = 0, & \mu(\{2,3\}) = 0.53, & \mu(\{3,4\}) = 0.49, & \mu(\{4,5\}) = 0.48, \\
\mu(\{5,6\}) = 0.45, & \mu(\{6,7\}) = 0.44. & &
\end{array}$$

We then consider the following order of criteria: $C_1 \succ C_2 \succ C_3 \succ C_4 \succ C_5 \succ C_6 \succ C_7$.

Step 2 – Choquet integral calculations

In Table 9, we present some calculations obtained by using the bipolar Choquet integral for projects 1 and 2 and for projects 1 and 3. The same has been done for all combinations of two projects.

Table 9 – Some calculations for a two project selection of SEBRAE-RJ case study.

Criterion	Project #1	Project #2	Project #1	Project #3
C_1	$0.93\mu(\{1\})$ $= 0.93 \cdot 0.56$ $= 0.52$	$0.89\mu(\{1\})$ $= 0.89 \cdot 0.56$ $= 0.50$	$0.93\mu(\{1\})$ $= 0.93 \cdot 0.56$ $= 0.52$	$0.89\mu(\{1\})$ $= 0.89 \cdot 0.56$ $= 0.50$
C_2	$(0.93 - 0.85)\mu(\{1,2\})$ $= 0.14$	$(0.89 - 0.79)\mu(\{1,2\})$ $= 0.19$	$(0.93 - 0.85)\mu(\{1,2\})$ $= 0.14$	$(0.89 - 0.02)\mu(\{1,2\})$ $= 1.61$
C_3	$(0.85 - 0.6)\mu(\{2,3\})$ $= 0.13$	$(1 - 0.79)\mu(\{2,3\})$ $= 0.11$	$(0.85 - 0.6)\mu(\{2,3\})$ $= 0.13$	$(0.6 - 0.02)\mu(\{2,3\})$ $= 0.31$
C_4	$(0.9 - 0.6)\mu(\{3,4\})$ $= 0.15$	$(1 - 0.9)\mu(\{3,4\})$ $= 0.04$	$(0.9 - 0.6)\mu(\{3,4\})$ $= 0.15$	$(0.9 - 0.6)\mu(\{3,4\})$ $= 0.15$
C_5	$(0.9 - 0.9)\mu(\{4,5\})$ $= 0$	$(0.9 - 0.1)\mu(\{4,5\})$ $= 0.38$	$(0.9 - 0.9)\mu(\{4,5\})$ $= 0$	$(0.9 - 0.9)\mu(\{4,5\})$ $= 0$
C_6	$(0.9 - 0.55)\mu(\{5,6\})$ $= 0.15$	$(0.66 - 0.1)\mu(\{5,6\})$ $= 0.25$	$(0.9 - 0.55)\mu(\{5,6\})$ $= 0.15$	$(0.9 - 0.66)\mu(\{5,6\})$ $= 0.11$
C_7	$(0.77 - 0.55)\mu(\{6,7\})$ $= 0.09$	$(0.66 - 0.45)\mu(\{6,7\})$ $= 0.09$	$(0.77 - 0.55)\mu(\{6,7\})$ $= 0.09$	$(0.77 - 0.66)\mu(\{6,7\})$ $= 0.05$
Min	Min(0.52, 0.14, 0.13, 0.15, 0, 0.15, 0.09, 0.50, 0.19, 0.11, 0.04, 0.38, 0.25, 0.09, 0.24, 0.24) = 0		Min(0.52, 0.14, 0.13, 0.15, 0, 0.15, 0.09, 0.50, 1.61, 0.31, 0.15, 0, 0.11, 0.05, 0.10, 0.10) = 0	
Choquet integral (Max)	Max(0.52, 0.14, 0.13, 0.15, 0, 0.15, 0.09, 0.50, 0.19, 0.11, 0.04, 0.38, 0.25, 0.09, 0.24, 0.24) = 0.52		Max(0.52, 0.14, 0.13, 0.15, 0, 0.15, 0.09, 0.50, 1.61, 0.31, 0.15, 0, 0.11, 0.05, 0.10, 0.10) = 1.61	

These calculations are performed by using the Min and Max operators along considering the respective two columns projects. The MaxMin operator has been used to calculate the Choquet integral. The ranking of the alternatives that are provided by the Choquet integral is then obtained by ordering these alternatives from the highest to the lowest values.

The results are presented in Table 9.

A two-combination choice

The Choquet integral was calculated for all two portfolios combinations and all two criteria combinations that is, the MinMax operator was used for each two of them.

In Table 10 the ordering for two project combination portfolios is shown for the three most important criteria, C_1 (cost of project with mean 1193.88), C_2 (generated revenue/total cost of project with mean 0.17) and C_3 (degree of synergy in the use of SEBRAE-RJ's products in the project with mean 6.18).

In this paper values higher that mean values are considered as high; values near mean values are considered as mean; and values lower that mean values are considered as low. This holds for Tables 10, 11 and 12.

It can be observed that project P_3 is present in all combinations for criteria that are related with minimum cost. high generated revenue and high synergy.

Table 11 presents a two combination choice for the criteria C_4 (capacity to contribute to the sustainable development of the region with mean 9.45), C_5 (capacity to interact with other sectors of the economy with mean 9.64) and C_6 (capacity to generate employment and income with mean 9.1).

It can be observed that most of combinations have high capacity to contribute to the sustainable development of the region, high capacity to interact with other sectors of the economy and high capacity to generate employment and income. It also can be seen that project P_3 is present in all of them. Table 12 presents a two combination choice for the criteria C_7 (degree of adherence of the partnerships) in the management as well as governance of the project with mean 9.3. In Table 12 criteria 8 and 9 were not considered since they were found to be redundant.

It can be observed that all combinations have high degree of adherence of the partnerships in the management as well as governance of the project, high chance of success and high degree of visibility that the project would bring to SEBRAE-RJ.

It also can be seen that project P_3 is present in all of them. In conclusion, we now reach the ordering by the bipolar Choquet integral for alternative two-project portfolios, as shown Table 13 presents the results obtained by using the bipolar Choquet integral for two-combination portfolios. Table 14 presents the results obtained by using the bipolar Choquet integral for three-project portfolio alternatives with similar calculations.

In the case of three-project portfolio alternatives, the Choquet integral was calculated for all three portfolios combinations and all three criteria combinations that is, the MinMax operator was used for each.

Table 10 – A two combination choice based on the first three most important criteria C_1, C_2 and C_3.

A two combination choice	C_1	C_2	C_3	Observations
$P_3 - P_7$	343.93 and 2354.2 mean = 1349.07	0.63 and 0.092 mean = 0.36	4 and 10 mean = 7	low cost; high generated revenue; high synergy
$P_3 - P_5$	343.93 and 327.55 mean = 335.74	0.63 and 0.10 mean = 0.36	4 and 6 mean = 5	low cost; high generated revenue; medium synergy
$P_3 - P_6$	775.79 and 343.92 mean = 559.86	0.08 and 0.63 mean = 0.36	4 and 8 mean = 6	low cost; high generated revenue; medium synergy
$P_3 - P_9$	343.93 and 929.54 mean = 636.74	0.63 and 0.122 mean = 0.375	4 and 10 mean = 7	low cost; high generated revenue; medium synergy
$P_3 - P_8$	343.93 and 884.39 mean = 1349.07	0.63 and 0.096 mean = 0.362	4 and 4 mean = 4	low cost; high generated revenue; low synergy
$P_2 - P_3$	393.06 and 343.93 mean = 368.5	0.061 and 0.09 mean = 0.345	6 and 4 mean = 5	low cost; high generated revenue; low synergy
$P_3 - P_4$	343.93 and 1572.24 mean = 958.09	0.63 and 0.1 mean = 0.36	4 and 6 mean = 5	low cost; high generated revenue; low synergy
$P_1 - P_3$	343.93 and 343.93 mean = 614.16	0.63 and 0.192 mean = 0.41	4 and 4 mean = 4	low cost; high generated revenue; low synergy
$P_3 - P_{10}$	343.93 and 982.67 mean = 663.3	0.63 and 0.305 mean = 0.47	4 and 6 mean = 5	low cost; high generated revenue; low synergy
$P_3 - P_{11}$	343.93 and 3684.94 mean = 2014.44	0.63 and 0.064 mean = 0.346	4 and 4 mean = 4	low cost; high generated revenue; low synergy

It can be observed that all projects are selected to compose a three-combination choice of portfolio for all nine criteria and the project P_3 is present in all.

5 CONCLUSIONS

We have concluded that the bipolar Choquet integral is adequate for solving multiple choice problems. As an application example, we have used the bipolar Choquet integral to show how SEBRAE-RJ can determine which two or three project combination choices should be formed.

Table 11 – A two combination choice based on criteria C_4, C_5 and C_6.

A two-combination choice	C_4	C_5	C_6	Observations
$P_3 - P_5$	10 and 10 mean = 10	10 and 10 mean = 10	10 and 10 mean = 10	high capacity to contribute to the sustainable development of the region; high capacity to interact with other sectors of the economy; high capacity to generate employment and income
$P_3 - P_7$	10 and 10 mean = 10	10 and 10 mean = 10	10 and 10 mean = 10	high capacity to contribute to the sustainable development of the region; high capacity to interact with other sectors of the economy; high capacity to generate employment and income
$P_3 - P_9$	10 and 10 mean = 10	10 and 10 mean = 10	10 and 10 mean = 10	high capacity to contribute to the sustainable development of the region; high capacity to interact with other sectors of the economy; high capacity to generate employment and income
$P_3 - P_{11}$	10 and 10 mean = 10	10 and 10 mean = 10	10 and 10 mean = 10	high capacity to contribute to the sustainable development of the region; high capacity to interact with other sectors of the economy; high capacity to generate employment and income
$P_1 - P_3$	10 and 10 mean = 10	10 and 10 mean = 10	8 and 10 mean = 9	high capacity to contribute to the sustainable development of the region; high capacity to interact with other sectors of the economy; medium capacity to generate employment and income

All of the selected two- and three-project combination portfolios do not cost too much and lead to high generated revenues/total project cost, high capacity to contribute to the sustainable development of the region and high capacity to create employment and income. For a two-choice problem, projects P_2, P_4, P_5, P_8 and P_{10} are not selected, and for a three-choice problem, all projects are selected. In essence, we have shown that the use of the bipolar Choquet integral could allow forming a portfolio of projects by considering the measures of interactions among criteria. However, one must keep in mind the limitations that are related to the use of the Choquet integral, such as the requirement to have the aggregation (e.g., utility) functions fixed a priori (Bouyssou et al., 2012). Nevertheless, when the bipolar Choquet integral can be used, the approach presented in this paper can be extended for $n = 3$ by induction.

An algorithm for the solution of a generic multiple choice problem based on the bipolar Choquet integral can be a generalization of a two portfolio choice. The steps to be followed should then be following: (1) determine the fuzzy measures to indicate the degree of evidence

Table 12 – A two combination choice based on criteria C_7.

A two-combination choice	C_7	Observations
$P_1 - P_3$	10 and 10 mean = 10	high degree of adherence of the partnerships in the management as well as governance of the project; high chance of success; high degree of visibility that the project would bring to SEBRAE-RJ; capacity to generate employment and income
$P_3 - P_4$	10 and 10 mean = 10	high degree of adherence of the partnerships in the management as well as governance of the project; high chance of success; high degree of visibility that the project would bring to SEBRAE-RJ; capacity to generate employment and income
$P_3 - P_6$	10 and 10 mean = 10	high degree of adherence of the partnerships in the management as well as governance of the project; high chance of success; high degree of visibility that the project would bring to SEBRAE-RJ; capacity to generate employment and income
$P_3 - P_7$	10 and 10 mean = 10	high degree of adherence of the partnerships in the management as well as governance of the project; high chance of success; high degree of visibility that the project would bring to SEBRAE-RJ; capacity to generate employment and income
$P_3 - P_9$	10 and 10 mean = 10	high degree of adherence of the partnerships in the management as well as governance of the project; high chance of success; high degree of visibility that the project would bring to SEBRAE-RJ; capacity to generate employment and income
$P_3 - P_{11}$	10 and 10 mean = 10	high degree of adherence of the partnerships in the management as well as governance of the project; high chance of success; high degree of visibility that the project would bring to SEBRAE-RJ; capacity to generate employment and income

that an element belongs to a set, considering a 2-additive model and the Shapley-Schubik index; (2) select a two portfolio alternative and calculate the bipolar Choquet integral for all two portfolio combinations and all two criteria combinations by using the MinMax operator for each combination; (3) for a three- project portfolio alternatives, the Choquet integral has to be calculated for all three portfolio combinations and all three criteria combinations that is, the MinMax operator has to be used for each of them; (4) and the same calculations can carried out for more than three projects selection. All these operations can be performed, as they were in this paper, by using a MATLAB program.

Table 13 – Ordering by the Choquet bipolar Integral for two-choice projects 2.

Two-combination Portfolios	Results obtained using the bipolar Choquet Integral
$P_3 - P_7$	1
$P_3 - P_9$	2
$P_3 - P_6$	3
$P_3 - P_9$	4
$P_3 - P_{11}$	5

Table 14 – Ordering obtained using the bipolar Choquet Integral for three-combination portfolios.

Three-combination Portfolios	Results obtained using the bipolar Choquet Integral
$P_1 - P_2 - P_3$	1
$P_1 - P_3 - P_6$	2
$P_1 - P_3 - P_8$	3
$P_1 - P_3 - P_9$	4
$P_1 - P_3 - P_{10}$	5
$P_1 - P_3 - P_{11}$	6
$P_2 - P_3 - P_4$	7
$P_2 - P_3 - P_7$	8

For future research, it is recommended to design and run detailed sensitivity analyses on using other types of membership functions and alternative values for the parameters.

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