

Primordial mathematical intuitions

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Abstract: This article presents an exploratory study on the quantitative and qualitative aspects of the number in Jung's work. Thus, aspects of the number were surveyed, from the word association tests to the concept of synchronicity. The article was divided into three parts: presentation of quantitative aspects (reaction time in the word association test; and concept of psychic energy); presentation of qualitative aspects (number as psychic content; number as an organizing element of the psyche; numinous character of number; number as unpredictable greatness between myth and reality; the relationship of number to synchronistic events; and number as an archetype of the order that has become conscious); and presentation of primordial mathematical intuitions (the infinite series of natural numbers; and the idea of a *continuum*). By emphasizing the qualitative aspects, Jung points to the archetypal base of the number as a factor of ordering and articulation between psyche and matter.

Keywords: C. G. Jung, analytical psychology, number, archetype.

Introduction

It is possible to think of Analytical Psychology as a field of knowledge in dialogue. In this sense, Jung undertook debates with researchers from several areas. From his field of work, he directed his attention to religious, mythical, anthropological, artistic, and pre-scientific phenomena – such as alchemy – and the various transformations of the science of his time, notably physics. With this, he did not intend, under any circumstances, to break with science and venture into metaphysical conceptions (Jung, 1935/2011). The observation of anomalous phenomena (Kuhn, 1962/2007) allowed the scientific scope to be broadened, taking it to the margins of psychological knowledge (Melo, 2015, 2019).

In this sense, the Eranos Conferences are exemplary. Organized by Olga Fröbe-Kapteyn, from 1933, and with a name suggested by Rudolf Otto – shared banquet or food exchange –, it had as principle the intense interdisciplinary debate (Ferreira & Silveira, 2015). Jung effectively participated in Eranos meetings from 1933 to 1951, giving fourteen lectures (Quaglino, Romano, & Bernardini, 2007). In this study, it is worth mentioning the 1940 Eranos Conference, whose general theme was the symbolism of the Trinity. At first, the only lecture of that year would be by mathematician Andreas Speiser, entitled “The Platonic Doctrine of the Unknown God and the Christian Trinity”. Jung listened attentively to the lecture and, at the end, asked the hostess for a Bible, made some notes in the shade of a tree and, in

response to Speiser, presented the lecture “A psychological approach to the Dogma of the Trinity” (Jaffé, 1989).

The lecture at Eranos took place amid the intense interdisciplinary dialogue established between Jung and the physicist Wolfgang Pauli. The exchange of knowledge between the two thinkers is quite significant in the refinement process on the quantitative and qualitative aspects of the number and measurement, leading to the expansion of the scientific scope. We can follow this dialogue in several books, articles and letters (Jung, 1939/2011, 1944/2011, 1946/2011e, 1950/2011c; Meier, 1992/2011; Miller, 2009; Pauli, 1952/1996). One of the most important points of this dialogue concerns the archetypal basis for theoretical conceptions. In this way, the two authors establish the necessary conditions to think about the relationship between psyche and matter, using the notion of archetype as a parameter.

From Jung's dialogue with Pauli, the concept of archetype acquires a new connotation. From a strictly psychological point of view, the archetype cannot be observed directly, but through psychic images of a collective character, that is, archetypal (Jung, 1944/2011). Without contradicting this aspect, based on studies on synchronicity phenomena, physical processes equivalent to psychic processes can occur in an acausal manner (Jung, 1950/2011c). In this interaction between physical and psychic realities, Pauli affirms that the quantitative and qualitative aspects must be recognized simultaneously and, in this sense, the analysis of the number as “the primordial mathematical intuition¹ is fundamental (Pauli, 1955/1999, p. 193, our translation), as the notion of

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1 In the original: “l'intuition mathématique primordiale”.



infinity present in the series of natural numbers (arithmetic) and in the idea of continuum (geometry).

Numbers are characterized as elements that enable quantification. Thus, we can order the world through numbers. However, as they are psychological contents, they also have qualitative aspects. In this sense, they make it possible to count and measure aspects of concrete reality (quantitative), but, on the other hand, they concern psychic content of an imaginary (qualitative) nature (Jung, 1954/2011). The ordering is not, therefore, limited to simple enumeration, and it is responsible for inserting the subject in the relation of producing and apprehending the order of the world, implying quantitative and qualitative aspects of the enumeration that complement each other (Franz, 1970/2012; Jung, 1950/2011c).

Taking these initial arguments into consideration, this article was divided into three parts with subdivisions, the first concerning the presentation of some quantitative aspects of the number and measurement in Jung's work, related thus to the reaction-time in the word association test and the concept of psychic energy. The second part presents some qualitative aspects of the number and measurement in Jung's work, namely: number as a psychic content; number as an organizing element of the psyche – also encompassing the compensatory character of the unconscious field and the mandala structure –, numinous character of the number; the greatness of God; relationship between trinity and quaternity; number as an unpredictable greatness between myth and reality; relationship of number to synchronistic events, and number as an archetype of order that has become conscious. Third and last parts present the so-called primordial mathematical intuitions, which are the infinite series of natural numbers and the idea of continuum. In this manner, the aim is to maintain the interdisciplinary debate and overcome the methodological monism (Feyerabend, 1975/2011).

Quantitative aspects of the number and measurement

In Jung's work, the quantitative aspects of the number and measurement are addressed, mainly, in two books: *Experimental Researches* (1905/2011) and *On Psychic Energy* (1928/2011). In these two cases, we have the quantitative aspects as a way to understand the phenomenology of feeling-toned complexes and psychic energy through measurement instruments and movement relations, respectively. In the word association test, what is expected is to specify attributes that can be measured (Jung, 1979/2011). The magnitude of a given object is denominated by the term *measurand* and, to establish its value, it is necessary to use appropriate measurement methods (Piacentini, Grandi, Hufmann, Lima, & Zimmermann, 2008). As for psychic energy, measurement is directly related to the strength of the nuclear element of

the complex, the frequency and intensity of the complexes and the intensity of affect (Jung, 1928/2011).

The reaction-time in the word association test

In his studies with the association test, Jung (1905/2011) starts from inductive words to which the subject of the experiment must react with a response-word. The interval between the enunciation of the stimulus-word and the verbal reaction of the experiment subject is the *reaction-time*, considered an important factor to be analyzed in the experiment. However, Jung observes other variables that interfere with the reaction-time, because in this interval the person may be flushed, tremble, stutter, sweat, etc. Thus, measurement of the reaction-time is associated with other emotional aspects, evidenced by physical changes. The reaction-time components are measured and analyzed. In addition to these data, pulse and heart rate measurements are also taken into account.

The instrument used by Jung (1905/2011) during the word association test is a stopwatch to one-fifth of a second, which is considered satisfactory for measuring time in successive association experiments. In addition to measuring each reaction, the mean of all response times is verified. However, this is not a simple arithmetic mean, as it considers that the response times are often excessively long, influencing the analysis of the experiment:

This can be avoided by using the method of the probable mean, which consists in arranging the figures in the order of their numerical value and taking that nearest the middle. By this means the influence of excessively highly values is eliminated. (p. 253)

In this context, the number is a data sorter, a measurer, that is, it fulfills a strictly quantitative function. This concern with quantification does not mean, however, the search for absolute means, but approximate mean values that create a basis for understanding pathological processes. Jung's experimental works show the primacy of the quantitative aspects of the number, aiming to sorter and measure data in the field of psychiatry (Jung, 1905/2011).

The concept of psychic energy

According to Jung (1928/2011), Wundt approaches physical phenomena from two models: mechanistic and energetic. The word association test is mechanistic, as it "conceives an event as the effect of a cause" (p. 13). On the other hand, the energetic point of view is of a finalist nature, that is, phenomena are understood from effect to cause. This means that, at the root of the changes that occurred in a given phenomenon, there is an energy that is kept constant, producing, in an entropic way, a general state of equilibrium at the core of these mutations.

The dynamics of the energetic process have a defined direction that categorically obeys the potential difference: “The idea of energy is not that of a substance moved in space; it is a concept abstracted from relations of movement” (Jung, 1928/2011, p. 14). In the same line of reasoning, physicist Paul G. Hewitt (2011) states that, although familiar, it is difficult to define energy, because: “We sense the energy in things only when it is being transferred or being transformed” (p. 104).

In this relation of movement – observation of energy in the process of transference or transformation –, it is possible to approach the phenomenon through the quantitative bias. Jung (1928/2011) thus argues that “there are no grounds for excluding psychic events from the field of objective experience” (p. 16). Then, assuming that energy is a measurable factor and that it is possible to verify the quantitative aspect of psychic energy, Jung defines the psyche as a “*relatively* closed system” (p. 18). But how to measure the quantity of psychic energy?

As the energetic approach is linked to quantitative assessments and does not concern the substances themselves, but the relationships they establish, Jung (1928/2011) notes that psychic energy needs parameters for an objective quantitative assessment. At this point, some considerations about the feeling-toned complex are important: the complex has a nuclear element and secondary associations are grouped around it; the nuclear element has a factor defined by experience related to the environment and another factor immanent to the individual; the main characteristic of the nuclear element is to present accentuated affect. Thus, from an energetic point of view, the feeling-tone is “value quantity” (p. 21).

The measurement of this quantity of affect must take into account, therefore, the constellating power of the nuclear element of the complex. This constellating power is directly proportional to its energy value. The quantity of psychic energy can be estimated from three aspects: (1) the relative number of constellations, that is, the trend is that the greater the chain of ideas associated with the nuclear element of the complex, the greater the psychological value; (2) the frequency and intensity with which the complexes cause disturbances; (3) the intensity of other affects, which can be verified through the pulse curve, the respiratory curve and psychogalvanic phenomena (Jung, 1928/2011; Peterson & Jung, 1907/2011; Ricksher & Jung, 1907/2011).

Quantitative aspects of the number and measurement

Jung (1979/2011) notes that both points of view – quantitative and qualitative – have their merits. Supported by Jung’s conceptions, physicist Wolfgang Pauli opposes the assumptions of modern Western science, which combines empirical induction with

logical-mathematical thinking. Pauli states that, in the first half of the seventeenth century, the new way of doing science (mathematical and quantitative), which led to the modern thought, collides with, for example, the alchemical tradition (of symbolic and qualitative images). This split between quantitative and qualitative was revisited in the dialogue established between Jung and Pauli, notably through archetypes, “as order operators and image makers”² (Pauli, 1952/1996, p. 280).

The position defended by Jung is often associated with the resumption and romantic resistance to scientism (Bishop, 2007). In this case, there is a tendency to align Analytical Psychology with the “romantic axiological axis... always of a restorative nature” (Figueiredo, 2015, p. 88). Jung’s reflections can be seen, however, as a way to combine scientific rigor with romanticism (Wahba, 2019). This second position makes us think of Analytical Psychology as a proposal that cultivates *romantic universalism* as a method (Duarte, 1999). The proposal arising from the dialogues between Jung and Pauli intends the alliance between the two positions – scientific and romantic – that prioritize, respectively, the quantitative and qualitative aspects of the number and measurement.

The number as psychic content

Marie-Louise von Franz (1980/1985) reaffirms the idea that the numbers have quantitative and qualitative aspects. As psychic content, numbers can arise in dreams, thoughts, fantasies and, also, in the production of knowledge in the field of mathematics. In this last case, there is the double function, pointed out by Pauli (1952/1996), of ordering empirical and psychic phenomena, with psychic images functioning as an archetypal foundation (qualitative aspect) for mathematical elaborations (quantitative aspect). In the other examples, the number appears only as an archetypal psychic image, therefore as a symbol, in its qualitative aspect (Vale & Melo, 2019). However, even mathematicians recognize that the number has irrational aspects, and it is much more than a simple and understandable element of measurement (Kasner & Newman, 1968).

Still in his psychoanalytic period, Jung (1910/2011) gave importance to the number as a symbolic image, analyzing a middle aged man’s dream, which had the number of a train ticket as a central element. But, in addition to the appearance of the number in a psychological manifestation, we can perceive the number as psychic content that is correlated to the other aspects presented in this section: organizing element of the psyche, numinous character, unpredictable greatness, synchronistic events, and archetype of order.

2 In the original: “en tanto que operadores de *orden* y formadores de imágenes”.

The number as an organizing element of the psyche

According to Jung (1921/2011, 1944/2011, 1950/2011b, 1961/2008), the psyche is organized based on quaternary arrangements, either in the archetypal aspects or in the functions of consciousness. The unconscious quaternary base is formed by two pairs of opposites: good/evil (Jung, 1952/2011), and male/female (Jung, 1916/2011). The field of consciousness is, likewise, composed of two pairs of opposing functions that are arranged in a quaternary manner: thinking/feeling, and sensation/intuition (Jung, 1921/2011).

Compensatory character of the unconscious field

The relationship of the number and measurement to the compensatory character of the unconscious field can be approached from different points of view, among which: more broadly, the modern emphasis on the statistical method that neglects ways of producing knowledge based on qualitative aspects; more specifically, the relation between the dynamics of the functions of consciousness, with unilateral specializations (higher function) and the opening for unconscious contents to emerge (lower function).

In the first case, we have the scientific method based on the cause-effect relationship, data quantification, and the statistical method. In comparison with the knowledge produced in the East, alchemy is characterized as a Western example, prior to the mathematization process. The position adopted by Jung goes in this direction, notably in his dialogues with Pauli (Jung, 1950/2011c; Meier, 1992/2011; Pauli, 1952/1996) and with sinologist Richard Wilhelm (Jung & Wilhelm, 1923/2013).

In the second case, thinking/feeling (rational nature) and sensation/intuition (irrational nature) dyads form opposite pairs. One of these functions of the consciousness occupies the superior position, and it is generally more used and specialized. Let us suppose that the function of thinking is the superior one. In this case, the function of feeling will be inferior, and it is unconscious. From specialization and habit, the superior function is different and the individual tends to become unconscious of his/her feelings. From an enantiodromia process, the tendency is that there is a compensation of the unconscious field and the inferior function comes to the surface, bringing with it unconscious contents loaded with affect (Jung, 1921/2011).

The structure of the mandalas

The primary manifestation of the attempt at psychic organization is often presented through circular images or tending to the circle: the mandalas. These configurations

are, above all, an arithmetic problem (Jung, 1954/2011). Although they can present variations and irregularities (Jung, 1950/2011b; Silveira, 1981) – mandalas structured in number two, three, five, seven, etc. –, these psychic configurations, present in dreams and artistic expressions, generally maintain a pattern of quaternary divisions or multiples of four, finding parallels in symbols of alchemy and/or mathematics, such as *quadratura circuli* (Jung, 1946/2011, 1950/2011b 1954/2011);

The observations of Nise da Silveira (1981) confirm Jung's proposal and point out that, generally, mandalas initially appear spontaneously in states of disorientation and psychic dissociation, in an attempt at reorganization. The strict order of the mandala system tries to compensate the disorder of the psychic apparatus and this can happen through a central point, a generatrix, which establishes the coordinates of orientation. The mandala is, therefore, a manifestation of the archetype of order, the same archetype from which the number comes.

The numinous character of the number

Mathematical propositions and discoveries are, in general, permanent. Problem-solving methods developed by the Babylonians thousands of years ago, for example, are still taught in school. The notation, obviously, is not the same as that from such remote times, but the historical link remains undeniable. The history of mathematics begins with the notion of number and counting, two mysterious factors (Stewart, 2014), and the origin of the whole number is so remote that it is lost in the mists of prehistory (Boyer, 1974).

The assumption of a practical origin for mathematics and for the development of counting systems is commonly the most usual. Carving on sticks or pieces of bones is the oldest and most immediate way of visibly expressing the idea of number, as well as one of the oldest ways of communicating and narrating, that is, of "telling" something (Burton, 2011). As enumerating and narrating are often correlated, it is possible to assume that, alongside practical origins, mathematics is linked to religious ceremonies (Boyer, 1974). Even though it is the "science of common sense" (Kasner & Newman, 1968, p. 21), the mythical aspect and divinities are linked to numbers and mathematics, as it "is abstract and it often seems absolute, universal, eternal and pure. More than other kinds of knowledge, it possesses characteristics that we associate with the divine" (Koetsier & Bergmans, 2005, p. 4).

Greatness of God

The correlations between number and divinity are not restricted to prehistoric times and religious rites, but are also emphasized by modern mathematicians. In the eighteenth century, for example, Carl Frederick

Gauss attributed one of his discoveries not to the effort and rigor of research, but to the grace of God (Franz, 1980/1985). This connection arose, even more evident, in Georg Cantor's elaborations: transfinite mathematics and its concepts – infinite, transfinite and absolute infinite – were conceived by the breath of God in his ears, as a divine product and, at the same time, as a path to understanding God (Krajewsky, 2016; Naylor, 2018; Recalde & Beltrán, 2017).

The relationship between the trinity and the quaternity

As we said at the beginning of this study, Jung (1940/2011) approached the main symbol of the Christian faith, the Trinity, from a psychological point of view. After presenting parallels between several divine triads – in Babylon, Egypt and Greece –, Jung affirms that the Trinity also represents a “symbolic number” (p. 21). On the one hand, the Trinity is analyzed as a result of thought as an “evolution required by the emancipation of the human spirit” (p. 103) and, on the other, as a psychological symbol that develops in three stages, each related to a personification of the Trinity (Father, Son and Holy Spirit). In this sense, it is a process of “becoming aware” (p. 103), which is not guided by the mutation of three different representations, but of a single substance.

Thus, the Trinity (Father, Son and Holy Spirit) is analyzed in its numerical progression, with symbolic meanings: the Father, as the primordial unity, is questioned for the imperfection of His work; the breaking of the unity makes possible the appearance of otherness, the dyad, the world of the Son; with the death of the Son, the third element, the Holy Spirit, appears. In this way, the numerical succession (1, 2 and 3) corresponds to the unfolding of the divinity: “Thus the history of the Trinity presents itself as the gradual crystallization of an archetype that moulds the anthropomorphic conceptions of father and son, of life, and of different persons into an archetypal and numinous figure, The ‘Most Holy Three-in-One’” (Jung, 1940/2011, p. 57).

In the event that the numerical-symbolic unfolding extends from the trinity to the quaternity, that is, to the totality, it is necessary to consider which contents and aspects are excluded and may be potentially involved in this process. The Christian Trinity excludes the feminine and evil, represented, for example, by Mary and the devil. Finally, the trinity does not have the presence of matter, but only spiritual aspects. In this sense, Jung (1940/2011) characterizes the Dogma of the Assumption of the Virgin Mary as the possibility of the body being introduced into heaven, preparing the divinization of Mary and, at the same time, the process towards quaternity. The transposition of the body into the metaphysical kingdom also points to the approach of another factor neglected in and by the Christian Trinity: the corrupting principle (evil).

The number as unpredictable greatness between myth and reality

Counting consists of establishing one-to-one relationships between elements (Smullyan, 1992). This principle is present in any context involving the act of counting, from the primitive practice of pairing calculations, pebbles and chopsticks, to modern counting and enumeration systems. Thus, people count until they name “many.” This is the critical point that marks one of the most severe problems in counting: the notion of infinity. Through the word “many” there is a perception that we all have, even if implicitly, the notion of infinitude of natural numbers (Franz, 1980/1985).

But the notion of the infinite escapes the strictly quantitative aspects of the field of mathematics. This is exactly the impossibility of counting progressing. In its mythical aspects, however, the notion of infinity starts to count on an unpredictable greatness. In some religions, the only being able to count infinitely is divinity. In the New Testament, for example, the God of great numbers, or rather, of the infinite, is able to count the hairs on our heads (Franz, 1980/1985), as shown in the Gospel of Luke (XII: 6-7).

Thus, the infinite is possible for God; there is a limit for humans, and, from there, it is said “many.’ Some aborigines in Australia count up to two, and South American Indians count up to six, after reaching their respective numerical limits, they enunciate “many” (Burton, 2011). In Queensland, also in Australia, the natives count “one, two, two and one, two and two, many” (Eves, 2011). The examples multiply: African pygmies, natives of Tierra del Fuego, the Hottentots. By varying the limit number, they all carry the same principle: “many” belongs to divinity. From the moment the mankind learned to count – one, two, then three, four, five and so on – to this day, men gradually removes “a little bit of territory from that all-counting god” (Franz, 1980/1985, p. 31).

The relationship of the number to the synchronistic events

From an intense and prolonged debate with physicist Wolfgang Pauli (Meier, 1992/2011), Jung (1950/2011c) developed the hypothesis of the acausal relationship between psychic and physical phenomena, the so-called significant coincidences or synchronicity. Modern Western science is based on causally related phenomena, that is, events in which every effect presupposes a cause. Nevertheless, observation of events related not by causality, but by meaning, makes Jung create arguments about the relationship between two phenomena, without one being the cause of the other. These events do not, therefore, have an explanatory basis in causality, nor are they casual, given that there is an

impact on the individual and make sense; they may be correlated in an acausal manner.

Jung (1950/2011c) identifies three ways that characterize the occurrence of synchronistic events: (1) simultaneity between a psychic and a physical event; (2) psychic and physical events close to simultaneity, and (3) coincidence between a psychic event and a future event. In all three cases, in order to speak of synchronicity, the slightest possibility of a causal relationship has to be ruled out and the phenomena must be related by meaning. As events are linked to the time factor, but do not necessarily happen synchronously, there is an option to create the concept of synchronicity.

Synchronicity phenomena can involve numerical series and/or establish significant coincidences with a given number. In both cases, we are talking about qualitative aspects of the number and measurement. We will highlight just one example presented by Jung (1950/2011c): a person buys a subway ticket and then a theater ticket and both have the same numbering; later, he receives a phone call and the interlocutor's telephone number is the same as that of both tickets. It is clear that this sequence of three events that present the same numbering cannot be thought of in terms of causality, but it creates a strong impression on the person involved.

Synchronicity phenomena and numbers establish connections, as they are shrouded in mystery and have numinosity (Jung, 1950/2011c). In the example shown, there is the number both in the series of events (three) and in the significant coincidence (the same numbering regarding the subway ticket, the theater ticket and the interlocutor's telephone number). In addition, the number is directly related to synchronicity because it appears as the property of matter and also as an unconscious ordering base. In this way, the number brings, in addition to the evident quantitative character, qualitative characteristics that enable its appearance in events of coincidences between psychic images and events of concrete reality, as "the element capable of unifying the domain of matter and the psyche"³(Franz, 1970/2012, p. 71).

Retaking the relationship between number three and number four, present in several of Jung's observations on various topics: in typology, there is an inferior function (Jung, 1921/2011); the Christian Trinity does not include the female element (Jung, 1940/2011); and in the dialogues between Jung and Pauli (Meier, 1992/2011), the synchronicity hypothesis is elaborated as the fourth element that become part of the triad space, time and matter, forming a quaternion with two pairs of opposites: indestructible energy/continuous space-time; constant connection by effect (causality)/inconstant connection through contingency or equivalence or "meaning" (synchronicity) (Jung, 1950/2011c).

The number as an archetype of order which has become conscious

From the conceptions of Analytical Psychology, we can say that the counting of N, that is, of everything, of the infinite, is an activity inherent to the Self. Thus, order is in the world and we can count the population of a city, the apples of a tree, the trees of a garden and, who knows, the hairs. But order is also implicit in each of us, having an archetypal basis (Jung, 1940/2011, 1950/2011a, 1954/2011).

For Jung (1954/2011), the formulation of knowledge is ordering and, when possible, it manifests itself in numerical sequences, apprehending some data about the nature of things. This ordering is not limited to a simple enumeration, but inserts the subject in the relation of reproducing and apprehending order in the world, also implying qualitative aspects of enumeration in the formulation of knowledge. Knowing, therefore, is a psychophysical formulation mediated by a psychoid resource, namely, the number, and therein is its psychophysical aspect: the possibility of manifesting physically in the patterns of nature and psychically in mental phenomena (Jung, 1950/2011c).

The coincidence between an objective fact and a mental image presupposes connections between the subject's mind and the material world. It is, therefore, the genuine encounter between psyche and matter, as if the functioning of the world and that of the psyche started to walk in consonance playing a single melody – the melody of formulated knowledge (Pauli, 1952/1996). This knowledge can be a complex equation of motion, the determination of joint cardinality or the descriptive narrative of an event that orders phenomena in a timeline. This last idea may seem strange at first, but it is easy to understand.

Comparative philology allows us to perceive the link between number and narrative through ordering. When a scientist reports an observed fact, he "tells us" something; the English version of that verb is *to tell* (to narrate or tell a story or event) which corresponds to the German *erzählen*, which derives from the noun *Zahl* (*number*), also linked to *erzählen* (*to enumerate*). Here there is an archetypal principle, that is, a primordial mathematical intuition, because *Erzähler* (*enumerator*, if translated literally) is the narrator, the storyteller, while *Erzählung* is a narrative, a story. In French, the verb *to tell* turns into *raconter*, related to the verb *compter* (*to count* or *to enumerate*), coming from the Latin *computare* (Franz, 1980/1985). A similar phenomenon occurs in the Portuguese language. When the verb *contar* is used, it is possible to refer to a static set or to the ordering of facts in time, and we are invariably referring to a psychoid ordering. This linguistic phenomenon is not restricted to Western languages. In Chinese and Japanese, there is the following ideogram: 契 – *chigiri* in the Japanese

3 In the original: "l'élément propre à unifier le domaine de la matière et celui de la psyché".

language. It is a *kanji* composed of three characters: the first, upper left, represents an engraving with carvings and is also the archaic way of expressing the number three (*san*, in the Japanese language) and, in general, expresses the idea of counting; the second, upper right, represents a knife or blade (*hā*); the last, at the bottom, means large (*ō*). A contract or agreement, in Japanese, is etymologically “an engraving with large carvings,” the narrative record of an agreement, or rather, telling an agreement (Almeida, 2013).

These examples show that the notion of one-to-one parity sustains and originates all kinds of more complex communication, whether arithmetic, narrative, folk or scientific. At some point, humanity must have noticed that, when telling a story, it is as if it were telling, ordering events in time, and thus obeying the predetermined rhythm of facts, the physical pulse of nature (Franz, 1980/1985). This whole harmonic process between psyche and matter is based on an archetypal image, that of the hero who unfolds the unknown and apprehends the rhythms of nature, gaining more and more space in the game of which only God, initially, knew the rules.

When enumerating, measuring or arranging geometric shapes, the human being orders the phenomena. The number is, therefore, a product of the human spirit and, as such, carries a series of symbolic aspects. Currently, we deal with numbers on a daily basis: when we draw up a shopping list, when we have to pay a bill, when writing down a recipe, in the interest for a team's ball possession, in difference of milliseconds established by a new world record or, who knows, when we count sheep. These banal examples point to the conclusion that the number is, in short, the “*archetype of order* which has become conscious” (Jung, 1950/2011c, p. 51).

The primordial intuitions

As seen so far, the number is a fundamental unit of the human psyche. Thus, any phenomenon confronted by man will have it as the main mediator of the process. Counting something is a way of assimilating events and the most elementary way to do it is through natural numbers. They are called natural because they are precisely those used to count. The sequence of icons 1, 2, 3, 4, 5, 6, 7, 8 and 9 are the signs, representations that make up the whole set of possible systems of organization of cognition around the elementary faculty of ordering and counting (Mendes, 2006).

The infinite series of natural numbers

This continuum or sequence of numbers that tends to infinity can be represented in several ways. Orally, these numbers are evoked through the names one, two, three, four etc., which represent the lowest

categories (or numbers) in the series up to a limit evidently determined by the resources of local languages. It is worth remembering, however, that these resources can be increased as needed. Regardless of the language, these words are almost always differentiated and autonomous in relation to the lexical system. In general, children are familiarized for the first time with such terms through an activity of symbolic calculation, usually present in rhyme narrative or riddles (Mendes, 2006).

From rhymes or numerical games, the ability to count must develop, later, in relation to the environment and events in the environment. Numbering is the most archaic intellectual act and, to operate it, it is necessary to establish a reference on which a numerical system will be conceived and developed (Mendes, 2006).

The whole theory that underlies this referential system derives from three basic Peano axioms. To understand mathematically the construction of the series of natural numbers, we have to think of them as undefined elements. Let us take an \mathbb{N} set, whose elements we will call natural numbers. We will then resort to a function $s: \mathbb{N} \rightarrow \mathbb{N}$. For all $n \in \mathbb{N}$, the number $s(n)$, the value that the function s assumes at the point n will be understood as a *successor to n* . Thus, the function must satisfy the following axioms enunciated by Peano: (I) $s: \mathbb{N} \rightarrow \mathbb{N}$ is *injective*. That is, $m, n \in \mathbb{N}$, $s(m) = s(n) \Rightarrow m = n$. In Portuguese, we say that two numbers that have the same successor are the same. (II) $\mathbb{N} - s(\mathbb{N})$ *calculation of a single element*. In other words, there is only one natural number that is not the successor to any other, it is called “one” and represented by the sign 1. (III) If $X \subset \mathbb{N}$ is a *subset* that $1 \in X$ and, for all $n \in X$, there is also $s(n) \in X$, then $X = \mathbb{N}$. The last principle, called the *Principle of Induction*, can be verbally stated as follows: let α be a property of natural numbers; if 1 has such a property and if, in fact, another natural number has the property p , it is possible to conclude that $p+1$ will also present the property α , and therefore all natural numbers will have this property (Lima, 2009).

These three axioms give life to the simple process of counting numbers one after the other. And it is precisely this perennial presence of a successor that guarantees the existence of an infinite series of natural numbers. And from this mere act of counting, several other events of greater or lesser complexity derive, one of them is Mathematics of Animals, a very peculiar phenomenon that says about the ability of animals to perceive and even operate in numerical or geometric mathematical devices. It is a special type of non-verbal and innate mathematics (Almeida, 2013). Several species in the animal kingdom enjoy this type of numerical perception. For now, it is known that among them are primates – including humans (Dehaene, 2005) –, insects (Nieder, 2018), fish (Matsuura, 2014) and even salamanders (Krusche, Uller, & Dicke, 2010).

The idea of continuum

The set of data evidenced by mathematics of animals are outstanding, as they show us that those fundamental Peano axioms, briefly stated previously, are alive and operating not only in humans, but also in several animals. Obviously, mathematics of animals has its limitations. When they count, animals do so in a reduced proportion. Humans, already endowed with cognitive and symbolic mathematics, go a little further: some count to five, others up to six, even ten, twelve or even gigantic numbers. As we pointed out, this amount varies according to customs and ethnic-cultural needs (Almeida, 2013).

But what would happen if this continuum of natural numbers was explored to its end? That was the question asked by Georg Cantor. The “mathematician of the infinites” developed several mathematical devices that allowed him to exceed the limits of counting. In short, Cantor counted until infinity was not enough and then jumped into the transfinite. And, in this new dimension, what he discovered was the existence of a new continuum, which led him to elaborate the *continuum hypothesis*. Cantor, evidently, did not reach the last number of the infinite series of natural numbers, because it is infinite. Through comparison between sets, the mathematician realized that there were infinities of different sizes and raised the hypothesis that these infinites were organized in a new series: the transfinite series (Vale, 2018).

What matters in this study is the psychological meaning unveiled by the hypothesis and the idea of a transfinite continuum. Cantor counted until the infinite and beyond it, touched the God that counts everything. Cantor’s mathematics, when explained by himself, is full of divine allusions and archetypal figures. When we

speak of a count that goes beyond finiteness, we speak of a direct contact with the Self, the archetype of order of everything that lies in the psyche (Vale, 2018).

Being the genic point of the psyche, the fundamental archetypal instance and the source of libido, the Self cannot be assimilated by the field of consciousness in its entirety. Thus, with each new assimilation of unconscious contents, a new infinite of possibilities opens up, a new series or a new continuum (Neumann, 1949/1995). In this way, even if a mathematician jumps to instances of the infinites, there will still be a new infinite series of infinites to traverse and the number, as an archetypal and numinous reality, will demonstrate its fatal irreducibility (Vale, 2018).

Final considerations

When analyzing a wide spectrum of conceptions of number and measurement in their quantitative, qualitative aspects and as a primordial mathematical intuition, we developed an exploratory study (Minayo, 2009). The central axis is to raise, throughout Jung’s work, the different aspects of the number. This survey makes it possible to deepen each of the topics presented and crossed analyzes between several of these aspects. We can consider that the number and measurement were present in the various works developed by Jung, from the word association test to the elaboration of the synchronicity hypothesis. Sometimes, the quantitative aspect prevails; at other times, the qualitative aspect is more important. It is evident in this paper that Jung emphasizes the qualitative aspects of number and measurement. And, even more, the psychological exploration on the theme points to the archetypal basis of the number as a factor of ordering and articulation between psyche and matter.

As intuições matemáticas primordiais

Resumo: Este artigo apresenta um estudo exploratório sobre os aspectos quantitativos e qualitativos do número na obra de Jung. Assim, foi efetuado levantamento de aspectos do número, desde os testes de associação de palavras até o conceito de sincronicidade. O artigo foi dividido em três partes: apresentação de aspectos quantitativos (tempo de reação no teste de associação de palavras e conceito de energia psíquica); apresentação de aspectos qualitativos (número como conteúdo psíquico, número como elemento organizador da psique, caráter numinoso do número, número como grandeza imprevisível entre mito e realidade, a relação do número com os eventos sincronísticos e o número como arquétipo da ordem que se tornou consciente); e apresentação das intuições matemáticas primordiais (a série infinita de números naturais e a ideia de *continuum*). Ao enfatizar os aspectos qualitativos, Jung aponta para a base arquetípica do número como fator de ordenação e de articulação entre psique e matéria.

Palavras-chave: C. G. Jung, psicologia analítica, número, arquétipo.

Les intuitions mathématiques primordiales

Résumé : Cet article présente une étude exploratoire sur les aspects quantitatifs et qualitatifs du nombre dans l’œuvre de Jung. On a réalisé une enquête sur les aspects du nombre, des tests d’association de mots au concept de synchronicité. L’article est divisé en trois parties : présentation des aspects quantitatifs (temps de réaction dans le test d’association de mots

et concept d'énergie psychique) ; présentation des aspects qualitatifs (le nombre en tant que contenu psychique et élément organisateur de la psyché ; le caractère numineux du nombre ; le nombre en tant que grandeur imprévisible entre mythe et réalité ; la relation du nombre avec les événements synchrones ; et le nombre en tant qu'archétype de l'ordre devenu conscient) ; et présentation des intuitions mathématiques primordiales (la série infinie de nombres naturels et l'idée de continuum). En mettant l'accent sur les aspects qualitatifs, Jung souligne la base archétypale du nombre en tant que facteur d'ordre et d'articulation entre la psyché et la matière.

Mots-clés : C. G. Jung, psychologie analytique, nombre, archétype.

Intuiciones Matemáticas Primordiales

Resumen: Este artículo hace un estudio exploratorio sobre los aspectos cuantitativos y cualitativos del número en la obra de Jung. Se examina aspectos del número desde pruebas de asociación de palabras hasta el concepto de sincronicidad. El artículo se divide en tres partes: presentación de aspectos cuantitativos (tiempo de reacción en la prueba de asociación de palabras y concepto de energía psíquica); presentación de aspectos cualitativos (número como contenido psíquico, número como elemento organizador de la psique, carácter numinoso del número, número como grandeza impredecible entre mito y realidad, relación del número con eventos sincronísticos, y número como arquetipo del orden en que se ha convertido consciente); y presentación de intuiciones matemáticas primordiales (la serie infinita de números naturales y la idea de un continuo). Al enfatizar los aspectos cualitativos, Jung señala la base arquetípica del número como un factor de orden y articulación entre la psique y la materia.

Palabras clave: C. G. Jung, psicología analítica, número, arquetipo.

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