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The relationship between demand and performance of magistrates: investigation of a functional model in the form of an inverted U

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The hypothesis of exogenous productivity points to the relationship between the number of judges and performance as linear. However, several weaknesses have been pointed out, such as the production limit of the judges. Therefore, this work aims to test if the performance of judges related' to demand has an inverted U functional model. The data used in the research comprise the years from 2009 to 2019. The results were statistically significant for all variables analyzed for both state and labor courts, confirming the hypothesis that performance and judicial demand may respond to an inverted U functional model. Confirming the research hypothesis sheds light on the discussion of how demand affects performance and how they relate. The contribution of this work lies in the discussion and endorsing the conclusions of other authors concerning weaknesses and incongruities of the hypothesis of exogenous productivity for the judiciary. There is empirical evidence that the increase in judicial demand pressures the increase in the performance of judges, but this increase has limits, and after a certain point, this can decline and negatively affect performance.
Keywords: judiciary; performance; quantitative methods; workload; public administration.

A relação entre demanda e desempenho dos magistrados: investigação de um modelo funcional em forma de U invertido

A hipótese da produtividade exógena aponta a relação entre quantidade de juízes e desempenho como linear, porém várias fragilidades vêm sendo apontadas, como o limite de produção dos juízes. Diante disso, o objetivo deste trabalho é testar se o desempenho dos juízes em relação à demanda tem um modelo funcional em forma de U invertido. Os dados usados na pesquisa compreendem os anos de 2009 a 2019. Os resultados foram estatisticamente significantes para todas as variáveis analisadas, tanto para a justiça estadual quanto para a trabalhista, confirmando a hipótese de que o desempenho e a demanda judicial podem responder a um modelo funcional em forma de U invertido. A confirmação da hipótese de pesquisa traz luz à discussão sobre como a pressão da demanda afeta o desempenho e como elas se relacionam. Este artigo discutiu e endossou as conclusões de outros autores sobre fragilidades e incongruências da hipótese da produtividade exógena para o Judiciário. Há evidências empíricas de que o aumento da demanda judicial pressiona o aumento de desempenho dos juízes, mas esse crescimento tem limites, e a partir de determinado ponto isso pode declinar e afetar negativamente o desempenho.
Palavras-chave: judiciário; desempenho; métodos quantitativos; carga de trabalho; administração pública.

La relación entre la demanda y el desempeño de los magistrados: investigación de un modelo funcional en forma de U invertida

La hipótesis de la productividad exógena indica que la relación entre el número de jueces y el rendimiento es lineal, pero se han señalado varios puntos débiles, como el límite de producción de los jueces. En vista de ello, el objetivo de este trabajo fue comprobar para los tribunales estatales y laborales si la actuación del juez en relación con la demanda tiene un modelo funcional en forma de *U* invertida. Los datos utilizados en la investigación abarcan los años 2009 a 2019. Los resultados fueron estadísticamente significativos para todas las variables analizadas tanto para los tribunales estaduales como para los laborales confirmando la hipótesis de que el rendimiento y la demanda judicial pueden responder a un modelo funcional en forma de *U* invertida. La confirmación de la hipótesis de la investigación arroja luz sobre la discusión sobre cómo la presión de la demanda afecta al rendimiento y cómo se relacionan. La contribución de este trabajo fue discutir y refrendar las conclusiones de otros autores sobre las debilidades e incongruencias de la hipótesis de la productividad exógena del poder judicial. Hay pruebas empíricas de que el aumento de la demanda judicial ejerce presión sobre el aumento del rendimiento de los jueces, pero este crecimiento tiene límites y a partir de cierto punto puede disminuir y afectar negativamente al rendimiento.
Palabras clave: poder judicial; desempeño; métodos cuantitativos; carga de trabajo; administración pública.

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1. INTRODUCTION

The Judiciary is responsible for resolving social conflicts by way of legal proceedings, but it has been suffering from slowness and an accumulation of lawsuits that are pending trial, as evidenced by the Justice in Numbers' report of the *Conselho Nacional de Justiça* (CNJ, 2020). At the end of 2019, more than 76 million cases were awaiting a court decision (CNJ, 2020), a number that shows the scale of the problem, which is related to efficiency in providing citizens with legal services.

The poor performance of the Judiciary is not just a problem in Brazil; various countries around the world are experiencing this same difficulty. Studies evaluating the performance of the courts have been carried out in Portugal (Teixeira, Bigotte, Repolho & Antunes, 2019), the Netherlands (Blank & Heezik, 2020), Italy (Falavigna, Ippoliti & Ramello, 2018), Argentina (Ferro, Romero & Romero-Gómez, 2018), Bulgaria (Dimitrova-Grajzl, Grajzl, Slavov & Zajc, 2016), Japan (Ramseyer, 2012), the United States (Christensen & Szmer, 2012), Spain (Rosales-Loópez, 2008), Israel (Beenstock & Haitovsky, 2004) and Brazil (Gomes, Alves & Silva, 2018; Gomes, Guimarães & Akutsu, 2017; Procopiuck, 2018; Sousa & Guimarães, 2018).

The number of works in different countries illustrates that concern with the efficiency and slow pace of justice systems around the world is recurrent and very important socially. In the case of Brazil, the low efficiency of this system is not only due to the legal framework, it is also linked to administrative, political and cultural problems (Ribeiro & Rudiniki, 2016).

The high demand for justice and the constant pressure on the judicial system to improve its performance need to be better understood. It is already known that the increase in demand on the courts has a positive effect on their performance (Beenstock & Haitovsky, 2004; Rosales-Loópez, 2008), but what is not yet known is whether this effect is linear and produces constant effects, or has a known limit (Sousa & Guimarães, 2018).

Although it is counterintuitive to imagine that increasing the workload will also increase a judge's performance, their production and efficiency have been shown to be exogenous, that is, an increase in their workload operates as an external stimulus for them to produce more, and so the relationship between the input and output of cases is balanced. This phenomenon has become known as the "hypothesis of exogenous productivity" and is based on several studies (Beenstock & Haitovsky, 2004; Dimitrova-Grajzl et al., 2012, 2016; Rosales-López, 2008) that indicate that the relationship between demand and the performance of the judiciary has no limits and a linear correlation.

Discussion currently revolves around whether demand and judicial performance establish a linear functional relationship, as proposed by the hypothesis of exogenous productivity (Beenstock & Haitovsky, 2004), or for example some other behavior, that is related to a growth in performance up to a critical point and, from there, to a decline in the productivity of the justice system (Sousa & Guimarães, 2018).

There are flaws in the premises of the hypothesis of exogenous productivity, such as the linearity between the "demand" and "judicial performance" variables, and the perspective of a constant increase in the productivity of judges. It must be remembered that the increase in the number of lawsuits has no known limits, as it depends on society as a whole. On the other hand, the judge's production capacity has a natural limit, given their physical and mental fatigue, and lack of time (Gomes, 2018; Gomes et al., 2017; Jonski & Mankowski, 2014).

There is, therefore, a gap in the literature when it does establish a dimension for what this performance limit might be given the increase in demand. As gaps to be studied, and in an attempt to verify the linearity assumption and the productivity limit, Sousa and Guimarães (2018) suggest analyzing the demand for judges, which may indicate an inflection in the increase in the performance of judges.

This paper questions the premises of the hypothesis of exogenous productivity by trying to verify whether the relationship between demand and judicial performance corresponds to a quadratic functional model. Demand was defined as an independent variable, and performance squared as a dependent variable. An attempt was made to predict an inverted U-shaped relationship, in which performance grows up to a certain critical point, after which there is a possible drop in productivity. In this way, linearity and productivity growth can be verified in a constant manner.

Another important point to be remembered is that the topic of "performance in the Judiciary" started developing recently because of what is known as the "law and economics" movement. Despite a constantly growing interest in this area, there are few Brazilian researchers dedicated to this field of knowledge when compared to other areas that study performance in the public sector (Gomes & Guimarães, 2013; Louro, Santos & Zanquetto, 2017; Sousa & Guimarães, 2014) few theoretical and empirical studies are found in Brazilian Judiciary Courts' literature. In order to empirically identify which variables (IT investments, own or outsourced human capital).

Given the situation presented above, we perceived the need to study a functional model that explains the relationship between demand and performance as applied to the judiciary and to the reality of Brazil. Based on the problem we identified in the literature, the aim of this work is to test whether the judge's performance in relation to demand has a functional inverted U-shaped model.

2. RELATIONSHIP BETWEEN DEMAND AND JUDICIAL PERFORMANCE

A discussion found in studies on performance in the judiciary refers to which variables determine the productivity results of judges and the courts. Roussey & Soubeyran (2018) describe three sets of judicial performance output indices: the first is linked to time and the number of cases dispatched by judges; the second to the quality of the decisions made; and the third to the terms of current legislation.

Of these indexes, the first stood out because of the growing number of studies in recent years that have tried to find the relationships between the variables that interfere with the productivity of the courts. These studies include, for example, works by Bielen, Peeters, Marneffe and Vereeck (2018), Ferro et al. (2018), Gomes et al. (2018), Procopiuck (2018), and Sousa and Guimarães (2018).

One relationship in particular between two variables, however, has been attracting a lot of attention and gaining space in studies: this is the link between lawsuits, characterized by the number of cases that enter the system, and the number of personnel, characterized by the number of people, especially judges, who deal with the cases. Examples of studies that verified the relationship of these variables in the literature are those by Beenstock and Haitovsky (2004), Dimitrova-Grajzl et al. (2012), Ferro et al. (2018), Gomes et al. (2017), Rosales-López (2008), and Sousa and Guimarães (2018).

The results found in these analyses indicate that there is a positive relationship between increase in demand and the productivity of judges, that is, an increase in the workload of judges leads to a positive variation in their performance. Beenstock and Haitovsky (2004) state that this happens because, when judges are pressured by the increase in demand they reduce the time dedicated to each case, thereby increasing their production and initially avoiding an indiscriminate increase in the number of lawsuits they have to deal with.

Beenstock and Haitovsky (2004) were among the first authors to create a mathematical model to explain the relationship between productivity and the workload of judges, and the results they found became known as the "hypothesis of exogenous productivity of judges". This idea argues that external pressures that are not under the control of judges exert a strong influence on their productivity. According to the authors, this pressure, which is characterized by the constant increase in the demand for legal services and a quick solution of cases by parties involved in litigation, influences judges to increase their production so the number of lawsuits does not rise to alarming levels. This notion has been confirmed by several authors, such as Dimitrova-Grajzl et al. (2012, 2016), Ferro et al. (2018), Gomes et al. (2017), Rosales-López (2008), Sousa and Guimarães (2018), as reported above.

Despite being the dominant theoretical strand in quantitative studies of court performance, the hypothesis of the exogenous productivity of judges has been criticized and contested by some authors. Jonski and Mankowski (2014) questioned it and pointed out an error in the model developed by Beenstock and Haitovsky (2004). They say that this relationship is not linear, which is why the analysis is methodologically inaccurate. Gomes (2018) points out that there is no linearity in the relationship between demand and productivity in the Brazilian judiciary, indicating that the performance of judges has its limits, as did Jonski and Mankowski (2014).

The demand for court services is increasing worldwide (Bielen et al., 2018). In some places, such as Brazil, the number of cases awaiting trial runs into millions, as noted in the CNJ's report, Justice in Numbers (2018). This shows that the workload of Brazilian judges is very high and there is no prospect of improvement. Every year the number of cases pending trial increases, as can be seen in the CNJ reports from 2003 to 2018, and this variable has no known limit so far. The productivity of judges, on the other hand, is limited, either by individual factors or by time limit.

Falavigna, Ippolite and Ramello (2018), Gomes (2018), Gomes et al. (2017), Jonski and Mankowski (2014) and Sousa and Guimarães (2018) point out that there is a limit to the increase in judges' productivity, and that from now on there may be an inflection in production. Based on data from the CNJ (2016), Gomes et al. (2017) point out that the productivity of judges has begun to decrease, showing that the limit may have already been reached.

Another point to highlight is the attempt by governments to increase the number of fulltime judges to try and reduce the increase in the number of cases awaiting trial. Authors such as Beenstock and

Haitovsky (2004), Dimitrova-Grajzl et al. (2012) and Gomes and Freitas (2017) show that increasing the number of judges does not increase the productivity of the courts. The explanation for this phenomenon is that the performance of overloaded judges decreases with the arrival of new judges, because individual workload is alleviated virtually. In addition to negatively affecting the productivity of the courts, if the increase in the number of judges is not accompanied by an increase in the number of administrative assistants, this might have an even greater negative impact, since these assistants do most of the bureaucratic judicial activities (Gomes, 2018).

Many factors have an influence on the performance of judges, such as the use of technology (Gomes et al., 2018), the cost of justice (Blank & Heezik, 2020), administrative management (Roussey & Soubeyran, 2018), the professional experience of judges (Ferro et al., 2018), or even the size of the institutional structure (Peyrache & Zago, 2016). For some of these variables, however, there are still no structured secondary data, or they do not fit the scope of this study, which focused on performance through the influence of a judicial demand that has already been consolidated in the literature of the area.

2.1 Research hypothesis

The hypothesis of this research was based on two assumptions taken from the literature about performance in the judiciary, and presented below. The first has as its reference Dimitrova-Grajzl et al. (2012, 2016), Ferro et al. (2018), Gomes et al. (2017); Rosales-López (2008), and Sousa and Guimarães (2018), who state that there is a positive relationship between the workload of judges and their productivity. This leads to the first hypothesis: "The production of judges increases with judicial demand".

Although this work assumes that productivity increases with demand, Gomes (2018), Gomes et al. (2017), Jonski and Mankowski (2014), and Sousa and Guimarães (2018) point out that this increase has a limit, that this relationship, as found in previous studies, is not infinite, and that there must be an inflection point in the performance level. The second hypothesis therefore, was thus formulated: "The production of judges has an upper performance limit".

Based on the preliminary hypotheses above, our intention was to investigate and test the following research hypothesis: "The productivity of judges follows an inverted U-shaped function (quadratic function) when they are subjected to an increased workload".

3. METHODOLOGY

From our review of the literature, we found no other study with a theoretical empirical model that had the same objective or used the methodology we chose. A quadratic model test to verify the correlation between performance and judicial workload is the result of the gaps in the research that were suggested by authors such as Sousa and Guimarães (2018), when they indicated that an optimization point for this relationship needs to be found, if in fact one exists. In theory, a quadratic model could help in this search. The closest thing to this is found in the studies of Peyrache and Zago (2016), and Santos and Amado (2014), who tried to find a value that was ideal for the size and number of courts (Voigt, 2012), or for both (Teixeira et al., 2019), but not for the relationship between workload and productivity.

In view of what was discussed in formulating the hypotheses, the model that best confirms the assumptions we made for achieving our research objective is quadratic regression. According to Agresti and Finlay (2012), ordinary regression models assume that the relationship between the studied variables is linear, although these relationships are not linear in the social sciences. Thanks to the low degree of non-linearity, however, they can be estimated approximately and studied as being linear. Also, according to the authors, this model may be inappropriate for modelling variables in which there is a relationship of non-linearity.

Agresti and Finlay (2012) state that undesirable consequences may appear when estimating a linear model in such a way as to explain a curvilinear phenomenon. Estimates of the means of the variables in linear regression may be biased and lead to a distancing from the line that best represents the relationship being studied. So, correlation models may not be estimated in the best possible way.

Wooldridge (2010) clarifies that adding quadratic terms to significant variables can be a way of verifying and finding the best functional model of a relationship between variables. This author states that, despite complicating the understanding of the functional model, quadratic terms may represent other functional problems that are not yet known.

Werkema (1996) demonstrates that a polynomial regression function for a response variable (y) and a single explanatory variable (x) has the following general form:

$$\gamma = \beta_0 + \beta_1 \mathrm{X} + \beta_2 \mathrm{X}^2 + \cdots + \beta_k \mathrm{X}^k \varepsilon$$

The value of k corresponds to the degree of the equation: if (k=1), there is a linear regression. For (k=2), there is a second-degree function, which is better known as quadratic regression. According to the author, the second-degree polynomial function has the following function:

$$\gamma = \beta_0 + \beta_1 \mathrm{X} + \beta_2 \mathrm{X}^2 + \varepsilon$$

In the above formula, β_0 is the mean of y when x=0, β_1 is the parameter associated with the linear effect x, β_2 is the parameter associated with the quadratic effect, and is the error term associated with the model failing to relate the data perfectly. With this statistical model, we propose the regression test between the productivity and workload of state and labor law judges in Brazil in order to verify their adhesion to the data made available by the CNJ (2018). The functional model we tested is indicated below:

$$Production = Workload\,[Carg] + Workload^2 + Administrative\ assistants[Admin] + Error\ term$$

$$Prod = \beta_0 + \beta_1 Carg + \beta_2 Carg^2 + \beta_3 Admin + \varepsilon$$

The main variables used in the most recent performance studies of the judiciary are: new cases, pending cases, resolved cases, number of judges, number of hearings, investment in communication and information technology, number of assistants and support staff, total cost. Due to the difficulty

of accessing data referring to all these variables in state and labor courts, or their non-existence, this work used the variables "workload", "administrative assistants" and "production of the judge". The variables and references that support this choice are shown in Box 1.

BOX 1 RESEARCH VARIABLES

Indicator	Variable	Bibliographic reference
Independent variable	Workload (carg.)	Ferro et al. (2018), Gomes et al. (2017), Gomes et al. (2018), Rosales-López (2008).
Independent variable	Administrative assistants (admin.)	
Dependent variable	Production of the judge (prod.)	

Source: Elaborated by the authors.

The "workload" variable represents the total number of lawsuits submitted to the courts divided by the number of judges each one has, thus enabling the total number of lawsuits or time bands to be analyzed. It comprises the sub-variables "new cases", which are inserted at the beginning of a certain period of analysis, and "pending cases", which had already been included before the analysis, and that are awaiting the judge's decision.

As Yeung and Azevedo (2011) pointed out, weighting the workload by the number of judges is indicated for controlling variations in the sizes of the courts, since the workload fluctuates freely, while the number of judges is fixed by law and remains relatively constant over time. Adopting the average that each judge in a given court is subjected to, therefore, minimizes problems related to differences in the size and structure of courts in the country. This same strategy was also adopted by Gomes et al. (2018) to relativize and minimize the differences between the courts that presented discrepancies in the demand for judicial services and the resources available (physical, financial and personal) for carrying out their activities.

The "administrative assistant" variable represents the number of people available for administrative activities who assist the courts divided by the number of judges in each court. There is no differentiation here in the type and degree of the difficulty of the activities, with only the number of people each judge has at their disposal being counted.

The "production of the judge" variable represents the number of lawsuits analyzed and judged by the courts, divided by the number of judges of each court, and the cases that were dispatched with in a certain period of time, that is, those that are no longer pending, having exhausted the competences of each judge and/or court.

The data used in this research are of secondary sources, collected from the Justice in Numbers database by the CNJ (2020), and made available for use and download on the institution's website. They have been collected since 2005, but from 2009 onwards a new collection methodology was established. Today, there is a panel database with information stacked and separated by court and by year, with numerical (continuous and discrete) and categorical variables.

This study has two variables, which correspond to three sub-variables in the CNJ database. The "workload" variable corresponds to the sum of the "cnl" sub-variable, which mean new cases for first degree jurisdiction, and the "cp1" sub-variable, which are pending cases for first degree jurisdiction. The "judge's production" variable corresponds to the "tbaix1" sub-variable, which is the total number of cases dispatched to first degree jurisdiction. The "administrative assistant" variable corresponds to the "sajud1" sub-variable, which is also related to first degree jurisdiction. The variables in this study were weighted by the number of judges provided ("magp1" subvariable) in each court, with the aim of reducing variations due to differences in the structure and size of the judiciary in each region and state of the federation.

All the variables are numeric and discrete, and there are no missing data, which makes the panel balanced. The data correspond to the years 2009 to 2019, because in previous years the methodology and variables collected by the study were different.

We used the functional model proposed in this work to analyze the data. As Hill, Griffiths and Judge (2006) found, there are no clear rules that help when choosing from the variables as to which functional model is the one that best represents the relationship being studied. This depends on the theoretical problems included in the discussion of the problem or research question, on the data available for analysis, or on the researcher's ability to understand which interaction responds to the objectives of the subject being tested.

Linear relationships, however, have already been found in the literature, as discussed in the theoretical section, and there is evidence of a possible quadratic relationship of these variables that could potentially represent this correlation better.

In regression analysis studies, one of the objectives for the functional elaboration of the relationship is to obtain statistically significant estimates of the values and signs of the model's regressors, which enable statistical inferences to be made, and serve the theoretical basis of the phenomenon being studied. (Gujarati & Porter, 2011).

The second step was to analyze how the panel data regression would be carried out. The three most common techniques for carrying out a regression with panel data are: pooled ordinary least squares (Pols), fixed effects model (MEF) and random effects model (MEA). The Hausman test was used to decide between the MEA and MEF techniques, the restricted F test to decide between Pols and MEF, and the Breusch-Pagan Lagrange multiplier test to decide between Pols and MEA. These tests are intended to establish whether or not the estimators of the two models being compared differ substantially from each other, and indicate the most appropriate (Fávero, Belfiore, Silva & Chan, 2009). The hypotheses and test result values for choosing the regression model will be presented in the results section.

The method used for calculating the estimators depends on the regression model selected. For the fixed-effect and pooled ordinary least squares model, we used the ordinary least squares (OLS) method, and the generalized least squares (GLS) method for the random-effects model, since OLS produces inefficient estimators for this model (Gujarati & Porter, 2011; Maddala, 2003).

We used "RStudio" software, version 1.2.5001, for the statistical calculations. We chose this software because of its free nature and the options it offers to create and run different tests and models that could be used in this study.

4. RESULTS

Tests were initially performed to assess the basic assumptions of the classic regression model. Box 2 systematizes the tests to assess whether there are problems of heteroscedasticity, autocorrelation, multicollinearity or normality of the residuals for data from the state and labor courts.

BOX 2 RESULTS OF THE ASSUMPTIONS FOR THE STATE AND LABOR COURTS

State courts			
Assumption	**Test**	**Result**	**Diagnosis**
Heteroskedasticity	Breusch-Pagan	BP=12.89; p-valor=0.004	Does not confirm the assumption
Autocorrelation	Durbin-Watson	DW=2.09; p-valor=0.801	Confirms the assumption
Multicollinearity	VIF	From 1.20 to 11.34	Does not confirm the assumption
Normality	Shapiro-Wilk	W=0.96; p-valor=0.001	Does not confirm the assumption
Labor courts			
Assumption	**Test**	**Result**	**Diagnosis**
Heteroskedasticity	Breusch-Pagan	BP=13.48; p-valor=0.003	Does not confirm the assumption
Autocorrelation	Durbin-Watson	DW=1.97; p-valor=0.409	Confirms the assumption
Multicollinearity	VIF	From 1.04 to 32.60	Does not confirm the assumption
Normality	Shapiro-Wilk	W=0.98; p-valor=0.002	Does not confirm the assumption

Source: Elaborated by the authors.

Although the models do not confirm the assumption of heteroskedasticity, this problem can be corrected by modifying the parameter estimation method. When there is heteroskedasticity, OLS estimators are not efficient, which can be corrected by adopting GLS estimators, which, in this situation, produce better linear unbiased estimators (Gujarati & Porter, 2011).

The data indicate the non-existence of autocorrelation of the error terms in the data and in the model proposed in this work, thus confirming the premise of the classic regression model.

Multicollinearity, in this case, was caused by the addition of the "workload squared" variable to the functional model, causing a perfect correlation between the two variables. The addition of polynomial terms to a regression model can cause multicollinearity to appear, and one way of dealing with this problem is to do nothing, because even so it is still possible to make estimates relatively efficiently (Gujarati & Porter, 2011).

We should emphasize that the abnormality of the residuals is due to the data structure itself. To solve this problem, we used the law of large numbers, by way of the probability theory theorem, that is, as the sample grows in size, residual error distribution gets closer to the normal distribution, producing valid estimators (Gujarati & Porter, 2011). For each variable in the study, we used 297 observations for state courts and 264 for the labor courts.

After testing the assumptions, we started the tests for choosing the estimation models. Box 3 systematizes the tests and indicates which model is most appropriate for the database we used in this study.

BOX 3 RESULTS OF THE CHOICE OF ESTIMATION MODEL

State courts				
Test	**Result**	**P-value**	**Hypothesis**	**Diagnosis**
F Test	1.3294	0.1366	H0 = Pols H1 = MEF	Pols
Breusch-Pagan's Lagrange multiplier test	0.9653	0.3258	H0 = Pols H1 = MEA	Pols
Hausman test	2.6167	0.4546	H0 = MEA H1 = MEF	MEA
Labor courts				
Test	**Result**	**P-value**	**Hypothesis**	**Diagnosis**
F Test	0.7845	0.7493	H0 = Pols H1 = MEF	Pols
Breusch-Pagan's Lagrange multiplier test	0.7950	0.3726	H0 = Pols H1 = MEA	Pols
Hausman test	4.8014	0.1869	H0 = MEA H1 = MEF	MEA

Source: Elaborated by the authors.

After this series of tests to choose the regression model, the pooled ordinary least squares model was the most suitable, and so it was used in the regressions to calculate the best estimators for the functional model we tested in this work, both for the state and labor courts. The data from both

justice systems pointed to heteroskedasticity, and one of the ways of correcting this problem is by using estimators and the generalized least squares technique (GLS), used only in the random effects model (*MEA*). Because of this limitation, we chose the random effects model (*MEA*) for data analysis in this work.

Table 1 presents the results of the regression of the functional model proposed in this study in relation to the state justice system. We used the random effects model (*MEA*), and the estimators were calculated by the generalized least squares method. As already mentioned, the panel data were balanced, and totaled 297 different pieces of information for each variable.

TABLE 1 RESULTS OF THE MULTIPLE REGRESSION OF THE STATE COURTS

$Prod = \beta_0 + \beta_1 Carg + \beta_2 Carg^2 + \beta_3 Adimin + \varepsilon$					
	Coefficient	Standard error	t-value	P-value	Sig.
Constant	169.84	92.83	1.8295	0.0673	
Workload	0.1531	0.0182	8.3889	<0.0001	***
Workload²	-0.0000029	0.0000009	-2.9195	0.0035	**
Assistant	26.694	6.266	4.2600	<0.0001	***
R^2: 0.5928			F statistic (3.293): 142.605		
R^2 adjusted: 0.5886			P-valor: <0.0001		

NB: Sig. = Significance level; *p-value <0.05; **p-value <0.01; *** p-value <0.001. R^2= Coefficient of determination. gl = degree of freedom.
Confidence interval of coefficient (99%): -0.0000055 > > -0.00000041.
Source: Elaborated by the authors.

The results were statistically significant for all the variables we analyzed. The value of the estimate of the "workload" variable was β_1=0.1531, and a p-value <0.0001, indicating significance for the model we analyzed. The value of the estimate of the "workload²" variable was β_2= -0.0000029, with a p-value of =0.0035, indicating significance for the model we analyzed. The value of the estimate of the "assistant" variable was β_3=26.694, with a p-value <0.0001, also indicating the statistical significance of the model analyzed. The F test was significant for the adherence of the model, with an estimate of F=142.60 and a p-value <0.0001. The measure that indicates the generalized adjustment of the model known as R^2 was 0.5928, with R^2 adjusted at 0.5886. The estimates found may indicate the possibility that the theoretical model we proposed in this work may find support in the data of the Brazilian state courts of first degree. The β_2 estimator is significant and negative, indicating a concave downwards functional curve, according to the model we tested.

Based on data from the CNJ (2020) and on the proposed model, the inference is that the hypothesis of the productivity of judges may have an inverted U-shaped function (quadratic function) when subjected to an increased workload. It cannot be rejected, therefore, when analyzing state court data.

Table 2 gives the estimation results for the labor courts. We used the random effects model (*MEA*),

and the generalized least squares method for calculating the estimators. As already mentioned, the panel data were balanced, totaling 264 different pieces of information for each variable analyzed.

TABLE 2 MULTIPLE REGRESSION RESULTS OF THE LABOR COURTS

$Prod = \beta_0 + \beta_1 Carg + \beta_2 Carg^2 + \beta_3 Adimin + \varepsilon$					
	Coefficient	Standard error	t-value	P-value	Sig.
Constant	-342.27	136.52	-2.5291	0.0114	*
Workload	0.8004	0.1063	7.5244	<0.0001	***
Workload²	-0.0001165	0.0000227	-5.1248	<0.0001	***
Assistant	31.281	11.180	2.7979	0.0051	**
R^2: 0.4936			F statistic (3.260): 84.495		
R^2 adjusted: 0.4878			P-valor: <0,0001		

NB: Sig. = Significance level; *p-value <0.05; **p-value <0.01; *** p-value <0.001. R^2= Coefficient of determination. gl = degree of freedom.
Confidence interval of coefficient (99%): -0.000175 > > -0.000057.
Source: Elaborated by the authors.

The results were statistically significant for all the variables we analyzed. The value of the estimate of the "workload" variable was β_1=0.8004, with a p-value <0.0001, indicating the statistical significance of the model we analyzed. The value of the estimate of the "workload²" variable was β_2=-0.0001165, with a p-value <0.0001, indicating significance for the model we analyzed. The value of the estimate of the "assistant" variable was β_3=31.281, with a p-value =0.0051, also indicating statistical significance for the model. The F test was significant for adherence of the model, with an estimate of F=84.49 and a p-value <0.0001. The measure that indicates the generalized adjustment of the model known as R^2 was 0.4936, and R^2 adjusted was at 0.4878. The estimates we found may indicate the possibility that the theoretical model we proposed in this work may also be supported by the data from the Brazilian labor court of first degree. The β_2 estimator is both significant and negative, indicating a concave downwards functional curve, according to the model we tested.

Based on data from the CNJ (2020) and the proposed model, the inference is that the hypothesis of the productivity of judges may have an inverted U-shaped function (quadratic function) when they are subjected to an increased workload. Thus, our proposed hypothesis cannot be rejected when the data from the labor courts are analyzed.

4.1 Discussion of the results

The results of this work confirm the hypothesis that judicial performance and demand may respond to an inverted U-shaped functional model, both for labor courts and Brazilian state courts. The non-rejection of the proposed research hypothesis sheds light on the discussion of how demand pressure affects the performance of the courts and how they are related. The discussion about the linearity between these variables takes on new contours with the indication that there may be a limit to the productivity of judges when they are subjected to the pressures arising from the increase in their workload.

Another finding that emerges from these results concerns the discussion about the extent to which judges' productivity can be increased, and whether or not there is a limit to this increase. Gomes (2018) started the discussion in Brazil about whether judges are already at this performance limit, or not. When analyzing the descriptive data of this research, there seems to be a convergence to conclude that state and labor court judges may be at the limit of their productivity, or very close to it, which is why demand pressure may have little influence on the performance of judges.

The "administrative assistant" variable proved to be very important in the composition of the judges' performance, as previously predicted by Ferro et al. (2018), Gomes et al. (2017), Gomes et al. (2018), Rosales-López (2008), and Santos and Amado (2014), showing that it is perhaps even more significant than workload in response to the productivity of judges, so we should think about changing the level of relevance of the variable in functional modeling. The importance of this variable in the productivity of judges is in line with what was pointed out by Gomes et al. (2017), who concluded that hiring assistants for judges is a more advantageous alternative for increasing their performance than hiring new judges. An excessive number of assistants, however, does not bring about a positive variation in performance, and there may be some point of equilibrium between performance and demand for this variable that still needs to be studied further.

The results indicate that the two main branches of Brazilian justice respond to a quadratic function between productivity and workload. Angular term estimates (β_1) were statistically significant, and the negative sign indicates a function curving downwards, showing that there may be a maximum point of the function, or a critical point. This possible critical point, which would be the optimal performance point, demonstrates the theoretical possibility of a real productivity inflection point (response variable) from which performance might even reduce, as assumed Sousa and Guimarães (2018).

Although statistically significant, the value of is of low magnitude, perhaps indicating that, after a certain point, the variation in the performance starts to vary marginally rather than as a direct relationship. Criticisms of the linear productivity model emphasize that linearity is not the most appropriate relationship, and the model we present here is a first step towards finding the one that best fits the empirical data.

The results suggest that there are limits to the productivity of labor and state judges. Even if judicial demand increases and brings positive pressure to bear on performance, this increase will not be linear. In this case, it will be quadratic, which is in line with what the hypothesis of exogenous productivity predicts. Gomes (2018) already indicated that the relationship between performance and demand in the Brazilian justice system could not follow a linear correlation, given the difference in the characteristics of each variable.

The linearity assumption of the relationship between the studied variables is also questioned in this study, because assuming that demand will positively influence performance and not predicting any limit is like assuming that judges will be able to increase their productivity *ad aeternum*. Authors such as Falavigna et al. (2018), Gomes (2018), Gomes et al. (2017), Jonski and Mankowski (2014), and Sousa and Guimarães (2018) have already indicated that this is impossible.

There are sufficient reasons to question the premises of the exogenous productivity hypothesis. Jonski and Mankowski (2014), some of the first authors to question the methodological validity of the premises of this hypothesis, have been joined by others in recent years. Falavigna et al. (2018), Gomes (2018), Gomes et al. (2017), and Sousa and Guimarães (2018) also question these assumptions and indicate that the productivity of judges may have human performance limits, which is in line with what we found in this study.

Based on these findings about the possible quadratic relationship between the "performance" and "demand" variables, it is possible to find mechanisms that allow a demand-based performance optimization point to be modeled. It would be possible, therefore, to define policies for hiring judges and their assistants according to more objective criteria, since recruiting and selecting judges with the idea of increasing performances and so reducing the number of cases awaiting trial will not have an effect on the productivity of the courts, as Beenstock and Haitovsky (2004), Dimitrova-Grajzl et al. (2012), and Gomes and Freitas (2017) all pointed out. The productivity of overworked judges reduces with the arrival of new judges, since the individual workload is virtually alleviated.

The general results of this research suggest that the exogenous productivity hypothesis needs to be reviewed and updated, as a series of studies have pointed out methodological and theoretical flaws in its assumptions. There is already enough evidence to point to the production limit of judges, and to the fact that the indefinite increase in demand from a certain point has little or no practical effect on increasing the performance of judges. The methodology we chose in this work was conducted in order to try and estimate a functional model that empirically assumes a limit to judges' productivity by way of a critical point of a quadratic function.

Finally, we suggest that the relationship between performance and the demand of the courts can be analyzed from the perspective of the non-linearity of the variables and the human performance limit of judges. This work suggests that this relationship be considered as an inverted U-shaped functional model.

5. CONCLUSION, LIMITATIONS AND RESEARCH AGENDA

Studies into performance in the Judiciary are increasing, and a new field of study is emerging in Brazil with the aim of finding answers to the various problems of access to and the resolution of conflict via the Judiciary. In the balance between the supply and demand of justice, there is, on the one hand, society, which demands a solution for its problems, and, on the other, the offer, which is the responsibility of the State, by way of a system of conflict mediation that concentrates on the role of the judge.

There is a direct relationship between the supply and demand of justice, and understanding how this correlation behaves was the problem we investigated in this work. Our objective was to test whether the correlation between performance and judicial demand responds to an inverted

U-shaped functional model. The results we found point to a possible functional model that can include quadratic variables. As a result, an inverted U-shaped functional model seems adequate for explaining the correlation between demand and judicial productivity, because the estimators were statistically significant for state and labor courts.

The contribution of this work was that it discussed and endorsed the conclusions of other authors, such as Falavigna et al. (2018), Gomes (2018), Gomes et al. (2017), Jonski and Mankowski (2014), and Sousa and Guimarães (2018), on the failures and inconsistencies of the hypothesis of exogenous productivity for the Judiciary. It is true that the increase in judicial demand puts pressure on the performance of judges, but this increase has its limits, and from a certain point onwards, it may even harm and negatively affect productivity. There are sufficient indications of the limits in the production of judges, and the results reported here corroborate this statement. The exogenous productivity hypothesis needs to be revised to predict the existence of this human limit, regardless of other technologies that may be added to this equation between demand and performance.

A limitation of the study is its reduced use of variables in the functional model, because the absence of other variables that influence the performance of judges can lead to inconsistencies in estimating parameters. This absence of variables is due to another limitation: the difficulty of accessing data over and above the variables mentioned by the CNJ. Variables such as "the experience of the judge", "the number of labor court districts" (Gomes et al., 2017), "the time allocated by the judge to administrative activities" (Rosales-López, 2018; Roussey & Soubeyran, 2018), and others that, admittedly, influence the judge's performance, still do not have structured data available in Brazil, so acquiring them often clashes with the bureaucracy of the courts, sometimes because there is still no adequate methodology for accessing this information.

Another limitation is in the scope of the study, because only the first instance of state and labor courts was analyzed, neither were data by category of case (civil, criminal, administrative), or judicial decision (injunction, administrative order, or final judgment) used. This prevents us from obtaining a more organic and complete look, and consequently any generalization of the results beyond the unit of analysis of this work.

As an agenda for future research, we suggest studying the limits and degree of importance that each variable has in the performance of judges, in order to identify the true impacts of each of them. In this way it would be possible to direct more adequate and efficient proposals and solutions to the problem of backlog of pending cases in providing a jurisdiction system in Brazil. Another suggestion is to carry out qualitative studies in courts that perform well and perform badly, in order to validate variables and provide evidence of others that are absent from the studies. We also suggest studies to identify the possibility of a functional response model for the performance of the judiciary that can predict the results, given the investments and the demand of society.

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