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# Analysis of multi-scale systemic risk in Brazil's financial market

**Adriana Bruscato Bortoluzzo**
*Insper Instituto de Ensino e Pesquisa – São Paulo/SP, Brasil*

**Andrea Maria Accioly Fonseca Minardi**
*Insper Instituto de Ensino e Pesquisa – São Paulo/SP, Brasil*

**Bruno Caio Fernando Passos**
*Insper Instituto de Ensino e Pesquisa – São Paulo/SP, Brasil*

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**RESUMO**

## Análise do risco sistemático multi-escalar no mercado financeiro do Brasil

Neste trabalho, é analisado se a relação entre risco e retorno prevista pelo *Capital Asset Pricing Model* (CAPM) é válida no mercado brasileiro de ações, com base na decomposição discreta de ondaletas em diferentes escalas de tempo. Essa técnica permite analisar a relação em diferentes horizontes de tempo, desde o curto prazo (2 a 4 dias) até o longo prazo (64 a 128 dias). Os resultados apontam que entre os anos de 2004 e 2007 há uma relação negativa ou nula entre risco sistemático e retorno para o Brasil. Como o retorno excedente médio da carteira de mercado em relação ao ativo livre de risco no período foi positivo, seria esperado que essa relação fosse positiva, ou seja, que um maior risco sistemático resultasse em um maior retorno excedente, o que não ocorreu. Portanto, não se observou nesse período uma remuneração adequada pelo risco sistemático no mercado brasileiro. As escalas que apresentaram a relação risco e retorno mais significativas foram as três primeiras, correspondendo a horizontes de mais curto prazo. Em outras palavras, ao se tratar diferentemente ano a ano e, em consequência, separar prêmios positivos e negativos, encontra-se em alguns anos alguma relevância na relação risco retorno prevista pelo CAPM, mas que não persiste ao longo de todos os anos. Portanto, não há evidência suficientemente forte de que o apreçamento dos ativos segue o modelo.

***Palavras-chave:*** apreçamento de ações, relação risco e retorno, CAPM, ondaletas, mercado acionário brasileiro.

*Adriana Bruscato Bortoluzzo*, Mestre e Doutora em Estatística pela Universidade de São Paulo, é Professora Assistente do Insper Instituto de Ensino e Pesquisa (CEP 04546-042 – São Paulo/SP, Brasil).
E-mail: adrianab@insper.edu.br
Endereço:
Insper Instituto de Ensino e Pesquisa
Rua Quatá, 300
Vila Olímpia
04546-042 – São Paulo – SP

*Andrea Maria Accioly Fonseca Minardi*, Mestre e Doutora em Administração de Empresas pela Fundação Getulio Vargas de São Paulo, é Professora do Insper Instituto de Ensino e Pesquisa (CEP 04546-042 – São Paulo/SP, Brasil).
E-mail: minardi@insper.edu.br

*Bruno Caio Fernandes Passos* é Graduado em Administração pelo Insper Instituto de Ensino e Pesquisa (CEP 04546-042 – São Paulo/SP, Brasil).
E-mail: bcfpassos@gmail.com

## 1. INTRODUCTION

One of the most often used models in modern finance is the Capital Asset Pricing Model (CAPM) developed by Sharpe (1964) and Lintner (1965). The model predicts the relation between the risk and the expected return on an asset according to **the expectations equilibrium** regarding the returns on risky assets. The CAPM predicts that the investor only prices the systemic risk, which is measured by the share's beta, and the investor demands a risk premium equal to the beta multiplied by the market portfolio risk premium. The share's beta corresponds to the regression coefficient for excess asset returns and excess market portfolio returns.

The risk-return relation predicted by the CAPM is widely used to estimate the rate of return demanded to adequately reward the risk that the shareholder assumes. It serves as a benchmark for the minimum required rate of return for implementing a project and is used to determine the fair value of assets. This relation is based on a theoretical market portfolio that is unobserved. In general, market indexes are used as proxies for the theoretical portfolio to estimate the share's betas.

Ross (1976) proposes the Asset Pricing Model (APT) as an alternative approach for asset pricing. This theory uses various factors to explain asset returns.

Fama and French (1992) analyze the cross-section relationship between betas and average returns, allocating the shares in portfolios according to the size of the company and the book to market ratio (the ratio of the book value of equity to its market value). The expected relationship between risk and returns is not observed; that is, it is true that the higher the beta value, the higher the portfolio return average. The authors conclude that there are other systemic risks that are not portrayed in the market portfolio and that size and book-to-market ratio are proxies for these risks.

According to Roll and Ross (1995) the exact relation between the average return and beta must be satisfied if the market index that serves as a proxy for the theoretical market portfolio is in the efficient part of the efficient frontier. Whenever we test the CAPM, we are actually testing the efficiency of the market index rather than the validity of the CAPM.

Studies such as Lakonishok and Shapiro (1984), Mankiw and Shapiro (1986), Breeden, Gibbons and Litzenberger (1989), and Cochrane (1996) did not find a significant relationship between systemic risk and return, as would be expected based on the CAPM. Therefore it is questionable if the CAPM is valid in Brazil.

Assuming that the CAPM is valid in Brazil, an issue often pointed in the literature is what procedures result in better betas (see for instance Blume [1971], Cohen, Hawawini, Maier, Schwartz & Whitcomb (1983), Cecco [1988], Gregory-Allen, Impson & Karafiath [1994]). Should we use higher length of data to produce more stable betas? What frequency (i.e. daily, weekly, monthly, etc) of time series is best indicated for calculating betas? Does the aggregation in portfolios improve beta's stability?

This work analyzes whether the **predicted** relation between risk and returns based on the CAPM when IBOVESPA is used, is valid in the Brazilian stock market. If market portfolio excess return is positive, we expect that the higher the beta, the higher the asset excess return. If it is negative, we expect that the higher the beta, the lower the asset excess return. This analysis is based on discrete wavelet decomposition at different time scales, which permits the use of different time horizons ranging from short-term (2 to 4 days) to long-term (64 to 128 days). Checking different scales horizon allows us to verify what frequency allows us to estimate better betas.

The study of the scalar relationship between systemic risk and returns in the Brazilian market offers the investor a more efficient way of pricing assets and measuring the cost of capital in a company, facilitating the analysis of investments according to national standards.

It has been observed that the short-term time scales generate betas that best explain the relationship between risk and returns. Therefore, using more frequent data (as for instance daily data instead of monthly data) is better to estimate betas in Brazil. However, during the period from 2004 to 2007, although the market premium was positive, a negative relationship between risk and returns was observed. That is, there was no evidence that Brazilian stocks were priced according to the relationship between risk and return predicted by the CAPM when using IBOVESPA.

This article is organized as follows. Section 2 contains a literature review; Section 3 presents the theoretical foundations of the CAPM, discusses its validity and contains a brief review of wavelets, followed by the methodology for estimating systemic asset risk using multi-scale decomposition; Section 4 presents the database; Section 5 contains the tables and results of the analysis and a comparison of the results with those of the international literature; and Section 6 presents the study conclusions and a brief summary of what was accomplished.

## 2. LITERATURE REVIEW

The use of wavelets in finance is recent in the Brazilian literature. Lima, Kimura, Assaf Neto and Perera (2010) use wavelets to decompose a time series, in conjunction with econometric and neural network models, to forecast the data for a 60 kg sack of soy. The results obtained were satisfactory when using a wavelet filter in a recurrent neural network. Morettin, Toloi, Chiann and Miranda (2010) introduce a copula estimator based on wavelet smoothing of empirical copulas for time series data and they study the correlation between daily returns of Ibovespa (Brazil) and IPC (Mexico), SP500 and DJIA, and CAC40 and DAX35. Pimentel and Silva (2011) analyze the correlation among financial indexes for the Brazilian, American and European markets, differentiating the long and short-run investment contribution to the energy of a time-series with wavelet decomposition. The use of wavelets to estimate the systematic risk in the Brazilian market, decomposing the beta on several time scales that represent the short to long term **is not yet published** in Brazil.

Ramsey and Zhang (1995) and Ramsey and Lampart (1998) used discrete wavelet decomposition to test models in which betas and/or risk premiums varied **as** time **went by**. Wavelets are functions with specific properties used to decompose a time series over time and in terms of frequency, allowing researchers to work with different time horizons. This, in turn, allows the study of correlations between markets using these time horizons. Thus, an analysis of the beta of an asset becomes more robust because the multi-scale decomposition of a series of returns on these assets allows one to observe the risk-return ratio according to the time scale and **as** from different points of view.

In this way, one can analyze which scale obtains betas that better explain the relationship between risk and returns and whether a more significant return is explained by a **higher** risk.

Gençay, Whitcher and Selçuk (2003) use wavelets to decompose a given time series on a multi-scale base to estimate the betas of assets. They use the methodology proposed for the stock markets of the United States, Germany, and the United Kingdom with the goal of finding the best time scale for measuring systemic risk.

Gençay *et al.* (2003) analyze the American economy using daily data from all of the stocks listed in the S&P 500 index from January 1973 until November 2000 and forming a database with 7,263 observations (28 years). The S&P 500 index is a proxy for the market portfolio, and the 10-year Treasury Bill is a proxy for the risk-free asset. The results indicate a positive relationship between beta and returns for all time scales, although this relationship is not linear (it forms a smile shape). The authors observe that the slope and the coefficient of determination ($R^2$) of the regression models grow as the scale increases (from high to low frequency). They conclude that the relationship between risk and return is multi-scale phenomenon and that the predictions associated with the CAPM are more relevant for an investor with medium to long-term horizons.

In the study of the stock market in Germany, a database was constructed from all stocks contained in the Xetra DAX (DAX30) index from January 2000 to December 2001 (except for three, for which, absent values were shown). This process yielded 499 observations. Gençay *et al.* (2003) consider the DAX30 as market returns and the daily Euro Interbank Offered Rate (EURIBOR) as the risk-free rate. They observe that the third scale (which considers a period from 8-16 days, a medium-term interval) best approximates the estimated average market premium, when compared to the actual one. The estimated market premium in Germany based on the best scale in the period is -18.5% (3rd level), and the actual premium in the period was -16% per year. The average excess market return is negative, and the assets with the **highest** beta have the lowest returns, resulting in a negative slope for asset returns and beta. These results are in accordance with those predicted by the risk-return ratio; during moments when the market is declining, the **higher** the systemic risk, the lower the return.

The database for the United Kingdom includes a random sample of thirty stocks listed in the Financial Time Stock Index (FTSE100) from January 2000 to December 2001 (491 observations). The proxy for the market portfolio is the FTSE100, and the proxy for the risk-free rate is the UK Treasury Bill middle rate with a one-month **length of time**. The relationship between risk and return is captured with **great** precision at the highest decomposition levels: that is, in the medium and long term. However, the observed relationship between risk and return is negative, and the actual market premium was -15.6% per year. Thus, the results are in accordance with those predicted by the CAPM risk-return ratio.

Fernandez (2006) analyzes the stock market in Chile using a sample of twenty-four stocks with liquidity of at least 85% that were traded in the Santiago stock exchange from January 1997 to September 2002. As a proxy for the market portfolio, Fernandez uses the Price Index of Selected Stocks (IPSA), whereas the proxy for risk-free assets is the **return rates** paid on bank deposits **within** 30 days. The average actual market premium during the period is -9.06% per year. Although the coefficients in the model are not significant at the 5% significance level, indicating that there is no relationship between risk and returns according to the CAPM, the author conclude that the CAPM model makes better predictions in scale 2 (4 to 8 days) because the estimated market risk premium for this scale (-11.5% per year) is the closest to the actual risk premium.

Rhaeim, Ammou and Mabrouk (2007) study twenty-six highly liquid stocks in the French stock market from January 2002 to December 2005 (1,044 observations, or 4 years). The CAC40 index is used as the market portfolio and the daily EURIBOR as the risk-free rate. The authors conclude that the predictions of the CAPM are more relevant in the short term than in the long term, which makes the French market different from those of the United States, Germany, and the United Kingdom. The relationship between risk and returns is negative for all the scales and linear only for scales 1, 2, and 6.

Aktan, Mabrouk, Ozturk and Rhaiem (2009) use a database composed of 98 stocks chosen randomly from those listed on the Istanbul Stock Exchange (ISE) during the period from January 2003 to October 2007. Their proxy for the market portfolio is the ISE-National100 index, and their proxy for risk-free asset returns is the returns paid daily on bank deposits. They find a positive relationship between risk and returns that is most significant at the 3rd level (8 to 16 days), concluding that the effect of market returns on an asset is stronger in this time horizon. In addition, the inclination rises according to the increase in the scale (from short to long-term), indicating that the CAPM is more relevant in long-term time horizons than other scales and that the relationship predicted by the model can be verified in the Turkish market.

Samaei (2012) analyzes the **multi-scale** systematic risk in Iran, using 15 selected stocks, listed on Tehran Stock Exchange (TSE) Actively traded over June 2004 and June 2009 (1,211 observations). He uses the annual interest rate of the investment bonds issued by the central bank as proxy for the risk-free asset returns and the total price index of the Tehran Stock Exchange (TEPIX) as proxy for the return of market portfolio. The relationship between the return of a stock and its beta is more robust at medium and short scales (2 to 32 days), indicating that the market is more efficient at first to fourth scales.

Consistent with works by Fernandez (2006), Rhaeim *et al.* (2007), Aktan *et al.* (2009), and Samaei (2012) this article uses the methodology proposed by Gençay *et al.* (2003), which uses wavelets as a tool for estimating the systemic risk of an asset **considering** the market portfolio as the systemic risk factor. Toward this end, we analyze the risk ratio for returns on assets Toward this end, we analyze the **risk ratio for** returns on assets listed in the São Paulo Stock Exchange (Bovespa) during the period from 2004 to 2007 at different levels. Our aim is to determine the scale that best reflects the beta of an asset in the Brazilian stock market.

## 3. METHODOLOGY

### 3.1. Capital Assets Pricing Model (CAPM)

The Capital Assets Pricing Model (CAPM, proposed by Sharpe [1964] and Lintner [1965]) emerges from the **problem maximization** for an agent in an environment of uncertainty. According to Blanchard and Fischer (1989), an agent with a horizon of T periods maximizes the function

$$\max E\left[\sum_{t=0}^{T-1}(1+\theta)^{-t}U(c_t)|0\right] \qquad \textbf{[1]}$$

where E represents the conditional expectation of the utility function for consumption ($U(c_t)$) during time t = 0 and where $\theta$ is the discount rate over time.

We assume that at time $t$, an agent chooses to allocate his wealth between a given risk asset $n$ with a stochastic rate of return (liquid) of $r_{it}$ ($i$=1,2,…,$n$) and a risk-free asset with a rate of return $r_{0t}$. The result of the maximization implies $n$+1 first-order conditions:

$$U'(c_t) = (1+\theta)^{-t}\,E[U'(c_{t+1})(1+r_{it})|t\,],\ i=1,2,\ldots,n, \qquad \textbf{[2]}$$

$$U'(c_t) = (1+\theta)^{-t}(1+r_{0t})\,E[U'(c_{t+1})|t\,] \qquad \textbf{[3]}$$

The agent should choose to consume in such a manner that his marginal utility is equal to the discounted marginal utility for the next period. This condition should be maintained independently of the asset, whether it is free from risk or not. For assets with risk, the marginal utility during $t$ depends on the expected value of the product of the marginal utility during $t$ +1 and its rate of return [2]. When the rate of return of the risk-free asset is known during time $t$, the rate can be determined based on the conditional expectation, resulting in equation [3].

Equations [2] and [3] result in a set of restrictions on returns on assets and the process of consumption. Thus, these equations provide the equilibrium condition for their returns given the process of consumption. Substituting [2] with [3], we obtain

$$E[U'(c_{t+1})(r_{it}-r_{0t})|t\,] = 0,\ i=1,2,\ldots,n. \qquad \textbf{[4]}$$

We can also simplify this notation, substituting $E[\cdot|t]$ for $E[\cdot]$

$$cov[U'(c_{t+1}),r_{it}] = E[U'(c_{t+1})(r_{it}-r_{0t})] - E[U'(c_{t+1})]E[U'(r_{it}-r_{0t})] \qquad \textbf{[5]}$$

$$E[U'(c_{t+1})]E[U'(r_{it}-r_{0t})] + cov[U'(c_{t+1}),r_{it}] = 0,\ i=1,2,\ldots,n. \qquad \textbf{[6]}$$

Thus, the expected return on asset $i$ satisfies the equation

$$E[r_{it}] = r_{0t} - \frac{cov[U\prime(c_{t+1}),r_{it}]}{E[U\prime(c_{t+1})]} = 0,\ i=1,2,\ldots,n. \qquad \textbf{[7]}$$

The **higher** the covariance of the return on an asset with marginal utility of consumption, the **higher** the expected return of the equilibrium asset will be. Given that, in equilibrium, the asset provides a *hedge* for consumption, the agents will be willing to obtain a lower return when the marginal utility of consumption decreases. On the other hand, agents will demand a higher rate of return when the marginal utility of consumption increases.

Taking an asset $m$ that is negatively correlated with $U'(c_{t+1})$, for any risky asset (for example, $U'(c_{t+1}) = -\gamma r_m$ for any positive $\gamma$),

$$cov[U'(c_{t+1}),r_{it}] = -\gamma cov(r_{mt},r_{it}) \qquad \textbf{[8]}$$

However, for the asset $m$, equation [7] implies that

$$E[r_{mt}] = r_{0t} - \frac{cov[U\prime(c_{t+1}),r_{mt}]}{E[U\prime(c_{t+1})]} = r_{0t} + \frac{\gamma var(r_{mt})}{E[U\prime(c_{t+1})]}. \qquad \textbf{[9]}$$

Substituting [8] and [9] with [7], we obtain

$$E[r_{it}] - r_{0t} = \left[\frac{cov(r_{it},r_{mt})}{var(r_{mt})}\right](E[r_{mt}]-r_{0t}). \qquad \textbf{[10]}$$

By definition, $\beta_i$ is known as the coefficient of regression (via ordinary least squares) for excess asset returns and excess market portfolio returns or those of the index. This coefficient may be obtained using the following equation:

$$\beta_i = cov(r_{it},r_{mt})/var(r_{mt}) \qquad \textbf{[11]}$$

It may also be derived using an ordinary least squares (OLS) estimator for the following equation:

$$r_{it} - r_{0t} = \alpha_i + \beta_i(r_{mt} - r_{0t}) + \varepsilon_{it}\ , \qquad [12]$$

where $\varepsilon_{it}$ is a random shock (white noise).

Thus, we obtain the relationship between systemic risk and the premium via the amount of market risk given by

$$E[r_{it}] - r_{0t} = \beta_i(E[r_{mt}] - r_{0t}). \qquad [13]$$

The Brazilian literature in general does not validate the CAPM relation in Brazil, and many papers suggest a multifactor approach. Oliveira and Carrete (2005) and Flister, Bressan and Amaral (2011) find that book to market is a relevant factor in explaining return. Minardi (2004) and Mussa, Trovão, Santos and Famá (2007) find evidence of momentum in Brazilian stocks. Lucena and Pinto (2008) observe that size and book-to-market are significant factors in Brazil. Machado and Medeiros (2011) verify that the inclusion of factors **such a**s size, book-to-market, moment and liquidity to the market portfolio improve the predictive power of risk and expected return in Brazil.

### 3.2. Wavelets

Wavelets are non-sinusoidal functions with limited duration and a mean equal to zero. They are used to simultaneously decompose time series in terms of time and scale. Unlike Fourier analysis, multi-scale decomposition divides the time-frequency plane into a set of components of high and low frequency. Thus, while Fourier analysis characterizes the global behavior of a time series, wavelets characterize the local behavior of the series.

First, one should consider the formation of an $L^2$ $(\Re)$ space for all integrable functions of the squared model; that is, $\int_{-\infty}^{\infty}|f(x)|^2\,dx < \infty$ based on the dilations and translations of the order $(j,k)$, respectively, of a function $\psi(\cdot)$. The wavelets $\psi_{j,k}(t)$ are formed from the function $\psi(t)$, also called the mother wavelet, using transformations and dilations and forming a orthogonal base for . These are given by

$$\psi_{j,k} = 2^{j/2}\psi(2^j t - k),\ j,k\ \in\ \mathbb{Z}. \qquad [14]$$

Each base function depends on two parameters, one of scale (j) and another found in (k). To obtain a representation, the binary dilations $2^j$ and dyadic translations $k2^{-j}$ of $\psi(t)$ should be considered.

The basic properties that characterize a wavelet are

$$\int_{-\infty}^{\infty}\psi(t)\,dt = 0\ and\ \int_{-\infty}^{\infty}\psi^2(t)\,dt = 1. \qquad [15]$$

Thus, the function should decrease rapidly towards zero when $|t|\rightarrow\infty$. If condition [15] is valid, for any $\Re$, $0<\epsilon<1$, there is an interval of a finite length [–T,T] such that

$$\int_{-\infty}^{\infty}\psi^2(u)\,du < 1-\epsilon. \qquad [16]$$

Therefore, the function should be practically null outside the interval [–T,T] for $\Re$ close to zero. The function $\psi(\cdot)$ also has the following properties:

$$\int_{-\infty}^{\infty}\frac{|\hat{\psi}(w)|^2}{|w|}dw < \infty, \qquad [17]$$

where $\hat{\psi}(w)$ is a Fourier transformation of $\psi(t)$.

The first M-1 moments of p(.) are null; that is,

$$\int_{-\infty}^{\infty}\mathrm{t}^{\mathrm{j}}\psi(t)\,dt = 0, j = 0,1,\dots,M-1, \text{ for some } M \geq 1 \text{ e } \int_{-\infty}^{\infty}|\mathrm{t}^{\mathrm{m}}\psi(t)|\,dt < \infty. \qquad [18]$$

Property [17] is known as the admissibility condition and guarantees that the function of interest $f(\cdot)$ can be reconstructed from the **wavelets transformation**. The value of M is related to the degree of smoothness of the wavelet where the greater the value of M, the more regular $\psi(\cdot)$. Some wavelets have compact support, a desirable property related to the fact that the wavelets are located in time. However, not all wavelets generate orthogonal systems. The advantage of working with orthogonal bases is that it allows **a** perfect reconstruction of the original signal based on the coefficients of the transformed wavelets.

Based on the first specification presented regarding the formation of a $L^2$ $(\Re)$ space, functions $\psi_{j,k}(\cdot)$ form an orthogonal base generated by $\psi(\cdot)$; then,

$$f(t) = \sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} c_{j,k}\,\psi_{j,k}(t), \qquad [19]$$

in which

$$c_{j,k} = \int_{-\infty}^{\infty} f(t)\psi_{j,k}(t)\,dt \qquad [20]$$

are the coefficients of the wavelets.

The scalar function $\phi(\cdot)$, known as the father wavelet, is the solution to the equation

$$\phi(t) = \sqrt{2}\sum_k l_k\phi(2t-k). \qquad [21]$$

The high-frequency detailed components are captured by the mother wavelets, whereas the low-frequency smooth components are captured by the father wavelet.

An orthonormal family in $L^2$ $(\Re)$ is formed via the dilations and translations of $\Re(\cdot)$; that is,

$$\phi_{j,k} = 2^{j/2}\phi(2^j t - k),\ j,k\ \in\ \Re. \qquad [22]$$

The wavelets $\psi(\cdot)$ may be obtained from the father wavelet as follows:

$$\psi(t) = \sqrt{2}\sum_k h_k \phi(2t-k), \qquad [23]$$

in which $h_k$=$(-1)^k\, l_{k-1}$ and $l_k$ and $l_k$ are coefficients of the high and low pass filters, respectively, given by

$$l_k = \sqrt{2}\int_{-\infty}^{\infty} \phi(t)\phi(2t-k)\, dt \qquad [24]$$

$$h_k = \sqrt{2}\int_{-\infty}^{\infty} \psi(t)\phi(2t-k)\, dt. \qquad [25]$$

As in this study, a number of applications that facilitate the analysis of wavelets use discrete wavelet transformation (DWT) to calculate the coefficients near a discrete signal. For more information on wavelets, see Chui (1992), Ogden (1997), Morettin (1999) and Percival and Walden (2000).

### 3.3. Multi-scale variance and covariance

Gençay *et al*. (2003) introduce the method of systemic risk estimation that uses wavelets to decompose a time series, transforming them to produce multi-scale coefficients. As indicated in Section 3.1, the estimation of the asset beta is obtained via equation [11] for each of the (j) levels of the wavelet.

Assuming that the market return structure $r_m$ is stationary, it is possible to define multi-scale variance independent of time, simply called "wavelet variance", for the market return *m* associated with level *j*, such that

$$\sigma_{mj}^2 = Var(c_{mj}), \qquad [26]$$

in which $c_{mj}$ are wavelet coefficients given by [20]. The wavelet variance at level *j* is the variance of the wavelet coefficients of level *j*.

Taking $r_{mt}$ and $r_{it}$ as market returns and the returns on a given asset *i*, respectively, and applying wavelet transformation, the vectors of the wavelet coefficients $c_{mj}$ and $c_{ij}$ are obtained via decomposition. The wavelet covariance of $r_{mt}$ and $r_{it}$ at level *j* is given by $Cov(c_{mj}, c_{ij})$. Note that the covariance and variance may be significantly different at certain levels, resulting in different betas for each scale.

Thus, the estimator for the systemic risk of an asset at scale is given by

$$\hat{\beta}_{i,j} = \frac{Cov(c_{mj}, c_{ij})}{Var(c_{mj})}. \qquad [27]$$

## 4. DATA AND METHODOLOGY

We collect in Bloomberg daily data of Brazilian stocks traded in BOVESPA from January 5th 2004 till January 17th 2007.

Because of the need for a number of observations, to the power of two, a condition imposed by the use of discrete wave decomposition, the missing data were completed with quotes from the following period, including some from the consecutive year. We included in the sample only assets with more than 60% liquidity per year, and the missing data for these assets were estimated using the Kalman filter to avoid the large-scale loss of information due to missing values for the selected assets. We need a balanced panel, and eliminated stocks that did not have shares traded in all years. Our final sample has 256 observations for each year.

We used the accumulated Selic and the daily Bovespa **indexes** (IBOV) as proxies for the risk-free rate and the market portfolio respectively. The sample contains 1000 trading days: that is, approximately 4 years.

The asset returns were calculated such that $r_{i,t}=Ln(P_{i,t}/P_{i,t-1})$ where $r_{i,t}$ and $P_{i,t}$ represent the **quote return** of the asset closing price *i* on date *t* and the closing price, respectively.

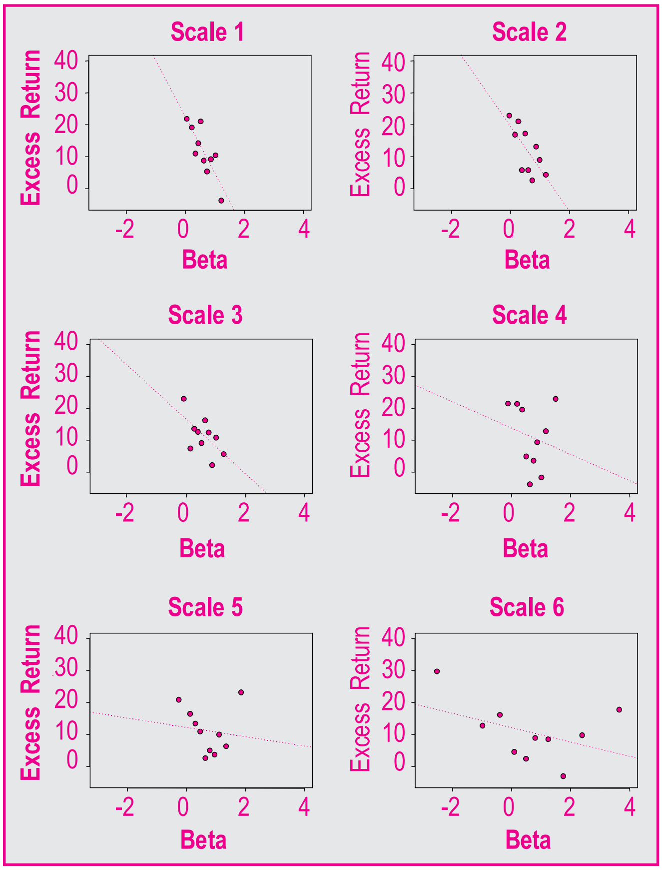
The betas of the assets were estimated according to formula [11] of Section 3.1, where $\hat{\beta}_{n,j}$ represents the estimator of systemic risk for asset *n* on scale *j*, $Cov(c_{mj}, c_{cnj})$ is the covariance between the wavelet coefficients of excess market returns ($c_{mj}$) and asset $n(c_{nj})$ and the $j^{th}$ level of decomposition, and $Var(c_{mj})$ is the variance in the wavelet coefficients of the risk premium index of the market at scale *j*.

The return series were decomposed in 6 levels via the discrete wavelet transformation method using wavelet D(8) such that the first scale is associated with an interval of 2-4 days, the second an interval of 4-8 days, the third an interval of 8-16 days, the fourth an interval of 16-32 days, the fifth an interval of 32-64 days, and the sixth an interval of 64-128 days.

Based on the method proposed by Reinganum (1981), 10 portfolios were formed for each year, all with the same number of equally weighted assets. The assets were ordered according to the estimated betas and placed in the 10 portfolios from the largest to the smallest beta. Thus, the first portfolio is formed from the assets with the lowest betas and the tenth portfolio from the assets with the highest betas. We then calculated the average representative beta of each portfolio. After calculating the portfolios for the years from 2004 **up** to 2007, we created a portfolio with the **returns and the betas** averages during the period under analysis.

## 5. RESULTS

Figure 1 shows the graphs of annualized excess return and the average betas from 2004 to 2007. The lowest scales represent short-term horizons, whereas the highest ones represent long-term horizons. For the Brazilian market, the relationship between risk and returns exhibits a negative correlation. In addition, the points become more dispersed as the scale increases.




**Source:** Prepared by the authors using a database collected from BLOOMBERG, 2004-2007.

**Note:** The scales 1 to 6 correspond to the following periods: (1) 2-4 days, (2) 4-8 days, (3) 8-16 days, (4) 16-32 days, (5) 32-64 days, and (6) 64-128 days.

***Figure 1: Wavelets Discrete Decomposition of Returns and Betas Average Between 2004 and 2007***

Table 1 presents the results estimated using ordinary least squares (OLS) regression for the period analyzed as per equation [28]. For demonstration purposes, the constants and slopes were multiplied by 100.

$$r_c - r_0 = \delta_j + \lambda_j \beta_{cj} + \nu_{cj},\; c = 1\ldots 10,\; j = 1\ldots 6, \qquad [28]$$

where $r_c$-$r_0$ corresponds to excess portfolio returns c; $\delta_j$ is the constant for the regression at level j of the wavelet, which, according to CAPM should be zero; $\lambda_j$ is the slope of the regression at level j of the wavelet, which corresponds to the risk premium of the market portfolio estimated at scale j; $\beta_{cj}$ is the beta of portfolio c estimated by OLS at level j of the wavelet; and $\nu_{cj}$ is the random error.

The better the relationship between risk and returns, the more coefficient λ should approach the historic premium of the market portfolio. This coefficient corresponds to the slope of the graphs presented in Figure 1. If the historic market premium is positive, then a positive relationship should be expected between the excess portfolio returns and the betas. If the historic market premium is negative, a negative relationship should be expected. It is also interesting to observe the constant $\delta_j$. According to CAPM, this constant should be zero.

***Table 1***

***Regression of Excess Returns and Systemic Risk from 2004 to 2007 in Different Decomposition Scales Using the Discrete Wavelet Transformation Method***

| Level (j) | Constant | Slope | $R^2$ |
|---|---|---|---|
| Scale 1 | 0.0807** | -0.0629** | 0.6772 |
| Scale 2 | 0.0693** | -0.0459* | 0.4903 |
| Scale 3 | 0.0592** | -0.0294 | 0.3648 |
| Scale 4 | 0.0491 | -0.0144 | 0.0375 |
| Scale 5 | 0.0444* | -0.0054 | 0.0180 |
| Scale 6 | 0.0432** | -0.0074 | 0.1715 |

**Source:** Prepared by the authors from a database collected from BLOOMBERG, 2004-2007.

**Notes:** **1% significance; * 5% significance. The constant and slope coefficients were multiplied by 100 for demonstration purposes.

According to Pettengill, Sundaram, and Mathur (1995), although the investor always demands an ex ante positive premium to invest, based on the ex post returns on assets, these premiums or excess returns may be negative. A positive $\lambda_j$ (the slope of the regression described in equation [28]) should be expected whenever the excess returns or premium observed ex post is positive, and it should be negative if the observed premium is negative. Therefore, the authors advise that one study the difference between the impact of observed positive premiums and that of negative premiums. Based on these authors' suggestion, the average year-by-year risk-return ratio was analyzed in different scales.

Detailed graphs and tables for the portfolios for all years and the average for the period can be found in the Appendix, along with the results of the regressions for each level of wavelet decomposition.

Based on the analysis of the results in Table 1, there is a larger coefficient of determination ($R^2$) from the first to the third scale, and the estimated risk premiums at lower levels (with higher frequency) have greater statistical significance.

However, one can observe that the constants of the regressions for various scales were significant and that the slopes were significant in the three first scales but negative overall, even though the average historic premium in IBOVESPA is 0.289% per day. That is, the assets and portfolios that possess the greatest systemic risk were not those with the largest positive premiums in the actual market, and the expected relationship between risk and returns was not observed.

Upon analyzing the year-by-year decompositions, listed in Figures A.1 and A.2 in the Appendix, we observe the expected risk and return relationship in 2004 and 2005 only. The expected risk and return relationship is not observed in 2006 and 2007(*).

In 2004, we observe a negative slope in all scales that was significant at 10% for scale 1, at 1% for scales 2, 3, and 6, and at 5% for scale 5. This is appropriate given that the historic market premium per year was -0.026 per day. The largest $R^2$ values were obtained for scales 2, 3, and 6. However, in all the scales, the intercept of the regression was positive. It can therefore be concluded that in that year, the market priced systemic risk but did not do so at the magnitude predicted by the CAPM.

In 2005, the slope was negative in the first scale but positive in the other five scales. It was significant at 5% only for scale 5, which was the scale with the greatest $R^2$ value. The sign of the slope was as expected; the average historic market portfolio premium is 0.06%. However, the results were significant only for scale 5. The constants of the regressions were significant at 10% for scales 2, 3, and 4 and at 5% for scale 5. Therefore, it can be concluded that in that year, the estimation horizon that produced the best results in terms of risk pricing was 32-64 days, although the magnitude of the premium explained by systemic risk was not as predicted by the CAPM.

Although the risk-return ratio has been observed for some years, it has not persisted along the entire time horizon, and therefore, we must reject the hypothesis that the Brazilian market has exhibited the pricing predicted by the CAPM. Other risk factors may exist beyond the market portfolio that should be taken into consideration in pricing Brazilian assets. The literature finds evidence that size, book to market, momentum and liquidity are significant factors in the Brazilian market.

Fernandez (2006) studies the Chilean asset market and indicates that as in the financial market in Brazil, in which the predictions were most significant for the first three scales (2-4, 4-8, and 8-16 days), the second scale (2-4 days) exhibits the greatest coefficients of determination (even though the relationship predicted by the CAPM is not substantiated in his work). Thus, Fernandez concludes that the predictions of the model are more relevant in the short term. However, studies performed in other countries differ in terms of the scale that best captures the relationship between risk and returns. Gençay *et al.* (2003) find that in the American market, the model predictions are more relevant for investors with medium and long-term time horizons, as in the United Kingdom. In Germany, on the other hand, the third scale is the most relevant (8-16 days), revealing that the medium-term horizon is the most appropriate for this market. This may be because more mature financial markets possibly present a different structure for risk and returns than do less mature markets. This idea requires more in-depth study.

(*) Tables with the results of the portfolios formed from discrete wavelet decomposition of each level and the results of the regression of risk and return are available with the authors.

Rhaeim *et al.* (2007), in a study of the financial market of France, observe that short and long-term horizons are the most relevant, contrary to observations in mature countries such as the United States, Germany, and the United Kingdom. Aktan *et al.* (2009), in analyzing the assets listed in the Istanbul stock exchange, find a positive relationship between risk and returns that is most significant at the 3rd level (8-16 days). Therefore, it is important to consider that the market analysis may differ when different time intervals are studied. We should consider structural breaks and exogenous factors not mentioned here when comparing the financial markets of different economies.

## 6. CONCLUSION

This work presents the results of the multi-scale decomposition of the assets listed in Bovespa between 2004 and 2007, using the method proposed by Gençay *et al.* (2003), with the goal of studying the relationship between systemic risk and returns for different time scales.

Our findings indicate that short term frequency produces better estimates of betas, but we do not validate the CAPM risk return relation for the Brazilian stock market. Although the average risk premium observed during the period analyzed was positive (7.68% per year), the estimations of the coefficients revealed a negative market premium for all levels of decomposition. Thus, the expected relationship between risk and returns according to the CAPM was not observed in the whole period 2004-2007.

When we investigate year by year, we observe that in 2004, the actual market premium was negative (-6.82% per year), whereas it was positive in 2005 (16.58% per year). The observed relationship was negative for all scales for 2004 and positive for practically all the scales except the first in 2005, but we did not observe the same consistency in 2006 and 2007. We can conclude that these predictions do not apply to the period as a whole.

Thus, keeping in mind that the estimates generated by the multi-scale decomposition model permit more robust analysis of the model for different time scales, we cannot support the hypothesis that Brazilian assets are priced according to CAPM.

The Brazilian literature identifies that other factors **such** as size, book-to-market, momentum and liquidity are significant in explaining Brazilian stock returns. We expect that the inclusion of these factors in the model should improve the risk return relation. An investigation of a multifactor model based on the wavelet approach should be the objective of future research. It should also be highlighted that **in order** to use the wavelet approach, it was necessary for us to fill in the days without trading in the price series with values estimated using the Kalman filter. This step may have generated a bias in the data, and should be considered a limitation of this study. ◆

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ABSTRACT

## Analysis of multi-scale systemic risk in Brazil's financial market

This work analyzes whether the relationship between risk and returns predicted by the Capital Asset Pricing Model (CAPM) is valid in the Brazilian stock market. The analysis is based on discrete wavelet decomposition on different time scales. This technique allows to analyze the relationship between different time horizons, since the short-term ones (2 to 4 days) up to the long-term ones (64 to 128 days). The results indicate that there is a negative or null relationship between systemic risk and returns for Brazil from 2004 to 2007. As the average excess return of a market portfolio in relation to a risk-free asset during that period was positive, it would be expected this relationship to be positive. That is, higher systematic risk should result in higher excess returns, which did not occur. Therefore, during that period, appropriate compensation for systemic risk was not observed in the Brazilian market. The scales that proved to be most significant to the risk-return relation were the first three, which corresponded to short-term time horizons. When treating differently, year-by-year, and consequently separating positive and negative premiums, some relevance is found, during some years, in the risk/return relation predicted by the CAPM. However, this pattern did not persist throughout the years. Therefore, there is not any evidence strong enough confirming that the asset pricing follows the model.

**Keywords:** stock pricing, risk-return ratio, CAPM, wavelets, Brazilian stock market.

RESUMEN

## Análisis del riesgo sistemático multiescala en el mercado financiero de Brasil

En este trabajo se analiza si la relación entre riesgo y rendimiento prevista por el Capital Asset Pricing Model (CAPM) tiene validez en el mercado de acciones brasileño, con base en la descomposición discreta de wavelet en diferentes escalas de tiempo. Esta técnica permite analizar la relación en diferentes horizontes temporales, desde el corto plazo (2 a 4 días) hasta el largo plazo (64 a 128 días). Los resultados muestran que entre los años 2004 y 2007 existe una relación negativa o nula entre riesgo sistemático y rendimiento en Brasil. Como el rendimiento en exceso medio de la cartera de mercado sobre el activo libre de riesgo en el período fue positivo, se esperaría que esta relación fuese positiva, es decir, un mayor riesgo sistemático se traduciría en un mayor rendimiento en exceso, lo que no ocurrió. Por consiguiente, no se observó en este período una remuneración adecuada por el riesgo sistemático en el mercado brasileño. Las escalas que presentaron una relación más significativa entre riesgo y rendimiento fueron las tres primeras, y corresponden a horizontes de más corto plazo. Se puede decir que – al tratar de manera diferente año a año y, por consiguiente, separar premios positivos y negativos – se encuentra en algunos años alguna relevancia en la relación riesgo rendimiento prevista por el CAPM, pero no persiste a lo largo de todos los años. Se concluye que no hay evidencia bastante fuerte de que la valoración de los activos siga el modelo.

**Palabras clave**: valoración de acciones, relación riesgo y rendimiento, CAPM – modelo de valoración de activos de capital, *wavelet*, mercado de acciones brasileño.

## APPENDIX A – GRAPHICS

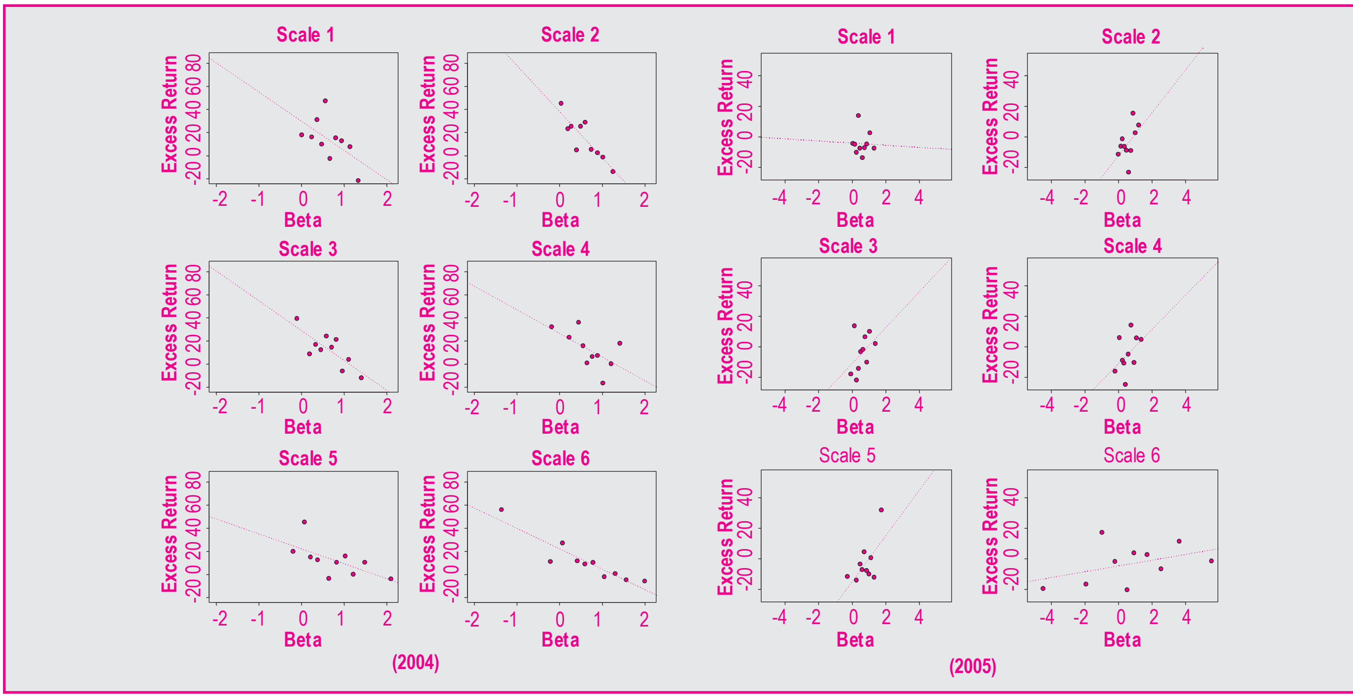


*Figure A.1: Discrete Wavelet Decomposition for the Years 2004 and 2005 (Annual excess portfolio returns versus systematic risk for different levels. Scales 1 to 6 correspond to the following periods: (1) 2-4 days, (2) 4-8 days, (3) 8-16 days, (4) 16-32 days, (5) 32-64 days, and (6) 64-128 days)*

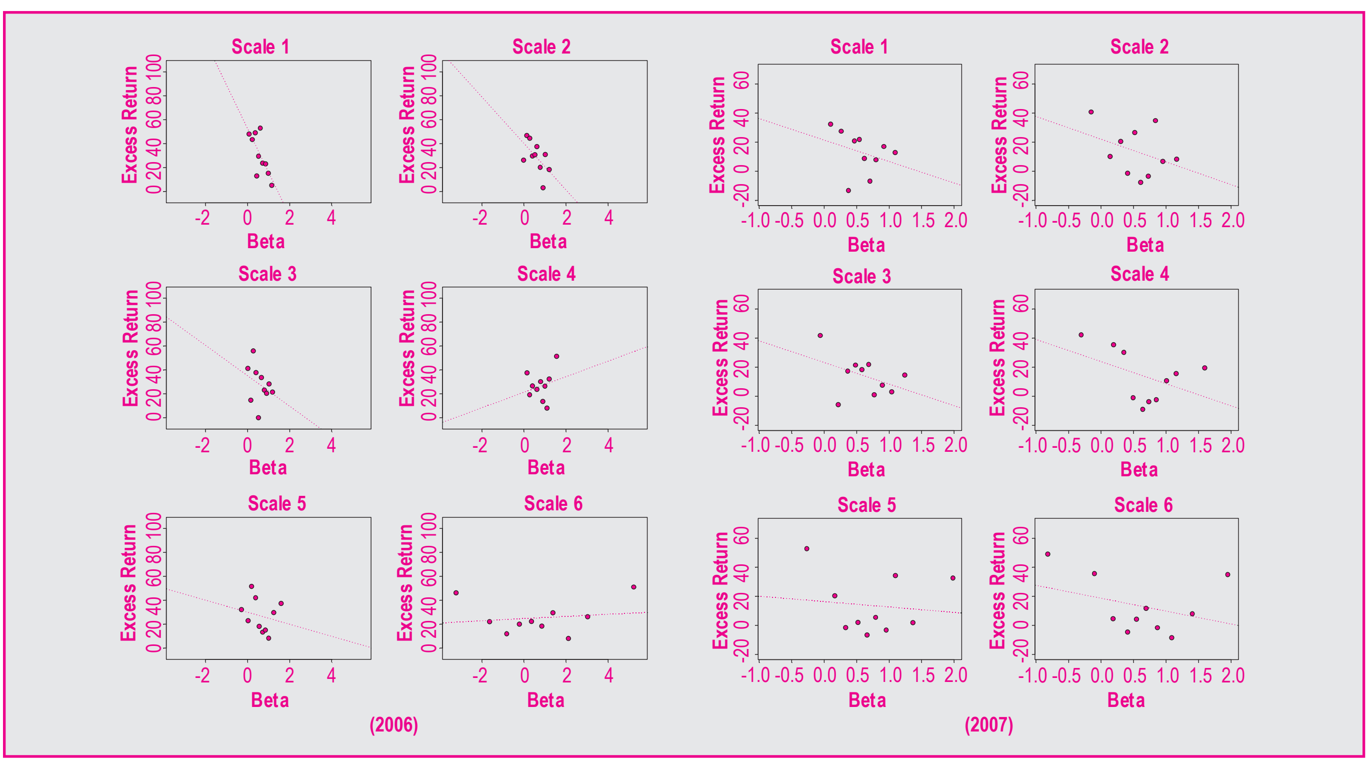


*Figure A.2: Discrete Wavelet Decomposition for the Years 2006 and 2007 (Annual excess portfolio returns versus systematic risk for different levels. Scales 1 to 6 correspond to the following periods: (1) 2-4 days, (2) 4-8 days, (3) 8-16 days, (4) 16-32 days, (5) 32-64 days, and (6) 64-128 days)*