Brazilian Journal of Poultry Science
Revista Brasileira de Ciência Avícola

ISSN 1516-635X 2022 / v.24 / n.1 / 001-010

http://dx.doi.org/10.1590/1806-9061-2021-1478

Original Article

Use of Mathematical Models in the Analysis of Growth and Commercial Performance of Brown Layers

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■**Keywords**

Brown laying hen; development; egg production; growth; mathematical model.

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Submitted: 28/May/2021
Approved: 29/September/2021

ABSTRACT

Small and medium-sized table egg producers have a large volume of data and information on daily production, which is not used in the decision-making process. It is important to define the best mathematical model for the prediction of both the growth and the egg production of brown layers to relate pullet growth traits with productivity in a commercial cycle. For the above purpose, growth and production data were obtained from 15 brown layer flocks, six mathematical models of growth and five of egg production were tested. Correlations were made between the parameters of growth and egg production, as well as between the parameters derived from the mathematical models. The prediction equations for the egg production indicators were estimated. As a result,the best model for predicting layer growth (Gompertz) and for egg production was obtained (Yang). The growth parameters with the greatest influence on egg production were the maximum growth rate (MGR) and the theta value (TV). Body weight at 8, 9, 10, 12, and 16 weeks of age had effects on egg production. As a conclusion maximizing the growth of the brown layer in key stages of its development has positive effects on egg production.

INTRODUCTION

In commercial conditions, egg producers have a large volume of data derived from production system records, which are partially used in decision-making processes, because the producer does not have enough analytical and computational tools. In this sense, such technological developments represent a professional challenge related to the analysis of large information bases and their relationship with the market (Arcila *et al*., 2016; Camargo *et al*., 2015).

Mathematical models are important to understand any biological dynamics and the most used in birds correspond to the functions of three parameters (Logistic, Gompertz and Von Bertalanffy) and four parameters (Richards, Weibull and Morgan-Mercer-Flodin) (Maruyama *et al*., 2001). Parameters are estimated by using empirical observations of body weight dynamics over several weeks (Ahmad, 2009), creating growth patterns over time. The scientific understanding of this phenomenon allows to establish a solid base to develop management and nutritional strategies oriented to optimize the production of eggs at a commercial level. On the other hand, the mathematical models to estimate the egg production curves allow to establish a standard pattern of expression of the genetic potential of layers under specific production conditions (Lokhorst, 1996). The application of mathematical models to systemically evaluate the growth and production of eggs represents the reality of the biological cycle of the layer (Oliveira *et al*., 2018), whose mathematical analysis is essential for the sustainability and competitiveness of egg production at a commercial level (Gómez *et al*., 2017).

MATERIALS AND METHODS

Data collection and management

The pullet growth and egg production data used were provided by a commercial brown egg producer from the Hy Line Brown strain. 15 commercial flocks with production between September 2008 and October 2018 were analyzed. The company is located in the municipality of Choachí, Colombia, whose ambient temperature and relative humidity meet production expectations in a thermoneutral environment (Institute of Hydrology, Meteorology and Studies Environmental [IDEAM], National University of Colombia [UNAL], 2018). The selection of the flocks was carried out by means of a non-probabilistic sampling with growth and egg production data, where the producer followed the strain management guide. A database was designed using the Microsoft Excel® software, organizing the variables of production for subsequent export to the statistical program SAS 9.4®. The estimated variables were: corporal weight (CW), egg production (P), flock uniformity (FU), feed conversion rate (FCR), cumulative egg mas/hen housed (CEM), number of hens housed eggs (HHE), age at 50% egg production/day, grams of feed per egg produced (GPE) and age at peak of egg production.

Fitting mathematical models

The mathematical growth models used are shown in Table 1 and egg production in Table 2.In the case of the growth models, these were selected by using the weekly average weights of the pullets from each of the flocks analyzed (average weight derived from weighing 4% of pullets in each flock; the average number of pullets per flock was 9337), regarding the modeling of egg production, the mathematical models were adjusted to the weekly egg production percentages presented in each flock, then the best model was selected. The flock is the replica and the weekly weight of the pullets in the flocks was averaged to choose the best mathematical model of growth, in turn the weekly percentage of egg production of the flocks was averaged to choose the best model of egg production. The best model was selected using the RMSPE methodology (Tedeschi, 2004).The modeling for each of the non-linear models and in each of the 15 flocks was performed using the NLIN procedure of the SAS 9.4 statistical program with the Levenberg-Marquardt algorithm with the selected model, and then each model in each flock was parametrized.

Table 1 – Animal growth models.

Growthmodel*	Equation	Reference
Logistic	$Y=\frac{a}{1+exp(-b(x-c))}$	Tjørve & Underhill, 2009
Gompertz	$Y=a\bullet exp(-exp(-b(x-c)))$	Ricklefs, 1968
Von Bertalanffy	$Y=a(1-exp(-b(x+c)))^3$	Tjørve & Tjørve, 2010
EVF	$Y=a(1-exp(-exp(b(x-c))))$	Williams, 1995
Richards	$1+b\bullet exp(-c\bullet x)^{-\left(\frac{1}{d}\right)}$	Narushin & Takma, 2003
Michaelis-Menten	$Y=\frac{a\bullet b^c+d\bullet x^c}{b^c+x^c}$	López *et al,* 2000

*For the Gompertz, logistic and EVF models, parameters a = asymptotic weight, b = growth or maturation rate and c = age at inflection point; Von Bertalanffy model a = asymptotic weight, b = maturation rate c = coefficient of proportionality; Richards model a = asymptotic weight, b = integration constant, c = maturity index, d = parameter controlling the inflection point; Michaelis-Menten model a = birth weight, b = age at inflection point or maximum growth, c = parameter that controls the inflection point and d = asymptotic weight.

Table 2 – Egg production models.

Production model*	Equation	Reference
Yang	$Y=\frac{a\bullet exp(-b\bullet x)}{1+exp(-c\bullet(x-d))}$	Yang*et al*, 1989
Mc Nally	$Y=\left(a\bullet x^b\right)\bullet exp\left(-c\bullet x+d\bullet x^{0.5}\right)$	Mc Nally, 1971
Adams-Bell	$Y=100\bullet\left(\left(\frac{1}{1+a^{x-b}}\right)-c\bullet x+d\right)$	Adams & Bell, 1980
Lokhorst	$Y=\left(\frac{100}{1+a\bullet b^x}\right)-\left(c+d^x+e\bullet x^2\right)$	Lokhorst, 1996
Compartmental	$Y=a\bullet(exp(-c\bullet x)-exp(-b\bullet x))$	Mc Millan, 1981

*Yang model a = asymptotic value of egg production, b = rate of decline in laying, c = reciprocal indicator of the variation in sexual maturity and d = mean age at sexual maturity of the flock; Mc Nally model a = initial production, b = rate of increase in production to peak, c = rate of decrease in production after peak, d = square root of time; Lokhorst model a and c = determine the egg production at the beginning of laying, b = influence the time between the start of laying and the peak of production, d and e = are parameters of decrease in laying; Compartmental model a = peak of egg production, b = rate of decrease in egg production and c = rate of increase in egg production; Adams-Bell model a = variation in sexual maturity, b = time to sexual maturity, c = rate of decline in laying and d = decline intercept.

The statistical criteria for the section of each model were:

Akaike information criterion (AIC):

AIC=-2L(Θ)+2P

Where P is the number of model parameters to be estimated and L (Θ) is the maximum value of the likelihood function of the model at point Θ (Akaike, 1974).

Bayesian information criterion (BIC):

BIC=Kln(n)-2ln(L)

Where K is the number of parameters, L is the maximum likelihood value and n is the number of data (Rubalcaba, 2017).

Adjusted coefficient of determination ($R^2_{adjusted}$):

R^2 adjusted $=1-((n-1)/(n-k-1))*(1-R2)$

Where R^2 is the coefficient of determination, n is the number of observations in the sample and k corresponds to the number of parameters of the model (Melillanca, 2018).

Mean square error of prediction (MSEP):

$MSEP= (\Sigma(Yi - f(X1,\ldots,Xp)i))^2/n$

Where Yi is the i-th observed value, f(X1,…,Xp)i is the i-th value predicted by the model and n is the number of observations (Tedeschi, 2004).

Mean bias (MB):

$MB = (\Sigma(Yi - f(X1,\ldots,Xp)i))/n$

Where Yi is the i-th observed value, f(X1,…,Xp)i is the i-th value predicted by the model and n is the number of observations (Tedeschi, 2004).

Growth and egg production parameters associations

Correlations were made between the variables of growth and egg production; additionally, multiple linear regression equations were estimated for the prediction of HHE, CEM and GPE using the stepway selection criteria.

RESULTS

Mathematical models selection and parameterization

Two mathematical models were highlighted (Gompertz & Richards), they presented the lowest values in the Akaike and Bayesian information criterion, the Gompertz model presented the lowest errors (RMSEP: 18.135) and together with the Michaelis-Menten model it stood out for its lowest biases (MB: -0.374 and 0.125 respectively) (Table 3). In general, all models present agood adjusted coefficient of determination (R^2_{adjs}: 0.99) and parameters with height statistical significance. The Gompertz growth model was chosen as the best predictor of brown layer growth in the present study, since it stood out in the statistical criteria as well as being a simpler model and easier to interpret biologically. Figure 1 graphically shows the fit of the growth models on the observed data.

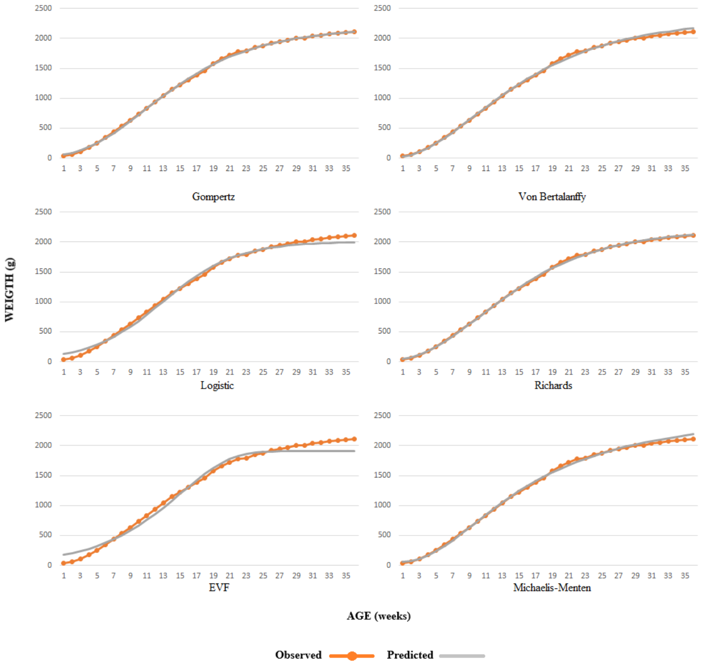

Figure 1 – Growth prediction curves in brown layers.

In the modeling of egg production (Table 4), the Yang and Adams-Bell mathematical models stand out, they presented the lowest values in the Akaike and Bayesian information criteria, on the other hand, the Yang model presented the highest adjusted coefficient of determination (R^2_{adjs}: 0.999), and the lowest error (RMSEP: 1.094); regarding the mean bias statistic, the Yang and McNally models were the most unbiased (MB: 0.269 and -0.118 respectively). Yang mathematical model was considered the best predictor of brown

Table 3 – Growth parameters and decision criteria in mathematical growth models.

MODEL[1]	PARAMETERS				AIC	BIC	R^2_{adjs}	RMSEP	MB
	a	b	c	d					
Gompertz	2179***	0.019***	68.130***		306.058	312.393	0.999	18.135	-0.374
Logístico	2004***	0.032***	83.500***		383.566	389.899	0.999	44.896	-7.440
EVF	1906***	0.024***	98.730***		424.805	431.139	0.999	72.913	-10.683
Von Bertalanffy	2317***	0.015***	15.610***		327.388	333.722	0.999	32.064	-21.520
Richards	2219***	-0.370	0.018***	-0.117	305.838	313.756	0.999	27.252	-19.324
Michaelis Menten	62.190***	104.800***	1.975***	2587***	338.217	346.135	0.999	22.322	0.125

1. For the Gompertz, logistic and EVF models the parameters a = asymptotic weight, b = growth or maturation rate and c = age at inflection point, Von Bertalanffy model a = asymptotic weight, b = maturation rate c = coefficient of proportionality, Richards model a = asymptotic weight, b = constant of integration, c = maturity index, d = parameter that controls the inflection point, Michaelis-Menten model a = birth weight, b = age at inflection point or maximum growth, c = parameter that controls the inflection point and d = asymptotic weight.AIC: Akaike information criterion; BIC: Bayesian information criterion; R^2_{adjs}: Adjusted determination coefficient; RMSEP: Square root of mean square error of prediction; MB: Mean bias. Significance codes: $p<0.0001$'***' $p<0.001$'**' $p<0.01$'*' $p<0.05$'.' $p<0.1$.

layer egg production in the present study due to its low AIC and BIC values, higher R^2_{adjs}, lower error and bias; in general, all the models presented good statistical validity in their parameters. Figure 2 graphically shows the fit of the egg production models on the observed data.

Table 4 - Egg production parameters and decision criteria in mathematical egg production models.

MODEL[1]	PARAMETERS					AIC	BIC	R^2_{adjs}	RMSEP	MB
	a	b	c	d	e					
Yang	99.140***	0.0005***	0.169***	21.130***		192.815	203.286	0.999	1.094	0.269
Mc Nally	1.172**	2.2609***	-0.014***	-0.734***		351.660	362.132	0.998	4.026	-0.118
Lokhorst	-157.60***	0.944***	-4.327	0.082***	-0.0001	281.539	294.105	0.978	3.375	-1.772
Compartimental	106.10***	0.039***	0.001***			389.824	401.133	0.997	4.962	-0.910
Adams-Bell	0.847***	20.737***	0.0004***	-0.013**		177.698	188.169	0.996	1.100	-0.529

1. Flock. 2. Yang model a = asymptotic value of egg production, b = rate of decline in laying, c = reciprocal indicator of the variation in sexual maturity and d = mean age at sexual maturity of the flock; Mc Nally model a = initial production, b = rate of increase in production to peak, c = rate of decrease in production after peak, d = square root of time; Lokhorst model a and c = determine the egg production at the beginning of laying, b = influence the time between the start of laying and the peak of production, d and e = are parameters of decrease in laying; compartmental model a = peak of egg production, b = rate of decrease in egg production and c = rate of increase in egg production; Adams-Bell model a = variation in sexual maturity, b = time to sexual maturity, c = rate of decline in laying and d = decline intercept. AIC: Akaike information criterion; BIC: Bayesian information criterion; R^2_{adjs}: Adjusted determination coefficient; RMSEP: Square root of mean square error of prediction; MB: Mean bias. Significance codes: $p<0.0001$'***' $p<0.001$'**' $p<0.01$'*' $p<0.05$'.' $p<0.1$.

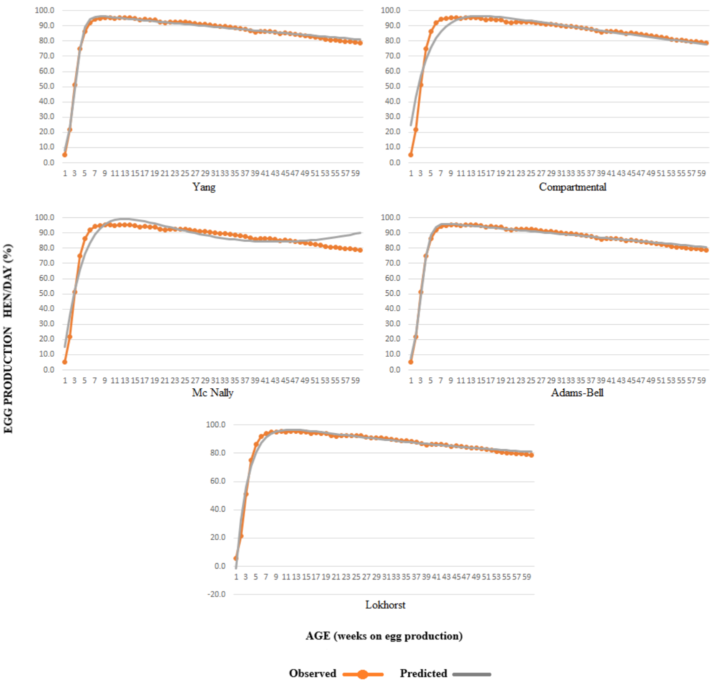

Figure 2 – Egg production prediction curves in brown layers.

The parameters derived from the adjusted Gompertz growth equation allow inferring characteristics that determine critical points in the development of the laying hen (Table 5). The parameterization derived from Yang's mathematical model allows us to infer at a commercial level the characteristics of: mean age at sexual maturity, the time it takes to reach the peak of egg production%, production percentage at the peak, and the decrease in the egg production after peak (Table 6).

The maximum adult body weight was 2156 g corresponding to flock 2 and the minimum of 1912 g for flock 4. The values of the maturation rate (%/day) varied between 0.0200 for flock 2 and 0.0235 for flock 1, these values showing the percentage of daily ripening after the inflection point. From the point of view of the expression of the genetic potential of the strain in the context of the study, lower maturation rates were observed and related to higher adult weights (r: -0.57, $p<0.05$), which suggests a better adaptation of the flocks with lower adult body weights. The delta value (end of structuring phase) of the different flocks showed a similar pattern with a maximum value at 21 days for flock 2 and a minimum of 16 days for flock 1. This response range reflects the dynamic adaptation of the strain to the environmental conditions and the quality of the chick at birth.

Yang's model established a mean asymptotic value of 99.14, which generated an average peak production of 96.5% for the 15 flocks studied. Flock 12 with an estimated production at the peak of 97.7% contrasted to flock 3, which only reached 95% production of eggs/hen per day. The average age of sexual maturity of the layers was 21.2 days after the start of lay, in flock 4 with 37.69 days compared to flock 11 with 15.23 days. The peak of production presented an average time of 51.6 days after the start of lay, with a maximum of 71 days for flock 4 and a minimum of 36 days for flock 14 (Table 6).

Growth and egg production parameter associations

The analysis of the correlations between the growth and production parameters showed that MGR affected the age at 50% of egg production (r: -0.78, $p<0.01$), the HHE at week 30 (r: 0.77, $p<0.01$) and week 50 (r: 0.81, $p<0.01$). The corporal weight (CW) of the pullets at 10

Table 5 - Gompertz model parameterization in growth of brown layers at a commercial level[1].

F*	a (g)	b (%/day)	c (Days)	Proportion of growth	Timefrom c to a (Days)	Postnatal growth duration (Days)	Maximum growth rate (g/day)	Delta value (Days)	Theta value (Days)
1	1968	0.0235	58	42.6	166	224	17.0	16	56
2	2156	0.0200	71	50.0	195	266	15.9	21	65
3	2047	0.0208	68	48.1	188	256	15.7	20	63
4	1912	0.0211	63	47.4	185	248	14.8	16	63
5	1937	0.0230	60	43.5	170	230	16.4	17	58
6	2129	0.0221	62	45.2	177	239	17.3	17	59
7	1990	0.0228	61	43.9	171	232	16.7	17	58
8	1933	0.0228	59	43.9	171	231	16.2	16	58
9	1977	0.0224	62	44.6	174	236	16.3	18	59
10	1969	0.0219	63	45.7	178	241	15.9	18	61
11	2147	0.0213	66	46.9	183	249	16.8	19	61
12	2053	0.0217	63	46.1	180	243	16.4	17	61
13	2020	0.0223	62	44.8	175	237	16.6	18	59
14	2074	0.0218	63	45.9	179	242	16.6	17	60
15	2057	0.0218	62	45.9	179	241	16.5	16	60

1. Gompertz model Y = a * exp (-exp (-b * (x-c)) where: a = asymptotic (Adult weight), b = Maturation rate, c = Age at inflection point, Proportion of growth = 1 / b. Days from inflection point (PI) to adult weight = Proportion of growth * 3.9020. Postnatal growth duration = age to inflection point + days from PI to a. Maximum growth rate = (a * b) / exp (1) Delta value = 1413.15 / maximum growth rate; Theta value = (Days 25 - 75% maturity) / $a^{0.25}$. * Flock.

weeks of age showed negative correlations (r: -0.68, *p*<0.01) with the age of sexual maturity (Parameter d of Yang equation) while CW at 12 weeks of age was negatively correlated with the mean age of sexual maturity (r: -0.67, *p*<0.01), while an age less than 50% of egg production was negatively correlated with CW at 16 weeks of age (r: -0.72, *p*<0.01). The CW at 16 weeks of age was positively associated with the number of HHE at the following weeks of age: 30 (r: 0.71, *p*<0.01), 50 (r: 0.70, *p*<0.01) and 80 (r: 0.68, *p*<0.01).

The variable FU during the growing period became more relevant at 9 and 12 weeks of age since these were correlated with some indicators of the egg production. A higher UF at week 9 was associated with the peak of egg production (r: 0.72, *p*<0.01), while FU at week 12 was correlated with the days at the peak of production (r: -0.73, *p*<0.01).

The FCR variable at 8 weeks of age had effects on: age at 50% of egg production (r: 0.66, *p*<0.01), days to reach peak production (r: 0.83, *p*<0.01) and the number of HHE at 30 and 50 weeks of age (r: -0.75, *p*<0.01). The cumulative egg mass per hen housed (CEM) at 30 and 50 weeks of age showed associations with TV (r: -0.67 and -0.65, respectively, *p*<0.01).

The FCR at 8 weeks of age of the pullets was associated with the CEM at week 30 (r: -0.72, *p*<0.01).

Table 6 - Yang model parameterization.[1]

F*	a	b (Eggs hen day/week)	c	d (Day)	Days to peak production	Peak production (%)
1	97.6	0.0004	0.16	16.34	54	95.2
2	99.6	0.0006	0.18	24.54	56	96.0
3	97.4	0.0004	0.19	26.45	58	95.0
4	98.9	0.0004	0.18	37.69	71	96.0
5	99.6	0.0005	0.23	19.42	46	97.1
6	100.5	0.0005	0.18	19.14	52	97.6
7	98.69	0.0004	0.19	22.29	54	96.4
8	99.01	0.0004	0.18	17.95	52	96.8
9	98.25	0.0004	0.19	18.41	51	96.1
10	99.4	0.0004	0.24	20.83	48	97.4
11	99.8	0.0006	0.16	15.23	51	96.6
12	100.3	0.0005	0.19	16.88	48	97.7
13	98.9	0.0004	0.23	21.04	49	96.9
14	100.3	0.0009	0.34	18.96	36	96.8
15	98.9	0.0004	0.26	23.14	48	96.9

1. Yang model Y = (a * exp (-b * x)) / (1 + exp (-c * (x-d))), where: a = asymptotic value of egg production, b = rate of decline of laying (eggs/layer/weekly day), c = reciprocal indicator of the variation in sexual maturity and d = mean age at sexual maturity of the flock (Days after start of lay). Days to peak production = d + (Ln (c-b) -Ln (b)) / c. Peak production = calculated by adjusting the days to peak production in Yang's equation. * Flock.

The maturation rate parameter was associated with FCR at 80 weeks (r: -0.71, *p*<0.01), while the MGR and TV parameters were associated with FCR at 30 weeks of age (r: -0.71 and r: -0.65, *p*<0.01, respectively). Likewise, the average age of sexual maturity of the layers showed highly significant correlations (*p*<0.01) with the FCR at weeks 30 (r: 0.89) and 80 (r: 0.80). Table 7 shows a summary of the principal correlations between growth and egg production parameters.

Multiple regression equations were estimated to predict HHE, CEM and grams of feed per produced egg in different weeks of the layer production period (Table 8). Layer growth parameters such as theta value, FCR at the 8th week, corporal weights and uniformities

Table 7 – Correlations between growth and egg production parameters in brown layers.

Laying period variables	Growth period variables	Association	Significance
Age at 50% de production	Maximum growth rate	-0.78	0.000
	Theta value	0.58	0.024
	Corporal weight at week 8	-0.56	0.030
	Corporal weight at week 9	-0.59	0.020
	Corporal weight at week 10	-0.61	0.015
	Corporal weight at week 12	-0.68	0.005
	Corporal weight at week 16	-0.72	0.002
	Feed conversion rate at week 8	0.66	0.006
Days at peak of lay	Corporal weight at week 12	-0.52	0.048
	Flock uniformity at week 12	-0.73	0.001
	Feed conversion rate at week 8	0.83	0.000
Production % at peak of lay	Flock uniformity at week 9	0.72	0.002
	Flock uniformity at week 12	0.60	0.017
	Flock uniformity at week 16	0.53	0.041
Cumulative egg mass at 30 week(Kg/hen)	Maximum growth rate	0.67	0.006
	Theta value	-0.67	0.006
	Maturation rate	0.58	0.022
	Corporal weight at week 9	0.53	0.041
	Corporal weight at week 10	0.58	0.022
	Corporal weight at week 12	0.59	0.021
	Corporal weight at week 16	0.67	0.005
	Feed conversion rate at week 8	-0.72	0.002
Cumulative egg mass at 50 week(Kg/hen)	Maximum growth rate	0.61	0.015
	Theta value	-0.65	0.009
	Maturation rate	0.58	0.022
	Corporal weight at week 10	0.58	0.022
	Corporal weight at week 16	0.61	0.015
	Feed conversion rate at week 8	-0.64	0.010
Cumulative egg mass at 80 week(Kg/hen)	Corporal weight at week 16	0.51	0.050
Hen housed eggs at 30 week	Maximum growth rate	0.77	0.000
	Theta value	-0.63	0.011
	Maturation rate	0.51	0.050
	Corporal weight at week 16	0.71	0.003
Hen housed eggs at 50 week	Maximum growth rate	0.81	0.000
	Theta value	-0.66	0.008
	Maturation rate	0.52	0.044
	Corporal weight at week 8	0.57	0.027
	Corporal weight at week 9	0.60	0.017
	Corporal weight at week 10	0.75	0.001
	Corporal weight at week 12	0.69	0.004
	Corporal weight at week 16	0.70	0.004
	Flock uniformity at week 9	0.54	0.037
	Feed conversion rate at week 8	-0.75	0.001
Hen housed eggs at 80 week	Theta value	-0.63	0.012
	Maturation rate	0.59	0.021
	Corporal weight at week 10	0.57	0.027
	Corporal weight at week 16	0.68	0.005

Table 8 – Production period parameters prediction equations.

PARAMETER	EQUATION[1]	$R^2_{Adjusted}$
HHE[2]	Y = 174.792 – (1.312*FCR W8)– (0.153*FU W16) – (4.507*Age50%P)	0.90
HHE[3]	Y = 184.664 – (0.139*CW W3) + (0.067*CW W10) – (4.334*Age50%P)	0.91
HHE[4]	Y = 764.184 + (0.597*FU W9) – (6.007*TV) – (0.377*CW W12) + (0.217*CW W16)	0.82
CEM[2]	Y = 21.595 – (0.118*TV) – (0.306* Age50%P) – (0.006*CW W9)	0.87
GPE[5]	Y = -131.971 + (2.189+VT) + (0.097*CW W7) + (0.398*P W19) + (4.030* Age50%P)	0.82

1.FU = Flock uniformity (%); P =Egg production (% hen/day); FCR = Feed conversion rate (kg/kg); W =Week; HHE = Hen housed eggs; CEM = Cumulative egg mass (kg/hen); GPE = Grams per egg;CW = Corporal weight (grams); TV = Theta value (days). Age at 50% production is expressed in weeks.

2. At 30 week of age.

3.At 50 week of age.

4.At 80 week of age.

5.Feed grams/produced eggs/Hen at 80 weeks of age.

stood out in the prediction of the described variables which are related to the productivity of the egg production system.

DISCUSSION

The mathematical models that appeared for their accuracy, precision and biological interpretation were Gompertz and Yang, but in general the applied models allowed to satisfactorily explain and describe the life cycle of the layer. In this regard, Aggrey (2002) found that the best models to simulate the growth of layers were those of: Gompertz and Richards. Later studies carried out by Oliveira *et al*. (2018) found that the Gompertz and Logistic models were the most appropriate. In both studies, the Gompertz model presented a greater fit (higher coefficient of determination R^2 and a lower Akaike information criterion), which agrees with the present study.

Regarding the egg production model, Narinc (2014) in a review article found that the Adams-Bell model presented the best fit to the poultry production curve, when it was compared with the models of: Mc Nally, Yang, Gamma and Adams-Bell, and he also found that all of them presented particularly good adjustments to the data according to the determination coefficients, however, the Adams-Bell and the Yang models presented the best adjustments to the egg production curve, such as it was also observed in the present study.

MGR fluctuated between 14.8 g/day and 17.3 g/day, these values being like those found by Dos Santos *et al*. (2017) of 15.6 g/day and 16.4 g/day for the Hy Line Brown strain. Alves *et al*. (2019) report an MGR of 16.5 g/day for the Lohman Brown strain and 15.9 g/day for the Dekalb Brown strain. The MGR was reached between 58 days (8.3 weeks) and 71 days (10.14 weeks). The TV value corresponding to the period between 25% and 75% of layer maturity fluctuated between 56 and 65 days, with an association with the growth rate (r: -0.98, $p<0.001$, data not shown).

Values of the maturity rate (%/day) ranged between 0.0200 and 0.0235 with a difference in the parameter over the third statistical decimal, which shows the% of daily maturity gained after the inflection point.

From the point of view of expression of the genetic potential of the lineage and in the context of the study, lower maturation rates were observed related to higher adult weights (r: -0.57, $p<0.05$, data not shown), which suggests a better adaptation of lots with lower adult weights to local commercial production conditions. In this regard, Alves et al. (2019) found maturity rates of 0.024%/day for the Lohman Brown and Dekalb Brown strains, while Dos Santos et al. (2017) observed maturity rates between 0.0226 and 0.0229%/day in layers in the Hy Line Brown strain, and while Sakomura et al. (2011) reported a maturity rate of 0.0230%/day.

The duration of postnatal growth, understood as the age at which the layer reaches its adult weight, fluctuated between 224 and 266 days, while in the study by Dos Santos *et al*. (2017) for the Hy Line Brown strain it was observed around 224 days of age.

The parameterization of the Yang model established a drop in post-peak production between 0.0004 and 0.0009 eggs bird/day per week. When analyzing that flock 1 was housed in 2008 while flock 15 was housed in 2018, a tendency to reduce the estimated time to reach peak production can be inferred; on the other hand, the drop in egg production has been slightly reduced, which is consistent with the observations made by Bedetti & Van De Braak (2021).

Regarding the association of the growth performance of the layers on their productive phase, variables such as the maturity rate, MGR and TV take relevance.

The optimization of the development of the layer in the above mentioned characteristics shows the importance of maximizing the gains in body weight of the layer in the exponential growth phase, which will culminate at the inflection point with a maximum body weight gain with notable effects in the layer production period, such as reductions in age to 50% of egg production, greater HHE and CEM.

That results indicates that a decrease in the period between 25 and 75% maturity of the layer will also have a relevant impact on the productive performance, making the layer a bird with the highest egg production during its commercial cycle, given by increases in chicks' daily body weight gain during the maturation process around the inflection point and during early sexual maturation.

It is important to maximize the body weights in the rearing period of the layer. CW at week 10 of life, in its association with performance in early sexual maturity and HHE agrees with the data obtained by Carrizo *et al*. (2007). On the other hand, the CW at weeks 12 and 16 stand out, showing the importance of reaching the upper growth thresholds described in the layer's management guide.

FCR takes great importance, especially at the 8th week of life, where the relevance of reaching higher growth thresholds at such age is observed, which is directly related to the use of feeding systems that allow ensuring these growth targets in an objective way, in accordance with the management guide for each particular strain. In general, the results show the importance of FCR throughout the productive period of the layer, since it will contribute to a greater quantity of eggs produced in terms of HHE and CEM.

It is concluded that the growth parameter that most influenced egg production was the MGR, because it was associated with a higher production at the beginning of the laying period, a decrease in age to 50% of production, a higher number of HHE and CEM at 30 and 50 weeks of age.

In addition, reductions in time between 25 and 75% of layer maturity (TV) will make the layer more productive, since this parameter was associated with CEM, and the HHE during the laying cycle. Maximizing the CW and FU during the growth and development of the layer (weeks 3, 8, 9, 10, 12 and 16) produce strategic effects on the production of eggs in variables such as: the reduction in the age of sexual maturity, age at 50% production and peak production, higher production peaks and higher HHE and CEM at 30, 50 and 80 weeks of age.

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