Revista Brasileira de Ciência do Solo

Division - Soil Processes and Properties | Commission - Soil Physics

Comparison of Methods for Determining Precompression Stress Based on Computational Simulation

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ABSTRACT: There are many methods for determining precompression stress (σ_p), whose value is affected by the slope of the soil compression curve. This study was designed to evaluate the hypothesis that for a certain compression curve all methods used to determine σ_p present the same value and accuracy. The aim of this study was to compare the accuracy and the relationship among seven of these methods by computational simulation of soil compression curves under nine scenarios. The following methods were used: Casagrande, Pacheco Silva, intersection of the initial void ratio with the virgin compression line (VCLzero), and the regression methods based on 2 (reg1), 3 (reg2), 4 (reg3), and 5 (reg4) points for modeling the elastic curve. Under each scenario, created by combining the swelling and the compression indices, 1,000 compression curves were computationally simulated via the Monte Carlo method. Subsequently, 95 % percentile confidence intervals were built using the 1,000 estimates of σ_p from each method under each scenario. Most of the differences among the methods were detected under scenarios consisting of high swelling and low compression indices. In general, Casagrande, Pacheco Silva, and reg4 were strongly correlated and presented the highest values of σ_p, as well as similar variability. The latter two can be considered as alternatives to the standard method of Casagrande, except for Pacheco Silva when the curve has a low compression index (≤0.2) and from medium to high swelling index (≥0.025), for which differences ($p<0.05$) were detected.

Keywords: soil stress, soil compression curve, soil compaction.

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Received: June 24, 2015
Approved: July 25, 2015

How to cite: Silva AR, Lima RP. Comparison of Methods for Determining Precompression Stress Based on Computational Simulation. Rev Bras Cienc Solo. 2016;40:e0150164.

INTRODUCTION

Soil compaction as an effect of agricultural machinery has been one of the great challenges of modern agriculture (Lima et al., 2015). Vast cultivated areas have received increasingly heavy and intensive machine traffic (Mosaddeghi et al., 2007; Lima et al., 2015), especially at crop harvest. This has adverse effects on crop production and the environment (O'Sullivan et al., 1999). According to Cavalieri et al. (2008), soil compaction has been a subject of study for many years due to its implications for crop yield.

Compaction can be understood by studying soil compressibility (Dias Júnior and Pierce, 1995). Compression is characterized by a mechanical process that describes the decrease in volume when soil is exposed to a mechanical load, which is defined by a soil compression curve. Three important parameters extracted from the soil compression curve were describe by Keller et al. (2011): the swelling or recompression index (C_s), the compressibility coefficient (C_c), and the preconsolidation or precompression stress (σ_p), where C_s is defined as the slope of the swelling line (SL) and C_c is the slope of the Virgin Compression Line (VCL).

The C_s is used as a measure of rebound and soil mechanical resilience, reflecting the first part of the curve subjected to historical stress, characterized by elastic deformation (recoverable). The second part is the VCL, for which plastic deformations are irreversible. This part can be verified by the C_c value, not subject stress (Keller et al., 2011). Finally, σ_p is mathematically defined as the point that divides the compression curve into the elastic and plastic parts of the soil compression curve (Casagrande, 1936).

There are many methods for determination of σ_p. The most widely used was proposed by Casagrande (1936), which is based on the maximum curvature point of the soil compression curve. Nevertheless, other methods have been developed, such as the Pacheco Silva (ABNT, 1990) method, based on the intersection of the VCL and the initial void ratio. Dias Júnior and Pierce (1995) showed the procedure for determination of σ_p by intersection of two linear regressions made for VCL and SL, which can have a different number of points considered for fitting VCL and SL (Cavalieri et al., 2008). Another method (VCLzero) consists of considering σ_p as the value on the *x*-axis defined by the intersection of the VCL with a horizontal line from the initial void ratio (Arvidsson and Keller, 2004).

The studies of Arvidsson and Keller (2004), Gregory et al. (2006), Cavalieri et al. (2008), Ajayi et al. (2013), and An et al. (2015) demonstrated there are variations in the methods used for determination of the σ_p and that the shape of curve is an important source of variation of indices (σ_p, C_s, C_c) extracted from the soil compression curve, showing that further studies are required. However, these studies have in common many soils, moisture contents, textures, and different conditions in soil physical properties, which makes it hard to define the parameters of the compression curve for a specific study and more accurate analysis. Under these conditions, simulations can help create scenarios for reproducing experimental data (Tagar et al., 2015), which formalize and analyze some error propagation methods for modeling; among them, the Monte Carlo method has general applicability and can be used in models with mathematical formulations (Ortiz et al., 2004). This procedure was used by Ortiz et al. (2004), and simulations based on other methods were used by Oliveira et al. (2013, 2014) for soil data in Brazil.

In this context, this study was designed to evaluate the hypothesis that for a certain compression curve all methods used to determine σ_p present the same value and accuracy. Hence, the aim of this study was to compare the accuracy and relationship among seven methods used to determine σ_p by simulating soil compression curves under nine scenarios.

METHODS

Methods for determination and calculation of indices

Seven methods for determination of σ_p were used: Casagrande, Pacheco Silva, intersection of initial void ratio with VCL (VCLzero), and linear regression methods based on 2, 3, 4, and 5 points for modeling of the elastic curve (swelling line, SL). An illustration of the regression method based on 2 points can be seen in figure 1.

The virgin compression line (VCL) was estimated through linear regression considering the last three points of the compression curve. The compressibility coefficient (C_c) was estimated as the slope of the linear regression fitted for VCL, determined as shown in equation 1, where *e* is the void ratio. The swelling index (C_s) was determined as the mean slope of the loading path up to 25 kPa (Equation 2), according to Keller et al. (2011). The C_c and C_s indices are identified on the compression curve and graphically represented in figure 1.

$$C_c = -\frac{e_{1600} - e_{400}}{\log_{10}(1600) - \log_{10}(400)} \qquad \text{Eq. 1}$$

$$C_s = -\frac{e_{25} - e_0}{\log_{10}(25) - \log_{10}(1)} \qquad \text{Eq. 2}$$

Scenarios of simulation

We created scenarios based on the values of the swelling (recompression index) and compression indices (Keller et al., 2011). This allowed us to reproduce different compression curves, which are associated with the results of a soil compressibility test. Simulation was based on the result of a simple uniaxial compression test with loads of 1, 12.5, 25, 50, 100, 200, 400, 800, and 1,600 kPa. In this case, the loading of 1 kPa corresponds to the initial void ratio or bulk density, which was only to calculate the swelling line associated with the initial state of the soil sample (Keller et al., 2011; An et al., 2015).

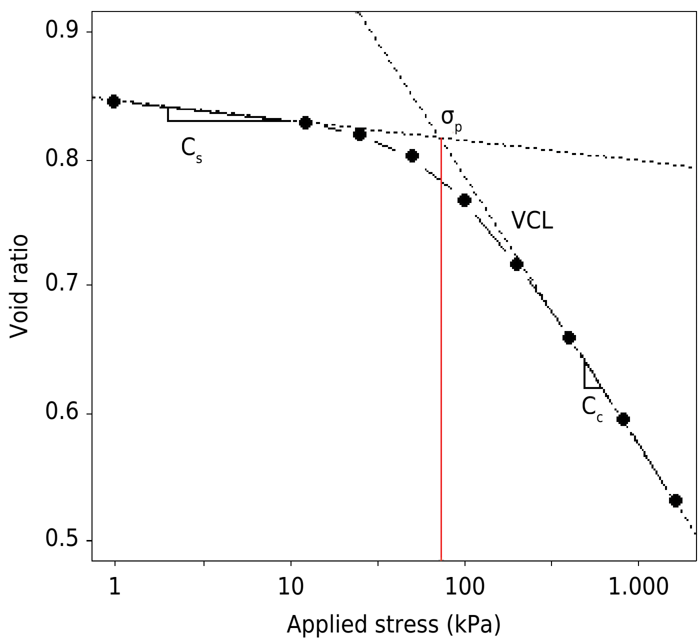

Figure 1. Determination of precompression stress (σ_p) by reg1, regression method based on 2 points, expressed in terms of void ratio, as a function of the logarithm of applied stress (kPa); Virgin compression line (VCL); the slope of the VCL is denominated as the compressibility coefficient, C_c; swelling index, C_s. Adapted from Keller et al. (2011).

We combined three values of C_c and three values of C_s in order to create nine scenarios (Figure 2). The slopes of the VCL and SL were generated on the $\log_{10}$ scale of the applied loads. The original data set and the boundary of the initial void ratio and the values of C_c and C_s used in simulations were based on data from Ajayi et al. (2009), Keller et al. (2011), Ajayi et al. (2013), and An et al. (2015).

Simulation

Monte Carlo simulation was performed to compute mean and standard deviation. For each one of the nine compression curves described earlier, a fourth degree polynomial model was fitted, $y = X\theta + \varepsilon$, where y is the vector of void ratio, X is the polynomial model matrix, and ε is a random vector representing the error of the model. After that, we computed the vector of estimates (Equation 3) and its (co)variance matrix (Equation 4).

$$\hat{\theta} = \left[\hat{\theta}_0 \quad \hat{\theta}_1 \quad \hat{\theta}_2 \quad \hat{\theta}_3 \quad \hat{\theta}_4\right]^T \qquad \text{Eq. 3}$$

$$C\hat{o}v(\hat{\theta}) = (X^T X)^{-1} s^2 \qquad \text{Eq. 4}$$

where s^2 is the estimate of residual variance.

Subsequently, we considered the vector of estimates to be normally distributed as $N_5\left[\hat{\theta}, Cov(\hat{\theta})\right]$ in order to simulate 1,000 other vectors of estimates, say $\hat{\theta}^*$. For every $\hat{\theta}_i^*$ (i = 1, 2, ..., 1,000), a corresponding predicted vector $\hat{y}_i = X\hat{\theta}_i^*$ was calculated. Finally, the pairs ($\hat{y}_i$,x) were used to determine 1,000 random estimates of precompression stress, σ^*_p, by each one of the seven methods.

Statistics for comparisons

We calculated the mean and coefficient of variation (%) of σ^*_p determined by each method. In addition, a confidence interval was built using the percentile method, i.e., taking the quantiles $\sigma^*_p\left(\frac{\alpha}{2}\right)$ and $\sigma^*_p\left(1-\frac{\alpha}{2}\right)$ as estimates of the lower and upper limits, respectively, of a 100(1 - α)% confidence interval.

Furthermore, we computed Pearson's correlation matrix in order to study the relationship among the methods.

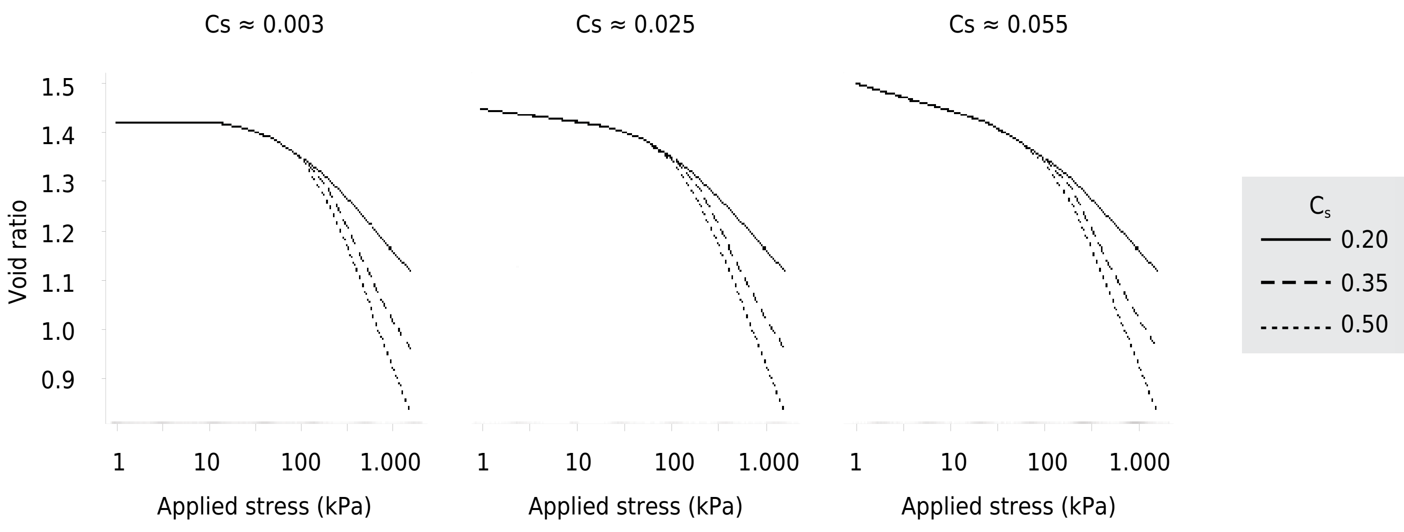

Figure 2. Simulation scenarios, created by combining three values of the compressibility coefficient (C_c) and three values of the swelling index (C_s).

Computing

All the simulations and data analyses were made using the software R 3.1.2 (R Core Team, 2015) *soilphysics* package (Silva and Lima, 2015). Calculation of σ_p was performed through the sigmaP() function. Simulations and percentile confidence intervals were performed using the simSigmaP() and plotCIsigmaP() functions, respectively.

RESULTS

Considering scenarios where $C_c = 0.2$, the reg4 and Casagrande methods were similar and showed the highest values of σ_p for all the C_s conditions (Figure 3). The methods of Pacheco Silva, reg3, reg2, and reg1 changed more than Casagrande and reg4 with variation in C_s. The Pacheco method was similar to Casagrande when considering the smallest C_s, but they were statistically different ($p<0.05$) when C_s increased. The regression method had higher values when using more points for modeling the swelling line (reg4>reg3>reg2>reg1).

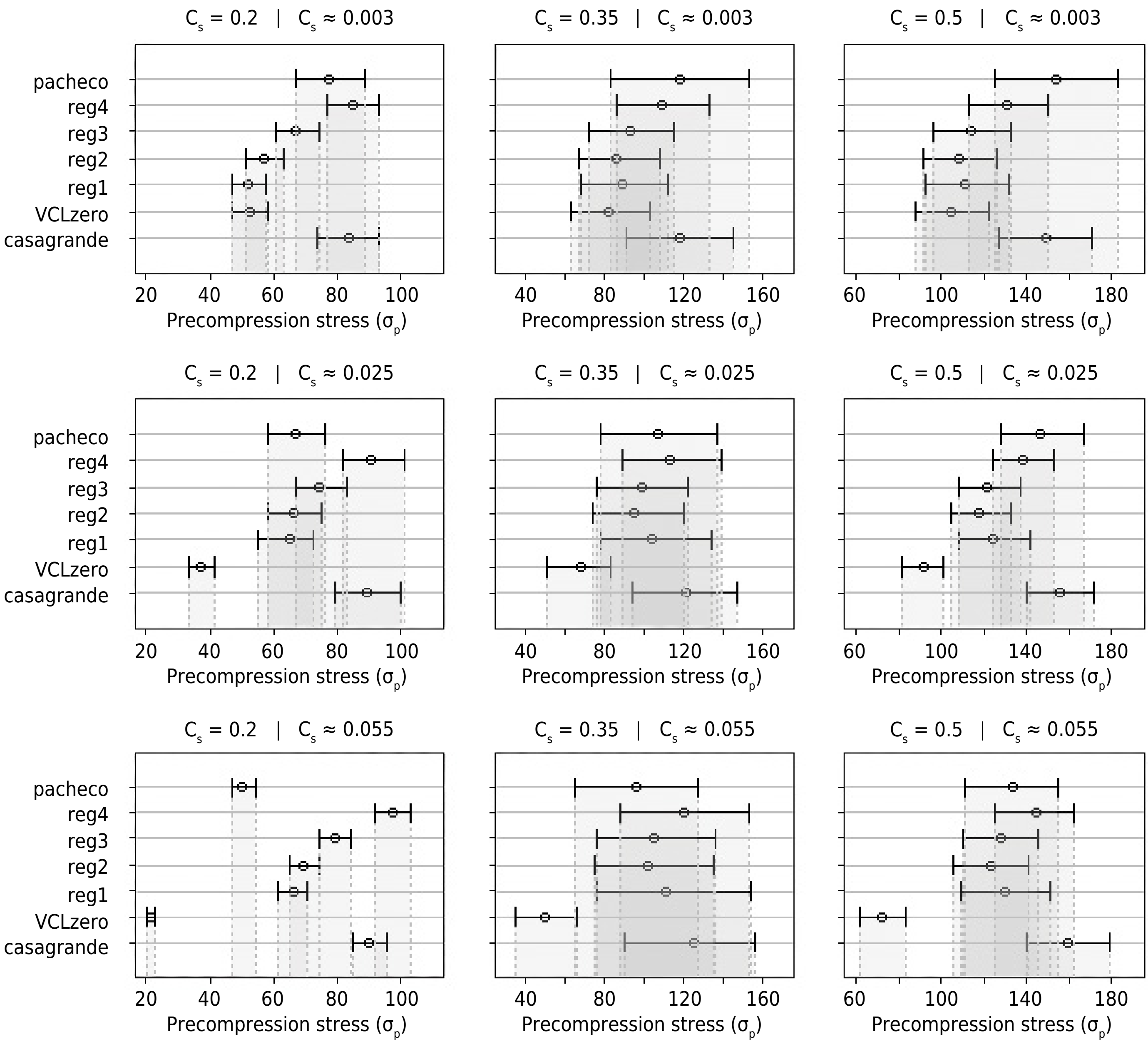

Figure 3. 95 % percentile confidence intervals for the mean of precompression stress (σ_p) determined by seven methods under nine scenarios designed according to the compression (C_c) and swelling (C_s) indices. Results based on 1,000 simulated soil compression curves.

For C_c = 0.35, the Casagrande and reg4 methods also tended to show the highest values of σ_p. The VCLzero method showed the lowest values, regardless of C_s (Figure 3). The methods of Pacheco, reg3, reg2, and reg1 changed more than the Casagrande and reg4 methods with variation in C_s. As C_s increased, VCzero was the only method that was statistically different ($p<0.05$) than Casagrande.

For C_c = 0.50, the methods of Casagrande, Pacheco Silva, and reg4 showed the highest σ_p values. The VCLzero was statistically different ($p<0.05$) from Casagrande for all C_s.

In general, the methods of Casagrande, Pacheco, and reg4 tended to show the largest values of σ_p. Specifically for C_c = 0.20, Casagrande and reg4 promoted the largest values, regardless of C_s. For C_c = 0.35 and 0.50, the general observation applies. The VCLzero method had the lowest values of σ_p for most scenarios. The Pacheco method was closer to Casagrande as C_s declined.

Under (C_c = 0.20, C_s ~ 0.055), we found the largest number of statistical differences ($p<0.05$) among the methods. In contrast, no difference ($p>0.05$) was found under the combination (C_c = 0.35, C_s ~ 0.003).

The variability shown by the methods in scenario (C_c = 0.20, C_s ~ 0.055) is noteworthy (Table 1). In fact, we observed ascending variability in the estimates according to the C_s, whatever the value of the C_c. Likewise, C_c = 0.20 tended to show the highest variability in the simulated means.

In all scenarios, the standard Casagrande method was more correlated ($r>0.90$) with the Pacheco method and regression method using 5 (reg4) and 4 (reg3) points (Table 2). Although VCLzero always tends to show the lowest value of σ_p, it had the same behavior as Pacheco and Casagrande. The reg1 and reg2 methods were related to each other in all scenarios.

DISCUSSION

Simulated soil compression curves

The curves simulated cover a wide range of soil compression curves, such as those obtained for soil samples in compression tests under different soil bulk densities, textures, and moisture contents (Arvidsson and Keller, 2004; Imhoff et al., 2004; Gregory et al., 2006; Cavalieri et al., 2008; Ajayi et al., 2009; Saffih-Hdadi et al., 2009; Ajayi et al., 2013; An et al., 2015).

A relationship between the simulated scenarios and situations of soil physical properties was observed. Ajayi et al. (2009) and Ajayi et al. (2013) obtained soil compression curves with lower values for C_c for samples with low water content. When water content increased, C_c also increased changing the shape of the soil compression curve.

For black and brown soils from Northeastern China under different bulk densities and water contents, An et al. (2015) obtained C_c values similar to those obtained by Ajayi et al. (2009) and Ajayi et al. (2013). They found that C_c was higher when water content in the

Table 1. Coefficient of variation for the simulated means of the seven methods in each scenario defined by C_c and C_s

C_s (approx.)	C_c		
	0.20	0.35	0.50
0.003	20.9	15.4	16.3
0.025	25.7	16.9	16.7
0.055	37.8	24.6	21.2

Table 2. Correlations among seven methods (C: Casagrande; V: VCLzero; r1, r2, r3, r4: regression methods; P: Pacheco Silva) of determination of σ_p under nine scenarios designed according to the compression (C_c) and swelling (S_s) indices. Results based on 1,000 simulated soil compression curves

	C_c = 0.2 | C_s = 0.003						**C_c = 0.35 | C_s = 0.003**						**C_c = 0.5 | C_s = 0.003**					
	V	r1	r2	r3	r4	P	V	r1	r2	r3	r4	P	V	r1	r2	r3	r4	P
C	0.89	0.77	0.91	0.97	0.99	0.98	0.85	0.62	0.85	0.94	0.98	0.98	0.79	0.48	0.77	0.91	0.96	0.96
V	-	0.70	0.81	0.85	0.87	0.96	-	0.52	0.71	0.79	0.81	0.95	-	0.36	0.58	0.68	0.72	0.93
r1	-	-	0.97	0.91	0.85	0.78	-	-	0.94	0.85	0.77	0.61	-	-	0.93	0.80	0.69	0.46
r2	-	-	-	0.98	0.96	0.90	-	-	-	0.97	0.94	0.82	-	-	-	0.96	0.91	0.73
r3	-	-	-	-	0.99	0.95	-	-	-	-	0.99	0.92	-	-	-	-	0.99	0.87
r4	-	-	-	-	-	0.97	-	-	-	-	-	0.95	-	-	-	-	-	0.91
	C_c = 0.2 | C_s = 0.025						C_c = 0.35 | C_s = 0.025						C_c = 0.5 | C_s = 0.025					
	V	r1	r2	r3	r4	P	V	r1	r2	r3	r4	P	V	r1	r2	r3	r4	P
C	0.93	0.78	0.92	0.97	0.99	0.98	0.85	0.58	0.83	0.94	0.98	0.98	0.82	0.51	0.78	0.91	0.96	0.97
V	-	0.78	0.88	0.91	0.92	0.98	-	0.49	0.69	0.78	0.81	0.94	-	0.40	0.61	0.71	0.75	0.93
r1	-	-	0.97	0.91	0.86	0.83	-	-	0.94	0.83	0.74	0.59	-	-	0.94	0.81	0.72	0.51
r2	-	-	-	0.98	0.96	0.94	-	-	-	0.97	0.93	0.82	-	-	-	0.97	0.92	0.76
r3	-	-	-	-	0.99	0.98	-	-	-	-	0.99	0.92	-	-	-	-	0.99	0.88
r4	-	-	-	-	-	0.98	-	-	-	-	-	0.95	-	-	-	-	-	0.93
	C_c = 0.2 | C_s = 0.055						C_c = 0.35 | C_s = 0.055						C_c = 0.5 | C_s = 0.055					
	V	r1	r2	r3	r4	P	V	r1	r2	r3	r4	P	V	r1	r2	r3	r4	P
C	0.90	0.70	0.88	0.96	0.98	0.95	0.83	0.49	0.78	0.92	0.97	0.97	0.82	0.47	0.76	0.90	0.96	0.97
V	-	0.66	0.79	0.85	0.86	0.96	-	0.43	0.65	0.75	0.78	0.93	-	0.42	0.63	0.73	0.76	0.93
r1	-	-	0.96	0.88	0.81	0.82	-	-	0.92	0.79	0.68	0.56	-	-	0.93	0.80	0.69	0.52
r2	-	-	-	0.98	0.94	0.93	-	-	-	0.96	0.91	0.82	-	-	-	0.96	0.91	0.78
r3	-	-	-	-	0.99	0.96	-	-	-	-	0.99	0.92	-	-	-	-	0.99	0.89
r4	-	-	-	-	-	0.96	-	-	-	-	-	0.95	-	-	-	-	-	0.93

soil samples increased. Variations in the shape of the curve when bulk density changed were also observed by Saffih-Hdadi et al. (2009) and An et al. (2015). They stated that C_c decreases for high bulk density values. According to the results obtained by An et al. (2015), the combination of high bulk density values and low water content minimizes the value of C_c.

The shape of the elastic line of a soil compression curve, represented here through C_s, varies mainly with water content, as observed by O'Sullivan and Robertson (1996) and Braida et al. (2008). However, the results found by O'Sullivan and Robertson (1996) show C_s values were higher in dry than in wet soils. However, Braida et al. (2008) found exactly the opposite, and attributed the results to the effects of the organic C and water content, which increasing the elastic proprieties of soil. Nonetheless, in any case, it is important to know that changing C_s increases the differences among the σ_p methods. This is most evident when comparing the methods under the same C_c value (Figure 3).

Behavior of the methods under each scenario

The slope of the elastic curve and VCL influenced the estimate of σ_p by the methods. Specifically, high values of C_s (0.055) and low values of C_c (0.2) increased differences among the methods (Figure 3). This can also be seen by the gradual increase in the coefficient of variation (Table 1). When the variation among methods is analyzed under the same C_s, differences mainly occur under low C_c (Table 1). Variation among methods is largely influenced by the slope of VCL, C_c (Rosa et al., 2011).

Considering all scenarios, Casagrande, Pacheco Silva, reg4, and reg3 showed strong correlation (Table 2). Casagrande was compared with regression methods based on 2 (reg3) and 3 (reg4) points for modeling VCL (Arvidsson and Keller, 2004). These authors found that VCLzero was best correlated with Casagrande, but also found that the correlation between the regression method and Casagrande increased with the number of points (regression using three points>regression using two points), corroborating the results in table 2. However, Arvidsson and Keller (2004) did not test regression with four and five points (reg3 and reg4 as specified here, respectively), which would probably increase similarity with the Casagrande method, as found here. Cavalieri et al. (2008) showed medium to high correlations between regression methods and Casagrande (Cavalieri et al., 2008), at least higher than those obtained by Arvidsson and Keller (2004). The regression method using 4 and 5 points was correlated with Casagrande, as well as the Pacheco method (Table 2). Similarity between Casagrande and Pacheco in terms of σ_p also was found by Rosa et al. (2011).

Applicability of the methods

The Casagrande method has been considered as standard in almost all comparison studies involving soil compressibility. However, its algorithm is relatively complex, since the point of maximum curvature of the compression curve must be determined. Regression methods are considerably simpler since they consist of intercepting two regression lines. However, evaluations of the regression methods, including comparison of their performance with the Casagrande method, can be found in the studies of Dias Júnior and Pierce (1995), Arvidsson and Keller (2004), and Cavalieri et al. (2008).

CONCLUSIONS

Most of the differences among the methods were detected under scenarios consisting of high swelling and low compression indices.

In general, Casagrande, Pacheco Silva, and reg4 were strongly correlated, showing the largest values of σ_p, and similar variability. The latter two can be considered as alternatives to the standard Casagrande method, except for Pacheco Silva when the curve has a low compressibility coefficient ($\leq$0.2) and medium to high swelling index ($\geq$0.025), for which differences ($p<0.05$) were detected.

ACKNOWLEDGMENTS

We thank the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES, Brazil) for granting scholarships, and the Instituto Federal Goiano (Brazil) for financial support.

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