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Estimating structural and frictional unemployment: An application of empirical mode decomposition

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The aim of this paper was to estimate Brazilian structural and frictional unemployment by the empirical mode decomposition (EMD) method, relying on the definitions of structural and frictional unemployment of Lipsey (1960). The advantage of the EMD method is that it does not suffer of the problems of approximation bias or unit root that are present in the OLS and ML estimates described in the literature.

Keywords. Unemployment, Structural, Frictional, EMD.

JEL classification. E24, C50.

1. Introduction

Economic theory classifies natural unemployment in two components: structural and frictional. By definition, structural unemployment is the permanent component and frictional unemployment is temporary. Both derive from the inability of the markets to find a condition of equilibrium of monetary or fiscal policy to eliminate unemployment (Salop, 1979).

Attempts to estimate structural and frictional unemployment are generally carried out based on linearization of the Lipsey model (Warren, 1991; Aysun et al., 2014) and the Burdett-Mortensen model (Koning et al., 1995). However, linearization can fail when assuming a first degree polynomial form and introducing the remaining terms of the polynomial in the error term. In particular, this causes the problem of approximation bias (Sun et al., 2011). Besides this, the empirical literature, when dealing with time series data, as if they were cross sectional data, to measure the components of unemployment, disregards the possible unit root problems. These anomalies can bias the OLS and ML estimates.

To overcome these drawbacks, this work will apply an approach that avoids approximation bias and unit root problems. The empirical mode decomposition (EMD) is designed to work well with nonstationary and nonlinear data (Huang et al., 1998). The EMD derives the components of any dataset in two signals, which can express the structural and frictional unemployment and produces results that are robust in relation to the

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OLS and ML estimators. Besides, the EMD method is comparable with other techniques (e.g., spectral analysis, Fourier and wavelet transformations). Thus, the EMD is a robust and powerful method to estimate the components of unemployment.

In addition to this introductory section, this study features four more sections. The second section provides an overview of unemployment. The third section presents the methodological aspects of the EMD technique. Section 4 shows the empirical results for the decomposition of unemployment in Brazil. Finally, the fifth section brings the conclusions.

2. Components of unemployment

2.1 *The model*

The labor market literature suggests that a constant flow of hiring and layoffs exists, or simply of people moving from one job to another. Therefore, all the job candidates will not be employed and all the job openings will not be filled. This movement defines frictional unemployment. Besides this, there is a set of workers who are not involved in this flow, but are looking for work persistently. However, for various reasons they cannot find work, mainly due to a mismatch of skills and needs of companies. These agents are faced with structural unemployment.

This situation is presented in the labor market model of Lipsey (1960), based on three identities:

$$V_t = J_t - E_t \tag{1}$$

$$U_t = L_t - E_t \tag{2}$$

$$E_t = \Delta E_t + E_{t-1}, \tag{3}$$

where V_t is the level of unfilled job vacancies, J_t is the number of jobs, E_t is the level of employment, U_t is the level of unemployment, L_t is the total labor force, and ΔE_t is the variation in employment.

These three identities are analogous to the equations of economic growth models. Equations (1) and (2) define the components of the production function for employment and equation (3) is a job accumulation function. Besides this, since the increase/decrease of resignations/layoffs varies during the business cycle, the sum of the components is a proportion of the level of employment, γ.

Assuming that a technology, A_t, exhibits constant returns to scale, then by (1) and (2) the following holds:

$$A_t\left(V_t, U_t, \beta\right) = \beta\left(V_t, U_t\right)^{\frac{1}{2}}, \tag{4}$$

where β is the rate of filling job vacancies.

Since (4) denotes the number of jobs filled and γE_t is the number of resignations/layoffs, it is possible to use equation (3) to redefine the job accumulation function as follows:

$$\Delta E_{t+1} = E_{t+1} + E_t = \beta\left(V_t, U_t\right)^{\frac{1}{2}} - \gamma E_t \tag{5}$$

Therefore, in a stationary state, $\Delta E_{t+1} = 0$,

$$\frac{\gamma}{\beta} = \frac{(V_t, U_t)^{\frac{1}{2}}}{E_t}$$

$$U_t = \left(\frac{\gamma}{\beta}\right)^2 \frac{E_t{}^2}{V_t} \tag{6}$$

Now let $u_t = \frac{U_t}{L_t}$, then:

$$u_t = \left(\frac{\gamma}{\beta}\right)^2 \frac{E_t{}^2}{V_t L_t}, \tag{7}$$

where u_t is the frictional unemployment rate.

The key element defined in (7) is that there is no long-term structural unemployment. This condition is based on the classical hypothesis of full employment; i.e., there are no cyclical components or inefficiencies in the labor market over the long run. Hence, the long-term component of unemployment will be totally frictional.

2.2 *Empirical strategy*

In general, the estimates of frictional and structural unemployment are based on cross-sectional data. The common empirical strategy rests on a measure of the job market inefficiency component obtained by a stochastic frontier model.

Since (5) is the cumulative unemployment function, it is possible to write:

$$\frac{\Delta E_{t+1}}{E_t} = \frac{\beta (V_t, U_t)^{\frac{1}{2}}}{E_t{}^{\frac{1}{2}}} - \gamma + \omega_t - \mu_t \tag{8}$$

$$\frac{\Delta E_{t+1}}{E_t} = \beta_0 + \beta_1 \frac{(V_t, U_t)^{\frac{1}{2}}}{E_t{}^{\frac{1}{2}}} + \omega_t - \mu_t, \tag{8'}$$

where $\beta_0 = -\gamma$, $\beta_1 = \beta$, ω_t represents the error term and μ_t the technical inefficiency term.

Thus, (8') can be rewritten following a linear specification, as proposed by the stochastic frontier model:

$$Y = \beta_0 + \beta_1 X + v, \tag{9}$$

where $Y = \Delta E_{t+1}/E_t$; $X = (V_t, U_t)^{\frac{1}{2}}/E_t{}^{\frac{1}{2}}$ and $v = \omega_t - \mu_t$.

The linear form of (9) will incur in a specification error whenever the polynomial components of higher order, treated as the remainder of the linearization process, are non-zero (Sun et al., 2011). This was also considered by Kmenta (1967) when reporting that approximations, via linearization of functions, can suffer of omitted-variable bias. In general, a production function like (9) can be written as follows:

$$Y = f(x) + r(x) + v, \tag{10}$$

where $f(x)$ is treated as a linear approximation and $r(x)$ is the remaining term of a Taylor expansion, i.e., *n-th* term of the polynomial associated with x. The deterministic terms, $r(x)$, can be non-zero and cause bias and inconsistency if (9) is estimated using maximum likelihood or least squares estimators.

Another problem of the strategies to measure the elements of unemployment is the use of time series data treating them as cross sectional data. The applications of time series like a cross-section ignore the stationarity condition. This procedure also produces bias in the OLS and ML estimators.

Therefore, the analysis of the components of unemployment obtained through these approaches reveal that their estimates are probably biased. The estimates of structural and frictional unemployment in Aysun et al. (2014) and Warren (1991), for instance, completely ignore the bias of the estimates produced by the ML estimator. These examples thus show that these approaches need to be revised.

A method that improves the robustness of the results can be found in Huang et al. (1998). This approach is called the empirical mode decomposition (EMD) model and it will be described below.

3. Empirical mode decomposition

The empirical mode decomposition can be applied to different types of series. This decomposition approach was proposed by Huang et al. (1998) and can be applied to nonlinear or nonstationary series.

Its is built on functions based on local maximum and minimum values involving the series. The decomposition is an element obtained through the arithmetic mean of the upper envelope, formed by the function that links the local maxima, and of the lower one, formed by the function that links the local minima. An intrinsic mode function (IMF) is derived for each decomposition process.

An intrinsic mode function must satisfy the following conditions:

(i) The number of extrema and the number of zero crossings must either be equal or differ at most by one; and

(ii) At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

Therefore, the intrinsic mode function is obtained to remove the high-frequency oscillations until the maximum and minimum locations become equal. In this case, the decomposition of a time series by the EMD method is replaced by the sum of the IMFs and a residual component. Formally:

$$x(t) = \sum_{k=1}^{n} \mathrm{IMF}_k(t) + r_n(t), \tag{11}$$

where $\sum_{k=1}^{n} \mathrm{IMF}_k(t)$ is the result of the process generated from the envelopes and $r_n(t)$ is the difference between the original series and its decomposition.

The EMD algorithm can be represented as follows:

a) Define $r_0(t) = x(t)$ and $\mathrm{IMF}_1(t)$;

b) Search for *n*-IMF:

   I. Assume that $h_0(t) = r_{k-1}(t)$ with $k = 1$;

   II. Find the local maxima and minima of $h_{i-1}(t)$

   III. Generate an upper envelope $E_u(t)$ and a lower envelope $E_l(t)$ by interpolating the local maxima and minima, respectively;

   IV. Compute the arithmetic mean of the envelopes;

   V. Let the residual component be the difference between the original series and the arithmetic mean of the envelopes;

   VI. Check whether $h_i(t)$ satisfies conditions (i) and (ii) to be an IMF.

c) If $r_k(t)$ is a constant or has a single extreme, stop the decomposition process. If not, repeat (a) and (b).

The steps of the EMD process can be observed in Figure 1. The decomposition eliminates the waves by applying steps (a) and (b) via the difference between the mean value obtained for the envelopes and the original values. Finally, the decomposition approach presents two components, structural and oscillatory.

3.1 *Database*

The decomposition of the unemployment series to measure the structural and frictional components is based on the number of unoccupied people. This information is provided by the Brazilian Institute of Geography and Statistics (IBGE). The series of unoccupied people has monthly frequency and starts in March 2012. Thus, the interval of the data will correspond to the numbers recorded between March 2012 and the latest measurement date (January, 2020).

An unoccupied person is an agent without a job in the reference month who took bona fide action to find work in that month and was available to accept a job if offered. The unoccupied contingent also includes unemployed individuals who have not taken effective action to find work in the reference period of 30 days because they already did so previously and will start working sooner than four months after the survey.

4. Results

The Empirical Mode Decomposition process was replicated three times in the unemployment application. Thus, the EMD algorithm found three IMF's (see Appendix A) which can be seen as oscillations of the labor market reflecting the mismatch between quits and layoffs.

It was then verified the inefficiency in the Brazilian labor market and over-employment at different times. The inefficiency of the labor market is a key element to explain frictional unemployment (Hosios, 1990; Pissarides, 1990). On the other hand, the expansion phase of the business cycle is responsible for over-employment

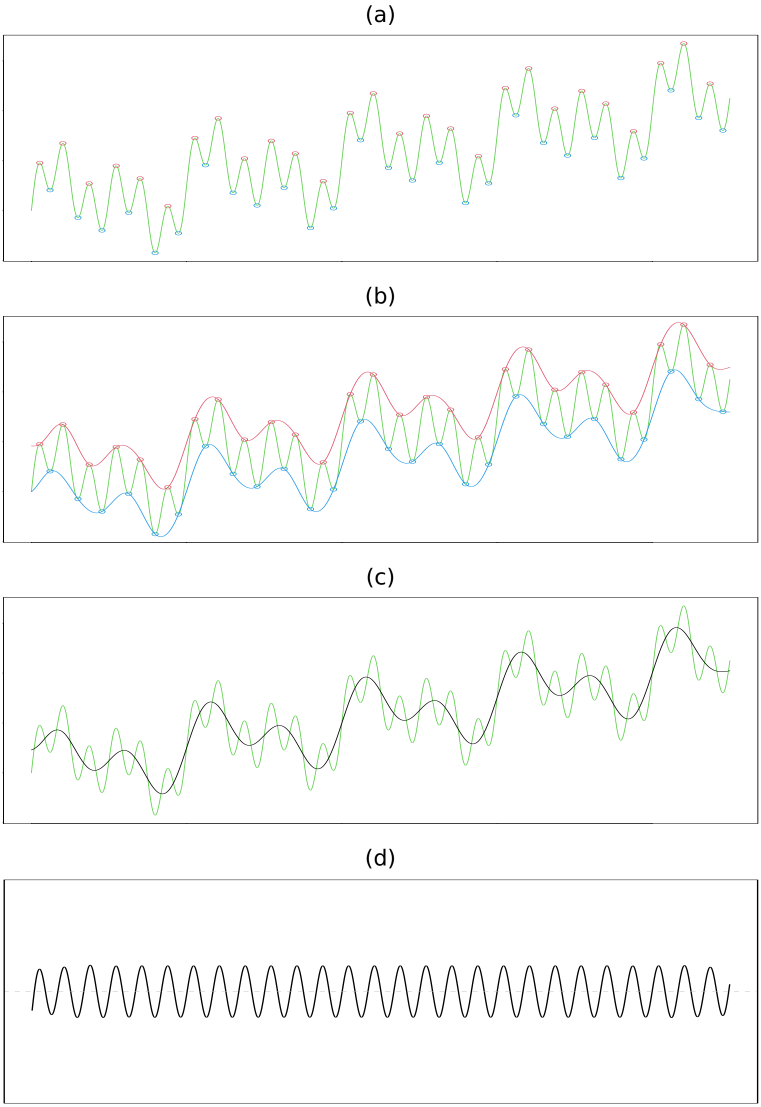

Figure 1. Decomposition process (adapted from Kim and Oh, 2009).

(Mortensen and Pissarides, 1994). Therefore, the measurements of the unemployment components by Warren (1991) and Aysun et al. (2014), which use a stochastic frontier approach, ignore the existing over-employment in the expansive stage of the business cycle.

The measurement of structural and frictional unemployment is underestimated as one defines the differential between potential employment and actual employment as frictional unemployment. Lipsey (1960) says that this is the steady-state condition. In cross-sectional data application, the technological changes happening along the time-

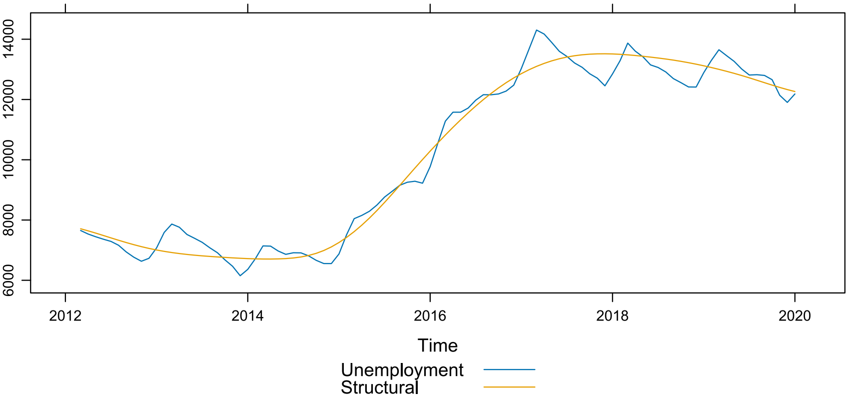

Figure 2. Unemployment and structural unemployment.

line are not captured. Such technological changes are necessary to verify points above potential employment. This fact is not considered in previous works.

Figure 2 shows that it is possible to capture both unemployment components by the Empirical Mode Decomposition at any time line point. This application identifies the moments of the business cycle expansion and the inefficiency of the labor market; i.e., the over-employment and over-unemployment (or its frictional components).

The over-employment possibility was first noted by Smith (1999). His finding suggests that firms and workers can break the wage-marginal productivity condition and generate inefficiency in the labor market. Hiring of labor is observed when the firms raise vacancies. New contracts are made through a bargaining process that induces the wage to decrease. Thus, each additional worker is hired at a cost lower than the value of his/hers marginal product, making the firm to hire inefficient excess labor.

This result is pointed out in Figure 2. When frictional unemployment is negative, i.e., always that unemployment is below its structural component, over-employment becomes present. Over-employment, the frictional form generally not computed and ignored in the specialized literature, is a current phenomenon in the Brazilian labor market, especially in the fourth quarter.

The traditional friction in the labor market, i.e., when actual unemployment is above structural unemployment, is extensively discussed in the literature as a flow of hiring and layoffs (Greenwald and Stiglitz, 1995), as a simple change from one job to another (Mortensen and Pissarides, 1994), as a consequence of seasonality (Maddison, 1980) and as imperfect mobility of labor (Dohmen, 2005; Mitra and Ranjan, 2010). This movement, or friction, is a phenomenon symptomatic of inefficiency in the labor market.

The business cycle stage leads the firms to hire or to layoff. The upper end of the business cycle leads to increasing hires and the lower end of the business cycle leads to increasing layoffs. Therefore, the labor market imbalance is related to how an unbalanced hire-layoff flow occurs in intermediate points of the business cycle (Greenwald and Stiglitz, 1995). This unbalanced flow generates friction in the labor market.

It is common for the labor market to present a flow of workers from one job to another. Such a flow can occur, for example, through the training of workers, the creation or destruction of jobs (Mortensen and Pissarides, 1994), and the mutual right of employees and employers to terminate the employment contract (Pearlman and Eskin, 1947).

Shifts in the demand for labor can also occur due to seasonal factors. Agriculture production is an example (De Neubourg, 1986). The inter-harvest period presents layoffs. Another example is the construction industry, which renegotiates labor contracts yearly, generating frictional unemployment. Therefore, the entry of new workers into the labor force, characteristic of the summer or spring season, can be associated with unemployment (Lilien, 1982).

Mobility in the labor market is limited for two reasons. First, the high costs of skill limit mobility. Low-skill workers receive fewer job offers than high-skill workers; therefore, the first ones are most likely to become unemployed (Davidson et al., 2008; Mitra and Ranjan, 2010). Second, housing tenure raises moving costs. The incentives to buy or build a house fix the owner in the long run. This has negative effects on job mobility (Dohmen, 2005).

Therefore, the frictional component of unemployment is made up of two parts. A first one, the over-employment, which is based on the incompatibility between wages and marginal productivity; and, the second one, which is the traditional friction concept, based on the mismatch in the labor market.

The estimate of the structural component of unemployment (see Figure 2) can be considered as a result of the institutional set up (Levine, 2013), mobility costs (Frey, 2009; Katz, 2010), laws (Paqué, 1996) and regulations (Betcherman, 2000). These factors affect the jobs demand-supply equilibrium, the price-wage relation, and the matching process.

There are differences in structural unemployment that depend on labor market institutions or settings. Levine (2013) addresses this issue mentioning three conditions. First, the degree of unionization makes it difficult to adjust wages to economic shocks. Second, generous unemployment insurance programs provide disincentives to search for jobs. Finally, public spending on labor market programs (for example, retraining and new skills) modifies the duration of unemployment (see also Scarpetta, 1996; Belot and Van Ours, 2000).

Geographic mobility is a source of structural unemployment. The mobility costs generate a geographic lock-in effect. This indicates that a geographic disparity between job seekers and job vacancies is a source of structural unemployment (Katz, 2010). Thus, the migration cost is a decisive factor in the mismatch. Furthermore, skill generates an additional cost that can affect inter-industry mobility; when there is a mismatch between the market's required skill and the worker's skills, there is a reduction in job creation and unemployment increases (Restrepo, 2015).

Employment laws may limit the number of vacancies offered, reducing the efficiency of job matching. For example, employment law determines minimum wage generating in many cases constraints to job contracts. It raises structural unemployment since the increases in minimum wage happen yearly (Werding, 2006). On the one hand, unemployed workers may not get jobs by decreasing their wages below the minimum wage

Table 1. Unemployment components estimates

			Frictional (%)		
Statistics	Unemployment	Structural (%)	Total	Standard	Over-Employment
Mean	9958	91.87	8.13	48.42	51.58
Standard Error	2763	5.31	5.31	50.24	50.24
Variance	7 636 001	28.18	28.18	2524.08	2524.08
Max	14 105	99.99	23.90	100.00	100.00
Min	6013	76.10	0.01	0.00	0.00

ceiling. On the other hand, the minimum wage law simply prevents the firm from keeping down wage costs, what lead it to hold back its demand for workers. As a consequence there is a increase in structural unemployment (Lindbeck, 1999).

The regulation of the labor market also influences the mechanism of hiring and firing. Labor legislation defines working contracts, layoff procedures, and payment norms, inflating consequently the costs of labor, working hours, and other labor standards. Thus, restrictive rules may obstruct hiring and regularization of collective layoffs (Betcherman, 2000).

Finally, the unemployment components and your respective rates in the Brazilian labor market are shown in Table 1.

Between 2012 and 2020, the Brazilian labor market presented a level of unemployment that was almost totally structural. Monthly, the average number of unemployed workers was 9,958,000; 9,590,000 were permanently unemployed and 182,000 temporarily unemployed and 186,000 temporarily employed. Therefore, it seems that the Brazilian labor market presents a weak evidence of frictional unemployment.

The structural unemployment rate wass 9.79% for the entire labor force between 2012 and 2020. Compared to other economies, such as the USA (2.37%) and Canada (2.2%), the level of structural unemployment is more than four times higher in Brazil (see Aysun et al., 2014; Osberg and Lin, 2000).

5. Conclusions

This paper proposed a new form to measure frictional and structural unemployment; the Empirical Mode Decomposition. Such approach is indicated when approximation bias and unit root problems are likely to be present in the data. Furthermore, this methodology does not generate the bias seen in the OLS and ML approaches.

Its application to the Brazilian labor market evidenced other advantages as well. First of all, the Empirical Mode Decomposition could simultaneously measure both unemployment components; i.e., structural and frictional unemployment. In the second place, the new frictional form (over-employment), proposed by Smith (1999), could be as well captured.

The estimates of unemployment components are very important pieces of information for policy makers. The unbiased estimate of structural unemployment would help to implement fiscal and monetary policies in order to counteract decreases or overheating of economic activity. Furthermore, the fiscal policy would need a reliable measure of the structural unemployment rate to calculate the GNP gap.

Moreover, measuring frictional unemployment in the labor market would also help to design efficient unemployment assistance programs. In the short run, the standard frictional unemployment level determines unemployment insurance expenses. Therefore, measuring over-employment, is a crucial contribution to the wage bargaining process and job legislation.

Bibliography

Aysun, U., F. Bouvet, and R. Hofler (2014): "An alternative measure of structural unemployment," *Economic Modelling*, 38, 592–603. [1, 4, 6, 9]

Belot, M. and J. C. Van Ours (2000): "Does the recent success of some OECD countries in lowering their unemployment rates lie in the clever design of their labour market reforms?" Tech. Rep. 147, IZA Discussion Papers. [8]

Betcherman, G. (2000): "Structural unemployment: How important are labour market policies and institutions?" *Canadian Public Policy / Analyse de Politiques*, 26, S131–S140. [8, 9]

Davidson, C., S. J. Matusz, and A. Shevchenko (2008): "Globalization and firm level adjustment with imperfect labor markets," *Journal of International Economics*, 75 (2), 295–309. [8]

De Neubourg, C. (1986): "Unidentified Floating Unemployment (UFU) and the Specification of the UV-curve," *Recherches Économiques de Louvain/Louvain Economic Review*, 52 (3-4), 227–255. [8]

Dohmen, T. J. (2005): "Housing, mobility and unemployment," *Regional Science and Urban Economics*, 35 (3), 305–325. [7, 8]

Frey, W. H. (2009): "The great American migration slowdown: Regional and metropolitan dimensions," Tech. Rep., Brooking Institution. [8]

Greenwald, B. C. and J. E. Stiglitz (1995): "Labor-market adjustments and the persistence of unemployment," *The American Economic Review*, 85 (2), 219–225. [7]

Hosios, A. J. (1990): "On the efficiency of matching and related models of search and unemployment," *The Review of Economic Studies*, 57 (2), 279–298. [5]

Huang, N. E., Z. Shen, S. R. Long, M. C. Wu, H. H. Shih, Q. Zheng, et al. (1998): "The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis," *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, 454 (1971), 903–995. [1, 4]

Katz, L. (2010): "Long term unemployment in the Great Recession," Testimony for the Joint Economic Committee, U.S. Congress, April 29. [8]

Kim, D. and H. S. Oh (2009): "EMD: a package for empirical mode decomposition and Hilbert spectrum," *The R Journal*, 1 (1), 40–46. [6]

Kmenta, J. (1967): “On estimation of the CES production function,” *International Economic Review*, 8 (2), 180. [3]

Koning, P., G. Ridder, and G. J. Van den Berg (1995): “Structural and frictional unemployment in an equilibrium search model with heterogeneous agents,” *Journal of Applied Econometrics*, 10 (S1), S133–S151. [1]

Levine, L. (2013): “The increase in unemployment since 2007: Is it cyclical or structural?” . [8]

Lilien, D. M. (1982): “Sectoral shifts and cyclical unemployment,” *Journal of Political Economy*, 90 (4), 777–793. [8]

Lindbeck, A. (1999): “Unemployment-structural,” . [9]

Lipsey, R. G. (1960): “The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1862-1957: A further analysis,” *Economica*, 27, 1–31. [1, 2, 6]

Maddison, A. (1980): “Monitoring the labour market: A proposal for a comprehensive approach in official statistics (illustrated by recent developments in France, Germany and the UK),” *Review of Income and Wealth*, 26 (2), 175–217. [7]

Mitra, D. and P. Ranjan (2010): “Offshoring and unemployment: The role of search frictions labor mobility,” *Journal of International Economics*, 81 (2), 219–229. [7, 8]

Mortensen, D. T. and C. A. Pissarides (1994): “Job creation and job destruction in the theory of unemployment,” *The Review Of Economic Studies*, 61 (3), 397–415. [6, 7, 8]

Osberg, Lars and Zhengxi Lin (2000): “How much of Canada’s unemployment is structural?” *Canadian Public Policy / Analyse de Politiques*, 26, S141–S157. [9]

Paqué, K. H. (1996): “Unemployment and the crisis of the German model: A long-term interpretation,” in *Fighting Europe’s Unemployment in the 1990s*, Berlin, Heidelberg: Springer, 119–155. [8]

Pearlman, L. M. and L. Eskin (1947): “Prospective labor supply on the west coast,” *Monthly Labor Review*, 64 (4), 563–575. [8]

Pissarides, C. (1990): “Unemployment and the persistence on employment shocks,” Tech. Rep., No. 377. [5]

Restrepo, P. (2015): “Skill mismatch and structural unemployment,” Tech. Rep., Massachusetts Institute of Technology Job Market Paper. [8]

Salop, S. C. (1979): “A model of the natural rate of unemployment,” *The American Economic Review*, 69 (1), 117–125. [1]

Scarpetta, S. (1996): “Assessing the role of labour market policies and institutional settings on unemployment: A cross-country study,” *OECD Economic Studies*, 26 (1), 43–98. [8]

Smith, E. (1999): "Search, concave production, and optimal firm size," *Review of Economic Dynamics*, 2 (2), 456–471. [7, 9]

Sun, K., D. J. Henderson, and S. C. Kumbhakar (2011): "Biases in approximating log production," *Journal of Applied Econometrics*, 26 (4), 708–714. [1, 3]

Warren, R. S. (1991): "The estimation of frictional unemployment: A stochastic frontier approach," *The Review of Economics and Statistics*, 73 (2), 373–377. [1, 4, 6]

Werding, M. (2006): *Structural unemployment in western Europe: Reasons and remedies*, MIT Press. [8]

Appendix: Unemployment, structural unemployment and frictional unemployment

In Figure A.3, the first panel shows the number of unemployed in thousands of people. The second panel shows structural unemployment and from the third to the fifth panel the process to compute frictional unemployment using the EMD method is presented.

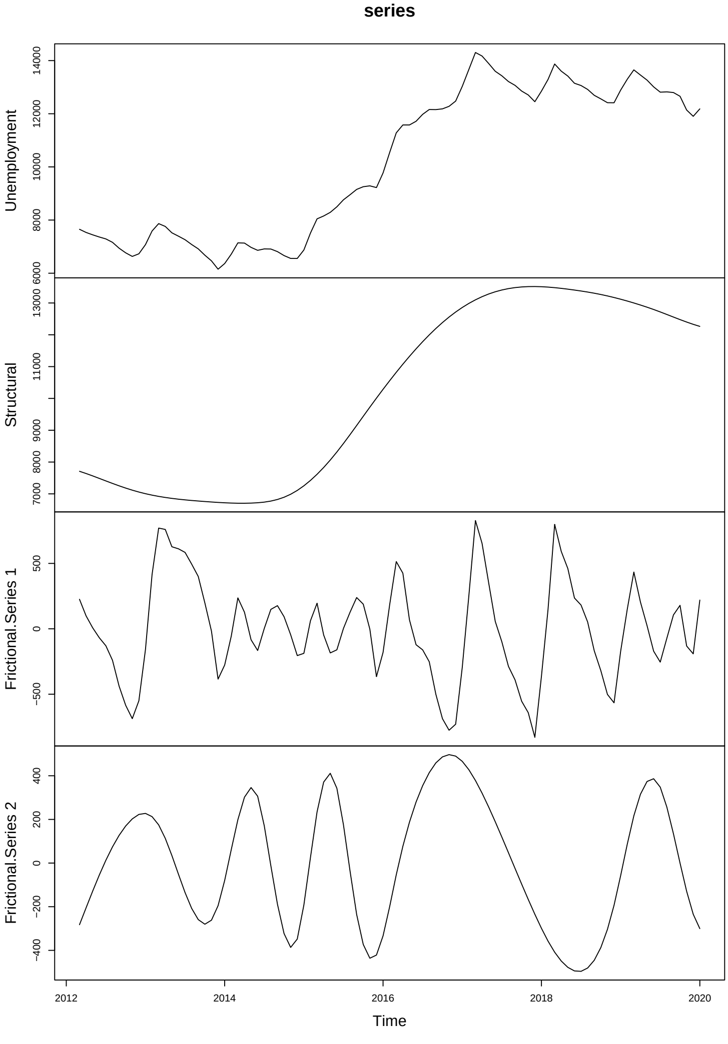

Figure A.3. Unemployment, structural and frictional unemployment.