

Information Exchanges in Cournot Duopolies*

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Summary: 1. Introduction; 2. The model; 3. Case 1: the case of unknown demand intercept; 4. Case 2: the case of unknown slope; 5. Conclusions.

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In this paper we analyze the profitability of information sharing among Cournot oligopolists receiving private information about random demand. We model the random demand as a linear demand having, 1) an unknown intercept, and 2) an unknown slope. In each of these two scenarios, firms observe private signals about the unknown parameter. We show that in the scenario-1, if the private signal observed by firms is accurate enough, information exchange is profitable and in the scenario-2, if there is a sufficiently large variation in the demand slope and private signals are accurate enough, firms earn strictly higher profits by sharing their information rather than keeping it private.

Neste artigo analisamos a rentabilidade dos intercâmbios de informação entre oligopolistas de Cournot quando recebem informação privada sobre uma demanda que é aleatória. Nós modelizamos a demanda aleatória como uma demanda lineal considerando, 1) Interseção da ordenada na origem desconhecida, e 2) Inclinação

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desconhecida. Em cada um destes cenários, as empresas observam sinais privados sobre o parâmetro desconhecido. Nós demonstramos que no primeiro cenário, se o sinal privado observado pelas empresas é suficientemente preciso, o intercâmbio de informação é rentável e no último cenário, se existe uma variação suficientemente ampla na inclinação das demandas e os sinais privados observados pelas empresas são suficientemente precisos, as empresas obtêm maiores lucros quando elas compartilham sua informação que quando a mantem privada.

1. Introduction

Models of information exchange among oligopolists have assumed that market uncertainty is due to either unknown constant marginal cost for the firms or unknown market demand.

There is a quantity of economic literature that deals with both cases of uncertainty. With respect to the uncertainty in market demand, the most important contributions were made in the 80's and 90's. Novshek and Sonnenschein (1982) studied the incentives of Cournot duopolists to share their private information about demand uncertainty. They found that firms would not benefit from sharing their information. Clarke (1983a,b) and Vives (1984) confirmed Novshek and Sonnenschein's results in Cournot oligopoly, but Vives (1985) found that allowing for price competition and differentiated products, exchanges of information on the common demand intercept may increase firms' profits. Li (1985), showed that Cournot oligopolists producing homogeneous goods would not benefit from exchanging their information on demand uncertainty. Gal-Or (1985), showed that firms will be more profitable when they share their information. Kirby (1988) found cases in which firms may have higher profit by sharing their information rather than by keeping it private, she considered perfect substitutes but assumed marginal cost to be sufficiently steep.

With respect to the uncertainty about firms' (constant) marginal costs of production, Gal-Or (1986) and Shapiro (1986) made the most important contributions. They found that if firms are Cournot competitors producing substitutive products, and the only uncertainty is each firm's (constant) marginal costs of production, then it will more profitable for the firms to share their private information about their costs. Other important contributions to the literature on information sharing, either considering unknown demand or unknown costs, are those of Sakai and Yamato (1989), Sakai (1990), Vives (1990), Sakai (1990, 1991), Ziv (1993), Hwang (1993), Hwang (1995), Jin (1996, 1998), Malueg and Tsutsui

(1996, 1998a,b), Novshek and Thomas (1998), Kultti and Niinimaki (1998), Doyle and Snyder (1999), Venayre (1999) and Reynolds and Wilson (2000).

In the models developed by these authors the fact that information exchanges were no profitable did not depend upon the accuracy of the firms' private information. In our paper however, we find that the accuracy of the firms' information turns out to be a key factor in the profitability or otherwise of exchanges in information. We go on to demonstrate that if each firm's private information is accurate enough, then said firms will wish to share their information, as a means of increasing profitability.

2. The Model

We consider a symmetric duopoly model in which two firms, firm 1 and firm 2 producing identical products face uncertain market demand. The inverse demand function is given by :

$$P(q^1 + q^2) = \alpha - \beta(q^1 + q^2)$$

where:

q^i denotes the amount of output produced by firm i ;

α and β denote the demand intercept and the slope of market demand respectively.

The inverse demand function is interpreted as net of costs. According to this demand function we study two cases:

Case 1: The uncertainty of market demand comes from the unknown demand intercept (α), then α is the random component and $\beta > 0$, is a positive parameter.

Case 2: The uncertainty of market demand comes from the unknown slope demand (β), then β is the random component and $\alpha > 0$ is a positive parameter.

We assume firms have no fixed costs and their marginal costs are constant and equal to c , $c \geq 0$. Before making their output decisions and depending on the case being studied, firms observe private signals about α (Case 1), or β (Case 2). Firm i 's privately observed signal is denoted by s^i , $i = 1, 2$. For the case 1, firm's i private signal s^i , takes on one of three values:

s_b^i , s_n^i and s_a^i , $i = 1, 2$, indicating that firms can receive a low, medium or normal and high signal about market demand. The conditional distributions of signal s^i , given α , are as follows:

$$\Pr(s_b^i|\alpha_b) = \Pr(s_a^i|\alpha_a) = \sigma, \quad \Pr(s_n^i|\alpha_b) = \Pr(s_n^i|\alpha_a) = 1 - \sigma$$

and

$$\Pr(s_b^i|\alpha_a) = \Pr(s_a^i|\alpha_b) = 0$$

where:

$\sigma \in (0, 1)$. Given the above assumptions for the conditional distributions of the signals, it is obvious that the distributions of the state of the demand conditioned by the signals ($\alpha|s$) are as follows: $\Pr(\alpha_b|s_b) = \Pr(\alpha_a|s_a) = 1$

Thus, the signal transmitted by a firm either, perfectly identifies the demand state (if s_b or s_a), or provides no information if the signal is (s_n) ¹.

As the parameter σ increases from 0 to 1, the signal becomes increasingly informative.

For the case 2, firm's i private signal s^i , takes on one of two values: s_b^i or s_a^i , $i = 1, 2$. The firms' private signals are equally accurate, with the conditional distribution of signal s^i given the actual demand slope, thus: $\Pr(s_b^i|\beta_b) = \Pr(s_a^i|\beta_a) = \sigma$, consequently we may deduce that $\Pr(s_b^i|\beta_a) = \Pr(s_a^i|\beta_b) = 1 - \sigma$, symmetry of distributions is assumed only for simplicity. Without loss of generality we assume $\sigma \geq 1/2$

We suppose that firms have common prior beliefs about the unknown random component of the market demand, α (Case 1) and β (Case 2).

Furthermore, we assume that the private signals received by the firms about α (Case 1) and β (Case 2) are conditionally independent given α and β .

Finally, we assume that the above description of the environment is knowledge which is common to all firms.

¹The reader should not think such a signalling technology pathological because a firm's realized signal is either perfectly informative or totally uninformative. We could have instead specified $\Pr(s_b^i|\alpha_b) = \Pr(s_a^i|\alpha_a) = \sigma$, $\Pr(s_n^i|\alpha_b) = \Pr(s_n^i|\alpha_a) = \lambda$ and $\Pr(s_b^i|\alpha_a) = \Pr(s_a^i|\alpha_b) = 1 - \lambda - \sigma$ where σ and λ are strictly positive and satisfy $\sigma + \lambda < 1$. In this case, the signal would not perfectly reveal the state. In this model, observation of "two opposite signal are completely offsetting in that the posterior beliefs coincide with the prior beliefs". Our finding in the case 1 that information sharing can be profitable as G approaches 1 would, by continuity, also be found by taking σ sufficiently close to 1 and λ sufficiently close to 0. We make the specification in case 1 purely to simplify computations-perfect revelation is inessential.

3. Case 1: The Case of Unknown Demand Intercept

3.1 Cournot equilibrium and information exchange

We use the Bayesian Cournot equilibrium concept to solve the model: each firm chooses its output to maximize its expected profit conditional on its information, given the output strategy of its rival. Let I^i denote the information available to firm i when it chooses its output. Firm i 's expected profit, given I^i is equal to:

$$E [(P (q^i + q^j)) q^i | I^i] = E [(\alpha - \beta (q^i + q^j)) q^i | I^i] \quad (1)$$

where:

$i \neq j, i, j = 1, 2$. The first order condition for profit maximization by firm i is, therefore:

$$E [\alpha | I^i] = 2\beta q^i (I^i) + \beta E [q^j | I^i] \quad (2)$$

The Cournot equilibrium is given by a pair of output strategies, one for each firm, each of which satisfies (2) for each possible realization of a firm's information. Given the conditions of this model the Cournot equilibrium is unique and symmetric. Eqs. (1) and (2) yield firm's i (ex ante) equilibrium expected profit:

$$E [(P (q^i + q^j)) q^i] = E [E [(\alpha - \beta (q^i + q^j)) q^i | I^i]] = \beta E [(q^i)^2] \quad (3)$$

3.2 The influence of forecast accuracy and profitable information exchange

In this section we built an index to characterize the degree to which information sharing can improve a firm's forecast of α , where:

$$e_{nc} = E [\alpha | s^1] - \alpha$$

denotes a firm's (random) forecast error when firms do not exchange information and,

$$e_c = E [\alpha | s^1, s^2] - \alpha$$

denotes the forecast error when they do. Letting $\text{var}(e)$ denote the variance of a random forecast error e , we define the index G by :

$$G = (\text{var}(e_{nc}) - \text{var}(e_c)) / \text{var}(e_{nc})$$

or alternatively

$$G = 1 - \text{var}(e_c) / \text{var}(e_{nc})$$

Index G measures the fraction of mean-squared forecasting error that can be eliminated by exchanging information; in other words, when index G is close to 1, mean-squared forecast error when firms share their information is much lower than when they don't, thus, index G shows when firms would have an incentive to share their information. In these cases, the second signal essentially removes all residual uncertainty about demand. Informally, we view values of G close to 1 as akin to a sufficient condition for profitability of information exchange. When G is close to 0, the mean-squared forecast error when firms share their information is similar to the mean-squared forecast error when firms don't share their private information. In this case firms don't find it profitable because there is no improvement in the precision derived from the exchange of information.

We make some assumptions about market conditions and we investigate how variations in the quality of firms' private information influence index G and therefore the profitability of information exchange.

We assume firms know that market demand can be high or low, i.e., firms know α takes on one of two values α_b and α_a ($0 < \alpha_b < \alpha_a$), indicating low demand and high demand respectively. In addition, we assume these parameter values are such that outputs and prices implied by eq. (2) are nonnegative.

The distributions of the demand intercept and signals are specified below:

$$\Pr(\alpha_b) = \Pr(\alpha_a) = 1/2.$$

High demand and low demand are equally likely.

Let $\bar{\alpha} = \frac{(\alpha_b + \alpha_a)}{2}$ denote the mean of the demand intercept α , and let $\text{var}(\alpha) = \frac{(\alpha_a - \alpha_b)^2}{4}$ denote the prior variance of α .

We solve eq. (2) to derive the equilibrium strategies, arriving at the amount of output that each firm offers in the case when firms share their private information about market demand (case a) and in the case when firms do not share this information (case b). The firm's equilibrium expected profit in a state of equilibrium is π_{nc} , when firms do not share their private information and π_c , when they do. The expressions for π_c and π_{nc} are obtained by substituting the equilibrium outputs

and appropriate probabilities into eq. (3).

Case a (when firms do not share their private information):

Let $I^i = \{s^i\}$, denote the information available to firm i when it chooses its output. At the equilibrium, the conditions of eq. (2), from firm 1's perspective, may be written as:

$$\left(\frac{1}{\beta}\right) \begin{pmatrix} E[\alpha|s_b^1] \\ E[\alpha|s_n^1] \\ E[\alpha|s_a^1] \end{pmatrix} = \begin{pmatrix} 2 + \Pr(s_b^2|s_b^1) \dots \Pr(s_n^2|s_b^1) \dots \Pr(s_a^2|s_b^1) \\ \Pr(s_b^2|s_n^1) \dots 2 + \Pr(s_n^2|s_n^1) \dots \Pr(s_a^2|s_n^1) \\ \Pr(s_b^2|s_a^1) \dots \Pr(s_n^2|s_a^1) \dots 2 + \Pr(s_a^2|s_a^1) \end{pmatrix} * \begin{pmatrix} q_b \\ q_n \\ q_a \end{pmatrix}$$

For the particular probabilities in this example, these conditions become:

$$\left(\frac{1}{\beta}\right) \begin{pmatrix} \alpha_b \\ \frac{\alpha_b + \alpha_a}{2} \\ \alpha_a \end{pmatrix} * \begin{pmatrix} 2 + \sigma \dots \dots \dots 1 - \sigma \dots \dots \dots 0 \\ \frac{\sigma}{2} \dots \dots \dots 3 - \sigma \dots \dots \dots \frac{\sigma}{2} \\ 0 \dots \dots \dots 1 - \sigma \dots \dots \dots 2 + \sigma \end{pmatrix} * \begin{pmatrix} q_b \\ q_n \\ q_a \end{pmatrix}$$

Solving this equation, we find the equilibrium outputs are:

$$q_b^c = \frac{(5 + \sigma) \alpha_b - (1 - \sigma) \alpha_a}{6(2 + \sigma) \beta}$$

This expression represents the Cournot equilibrium outputs produced by each firm when the signal received is low.

$$q_n^c = \frac{\alpha_b + \alpha_a}{6\beta}$$

This expression represents the Cournot equilibrium outputs produced by each firm when the signal received is normal.

$$q_a^c = \frac{(5 + \sigma) \alpha_a - (1 - \sigma) \alpha_b}{6(2 + \sigma) \beta}$$

This expression represents the Cournot equilibrium outputs produced by each firm when the signal received is high.

Case b (when firms share their information):

Let $I^i = \{s^1, s^2\}$, denote the information available to firm i when it chooses its output:

In this case, the equilibrium condition eq. (2) is simply:

$$q(s^1, s^2) = E[\alpha | s^1, s^2] / 3\beta \quad (4)$$

If either firm observes s_b or s_a , then the firms know for sure the value of α . If both firms observe s_n , then they will have gained no information about demand and will continue to assign probability 1/2 to each possible value of α . Let the Cournot equilibrium with information sharing be denoted by (q_b^c, q_n^c, q_a^c) , where q_b^c denotes each firm's output when at least one firm has observed s_b , q_a^c denotes each firm's output when at least one firm has observed s_a , and q_n^c denotes each firm's output when at least one firm has observed s_n . Then it immediately follows from eq. (4) that:

$$q_b^c = \frac{\alpha_b}{3\beta}$$

$$q_n^c = \frac{\alpha_b + \alpha_a}{6\beta}$$

$$q_a^c = \frac{\alpha_a}{3\beta}$$

Once we derive the equilibrium strategies from each case (sharing and non sharing), then from eq. (3) we find each firm's expected profit in the equilibrium state.

Expected profit when firms do not share their information:

$$\pi_{nc} = \frac{1}{9\beta} \left\{ \bar{\alpha}^2 + \frac{9\sigma}{(2+\sigma)^2} * \text{var}(\alpha) \right\}$$

Expected profit when firms share their information:

$$\pi_c = \frac{1}{9\beta} \{ \bar{\alpha}^2 + \sigma(2-\sigma) * \text{var}(\alpha) \}$$

Comparison of π_c and π_{nc} shows information exchange is profitable when private information is sufficiently accurate, i.e., there exists a $\sigma = \sigma^*$ such that $\pi_c > \pi_{nc}$ if and only if $\sigma > \sigma^*$. Solving $\pi_c - \pi_{nc} \geq 0$ and taking into account the expressions for $\bar{\alpha}$ and $\text{var}(\alpha)$ given in section 3.2 we get $\sigma^* \cong 0.303$. From which we obtain proposition 1

Proposition 1: There exists $\sigma^* \in (0, 1)$ such that $\pi_c > \pi_{nc}$ if and only if $\sigma > \sigma^*$.

The potential for profitable information sharing can be understood in terms of the index G and the effect of sharing on posterior beliefs. A firm's demand forecast is not perfectly accurate when and only when the signal (s) it observes is (are) equal to s_n . In this case, the firm's posterior variance for α is equal to the prior variance of $\text{var}(\alpha)$. Without sharing, the chance that a firm does not know demand, given its signal, is $1-\sigma$, so the expected posterior variance is $(1-\sigma)\text{var}(\alpha)$. With information sharing, the chance that there is any ex post forecast error is equal to $(1-\sigma)^2$, so the expected posterior variance of α is equal to $(1-\sigma)^2\text{var}(\alpha)$. Thus, as σ approaches 1, the expected forecast error approaches zero much faster when firms share their information than when they do not. Indeed, from the above calculations of expected posterior variance, it follows that $G = \sigma$:

$$G = \frac{(\text{var}(e_{nc}) - \text{var}(e_c))}{\text{var}(e_{nc})} = \frac{(1-\sigma)\text{var}(\alpha) - (1-\sigma)^2\text{var}(\alpha)}{(1-\sigma)\text{var}(\alpha)} = \sigma$$

Thus the improvement in forecasting accuracy resulting from information exchange runs the gamut from essentially no improvement, when, $\sigma \approx 0$, to the elimination of virtually all error, when $\sigma \approx 1$. Finally we can conclude that: *information sharing is profitable if and only if the accuracy gains as measured by G are sufficiently large.*

4. Case 2: The Case of Unknown Slope

4.1 Cournot equilibrium and information

We use the Bayesian Cournot equilibrium concept again to solve the model: each firm chooses its output in order to maximize its expected profit conditioned to its information, given the output strategic of its rival. Let I^i denote the information available to firm i when it chooses its output. Firm i 's expected profit, given I^i is equal to:

$$E [P(q^i + q^j)q^i | I^i] = E [(\alpha - \beta q^i - \beta q^j)q^i | I^i] \quad (5)$$

where:

$i \neq j$;

$i, j = 1, 2$. The first order condition for profit maximization by firm i is, therefore:

$$E [\alpha | I^i] = 2E [\beta | I^i] q^i (I^i) + E [\beta q^j | I^i] \quad (6)$$

When the demand intercept, but not its slope, is uncertain, β can be factored out of the last term on the right in (6), and may be written as follows:

$$\alpha = 2E[\beta|I^i]q^i(I^i) + E[\beta q^j|I^i] \quad (7)$$

A Cournot equilibrium in this model is given by a pair of output strategies, one for each firm, each of which satisfies (7) for each possible realization of a firm's information. Eqs. (3) and (7) yield firm i 's equilibrium expected profit:

$$E[P(q^i + q^j)q^i] = E[E[(\alpha - \beta q^i - \beta q^j)q^i|I^i]] = E[\beta(q^i)^2] \quad (8)$$

4.2 Cournot equilibrium without information sharing

If firms do not share their private information, then $I^i = s^i$, $i = 1, 2$. Therefore, firm i 's output strategy may be written as a pair of numbers, (q_b^i, q_a^i) , where q_x^i denotes firm i 's output, given that it has observed the signal $s^i = s_x^i$, $x \in \{a, b\}$, $i = 1, 2$.

The Cournot equilibrium may be denoted by the vector (q_b^c, q_a^c) , which denotes the output strategy for each firm. Firm 1's first-order condition (4), for (q_b^c) , at the Cournot equilibrium, is now given as

$$\begin{aligned} \alpha &= 2E[\beta|s_b^1]q_b^c + E[\beta q^c|s_b^1] \\ &= 2\{Pr(\beta_b|s_b^1)\beta_b\} + Pr(\beta_a|s_b^1)\beta_a q_b^c + Pr(\beta_b, s_b^2|s_b^1)\beta_b q_b^c \\ &+ Pr(\beta_b, s_a^2|s_b^1)\beta_b q_a^c + Pr(\beta_a, s_b^2|s_b^1)\beta_a q_b^c + Pr(\beta_a, s_a^2|s_b^1)\beta_a q_a^c \\ &= q_b^c[\beta_b\sigma(2 + \sigma) + \beta_a(1 - \sigma)(3 - \sigma)] + q_a^c(\beta_b + \beta_a)\sigma(1 - \sigma) \end{aligned} \quad (9)$$

(See footnote 3 for the whole understanding of the calculus on joint probabilities of signals and parameters (s, β) in eq. 9)²

Similarly, the first-order condition for q_a^c is given as

$$\alpha = q_b^c(\beta_b + \beta_a)\sigma(1 - \sigma + q_a^c[\beta_b(1 - \sigma)(3 - \sigma) + \beta_a\sigma(2 + \sigma)] \quad (10)$$

The solution of eqs. (9) and (10) provides the Cournot equilibrium outputs, which are then used in (8) to calculate each firm's expected profit when firms do not share their private information. Therefore,

²As s^i are conditionally independent given β , the joint probabilities (s, β) are as follows: $Pr(\beta_b, s_b^2|s_b^1) = Pr(\beta_b|s_b^1) * Pr(s_b^2|\beta_b) = \sigma * \sigma$; $Pr(\beta_b, s_a^2|s_b^1) = Pr(\beta_b|s_b^1) * Pr(s_a^2|\beta_b) = \sigma * (1 - \sigma)$; $Pr(\beta_a, s_b^2|s_b^1) = Pr(\beta_a|s_b^1) * Pr(s_b^2|\beta_a) = (1 - \sigma) * (1 - \sigma)$; $Pr(\beta_a, s_a^2|s_b^1) = Pr(\beta_a|s_b^1) * Pr(s_a^2|\beta_a) = (1 - \sigma) * \sigma$

$$q_b^c = \frac{\alpha [(1 - \sigma)(3 - \sigma)\beta_b + \sigma(1 + 2\sigma)\beta_a]}{6y(\beta_b + \beta_a)^2 + 9\beta_b\beta_a(1 - 4y)}$$

and

$$q_a^c = \frac{\alpha [\sigma(1 + 2\sigma)\beta_b + (1 - \sigma)(3 - \sigma)\beta_a]}{6y(\beta_b + \beta_a)^2 + 9\beta_b\beta_a(1 - 4y)}$$

where:

$$y = \sigma(1 - \sigma).$$

Each firm's expected profit at this equilibrium is equal to:

$$\begin{aligned} \pi_{nc} &= P_{bb}\beta_b(q_b^c)^2 + P_{ba}\beta_b(q_a^c)^2 + P_{ab}\beta_a(q_b^c)^2 + P_{aa}\beta_a(q_a^c)^2 = \\ &= \frac{\alpha^2(\beta_a + \beta_b)}{2D^2}(\beta_a + \beta_b)^2 y(9 - 20y) + 3\beta_b\beta_a(3 - 8y)(1 - 4y) \end{aligned}$$

where:

$D = 6y(\beta_a + \beta_b)^2 + 9(1 - 4y)\beta_a\beta_b y P_{ij} = Pr(\beta = \beta_i, s^1 = s_j^1), i, j \in a, b$. It is easy to prove that $P_{bb} = P_{aa} = \sigma/2y P_{ba} = P_{ab} = (1 - \sigma)/2$.

4.3 Cournot equilibrium with information sharing

If firms share their private information, then $I^i = \{s^1, s^2\}$, $i = 1, 2$. Therefore, firm i 's output strategy may be written as a vector of four numbers, $(q_{bb}^i, q_{ba}^i, q_{ab}^i, q_{aa}^i)$, where q_{xz}^i denotes firm i 's output, given that it has observed signals $s^1 = s_x^1 y s^2 = s_z^2$, $x, z \in \{b, a\}$. The Cournot equilibrium may be denoted by the vector $(q_{bb}^c, q_{ba}^c, q_{ab}^c, q_{aa}^c)$.

After information is exchanged, there is no uncertainty about a rival's output. Hence, the Cournot equilibrium output q_{xz}^c found from the first-order condition (4) satisfies

$$\alpha = 3E[\beta | s_x^1, s_z^2] q_{xz}^c \quad (11)$$

where:

$$x, z \in \{b, a\}.$$

Each firm's equilibrium output strategy in the Cournot equilibrium with information sharing is as follows:

$$q_{bb}^c = \frac{\alpha (\sigma^2 + (1 - \sigma)^2)}{3 (\beta_b \sigma^2 + \beta_a (1 - \sigma)^2)} \quad q_{aa}^c = \frac{\alpha (\sigma^2 + (1 - \sigma)^2)}{3 (\beta_b (1 - \sigma)^2 + \beta_a \sigma^2)}$$

and

$$q_{ba}^c = q_{ab}^c = \frac{2\alpha}{3(\beta_b + \beta_a)}$$

Each firm's expected profit at this equilibrium is equal to:

$$\pi_c = (P_{bbb}\beta_b + P_{abb}\beta_a)q_{bb}^2 + (P_{baa}\beta_b + P_{aaa}\beta_a)q_{aa}^2 + [(P_{bab} + P_{bba})\beta_b + (P_{aab} + P_{aba})\beta_a]q_{ba}^2 = \frac{\alpha^2(1 - 2y)^3}{18} \left\{ \frac{\beta_a + \beta_b}{(\beta_a + \beta_b)^2 y^2 \beta_a \beta_b (1 - 4y)} \right\} + \frac{4\alpha^2 y}{9(\beta_b + \beta_a)}$$

where:

$$p_{ijk} = Pr \left(\beta = \beta_i, s^1 = s_j^1, s^2 = s_k^2 \right) i, j, k \in \{b, a\}.$$

Therefore

$$P_{bbb} = P_{aaa} = \sigma^2/2;$$

$$p_{baa} = P_{abb} = (1 - \sigma)^2/2$$

and

$$P_{bba} = P_{bab} = P_{aba} = P_{aab} = \sigma(1 - \sigma)/2$$

4.4 Information exchange vs firms' profits

In this section we explore the firms willingness to share their information. Firms will want to share their information when the expected profit from sharing is greater when they don't ($\pi_c > \pi_{nc}$).

Therefore:

Proposition 2: if $\sigma \leq (17 + \sqrt{17}) / 34 \approx 0.62$, then $\pi_c < \pi_{nc}$.

Proposition 3: if $\sigma > (17 + \sqrt{17}) / 34$, then there exists \bar{r} (depending on σ) such that $\pi_c < \pi_{nc}$, for $\frac{\beta_a}{\beta_b} < \bar{r}$, and $\pi_c > \pi_{nc}$ for $\frac{\beta_a}{\beta_b} > \bar{r}$.

For proofs of these propositions and a specification of the threshold \bar{r} , see appendix).

If $\frac{\beta_a}{\beta_b}$ is relatively close to 1, then, in the range of outputs near the equilibrium levels, the demands are almost parallel, and the demand uncertainty is approximately that which would arise if instead the demand intercept (not the slope) had been uncertain. In this case, previous results in the framework of linear-expectations (Clarke (1983b); Vives (1984); Kirby (1988)) show that firms will prefer not to share their information. This is just the result found in proposition 2 and 3. However, when $\frac{\beta_a}{\beta_b}$ is sufficiently greater than 1, this reasoning is no longer valid and firms may actually prefer to share their information. In particular for sufficiently accurate signals, increasing $\frac{\beta_a}{\beta_b}$ eventually yields an environment in which profit is higher when firms share their information rather than keeping it private.

The results achieved above have been partially corroborated by Raith (1996—prop 4.4, pp. 274–275) which argue “. . . *that with perfect signals, complete pooling is always profitable, regardless of any other parameters of the model. . .*”. Our result goes further that (Raith, 1996). In this section, we study the profitability of information exchanges in the case of an unknown slope which, as far as we know, have not been studied by Raith. Further, we show that the profitability of information exchanges does not depend simply on the accuracy of firms’ signals, which are also crucially important, but on the ratio between the slopes of the demand function. In other words the signals’ accuracy in this case is a necessary condition for the profitability of information exchanges but not a sufficient condition³.

5. Conclusions

We have shown, in a simple linear Cournot model with uncertainty of demand, given by the uncertainty in the demand intercept (case 1) or in the uncertain slope

³We would like to thank to an anonymous referee for very valuable comments on the need to situate the above results within the context of current literature.

(case 2), that firms may obtain greater profits when sharing their information rather than by keeping it private when:

- their signals are sufficiently accurate (the case of uncertainty in the demand intercept)
- their signals are sufficiently accurate and the $\frac{\beta_a}{\beta_b}$ is sufficiently great (the case of uncertainty in the demand slope).

Of course, the results achieved in this article are conditioned by the assumptions made in the models, so, it is possible to made broad generalizations on the incentives to share information between competitors. As Raith (1996:260) pointed out “. . . According to the received view on the current state of this field, there is no general theory regarding the incentives of firms to share private information; rather, the results of the models depend delicately on the specific assumptions . . .”.

What is certainly true, however, is that the precision or quality of information es crucial to the profitability or otherwise of information exchanges.

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Appendix

Proofs of propositions 2 and 3 in section 4.4. Let

$$\pi_c = \frac{\alpha^2(1-2y)^3}{18} \left\{ \frac{\beta_a + \beta_b}{(\beta_a + \beta_b)^2 y^2 \beta_a \beta_b (1-4y)} \right\} + \frac{4\alpha^2 y}{9(\beta_b + \beta_a)}$$

$$\pi_{nc} = \frac{\alpha^2(\beta_a + \beta_b)}{2D^2} \left\{ (\beta_a + \beta_b)^2 y(9-20y) + 3\beta_b \beta_a (3-8y)(1-4y) \right\}$$

denote the profits of the firms when they share and do not share their information respectively. Let define $A = (\beta_a + \beta_b)^2$ and $B = \beta_a \beta_b$ so π_c and π_{nc} can be rewritten as:

$$\pi_{nc} = \frac{\alpha^2 \sqrt{A} [A(9-20y)y + 3B(3-8y)(1-4y)]}{2[6yA + 9B(1-4y)]^2}$$

and

$$\pi_c = \frac{\alpha^2}{9} \left\{ \frac{(1-2y)^3 \sqrt{A}}{2[y^2 A + B(1-4y)]} + \frac{4y}{\sqrt{A}} \right\}$$

Using the mathematica software for solving $\pi_c - \pi_{nc} \geq 0$ and simplifying the steps we obtain:

$$\begin{aligned} \pi_c - \pi_{nc} &= \frac{(A-4B)y(-1+4y)(18B^2(1-4y)^2)}{18\sqrt{A}(B(3-12y)+2Ay)^2(B-4By+Ay^2)} \\ &\quad + \frac{A^2y(-4+17y)-3AB(1-15y+44y^2)\alpha^2}{18\sqrt{A}(B(3-12y)+2Ay)^2(B-4By+Ay^2)} \end{aligned}$$

Therefore defining $A/B \equiv k$, we have:

$$\begin{aligned} \pi_c - \pi_{nc} &= \frac{\alpha^2(A-4B)(1-4y)yB^2}{18\sqrt{A}(2Ay+3B(1-4y))^2(Ay^2+B(1-4y))} \\ &\quad \{-18(1-4y)^2+3(1-4y)(1-11y)k+y(4-17y)k^2\} \end{aligned}$$

the first part of the difference between $\pi_c - \pi_{nc} \geq 0$ is

$$\frac{\alpha^2(A-4B)(1-4y)yB^2}{18\sqrt{A}(2Ay+3B(1-4y))^2(Ay^2+B(1-4y))}$$

which is always positive given the assumptions about the parameters in our model.

It must be noted that $(A-4B) = (\beta_a - \beta_b)^2$ is strictly positive for all $\beta_a \succ \beta_b$.

The signal of $\pi_c - \pi_{nc} \geq 0$ depends on the different values take by y and k in this expression $\{-18(1-4y)^2+3(1-4y)(1-11y)k+y(4-17y)k^2\}$.

Let define $g(y, k)$ be defined by:

$g(y, k) = -18(1-4y)^2+3(1-4y)(1-11y)k+y(4-17y)k^2$ as a function of y and k so that:

$$\pi_c > \pi_{nc} \Leftrightarrow g(y, k) > 0$$

To prove proposition 2, we suppose that $y \geq 4/17$, so the first and second terms of g are strictly negative, and the coefficient of k^2 nonpositive. Consequently, $\pi_c - \pi_{nc} < 0$, regardless of the value of $k > 4$ (note that $k \geq 4$ for any values of β_a and β_b). Since $y = \sigma(1-\sigma)$, we see that the condition $y \geq 4/17$ corresponds to $\sigma \leq (17+\sqrt{17})/34$.

Next suppose $y < 4/17$, since g is quadratic in k and the coefficient of k^2 is strictly positive, it follows that, with respect to $k \geq 4$, there exists \bar{k} such that $g(y, k) > (<)0$ if $k > (<)\bar{k}$.

We know that $A/B \equiv k = \frac{(\beta_a + \beta_b)^2}{\beta_a \beta_b} = \frac{(\frac{\beta_a}{\beta_b} + 1)^2}{\frac{\beta_a}{\beta_b}}$ that is increasing in $\frac{\beta_a}{\beta_b}$ for $\frac{\beta_a}{\beta_b} > 1$

Let $r = \frac{\beta_a}{\beta_b}$, we can rewrite the former expression as $k = (r + 1)^2 / r$.

Let \bar{r} denote the value of r that solves $\bar{k} = (r + 1)^2 / r = k$. Therefore if $r < (>)\bar{r}$ this implies that $\bar{k} > (<)k$, which means that \bar{r} has the property specified in proposition 3.

Moreover by \bar{r} depending on σ we mean that if σ is greater than a certain value (more specifically $\sigma > (17 + \sqrt{17}) / 34$), the relationship between the different possibilities in the slopes of the demand function needs to meet an specific ratio (\bar{r}) for the profitability of the information exchanges. If σ is lower than $(17 + \sqrt{17}) / 34$ information sharing is never profitable. So the existence of this \bar{r} depends on the value of σ .