# r b e

# Real Wage Rigidity and the New Phillips Curve: The Brazilian Case[*]

Antonio Alberto Mazali[†], José Angelo Divino[‡]

The new Keynesian Phillips curve has been criticized for not explaining the short-run inflation-output gap trade-off. Blanchard and Galí (2007) introduced real wage rigidity and derived a trade-off between stabilizing inflation and the gap between actual and efficient output. This paper estimates the new Phillips curve for the Brazilian economy, computes short-run trade-off, analyzes real wage rigidity, and tests theoretical restrictions imposed by the model. The GMM estimations fit the data very well and all theoretical restrictions are satisfied. There is strong real wage rigidity and a high output-gap cost to stabilize inflation in the short run.

*A curva de Phillips novo-keynesiana tem sido criticada por não explicar o trade-off de curto prazo entre inflação-hiato do produto. Blanchard e Galí (2007) introduziram a rigidez do salário real e derivaram um trade-off entre estabilizar inflação ou estabilizar hiato entre produto real e eficiente. Neste artigo estima-se essa nova curva para a economia brasileira, calcula-se o trade-off, analisa-se a rigidez do salário real e testa-se restrições teóricas do modelo. Estimações GMM adequam-se bem aos dados e todas as restrições teóricas são satisfeitas. Há forte rigidez do salário real e alto custo de hiato do produto para estabilizar inflação no curto prazo.*

[†]Catholic University of Brasilia and Thesis Consultoria Empresarial, SRTVN 701 Ed. Centro Empresarial Norte, Ala "A", Sala 609, Brasilia − DF, Zip: 70.719-903, Brazil. Phone +55(61)3327.7062. Fax: +55(61)3326.4898. E-mail: mazaliantonio@uol.com.br

[‡]Catholic University of Brasilia. Graduate Program in Economics, SGAN 916, office A-116, Asa Norte, Brasilia − DF, Zip: 70.790-160, Brazil. Phone: +55(61)3448-7192. Fax: +55(61)3347-4797. E-mail: jangelo@pos.ucb.br

## 1. INTRODUCTION

The objective of this paper is to provide empirical evidence on the new Phillips curve for the Brazilian economy in the recent period, characterized by relative price stability achieved after the edition of the Real Plan in June of 1994. The paper investigates how changes in the unemployment rate and occurrence of supply shocks impact on the country's inflation rate. Estimated parameters from the new Phillips curve are used to analyze real wage rigidity, estimate interval of price duration, and compute the short-run trade-off between inflation versus output gap stabilization faced by the monetary authority. This is an important issue because Brazil has historically experienced high levels of both inflation and unemployment. An estimate of the policy trade-off allows the Central Bank to evaluate the impacts of an anti-inflationary monetary policy on the economic activity.

The new Phillips curve is a recent contribution by Blanchard and Galí (2007), who introduced real wage rigidity and supply shocks in the standard framework of the New Keynesian model. They defined *welfare-relevant output gap* as being the gap between actual output and *efficient output* – the latter represented by the output that would be produced under perfect competition in both goods and labor markets. From this modified framework, it emerges a new Phillips curve, whose coefficients carry out distinct meanings when compared to the traditional curve.

The central issue refers to the standard New Keynesian Phillips Curve (NKPC), which might be represented by the following log-linearized equation:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left( y_t - \tilde{y}_t \right)$$

where $\pi_t$ is current inflation, $E_t \pi_{t+1}$ is expected inflation conditioned on period $t$ information set, $y_t$ is the observed output, $\tilde{y}_t$ is the natural output, and $(y_t - \tilde{y}_t)$ is the output gap.[1] The forward looking solution of this equation shows that current inflation depends only on current and expected future values of the output gap. As a result, the monetary authority would be able to stabilize both inflation and output gap in the economy. This absence of short-run trade-off is the main criticism to the NKPC when compared to the original Phillips curve.[2] It is in sharp contrast with the empirical evidence, as argued by Clarida et al. (1999), Woodford (2003), Galí et al. (2001), among others.

Blanchard and Galí (2007) called the lack of trade-off in the NKPC *divine coincidence* and showed that it is tightly linked to the fact that the gap between the natural level of output and the efficient level of output is constant and invariant to shocks in the new Keynesian framework. So, stabilizing the output gap is equivalent to stabilize the welfare-relevant output gap – the gap between actual and efficient output.

This undesirable property might be traced to the absence of non trivial real imperfections in the standard model. By introducing real wage rigidity, a trade-off between inflation and the welfare relevant output gap shows up in the resulting Phillips curve. The trade-off refers to choices that the monetary authority has to do in order to fight cost push shocks. Thus, given any supply shock, the monetary authority has to choose between stabilizing inflation, at a cost of higher output gap, or stabilizing output gap, allowing for higher inflation.

Historically, the empirical evidence on the inflation versus unemployment trade-off is due to Phillips (1958), who analyzed the case of United Kingdom from 1861 to 1957. Phelps (1967) and Friedman (1968) argued against a stable trade-off based on the definition of equilibrium unemployment rate, called *natural rate of unemployment*, and on the notion of inflation expectation. According to Friedman (1968), inflation expectation is an important variable to evaluate nominal wages and the traditional analysis by Phillips (1958) is misguided because he did not make a distinction between nominal and real wages. Both Phelps (1967) and Friedman (1968) recognized only the existence of a short-run trade-off, which does not come from inflation itself, but from unexpected inflation.

---

[1] For a formal derivation of the NKPC see, for instance, Woodford (2003).

[2] The relationship between output gap and unemployment is given by the Okun's law, originally proposed by Okun (1962).

This argument is reinforced by the Lucas (1972) incomplete information model. When there is an increase in price, the producer does not know whether there has been a change in relative prices or a rise motivated by inflation. The producer's rational response is to attribute one fraction of the increase to change in relative prices and another to aggregate inflation. Hence, his decision would be to raise production in the short run, allowing for a positively sloped supply curve in the short run.

The New Keynesian literature argues that both prices and wages show some rigidity due to sluggishness of the adjustment toward new market conditions. The existence of price and wage contracts is among the reasons commonly used to explain rigidity. Even in the absence of such contracts, firms can face menu costs or fear the distaste of customers for frequent changes in prices. Thus, one should expect some sluggishness of price adjustment.

The implication of rigidity is that the short-run supply curve is positively sloped, giving support to the hypothesis of short-run trade-off admitted by Friedman (1968) and Phelps (1967). As a consequence, monetary disturbs might affect real activity in the short-run.

Because of the controversy on standard NKPC, alternative proposals have appeared in the literature. A hybrid model, lacking from rigorous theoretical background, was proposed by Galí and Gertler (1999), who considered that some firms adopt a backward looking and others a forward looking behavior when adjusting prices. Their empirical evidence indicated that, besides statistically significant, price adjustment by the backward looking rule was not quantitatively important for the U.S. economy in the period from 1960 to 1997.

Another formulation uses the average real marginal cost, in percentage deviation from its steady state level, in substitution of the output gap. Galí et al. (2001) found a better fit to the Europe and U.S. data using this alternative version of the Phillips curve. However, under certain conditions, Woodford (2003) shows that it is equivalent to the standard one.

Mankiw and Reis (2002) introduced the concept of information rigidity instead of price rigidity in the derivation of the Phillips curve. This change was successful in generating inflation inertia. A criticism to this framework, however, argues that firms do not change prices continuously, as noted by Blanchard and Galí (2007).

A combination of both wage and price rigidity can be found in the model suggested by Erceg et al. (2000). However, they were not able to resolve the trade-off controversy. As stressed by Blanchard and Galí (2007), it just assumed a new format. In Erceg et al. (2000), to stabilize the gap between actual output and efficient output is equivalent to stabilize the weighted average of wage inflation and price inflation.

In light of the theoretical contribution brought by Blanchard and Galí (2007), this paper focuses on the estimation of the new Phillips curve for the Brazilian economy. Given the rational expectations nature of the new Phillips curve, the parameters were estimated by GMM with robust standard errors. The estimated parameters presented expected signs, were statistically significant, and fulfilled a set of theoretical restrictions imposed by the model. The empirical results revealed that the Central Bank faces a short-run trade-off between stabilize inflation versus output gap and there is a high degree of real wage rigidity in the Brazilian economy.

The contribution of the paper is to show that the new Phillips curve fits the Brazilian data very well, as opposed to alternative versions estimated by Schwartzman (2006), Minella et al. (2003), Areosa and Medeiros (2007), among others. One difficulty faced by those authors was with the *proxy* for marginal cost, which shows up with sign opposite to expected or statistically non-significant in the estimation. Areosa and Medeiros (2007), for instance, used share of labor in production and output-gap as proxies for the marginal cost and economic activity, respectively, in the estimation of Phillips curves for both closed and open economies. They found no statistical significance for the first and a negative estimated coefficient for the second variable, which is theoretically meaningless. In Minella et al. (2003), the marginal cost was represented by the unemployment rate. Nevertheless, it was statistically significant only at the 10% level in one of the estimated equations. Under the Blanchard and Galí (2007) framework, however, there is a significant role for the unemployment rate in the estimated new Phillips curve. As

accompanying outcome, we provide a measure of real wage rigidity and its impact on the Brazilian inflation rate.

We found that a 1% increase in unemployment leads to a statistically significant reduction in inflation of about 0.12%. In addition, our main results reveal a degree of real wage rigidity of 92% in the recent low inflation period. Apart from methodological and empirical differences, that high degree of wage rigidity is in line with other studies for the Brazilian economy. Arbache and De Negri (2004) investigated the inter-industry wages in Brazil and found high stability of wage awards, suggesting that the wage structure is rigid and insensitive to the economic cycle. Orellano et al. (2009) showed that the replacement rate of workers by Brazilian firms as a tool to reduce wages has decreased with inflation decline. They reported a reduction from 40 to 20% in the replacement rate from 1991 to 1998, leading to increases in the real wage rigidity in the same period. This view is shared by Reis and Camargo (2007), who argued that the inflation stabilization achieved after the Real plan has increased both real wage rigidity and unemployment among young people because low inflation imposes less flexibility to the real wage when the nominal wage cannot be legally be reduced.

Internationally, the Brazilian parameter is close to the 90% real wage rigidity calibrated by Blanchard and Galí (2007) for the U.S. economy. Holden and Wulfsberg (2009) used data for 19 OECD countries from 1973 to 1999 to analyze wage rigidity at the industry level. They found evidence of downward real wage rigidity in the core European countries, and in the Anglo group, but not for the southern European countries, arguing that the explanation is due to unemployment benefits, union density, and the degree of coordinated wage setting. However, differences in the theoretical and empirical approaches do not allow direct comparison of the major results. Other studies giving support to increasing real wage rigidity include Goette et al. (2007) and Campbell III and Kamlani (1997).

Finally, our results indicate that following a 1% supply shock, the Central Bank must choose between allowing a 0.29% decrease in the output gap, keeping inflation unchanged, or a rise of 0.31% in the inflation rate, maintaining fixed the output gap. Any combination between those two values is also a feasible choice for the monetary policy.

The paper is organized as follows. Next section briefly describes the theoretical model proposed by Blanchard and Galí (2007) to derive the new Phillips curve. The third section discusses the econometric procedure and the data set. The results are reported and analyzed in the fourth section. Finally, the fifth section is dedicated to the concluding remarks.

## 2. THEORETICAL MODEL

### 2.1. Firms and households

The Blanchard and Galí (2007) model assumes that there is a continuum of firms acting in monopolistic competition to produce a differentiated good. Firms face an isoelastic demand curve and are subject to a Cobb-Douglas production function given by:[3]

$$Y = M^{\alpha} N^{1-\alpha} \tag{1}$$

where $Y$ is output, $M$ is a non-produced input, which is subject to supply shocks, and $N$ is labor. There are constant returns to scale, so that the share of the non-produced input in the production is $0 < \alpha < 1$.

There is a large number of identical households who own the firms. Their preferences are separable and represented by:

$$U(C,N) = \log(C) - \exp\{\xi\} \frac{N^{1+\phi}}{1+\phi} \tag{2}$$

---

[3]Unless otherwise noticed, the absence of subscript t means a current period variable.

where $C$ is composite consumption, with elasticity of substitution between goods given by $\varepsilon$, $N$ is labor supply, $\xi$ is a preference parameter, which might be time varying, and $\phi$ is the slope of the labor supply curve.

The marginal rate of substitution $(MRS)$ between consumption and labor is derived from the solution of the household's problem. In logarithms, it is given by:

$$mrs = c + \phi n + \xi \tag{3}$$

where $\phi$ and $\xi$ follow the definitions given in equation (2) and hereon, for simplicity, small-case letter denote the natural logarithm of the original variable, expressed in capital letter.

## 2.2. First best level of output

The first best level of output (or efficient output) is defined as the optimal output of an economy working under perfect competition in all markets. There is efficient allocation of all production factors and no involuntary unemployment. From the production function (1) and utility function (2) this level of output (in logarithms) is:

$$y_1 = \alpha m + (1 - \alpha) n_1 \tag{4}$$

where

$$n_1 = \left( \frac{1}{(1+\phi)} \right) \left[ \log (1 - \alpha) - \xi \right]$$

and $y_1$ and $n_1$ are efficient levels of output and employment, respectively, $m$ is a non-produced input and $\alpha$, $\phi$ $and$ $\xi$ are the parameters already defined in (1) and (2).

## 2.3. Second best level of output

The second best level of output (or natural output) is defined by Blanchard and Galí (2007) as the optimal level of output under monopolistic competition. In this market structure, firms have market power and are able to set a mark up $\mu^p \equiv \log[\varepsilon/(\varepsilon - 1)]$ over the marginal cost, where $\varepsilon$ is the elasticity of substitution between goods. From equations (1) and (2), this level of output is:

$$y_2 = \alpha m + (1 - \alpha) n_2 \tag{5}$$

where

$$n_2 = \frac{1}{(1+\phi)} \left[ \log (1 - \alpha) - \mu^p - \xi \right]$$

and $y_2$ and $n_2$ are optimal levels of output and employment under monopolistic competition, but with flexible prices and wages. Again $m$ is a non-produced input and $\alpha$, $\phi$, $and$ $\xi$ were defined in (1) and (2). Notice that the difference between (4) and (5) is constant and equals to:

$$y_1 - y_2 = \delta = \frac{\mu^p (1 - \alpha)}{(1 + \phi)} \tag{6}$$

## 2.4. New Keynesian Phillips curve

The supply side of the standard New Keynesian model for a closed economy under monopolistic competition and flexible wages generates the well known NKPC:

$$\pi = \beta E \pi_{t+1} + \kappa (y - y_2) \tag{7}$$

where $\beta \in (0, 1)$ is the discount factor, $\kappa \equiv \lambda \frac{(1+\phi)}{(1-\alpha)}$, with $\phi$ and, $\alpha$ defined in equations (2) and (1), respectively, $\lambda \equiv \theta^{-1}(1-\theta)(1-\beta\theta)$, with $\theta \in (0, 1)$ representing the Calvo (1983) probability that a given firm does not change its price in period $t$, $\pi$ is current inflation, $E_t \pi_{t+1}$ is expected inflation conditioned on period $t$ information set, $y$ is the current period actual output, and $y_2$ is the natural output, obtained from equation (5).

Notice that supply shocks and preference shocks do not directly appear in (7). Indirectly, however, they affect $y_2$. But, as seen before, the difference between $y_1$ and $y_2$ is constant and equals to $\delta$. Thus, with flexible wages and price rigidity, it remains valid the result that stabilizing inflation is equivalent of stabilizing the gap between actual output and efficient output.

## 2.5. Real wage rigidity

In the previous framework, the real wage was always equal to the marginal rate of substitution. This might be changed by assuming that the (logarithm of) real wage, $w$, evolves according to the *ad-hoc* rule:

$$w = \gamma w_{t-1} + (1-\gamma)mrs \tag{8}$$

where $\gamma \in (0, 1)$ is a measure of real wage rigidity.

Equation (8) assumes that there is no change in preferences. Therefore, real wage adjustments are due to imperfections in the labor market. The first best level of output is still defined by equation (4), given that it is derived for a frictionless economy under perfect competition. The second best level of output, however, will be affected by wage rigidity, as described in the following section.

## 2.6. Second best level of output under real wage rigidity

The second best level of output (or natural output) for an economy under monopolistic competition and real wage rigidity defines its feasible optimal output. Blanchard and Galí (2007) showed that it can be expressed as:

$$[y_2 - y_1 + \delta] = \Theta[(y_2)_{t-1} - (y_1)_{t-1} + \delta] + \Theta(1-\alpha)[\Delta m + (1+\phi)^{-1}\Delta\xi] \tag{9}$$

where $\Theta \equiv \frac{\gamma\alpha}{[\gamma\alpha + (1-\gamma)(1+\phi)]} \in [0, 1]$ and all parameters and variables follow previous definitions.

Under real wage rigidity, the gap between the first and second optimal levels of output is no longer constant, but is affected by both supply ($\Delta m$) and preference ($\Delta\xi$) shocks. One can show that $\Theta$ is increasing in $\gamma$, implying that the size and persistency of the gap between $y_2$ and $y_1$ increase with the degree of real wage rigidity.

An adverse supply shock (decrease in $m$) generates a negative variation in the gap of equation (9). Gradually, however, the size of the gap converges to its stationary level, $\delta$, as the real wage is adjusted over time.

A preference shock that rises $\xi$ leads to a reduction in both $y_1$ and $y_2$. This reduction, however, must be smaller in $y_2$ because, according equation (9), without shock $y_2 - y_1 + \delta = 0$ and with this shock $y_2 - y_1 + \delta > 0$. That is only possible if $y_2$ falls by less than $y_1$, since $\delta$ is constant.

## 2.7. The new version of the Phillips curve

The framework proposed by Blanchard and Galí (2007) allows representing inflation as a function of the output gap as follows:

$$\pi = \beta E \pi_{t+1} + \frac{\lambda}{1-\gamma L}(x_2) \tag{10}$$

where $x_2 = (1-\alpha)^{-1}[(1-\gamma)(1+\phi)(y-y_2) + \gamma\alpha(\Delta y - \Delta y_2)]$, $\lambda = \theta^{-1}(1-\theta)(1-\beta\theta)$, $L$ is the lag operator, such that $LX_t = X_{t-1}$, and all other variables and parameters were defined in equation (7).

According to equation (10), stabilizing inflation is still equivalent to stabilizing output gap, that is, to keep $(y - y_2)$ constant. But now, it is no longer desirable to stabilize $(y - y_2)$ because the welfare relevant measure of output gap is $(y - y_1)$. The distance between the first best and second best levels of output is affected by disturbances and so is no longer constant.

One can obtain a relation between inflation and the gap between actual output and the first best level of output expressed as:

$$\pi = \beta E\pi_{t+1} + \frac{\lambda}{1-\gamma L}x_1 - \frac{\lambda\gamma\alpha}{1-\gamma L}[\Delta m + (1+\phi)^{-1}\Delta\xi] \qquad (11)$$

where $x_1 \equiv (1-\alpha)^{-1}[(1-\gamma)(1+\phi)(y-y_1+\delta) + \gamma\alpha(\Delta y - \Delta y_1)]$, with all variables and parameters as previously defined.

By equation (11), inflation depends on expected inflation, lagged distribution of the gap between actual output and first best level of output and lagged distribution of supply and preference shocks. It can be rewritten in terms of unemployment and change in the price of the non-produced input $M(\Delta\upsilon)$, yielding the new version of the Phillips curve:[4]

$$\pi_t = \frac{1}{1+\beta}\pi_{t-1} + \frac{\beta}{1+\beta}E\pi_{t+1} - \frac{\lambda(1-\alpha)(1-\gamma)\phi}{\gamma(1+\beta)}u_t + \frac{\alpha\lambda}{1+\beta}\Delta\upsilon_t \qquad (12)$$

where $\beta \in (0,1)$ is the discount factor, $\phi, \alpha,$ and $\lambda$ were defined in (2), (1), and (7), respectively, $\gamma \in (0,1)$ is a measure of real wage rigidity, $\pi_t$ is current inflation, $\pi_{t-1}$ is last period inflation, $E_t\pi_{t+1}$ is expected inflation conditioned on period $t$ information set, $u_t$ is the deviation of unemployment rate from its steady state level, or *natural rate of unemployment*, and $\Delta\upsilon_t$ is the change in the real price of the non-produced input $M$ at time $t$, a measure of supply shock.

## 3. EMPIRICAL EVIDENCE

### 3.1. Econometric model

The regression to be estimated for the Brazilian economy is directly derived from equation (12). It can be written as:
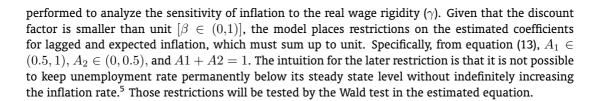
$$\pi_t = A_1\pi_{t-1} + A_2E\pi_{t+1} - A_3u_t + A_4\Delta\upsilon_t + \zeta_t \qquad (13)$$

where $A_1 = \frac{1}{1+\beta}$, $A_2 = \frac{\beta}{1+\beta}$, $A_3 = \frac{\lambda(1-\alpha)(1-\gamma)\phi}{\gamma(1+\beta)}$, $A_4 = \frac{\alpha\lambda}{1+\beta}$, with all variables and parameters defined as in (12), and $\zeta_t$ is the random error term.

According to equation (13), inflation depends on the last period inflation, next period expected inflation based on time $t$ information set, unemployment rate, and change in the real price of the non-produced input $M$. The expected signs for all variables but unemployment rate are expected to be positive.

Notice that it is not possible to identify all structural parameters from the estimated coefficients of equation (13). The only exception is $\beta$, which is over-identified in the model, since both $A_1$ and $A_2$ are single functions of $\beta$. After exogenously setting 2 parameter values, a calibration exercise will be

---

[4]Using the conventional NKPC, Cochrane (2007a,b) argues that the rational expectations (RE) equilibrium determinacy in the standard new Keynesian framework do not guarantee stable path for inflation. This point, however, is questioned by McCallum (2008), who shows that the model desired dynamics are preserved provided that the RE equilibrium satisfies the property of least-squares learnability.

performed to analyze the sensitivity of inflation to the real wage rigidity ($\gamma$). Given that the discount factor is smaller than unit $[\beta \in (0,1)]$, the model places restrictions on the estimated coefficients for lagged and expected inflation, which must sum up to unit. Specifically, from equation (13), $A_1 \in (0.5, 1)$, $A_2 \in (0, 0.5)$, and $A1 + A2 = 1$. The intuition for the later restriction is that it is not possible to keep unemployment rate permanently below its steady state level without indefinitely increasing the inflation rate.[5] Those restrictions will be tested by the Wald test in the estimated equation.

### 3.2. Data

The time series used in the estimation are quarterly for the period from 1995:1 to 2008:4. All variables were transformed by natural logarithm and tested for seasonality using seasonal dummies. Only unemployment rate presented a statistically significant seasonal component and was seasonally adjusted.

Inflation, $\pi_t$, is represented by the variation in the extended national consumer price index (IPCA), computed and published by IBGE. This is the official measure of inflation used by the Central Bank of Brazil to calibrate the inflation targeting regime.

Unemployment rate, $u_t$, is measured by the open unemployment rate at the metropolitan region of São Paulo, obtained from DIEESE and seasonally adjusted. It refers to individuals with 10 or more years of age that looking for employment. This series was used as *proxy* for the Brazilian unemployment because the national time series computed by IBGE, which refers to individuals with 15 or more years of age that were looking for a job, suffered a major methodological change in 2003 and has been interrupted.

Supply shocks, $\Delta v_t$, are measured by quarterly percentage change in the Brazilian real (R\$) per US dollar (US\$) nominal exchange rate, as published by the Central Bank of Brazil. It refers to the average exchange rate negotiated in the interbank exchange market. This measure of supply shocks is closely related to changes in the non-produced input prices, $M$, described in the equation (1), because changes in the US dollar exchange rate affect the input $M$ prices as well as other prices in the economy due to transmission channels. Formally, one has that:

$$\Delta v_t = \ln\left(\frac{(R\$/US\$)_t}{(R\$/US\$)_{t-1}}\right) \tag{14}$$

## 4. RESULTS

### 4.1. Unit root tests

The empirical analysis starts by testing the time series for the presence of unit roots. A common criticism of traditional unit root tests, primarily those based on the classic methods of Dickey and Fuller (1979, 1981) and Phillips and Perron (1988), is that they suffer from low power and size distortions. However, these shortcomings have been overcome by modifications to the testing procedures, such as the methods proposed by Elliot et al. (1996), Perron and Ng (1996), and Ng and Perron (2001).

Elliot et al. (1996) demonstrate that OLS de-trending is inefficient if the data presents high persistence, and suggest using GLS de-trended data, which is efficient. Ng and Perron (2001) show that, in the presence of a strong negative moving average coefficient, the unit root estimate is strongly biased if the lag truncation, $k$, is small because the residuals of the test equation are serially correlated. In order to select the optimal value of $k$ to account for the inverse non-linear dependence between the bias in the unit root coefficient and the selected value of $k$, Ng and Perron (2001) proposed a modified Akaike

---

[5]In this situation, a given variable is in steady state if it is constant over time.

Information Criterion (MAIC). Thus, the modified ADFGLS ($MADFGLS$) test uses GLS de-trended data and the MAIC in order to choose the truncation lag.

The modified Phillips-Perron test ($MPPGLS$), which also uses GLS de-trended data and the MAIC to select the optimal truncation lag, is due to Phillips and Perron (1988), Perron and Ng (1996) and Ng and Perron (2001). The asymptotic critical values for both the $MADFGLS$ and $MPPGLS$ tests are given in Ng and Perron (2001).

The presence of structural breaks, a common feature among Brazilian time series, can severely bias unit root tests. One should be aware that distortions can go in both directions, reducing statistical power of the tests (Perron, 1989) or leading to spurious stationarity (Franses and Haldrup, 1994). Thus, it is necessary to perform unit root tests that account for structural breaks. Perron (1989) proposes a test where the time of the break is exogenous and assumed to be known a priory. The break might affect both intercept and slope of the series.

A potential problem with Perron (1989) test is that it allows for just one break and assumes no structural break under the null hypothesis of unit root. Lee and Strazicich (2003) show that this assumption might result in spurious rejections. The two-break minimum LM unit root test, due to Lee and Strazicich (2003), is unaffected by whether or not there is a break under the null. Times of the breaks are endogenously chosen by points where $t$-statistic for the null of unit root is at a minimum. Critical values were tabulated by Lee and Strazicich (2003).

The results of the unit root tests are reported in Table 1. In the first panel of Table 1, one can see that only inflation rate, $\pi$, does not have unit root by the $MADFGLS$ and $MPPGLS$ tests at the 10% significance level. The time series of unemployment, $u$, and supply shocks, $\Delta v$, have unit root according to both tests. This result, however, might be due to the presence of structural breaks.

The results for the Perron (1989) test, reported in the second panel of Table 1, confirm that inflation rate is stationary and find that the variable supply shocks is also $I(0)$ once the change in monetary policy occurred in the beginning of 1999 is accounted for. The unemployment rate, however, remains as non-stationary. That might be because the series of unemployment has more than one structural break in the period.

The last panel of Table 1 shows that, in fact, unemployment rate is also stationary. This result was obtained after allowing for a second break in the time series, generated by economic instability following the electoral process and election of a left wing party candidate for the presidency of Brazil. Besides endogenously chosen, the time of the two breaks coincide with the change in the monetary policy regime to inflation targeting coupled with floating exchange rate in 1999 and the electoral process of 2002. Thus, based on the results of Table 1, one can conclude that inflation rate, unemployment rate, and supply shocks are stationary in the period under consideration.[6]

## 4.2. GMM estimation of the new Phillips curve

Initially, it was estimated the original equation (13) by GMM using quarterly time series for the Brazilian economy. Given that the theoretical model assumes that agents are rational, a natural way to form current expectations of future inflation is based on lagged variables included as instruments in the regression. As in Blanchard and Galí (2007), who used a naive instrumental variables estimator, the instrument list was composed by lagged variables according to the following distribution: lags 2 and 3 for inflation and lags 1 to 3 for unemployment and supply shocks[7]. The validity of the instruments

---

[6]In Lee and Strazicich (2003), the break point is chosen such that the $t$-statistic for testing the null hypothesis of a unit root is smallest among all possible break points. Usually, this implies no coincidence with any exogenous break selection criteria. The endogenous break might be hard to justify when it does not match any relevant economic event. This feature is one of the concerns in the endogenous break selection literature. See Perron (1997) for a detailed discussion.

[7]Notice that the number of instruments in the estimated regression satisfies the Hall's (2005) condition because it is smaller than the number of parameters plus 4 (8<10).

Table 1: Unit root tests

| Variables | $MADF^{GLS}$ | | | $MPP^{GLS}$ | | |
|---|---|---|---|---|---|---|
| | $Z = \{1\}$ | $Z = \{1,t\}$ | lags | $Z = \{1\}$ | $Z = \{1,t\}$ | lags |
| $\pi$ | -1.81* | -2.71* | 2 | -1.49 | -2.27 | 2 |
| $u$ | -1.07 | -1.04 | 0 | -0.97 | -0.85 | 0 |
| $\triangle v$ | -1.47 | -1.23 | 8 | 1.11 | 1.12 | 8 |
| Perron (1989) | | | | | | |
| | Model 2 | lags | Break | 5%cv | | |
| $\pi$ | -4.77** | 0 | 1999:01 | -3.87 | | |
| $u$ | -1.92 | 0 | 1999:01 | -3.87 | | |
| $\triangle v$ | -4.71** | 0 | 1999:01 | -3.87 | | |
| Lee and Strazicich (2003) | | | | | | |
| | Model 2 | lags | Break1 | Break2 | 5%cv | |
| $\pi$ | -6.51** | 1 | 1998:03 | 2003:02 | -5.29 | |
| $u$ | -6.34** | 9 | 1999:02 | 2002:04 | -5.29 | |
| $\triangle v$ | -6.62** | 0 | 2001:03 | 2002:03 | -5.29 | |

Notes: *,**,*** the null hypothesis of unit root is rejected at the 10, 5, and 1% significance level, respectively.

Critical values at the 5% (10%) level for the MADFGLS and MPPGLS tests are $-1.98$ ($-1.62$) and $-2.91$ ($-2.62$) for the models

with constant, $Z = \{1\}$, and constant and trend, $Z = \{1, t\}$, respectively. Model 2 of Perron (1989) is the "changing in

growth model", which includes dummy for change in the slope of the time series.

Model 2 of Lee and Strazicich (2003) includes two changes in intercept and trend slope of the time series.

was tested by the Hansen (1982) over-identifying restrictions test. The results are reported in equation (15). In parenthesis are the estimated standard deviations, which are robust to heteroskedasticity and autocorrelation of unknown form.

$$\pi_t = \underset{(0.0047)}{0.0106} + \underset{(0.0512)}{0.5999}\,\pi_{t-1} + \underset{(0.0795)}{0.4415}\,E_t\pi_{t+1} - \underset{(0.0522)}{0.1365}\,u_t + \underset{(0.0084)}{0.0662}\,\Delta v_t + \underset{(0,0012)}{0.0029}\,D_t \qquad (15)$$

with $H$-test=5.5848    [0.586].

Given the results of the unit root tests from subsection 4.1, equation (15) included a dummy variable, $D_t$, for the change in monetary policy occurred in the first quarter of 1999[8]. One can see that all estimated coefficients present signs according to the expected from equation (13). Robust standard deviations, in parenthesis, indicate that all coefficients are statistically significant at 5% significance level. The Hansen's test ($H$-test) does not reject the null hypothesis that the over-identifying restrictions are satisfied[9]. In addition, the theoretical restriction imposed by the model on the coefficients of $\pi_{t-1}$ and $E_t\pi_{t+1}$, which should sum up to unity, was not rejected according to the Wald test[10].

The restricted regression was, then, estimated and the results reported in Table 2. It is remarkable the stability of all estimated coefficients between these two models, with and without restrictions. For

[8]A second dummy variable, for the pre- and post-electoral period of 2002, was also included but did not adjust well to the model.

[9]The critical value of $\chi^2_{(7)}$ at 5% of significance is 14.07. The $p$-value is within brackets.

[10]Both the $F$ and $\chi^2$ versions of the test yielded $p$-values of 0.4.

comparisons purposes, Table 2 also includes the restricted estimation obtained by Blanchard and Galí (2007) for the U.S. economy using an instrumental variables technique.

Table 2: Restricted new Phillips curves for Brazil and USA

| Coefficients | $A_0$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | Dummy | $H$-test |
|---|---|---|---|---|---|---|---|
| Brazil | 0.13 | 0.59 | 0.41 | -0.12 | 0.07 | 0.0031 | 5.61 |
| (this study: GMM) | (0.04) | (0.05) | (0.05) | (0.04) | (0.01) | (0.0011) | (0.691) |
| USA | not | 0.52 | 0.48 | -0.08 | 0.014 | – | – |
| (Blanchard-Galí: IV) | reported | (0.05) | (0.05) | (0.05) | (0.009) | – | – |

Note: Blanchard and Galí (2007) used quarterly data for the U.S. economy from 1960 to 2004.

This study used quarterly data for the Brazilian economy from 1995 to 2008.

Robust standard errors are in parenthesis. The critical value of at the 5% level of significance is 14.07.

The $p$-value is within brackets.

One can see that the estimated coefficients for lagged inflation ($A_1$) and expected inflation ($A_2$) are very similar between the two economies. The effect of unemployment on current inflation ($A_3$) is, in absolute value, higher for Brazil than for US. The same is true for the coefficient of supply shocks ($A_4$). All coefficients are highly significant in the Brazilian case. As in the unrestricted model, the $H$-test does not reject the over-identifying restrictions. In general, the new Phillips curve has fitted very well the Brazilian data.

The estimated coefficients for $A_1$ and $A_2$ imply that the discount factor is 0.69 ($\beta = 0{,}69$), smaller than the 0.92 found by Blanchard and Galí (2007) for the US economy. The lower value of $\beta$ is explained by the fact that the Brazilian economy has had one of the highest real interest rate in the world. The US economy, on the other hand, is among the countries with the lowest real interest rate. Thus, individuals discount more future economic variables in the Brazilian than in the US economy.

For the US, the adjustment was not as good given that the estimated coefficients of unemployment and supply shocks were not statistically significant at the standard 5% level of significance. In addition, the authors did not provide any information on the model's over-identifying restrictions. A discussion on the estimated coefficients *vis-a-vis* the theory for the Brazilian case is left to the next section.

## 4.3. Identification and calibration

As a result from the theoretical definitions given by equation (12), the estimated coefficients $A_1$ and $A_2$ are subject to restrictions. The first one is that $0.5 \leq A_1 \leq 1$, because $A_1 = \frac{1}{1+\beta}$ from equation (13), with $\beta \in (0, 1)$. The estimated value of $A_1 = 0.59$ satisfies this restriction. The second one comes from $A_2 = \frac{\beta}{1+\beta}$, also implied by equation (13), imposing the restriction that $0 \leq A_2 \leq 0.5$. The estimated value of $A_2 = 0.41$ is also consistent with that theoretical restriction. The third, and final, restriction is that $A_1 + A_2 = 1$, because $\frac{1}{1+\beta} + \frac{\beta}{1+\beta} = 1$, which was tested and not rejected by the Wald test.

Intuitively, the latter restriction implies that the only way of keeping unemployment rate permanently below its steady state level is allowing for an indefinite increasing in the inflation rate. Thus, any monetary policy which seeks artificially increasing the level of employment might have its counterpart in a higher inflation rate.

The other two coefficients, $A_3$ and $A_4$, are functions of structural parameters, which cannot be individually identified. The parameters $\lambda$ and $\gamma$, defined earlier, are of special interest. Let's, first, examine the compound coefficient $A_4 = \frac{\alpha \cdot \lambda}{1+\beta}$, defined in equation (13).

By using the estimated value of $A_4 = 0.07$ from Table 2 and the discount factor $\beta$ computed from $A_1$ (or $A_2$), one can find the product $\alpha\lambda \cong 0.12$, where $\alpha$ is the share of the non-produced input, $M$, in the Cobb-Douglas production function. One can, then, analyze the wage rigidity appearing in the coefficient $A_3 = -\frac{\lambda\phi(1-\alpha)(1-\gamma)}{\gamma(1+\beta)}$, which may be rewritten as:

$$\gamma = \frac{-\lambda\phi(1-\alpha)}{A_3(1+\beta) - \lambda\phi(1-\alpha)} \tag{16}$$

Assuming a labor supply slope of $\phi = 1$, as Blanchard and Galí (2007), and that the fraction of firms that do not change prices in the current period is 0.25, i.e. $\theta = 0.25$, obtained from micro-data,[11] one gets $\lambda \equiv \theta^{-1}(1-\theta)(1-\beta\theta) = 2.48$. This implies that the real wage rigidity in the Brazilian economy is $\gamma = 0.92$. From equation (8), it means that about 92% of the current wage is explained by previous wage and only 8% is due to changes in the marginal rate of substitution.

The explanation for this high real wage rigidity follows Reis and Camargo (2007), who argued that the inflation stabilization achieved after the Real plan has increased the real wage rigidity because inflation imposes more flexibility to the real wage in situations where there is rigidity to decreases in the nominal wage. This is the typical scenario of the Brazilian economy in post 1995 period. Inflation has considerably decreased and reductions in the nominal wage are limited by the law.

## 4.4. Analysis of the Trade-off

In the occurrence of a supply shock, the monetary authority faces a trade-off between stabilizing inflation or stabilizing the gap between actual output and efficient output. Supposing that the goal of the monetary authority is to keep inflation constant when faced with a supply shock, the gap between actual output and efficient output shall suffer a higher decrease the higher is the real wage rigidity. This effect is given by:[12]

$$\frac{\partial(y-y_1)}{\partial v} = \frac{d(y-y_1)/d\varepsilon_m}{dv/d\varepsilon_m} = -\frac{(1-\alpha)\Theta}{(1-\alpha)(1-\Theta)} = -\frac{\alpha\gamma}{(1-\gamma)(1+\phi)} \tag{17}$$

Substituting values for $\gamma = 0.92$, the corresponding $\lambda = 2.48$, and using $\alpha\lambda = 0.12$ to estimate $\alpha \cong 0.05$, one gets:

$$\frac{\partial(y-y_1)}{\partial v} = -\frac{0.05(0.92)}{0.08(1+1)} = -0.29$$

Thus, after a 1% increase in non-produced input prices, as long as the inflation is kept constant, the gap between actual output and efficient output shall experience a negative variation of 0.29% on the first quarter, implying that the actual output will depart from $y_1$.[13] The persistence of this effect is measured by the auto-regressive parameter $\Theta = 0.13$, given by equation (9).

On the other hand, if the objective is to keep the gap between actual output and efficient output constant, then the inflation shall rise, as indicated below:

$$\frac{d\pi_t}{dv} = \frac{d\pi/d\varepsilon_m}{dv/d\varepsilon_m} = \frac{\lambda\alpha\gamma}{(1-\beta\gamma)[\alpha\gamma + (1-\alpha)]} \tag{18}$$

Substituting the estimated values:

---

[11] This value of $\theta = 0.25$ implies a price duration of 4 months, which is consistent with the 3.8 months found by Gouvea (2007) and 4.2 months estimated by Barros and Matos (2008).

[12] The solution of the differential equations that yielded (22) and (23) were taken from Blanchard and Galí (2007). It was assumed $\xi = 0$ and a random walk for $M$, such that $\Delta m = \varepsilon_m$ is a white noise.

[13] The output gap rises in absolute value, provided that $y < y_1$.

$$\frac{d\pi_t}{dv} = \frac{(2.48)\,(0.05)\,(0.92)}{[1 - 0.69(0.92)][(0.05)(0.92) + (1 - 0.05)]} = 0.31$$

A rise of 1% in the non-produced input prices leads to 0.31% increase in next period inflation, keeping the output gap constant. Therefore, under a 1% supply shock, the monetary authority might decide between allowing 0.31% increase in next period inflation, a negative variation of 0.29% in the gap between actual output and efficient output, or any combination of these two results.

The estimated short-run trade-off between stabilize inflation versus output gap offers a guide to the monetary policy, allowing the monetary authority to foresee potential social costs of alternative anti-inflation policies.

## 5. CONCLUDING REMARKS

The standard version of new Keynesian Phillips curve has been criticized for not being able to explain the trade-off between stabilizing inflation versus stabilizing the gap between actual output and natural output. Blanchard and Galí (2007) proposed a new approach to deal with this issue, showing that when real wage rigidity is incorporated into the new Keynesian framework, stabilizing the gap between actual and natural output is not the same as stabilizing the gap between actual and efficient output. From their model, it emerges a new analytical version of the Phillips curve, according to which there is a short-run trade-off between stabilizing inflation and stabilizing the gap between actual and efficient output. The latter is defined as the optimal level of output for an economy working in perfect competition in the goods and labor markets. Theoretically, this output gap is directly linked to unemployment in their framework. Thus, the new Phillips curve has current inflation explained by past and next period expected inflation, unemployment rate, and changes in price of the non-produced input – or supply shocks.

This new Phillips curve was confronted with Brazilian data to estimate the effects of unemployment and supply shocks on the inflation rate, compute the short-run trade-off between inflation and output gap faced by the Central Bank, and analyze real wage rigidity. Due to the rational expectations nature of the model, the equation was estimated by GMM with robust standard errors. The estimated coefficients satisfied a set of restrictions imposed by the theoretical model and over-identifying restrictions implied by the excess of instrumental variables were not rejected by the Hansen test. Thus, the estimated new Phillips curve adjusted very well to the Brazilian data in the period under consideration.

The novelties of the paper are in the empirical side. The results showed that there is a negative and statistically significant relationship between inflation and unemployment, overcoming a common empirical difficulty with the appropriate measure of marginal cost to be included in the Phillips curve [e.g. Schwartzman (2006), Minella et al. (2003), Areosa and Medeiros (2007)]. In addition, we found evidence of high real wage rigidity in the recent low inflation period following the edition of the Real plan. About 92% of current real wages come from past real wage while only 8% is explained by the marginal rate of substitution. This finding agrees with Reis and Camargo (2007), who argue that the recent inflation stabilization has increased the real wage rigidity because inflation imposes more flexibility to the real wage in situations where the nominal wage is not legally allowed to be reduced. Results by Arbache and De Negri (2004) and Orellano et al. (2009), using micro-data, also suggest that the Brazilian wage structure is rigid and insensitive to the economic cycle.

Finally, the estimated short-run trade off indicated that the Central Bank, following a 1% supply shock, must choose between allowing a 0.29% decrease in the output gap, keeping inflation unchanged, or a rise of 0.31% in the inflation rate, maintaining fixed the output gap. Alternative combinations within those limits are also feasible choices for the monetary policy.

The estimated coefficients satisfied all theoretical restrictions imposed by the model. Thus, the estimated coefficient for lagged inflation and expected future inflation were found in the intervals $[0.5, 1]$

and $[0, 0.5]$, respectively. Yet, they sum up to unity according to the Wald test. Intuitively, the latter restriction means that the cost of any monetary policy which seeks artificially to keep unemployment rate permanently below its natural level is an indefinite increase in the inflation rate.

Interestingly, the lower the real wage rigidity parameter, $\gamma$, the less persistent will be the short-run trade-off. For the special case of $\gamma = 0$, which corresponds to fully flexible wages, the real wage is given by the marginal rate of substitution and there is no short-run trade-off. In this case, the *divine coincidence* identified by Blanchard and Galí (2007) is still observed.

## BIBLIOGRAPHY

Arbache, J. S. & De Negri, J. A. (2004). Filiação industrial e diferencial de salários no Brasil. *Revista Brasileira de Economia*, 58:159–184.

Areosa, W. D. & Medeiros, M. (2007). Inflation dynamics in Brazil: The case of a small open economy. *Brazilian Review of Econometrics*, 27:131–166.

Barros, R. W. S. & Matos, S. M. (2008). Price behavior in Brazil: Evidences using micro data. In *Proceedings of the XXXVI Meeting of the Brazilian Economic Association*. ANPEC.

Blanchard, O. & Galí, J. (2007). Real wage rigidities and the new keynesian model. *Journal of Money, Credit and Banking*, 39:35–66. Supplement.

Calvo, G. (1983). Staggered prices in a utility maximizing framework. *Journal of Monetary Economics*, 12:383–98.

Campbell III, C. M. & Kamlani, K. (1997). The reasons for wage rigidity: Evidence from a survey of firms. *Quarterly Journal of Economics*, 112:759–89.

Clarida, R., Galí, J., & Gertler, M. (1999). The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature*, XXXVII:1661–1707.

Cochrane, J. H. (2007a). Identification with Taylor rules: A critical review. NBER Working Paper 13410.

Cochrane, J. H. (2007b). Inflation determination with Taylor rules: A critical review. NBER Working Paper 13409.

Dickey, D. A. & Fuller, W. A. (1979). Distribution of the estimates for autoregressive time series with unit root. *Journal of the American Statistical Association*, 74:427–31.

Dickey, D. A. & Fuller, W. A. (1981). Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica*, 49:1057–72.

Elliot, G., Rothenberg, T. J., & Stock, J. H. (1996). Efficient tests for an autoregressive unit root. *Econometrica*, 64:813–36.

Erceg, C. J., Henderson, D. W., & Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, 46:281–313.

Franses, P. H. & Haldrup, N. (1994). The effects of additive outliers on tests for unit roots and cointegration. *Journal of Business and Economic Statistics*, 12:471–79.

Friedman, M. (1968). The role of monetary policy. *American Economic Review*, 58:1–17.

Galí, J. & Gertler, M. (1999). Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics*, 44:195–222.

Galí, J., Gertler, M., & López-Salido, J. D. (2001). European inflation dynamics. *European Economic Review*, 45:1237–70.

Goette, L., Sunde, U., & Bauer, T. (2007). Wage rigidity: Measurement, causes and consequences. *The Economic Journal*, 117:F499–F507.

Gouvea, S. (2007). Price rigidity in Brazil: Evidence from micro data. Working Paper 143, Central Bank of Brazil.

Hall, A. R. (2005). *Generalized Method of Moments*. Oxford University Press, Oxford.

Hansen, L. (1982). Large sample properties of generalized method of moments estimators. *Econometrica*, 50:1029–1054.

Holden, S. & Wulfsberg, F. (2009). How strong is the macroeconomic case for downward real wage rigidity? *Journal of Monetary Economics*, 56:605–15.

Lee, J. & Strazicich, M. C. (2003). Minimum Lagrange multiplier unit root test with two structural breaks. *Review of Economics and Statistics*, 85:1082–1089.

Lucas, R. E. J. (1972). Expectations and the neutrality of money. *Journal of Economic Theory*, 4:103–124.

Mankiw, N. G. & Reis, R. (2002). Sticky information vs. sticky prices: A proposal to replace the new keynesian Phillips curve. *Quarterly Journal of Economics*, CXVII:1295–1328.

McCallum, B. T. (2008). Inflation determination with Taylor rules: Is new keynesian analysis critically flawed? NBER Working Paper 14534.

Minella, A., Freitas, P. S., Goldfajn, I., & Muinhos, M. K. (2003). Inflation targeting in Brazil: Constructing credibility under exchange rate volatility. *Journal of International Money and Finance*, 22:1015–1040.

Ng, S. & Perron, P. (2001). Lag length selection and the construction of unit root tests with good size and power. *Econometrica*, 69:1519–54.

Okun, A. M. (1962). Potential GNP: Its measurement and significance. In *Proceedings of the Business and Economics Statistics Section*, pages 98–103. American Statistical Association, Washington-DC.

Orellano, V., Mattos, E., & Pazello, E. (2009). A substituição de trabalhadores como instrumento para redução de gastos com salários: Evidências para a indústria paulista. *Revista Brasileira de Economia*, 63:135–152.

Perron, P. (1989). The great crash, the oil price shock and the unit root hypothesis. *Econometrica*, 57:1361–1401.

Perron, P. (1997). Further evidence on breaking trend functions in macroeconomic variables. *Journal of Econometrics*, 80:355–85.

Perron, P. & Ng, S. (1996). Useful modifications to unit root tests with dependent errors and their local asymptotic properties. *Review of Economic Studies*, 63:435–65.

Phelps, E. S. (1967). Phillips curve, expectations of inflation and optimal unemployment over time. *Economica*, 34:254–81.

Phillips, A. W. (1958). The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861-1957. *Economica*, 25:283–99.

Phillips, P. C. & Perron, P. (1988). Testing for unit root in time series regression. *Biometrika*, 75:335–46.

Reis, M. C. & Camargo, J. M. (2007). Desemprego dos jovens no Brasil: Os efeitos da estabilização da inflação em um mercado de trabalho com escassez de informação. *Revista Brasileira de Economia*, 61:493–518.

Schwartzman, F. F. (2006). Estimativa de curva de Phillips para o Brasil com preços desagregados. *Economia Aplicada*, 10:137–155.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, New Jersey.