Analyzing Atmospheric Pressure Variations in Time and Height: a Didactic Proposal Employing a Smartphone Barometer

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This work presents a didactic proposal to study atmospheric pressure. The introduction of technological resources in physics teaching is ever-increasing today. In this context, access to smartphones proves an accessible tool due to their massification and the possible uses of their built-in sensors. Therefore, we choose to use a smartphone as a barometer to measure the atmospheric pressure. We suggest experiments to measure atmospheric pressure as a function of height and time that can be easily reproduced by students in environments outside laboratories. The experimental results are compared with those provided by a meteorological station of the Company of Environmental Sanitation Technology of the State of São Paulo. Determining the atmospheric pressure variation as a function of height also allowed us to estimate the Boltzmann constant, thus establishing an experiment that can be introductory in statistical physics. Other than stimulating the practice and understanding of graphical analyses of experimental results, the proposed experiments serve as a deeper understanding of the atmospheric pressure phenomenon than that usually found in textbooks, proving to be a robust and suitable tool for physics teaching at an undergraduate level.
Keywords: Halley's Law, Boltzmann Constant, Arduino Science Journal.

1. Introduction

The use of technological resources in physics teaching and other disciplines occupies an ever-increasing important space in education, especially in laboratory practices. Their use started about 30 years ago with the introduction of computers in laboratory educational practices [1, 2] and, more recently, with the popularization of electronic platforms such as Arduino [3–6], programmable logic devices [7], and microcontrollers [8–10]. Currently, familiarity and access to smartphones are the main contributing factors for their educational use in physics laboratories, mainly due to the possibility of installing applications (many free) that allow to instrumentalize the device and render it useful for teaching, as can be verified in many physics topics [11–16].

Depending on the model, a smartphone can have many sensors installed internally: accelerometers, barometers, magnetometers, sound sensors (of frequency and sound intensity), light sensors, among others. One of the advantages of having these sensors available in a device that the student can carry with them is the possibility of performing experiments, both in the school environment and elsewhere, including at home.

In times of restricted social contact (2020/2021 – COVID-19 pandemic), in which laboratory activities can only be done remotely, or even for practices related to distance learning, technological resources allow experimental activities to be carried out autonomously and in environments outside the laboratories. The opportunity of conducting a homemade experiment with scientific rigor contributes to increasing the students' creativity, helping them relate and visualize the contents studied in their daily life, besides offering the possibility of involving family and friends in scientific studies, thus facilitating the dissemination of science.

Among the many sensors, we highlighted as present in smartphones are the barometers. Not all smartphones have a built-in barometer, but since atmospheric pressure changes with height, some of these can be to indicate the location of its bearer inside a building, determining whether someone is below or above a bridge, or even identifying a position along with the Earth's terrain variations. Users can access their smartphone's barometer from applications, such as the free and open-source Arduino Science Journal [17]. In its operation, the barometric sensor allows for vertical location through the direct and technological use of the concept of atmospheric pressure, defined as the force that an air column exerts on a surface. In the International System of Units, the unit of pressure is Newton per square meter (N/m^2), which received the special name "Pascal" (symbol Pa) after the 14th General Conference on Weights and Measures [INMETRO]. The designation is a tribute to

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the French scientist and philosopher Blaise Pascal (1623–1662).

Historically, the concept of atmospheric pressure was posited in Ancient Greece; but it was only in the mid-17th century that Galileo Galilei and Evangelista Torricelli revised it [18]. Torricelli created the well-known mercury barometer and, through experiments, established the causal relationship between the atmosphere (which he called a sea of air) and the height of a mercury column. Pascal deepened Torricelli's investigation by conducting experimental proof that the air weights less when the barometer is set at the top of a mountain, which was verified by observing the drop in the height of the mercury column. More aspects of the rich history of Pascal's experiment and other 17th century thinkers who investigated aspects of the nature of the atmosphere are found in [19–21].

Aside from its historical aspects, the topic of atmospheric pressure is often oversimplified in physics textbooks [22–24]. Usually, pressure at sea level and their dependence on altitude is discussed [25, 26]. However, besides altitude, atmospheric pressure also varies according to temperature, latitude, and time of day. More specific investigations of its variations date back to the 19th century and show that atmospheric pressure can vary periodically and occasionally be subject to variations whose causes cannot always be easily explained [27]. Thus, atmospheric pressure involves much more complex phenomena than is usually discussed in physics textbooks.

Changes in atmospheric pressure with altitude are determined by Halley's law [25], also known as the barometric equation. Berberan-Santos, Bodunov, and Pogliani [28] present a historical overview of the determination of atmospheric pressure and address Halley's law in greater depth. Bolanakis and Dias [29] present studies on the dependence of atmospheric pressure on temperature. Derivations of the barometric formula from mechanical, thermomechanical, and statistical physics points of view are discussed in [30]. Pressure variations with latitude and altitude are also analyzed in [31], using an approximate formula proposed by Laplace. Another work shows the relationship between Halley's law and Boltzmann constant [32].

Another aspect to be considered in changes of atmospheric pressure is its dependence on the time of day. The fluctuations in atmospheric pressure throughout the day characterize the so-called barometric tides, in allusion to ocean tides. It should be clarified that ocean tides result from the effects produced by the gravitational pull of the Moon and Sun on the Earth. In atmospheric pressure variations, however, the thermal effect prevails over the gravitational one [33, 34].

Despite the complexity of the topic discussed in the preceding paragraphs and in the cited references, the study of atmospheric pressure can be explored didactically considering the ease of taking measurements outside specific environments and with popular devices such as smartphones. Exploring the electronic instrumentation in the process of measuring atmospheric pressure can also be seen as complementary to the teaching activity. Moreover, the differential formulation of Halley's law is also appealing for interdisciplinary activities, encompassing concepts of physics and mathematics at undergraduate level. As an example, a valuable analysis of physics concepts and the mathematical development of the ordinary differential equation of atmospheric pressure is made in [35]. The studies in [36–38], in turn, explore the technological aspects of atmospheric pressure measurement and its correlation with vertical movements. Using atmospheric pressure and temperature sensors on an Arduino platform, Carvalho and Amorim [33] reported the tidal behavior of barometric pressure throughout the day. Taking advantage of a built-in smartphone barometer sensor, Salinas *et al.* [39] propose two valuable experiments to explore vertical changes of pressure in water, allowing for the determination of the density of salty water and the study of pressure oscillations due to mechanical damped oscillations.

Regarding devices for determining atmospheric pressure, physics textbooks cite pressure measurements from mechanical artifacts based on fluid movement or mechanical deformation of some elements. These devices, despite their historical importance, are bulky, sensitive to the influence of other physical quantities, and reach satisfactory levels of accuracy only when built using refined precision mechanics techniques. Currently, though, pressure meters are inserted in electronic instrumentation systems [40, 41] and, therefore, instead of responding visually, they respond via electrical signals. For this purpose, there are at least three types of sensors that can be used for pressure measurement: piezoelectric, capacitive, and piezoresistive. With the development of microelectronics, these sensors are implemented in semiconductor devices, called MEMs (microelectromechanical systems). These solid-state sensors have diminutive dimensions (of the order of mm) and are implemented on chips together with electronic circuits, which ensure their stability and compensation on temperature variations.

Given that atmospheric pressure is a static measurement, capacitive and piezoresistive sensors are the most suitable, since piezoelectric sensors respond only with electrical voltages during a strain variation. Capacitive and piezoresistive sensors operate based on the deformation of a membrane that changes the electrical response of one of these components or a bridge formed by them. Examples of construction and characterization processes for capacitive and piezoresistive sensors are given in [42] and [43], respectively.

Based on the above, this study explores the use of smartphones in conducting experiments that allow us to measure atmospheric pressure and evaluate its possible dependencies on external factors. Thus, we intend to

encourage students to expand their knowledge on the topic and develop experiments outside the teaching laboratory, in their own homes. To this end, after discussing the theoretical basis in the following section, we essentially present two experiments. In the first one, we investigate the dependence of atmospheric pressure on height and estimate the Boltzmann constant; in the other, we explore the dependence of atmospheric pressure on time, that is, as a function of the time of day. Finally, we present and discuss the experimental results and their comparisons with pre-existing models and reference values measured in a meteorological station.

2. Theory

Used in this work to study the dependence of atmospheric pressure on height, Halley's law is developed based on basic concepts of differential calculus, although its original formulation was based on geometric aspects [20, 44]. Considering the former, let us consider an ideal fluid element in equilibrium. Figure 1 presents a schema of this element. Each particle of this fluid is under the action of the gravity force. Thus, the resulting force on the volume is the net force of all particles, which is, in turn, proportional to the element volume.

In this problem, the fluid is atmospheric air. The forces acting on the particles of a given volume define a force density given by Equation (1):

$$\boldsymbol{f} = \rho \boldsymbol{g} \tag{1}$$

where:
$\boldsymbol{f}$ is the force density (N/m^3);
$\rho = m_T/V$ is the volumetric density (kg/m^3), where m_T is the total mass of air enclosed in the volume element V;
$\boldsymbol{g}$ is the gravitational acceleration (m/s^2).

In the element (Figure 1), between the surfaces z and $z+dz$, both perpendicular to vector $\boldsymbol{n}$, there is a pressure difference given by Equation (2):

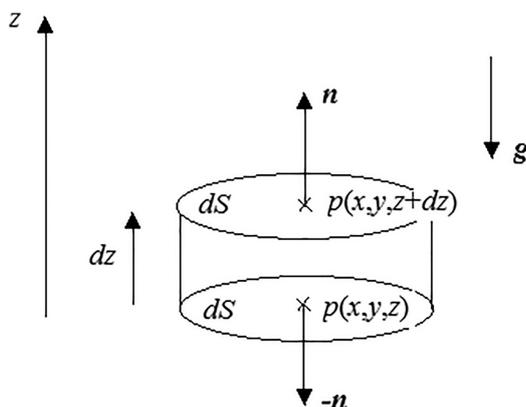$$p(x,y,z+dz) - p(x,y,z) = \frac{\partial p(x,y,z)}{\partial z}dz. \tag{2}$$

Figure 1: Illustration of the volume element of the fluid.

Hence, the forces on these surfaces (F_s) are given by Equation (3):

$$F_s = \frac{\partial p}{\partial z}dSdz \tag{3}$$

The equilibrium of the element is established by adding the forces defined in Equations (1) and (3):

$$\left(f_z - \frac{\partial p}{\partial z}\right)dSdz = 0, \tag{4}$$

therefore,

$$f_z = \frac{\partial p}{\partial z}. \tag{5}$$

As f_z is the vertical component of the force density produced by gravity, which appears exclusively in this direction ($f_x = f_y = 0$), then

$$\frac{\partial p}{\partial z} = -\rho g. \tag{6}$$

It is worth mentioning the physical interpretation of the precedent differential equation. In it, the rate of change of pressure is negative, which means that the atmospheric pressure decreases towards higher heights. This is also consistent with the fact that the air gradually becomes thinner as the height increases, so that $\rho = \rho(z)$ in Equation (6).

By considering an isothermal atmosphere, Boyle's law ($p_1V_1 = p_2V_2$) can be used to show that the $\left(\rho = \frac{m_T}{V}\right)$ density is proportional to pressure; hence,

$$\frac{\rho(z+dz)}{p(z+dz)} = \frac{\rho(z)}{p(z)} = \frac{\rho_0}{p_0} \tag{7}$$

therefore,

$$\rho(z) = \frac{\rho_0}{p_0}p(z), \tag{8}$$

where p_0 and ρ_0 are atmospheric pressure and air density at sea level, respectively.

By replacing (8) in (6), one gets the differential equation

$$\frac{dp}{p} = -\lambda dz \tag{9}$$

where

$$\lambda = \frac{\rho_0 g}{p_0}, \tag{10}$$

whose solution is the so-called Halley's law or barometric formula:

$$p(z) = p_0 e^{-\lambda z}. \tag{11}$$

Equation (11) shows that in an isothermal atmosphere, pressure variation with height is described by

an exponential function. In physical terms, the pressure then exponentially decreases with height.

Moreover, air density can be written as

$$\rho_0 = \frac{Nm}{V}, \tag{12}$$

where:

m is the average mass of an "air molecule"[1] (kg);
N is the number of air "molecules" present in volume V (expressed in m^3).

By replacing (12) in (10), using the ideal gas law ($pV = NkT$) and the definition of molar mass ($M = N_0\, m$), where N_0 is the Avogadro constant, one gets:

$$\lambda = \frac{Mg}{k_B N_0 T} \tag{13}$$

where:

M is the average molecular mass of air in kg/mol;
k_B is the Boltzmann constant in J/K;
T is the temperature in K.

By means of Equations (13) and (11) the atmospheric pressure can be expressed as

$$p(z) = p_0 e^{-\frac{(mgz)}{(k_B T)}}, \tag{14}$$

in which mgz is the potential energy. Alternatively, Equation (13) can also be expressed as a function of the ideal gas constant, given by $R = N_0\, k_B$.

In terms of the density of particles $n = N/V$, and by using the ideal gas law, Equation (14) becomes

$$n(z) = n_0 e^{-\frac{(mgz)}{(k_B T)}}. \tag{15}$$

As pointed out in the by Richard Feynman in his *Lectures on Physics*[2], Equation (14) is the Boltzmann law of statistical physics. Historically, the analogy with the exponential behavior of the pressure with height was used to Jean Perrin to derive the law of the vertical distribution of an emulsion [45, 46], by which he was laureate with the Nobel Prize in Physics in 1926[3]. Perrin's results were on the height distribution of particles of resin suspended in water.

The following section details both practical and theoretical methodological aspects to experimentally explore variations of pressure over time and height. For the latter, we propose a teaching activity to explore the subject in a deeper level than the one usually found in textbooks.

3. Materials and Methods

The experimental development of this work consisted of validating and using a smartphone to determine atmospheric pressure as a function of time and height, using the Arduino Science Journal application.

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[1] Quotation marks are used here because air is composed of different chemical elements.
[2] "The exponential atmosphere", https://www.feynmanlectures.caltech.edu/I_40.html
[3] Nobel Lecture, December 11, 1926: https://www.nobelprize.org/prizes/physics/1926/perrin/biographical/

3.1. Preliminary measurements of atmospheric pressure with the smartphone

The experiment used an Apple's iPhone 8 Plus, with the free application Arduino Science Journal in it [17]. Figure 2 shows an illustration of the smartphone and a preview of the app screen. Among other features, the Arduino Science Journal allows us to use the phone's atmospheric pressure sensor, as explored in this work. The use of the application is very simple, one needs to only tap on the screen and select the sensor to get the measures (Figure 2).

To validate the measurements, we compare the application's first atmospheric pressure readings with respective data provided by the Air Monitoring Station of the Company of Environmental Sanitation Technology of the State of São Paulo (CETESB) located in the city of Limeira, São Paulo, Brazil. These reference values were obtained in one-hour intervals from CETESB's QUALAR system [47]. Regarding the time interval between measurements taken with the smartphone, in our experiments, we observe that the Arduino Science Journal provides atmospheric pressure measurements at a frequency of 15.0 Hz. To proceed with the validation, atmospheric pressure was continuously measured for 48 hours, starting from 0h00 on the day February 13,
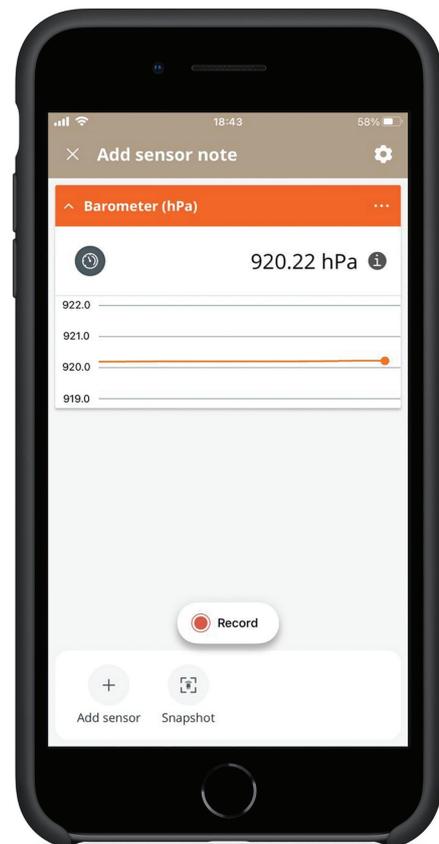

Figure 2: Image of the smartphone used in the experiments, with a snapshot of the Arduino Science Journal screen.

2021, and ending at 23h59 on February 14, 2021. As we have described, we remind the reader that the atmospheric pressure presents a daily cyclic behavior, with an approximate 12-hour period [48].

To evaluate the sensitivity and stability of the barometer, we experiment with small vertical displacements of the smartphone inside a room with a ceiling height of about 3.0 m. Atmospheric pressure was continuously measured for 60 seconds at each height, namely, at 0.9 m, 1.8 m, and 2.7 m. From the stored data, we compute the mean and standard deviation of the repeated measurements in time, for each height.

These two preliminary investigations were carried out to validate and inform static and dynamic measurements of atmospheric pressure before investigating its dependence on height, and then proceed with determining the Boltzmann constant.

3.2. Pressure variation with height

The experiments to determine atmospheric pressure as a function of height were performed in the city of Limeira, São Paulo, Brazil (22°33'53" south latitude, 47°24'06" longitude, 588 m altitude and 9.7861 m/s² local gravity [49]) in a 15-story building that, added to two basements and the ground floor, totals 18 levels or levels of measurements, making up an estimated height of 51.0 m. In the procedure, to measure the atmospheric pressure the experiment operator went up to the building's 15th floor and down the stairs, measuring the atmospheric pressure at each level, to the second basement. The experiment was repeated three different times throughout May 2, 2021. We previously defined the times for the experiments after observing the results of pressure variation throughout the day. According to these results, we chose three times for measurement: first at 10h00, time of day when the pressure values are usually close to a maximum value; second at 16h00, when the atmospheric pressure usually approaches a minimum value; and finally, at 22h00, when intermediate values of atmospheric pressure are generally observed. The purpose of taking measurements at different times is to check if there are any changes in the rate of pressure variation with height at different times and pressure levels. To determine the Boltzmann constant, we also measure the local temperature for each realization of the experiment.

To investigate the rate of pressure variation with height, we observe that the variation of atmospheric pressure is relatively small when the height varies by a few tens of meters. In this paper, therefore, we propose that the exponential function of Halley's law (11) can be approximated by a straight line given by the Taylor series around $z = 0$, truncated into first order:

$$e^{-\lambda z} \approx 1 - \lambda z. \tag{16}$$

Thus, Equation (11) is now written as a linear function in which the atmospheric pressure value can be related to height,

$$p(z) = p_0 - p_0 \lambda z, \tag{17}$$

and the experimental results can be adjusted by straight lines using the least-squares method [50, 51]. The z values considered in this experiment ranged from 0.0 to 51.0 m. The linear coefficient p_0 is not the atmospheric pressure at sea level, but at the local altitude level redefined as p_1:

$$p(z) = p_1 - p_1 \lambda z. \tag{18}$$

3.3. Determination the Boltzmann constant

Here we expand the didactic appeal of our work, linking the experiment to the study of statistical physics and kinetic theory of gases. First, to determine the value of λ, we divide the absolute value of the angular coefficient $a = p_1 \lambda$ by the linear coefficient $b = p_1$ in Equation (18) to eliminate the dependence of λ on p_1. To estimate the value of the Boltzmann constant k_B, the following parameters were used: local gravity acceleration equal to $g = 9.7861$ m/s², Avogadro constant $N_0 = 6.02 \times 10^{23}$, and average molecular mass of air equal to $M = 0.02895$ kg/mol, which remains approximately constant throughout the entire altitude range [32]. Under the isothermal hypothesis, the local temperature was also taken using the smartphone and its value was used in Equation (13) to estimate the Boltzmann constant.

4. Results and Discussion

In this section, we present the experimental results and discuss the experiments previously described in Section 3.

4.1. Barometer sensitivity and stability under small vertical displacements

Table 1 shows the results of the mean atmospheric pressures and their standard deviations. We observe that the behavior of the measuring system is quite stable. Regarding height, using a linear fitting, we found that the atmospheric pressure decreases by approximately 0.1 hPa with each 1.0-meter elevation. The standard deviation of each measurement, however, indicates that atmospheric pressure can be measured with an accuracy of tenths of hPa. Thus, the estimated accuracy of the height location given by the barometer is 0.01 hPa, corresponding to a height variation of 0.1 meters, or 10 cm, which defines the sensitivity of the barometer.

Table 1: Atmospheric pressure as a function of height.

Height (m)	0.90	1.80	2.70
Pressure (hPa)	920.045 ± 0.006	919.965 ± 0.006	919.862 ± 0.004

4.2. Atmospheric pressure as a function of time

Regarding the periodicity of the atmospheric pressure throughout the day, Figure 3 shows the results of the measurements obtained with the smartphone compared with data from the CETESB Meteorological Station. By visual inspection, one verifies that the curves are satisfactorily superposed. Table 2 presents a quantitative comparison between the maximum pressure magnitudes and their respective times, also labeled on the graph of Figure 3.

As shown in Table 2, the percentage difference between the values from CETESB and the measurements taken with the smartphone is very small, which attests to the reliability of the device's measurements. Regarding the results, according to the literature, tropical regions have maximum pressure values around 10h00 and around 22h00, therefore within an approximately 12 hours period [32]. These periods and times are observed approximately not only in our measures but also in the data from CETESB (from $t2 - t1 \approx 12$ hours and $t3 - t2 \approx 11$ hours). This small change in the pattern of expected local maximum (and also minimum) may have been caused by weather interferences of humidity, temperature, etc, which reveals that atmospheric pressure is a topic much more complex than usually is covered in physics textbooks. Two other factors may have perturbed the results: the linear distance of approximately 1.7 km between the CETESB station and the place of the smartphone experiment (at home), and an estimated difference in altitude of approximately 20 meters less
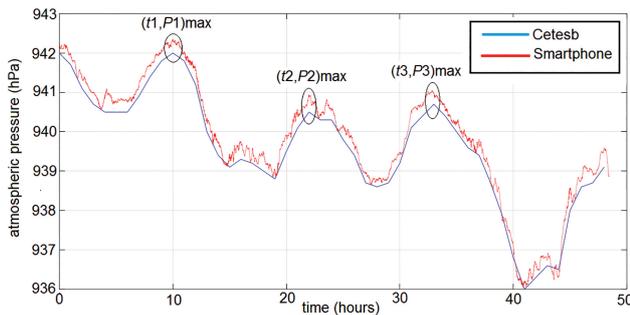
in our experiment. Despite this, we emphasize that the small differences of about 1% (or less) observed in Table 2 are reasonably acceptable for teaching purposes.

4.3. Variation of the atmospheric pressure with height

As for the variation of atmospheric pressure regarding height, Figure 4 shows the data obtained at the three times of the experiments and their respective linear fittings via the least-squares method. Table 3 presents the angular (a, in hPa/m) and linear (b, in hPa) coefficient values of the fitted lines, with respective errors, according to our proposed linearization of Halley's law introduced in Section 2. This same table also presents the theoretical (predicted) and experimental (gathered) values for λ, defined by Equation (13).

From the results, we observe that the values of λ are sufficiently close to each other, both in relation to the theoretical and respective experimental values at each time as well as among experiment repetitions at different times. This indicates that the pressure variation is kept constant regardless of the atmospheric pressure reference
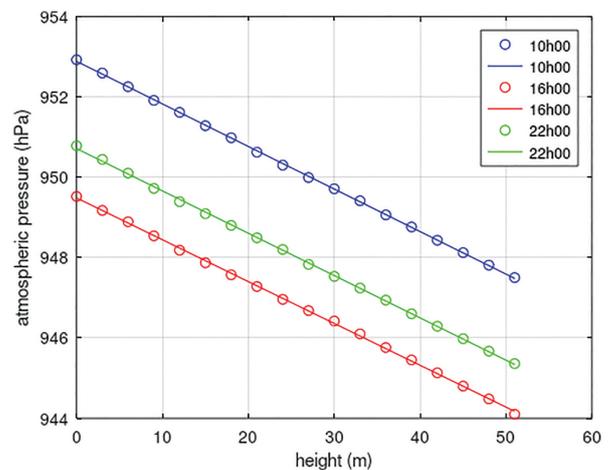

Figure 4: Atmospheric pressure variation with height at three different times (10h00 – blue, 16h00 – red, 22h00 – green). The circles and straight lines represent the experimental data and the fitted curves, respectively.

Figure 3: Atmospheric pressure throughout February 13 and 14, 2021; CETESB data (blue curve), data measured with the smartphone (red curve).

Table 2: Times and their maximum local values of atmospheric pressure, using data from CETESB for validation of the measures with the smartphone.

	$t1$max (h)	$P1$max (hPa)	$t2$max (h)	$P2$max (hPa)	$t3$max (h)	$P3$max (hPa)
CETESB	10.05	942.00	21.80	940.52	32.97	940.69
Smartphone	9.93	942.33	21.91	940.95	32.86	941.03
Relative Difference (%)	1.19	0.03	0.50	0.04	0.33	0.04

Table 3: Values of the temperature, angular coefficient (a) and linear coefficient (b), λ and their respective errors.

Time	10h00	16h00	22h00
Temperature (K)	295.5 K	298.4 K	290.0 K
a (hPa $\cdot$ m^{-1})	-0.1059 ± 0.0003	-0.1041 ± 0.0006	-0.1053 ± 0.0005
b (hPa)	952.88 ± 0.04	949.48 ± 0.07	950.71 ± 0.06
Theoretical Value λ (m^{-1})	1.1539×10^{-4}	1.1427×10^{-4}	1.1721×10^{-4}
Experimental Value λ (m^{-1})	1.1115×10^{-4}	1.0968×10^{-4}	1.1079×10^{-4}
Percentage Relative Error λ	3.8 %	4.2 %	5.8 %

level, which is determined by the time of day and weather conditions.

4.4. Experimental determination of the Boltzmann constant

Regarding the determination of the Boltzmann constant, Table 4 shows estimated values for k_B and respective relative errors in relation to the reference value $k_B = 1.38 \times 10^{-23}$ J/K. Note that the results obtained for the Boltzmann constant are very close to the theoretical value, with relative errors on the order of 5 %, as obtained for the relative errors of λ. Imposing $T = 0°$C, the errors for the measurements at 10h00, 16h00, and 22h00, would be 12.4%, 13.9%, and 12.7%, respectively. Thus, comparing the results obtained from a large temperature variation (between 0°C and around 22°C) we note that the relative error of k_B is modified. On the other hand, small temperature variations have a little influence on this error. For this reason, when experimenting at home the student does not need to worry about having an instrument for measuring room temperature with high precision or accuracy. By the set of results discussed, the didactic proposal presented here is suitable to explore the determination of the Boltzmann constant in teaching activities. To promote them, the spreadsheet tool of our analysis is freely available at: https://github.com/diegosrodrigues/b arometer

Table 4: Experimental values for the Boltzmann constant.

Time of the day (h)	10h00	16h00	22h00
Temperature, T (K)	295.5 ± 0.10	298.4 ± 0.10	290.9 ± 0.10
Boltzmann constant, k_B (J/K) – experimental	1.43×10^{-23}	1.44×10^{-23}	1.46×10^{-23}
Percentage Relative Error k_B	3.8%	4.2%	5.8%

To conclude our discussion, as a complement, by means of historical aspects of science, one could use the original Perrin's work and dataset [45, 46] to explore and discuss the full exponential behavior of a vertical distribution law in a teaching context.

5. Conclusion

The atmospheric pressure values obtained in the experiments proved to agree with the reference values from the CETESB Meteorological Station, showing the capability of smartphones to measure the atmospheric pressure in teaching and learning activities performed by teachers and students. The proposed experiments can be performed at home and without the need for additional devices or resources, contributing to the students' autonomy and having the potential for attracting the curiosity of their families to a scientific topic. We also believe that a first contact with the Arduino Science Journal

can work as a motivation for stimulating undergraduate students to freely explore their smartphones to measure other physical quantities using other sensors as well.

It is worthy to note that we have considered the air as an ideal gas to determine the Boltzmann constant. The studies of an ideal gas [22] are based on the premise that the gas occupies a large volume and the molecules interactions are due only to the collision between them. Although an ideal gas does not exist, a real gas on low pressure and high temperature has behavior that can be described in the same way. The atmospheric conditions where the experiments were performed guarantee the assumption that the air, though be a gases mixture, can be considered as ideal gas. Conditions that extrapolate the ideal gas definition may produce situations in which the product of pressure and volume is not linearly dependent on the temperature. Thus, this proportionality would not be described in terms of a constant.

The approaches presented here are accessible to first-year undergraduate students in the exact and technological sciences. Except for the least-squares method, and lacking why the linear approximation is suitable, the proposed experiments are also accessible to high school students. In this case, the use of basic spreadsheet tools allows for fitting a linear trend line to a dataset, which is sufficient to estimate the Boltzmann constant, including its error. In a high school setting, instead of the Boltzmann constant, we suggest addressing the ideal gas constant, as it is more adequate for this level of education.

The didactic experiments proposed here also allow us to explore the concept of atmospheric pressure in greater depth than is usually found in textbooks. They can be replicated in locations with different altitudes, allowing assessment of the differences in terrain and their influence on atmospheric pressure. Nonetheless, one must consider that the tidal behavior of the atmospheric pressure curve can be influenced by weather conditions, humidity, temperature, etc. As a complement to this study, one could consider measuring the atmospheric pressure in buildings much higher than the ones we have performed, exploring the results beyond the 50.0-meter approximation range that we have reported here. Beyond the Perrin's original dataset we have reported here, the use of readily other ones available on the internet could also be useful to fully explore the exponential behavior of vertical distributions of particles in fluids such as Halley's law.

Acknowledgments

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