Revista Brasileira de Ensino de Física, v. 37, n. 3, 3501 (2015)
www.sbfisica.org.br
DOI: http://dx.doi.org/10.1590/S1806-11173721809

Desenvolvimento em Ensino de Física

Calculation of vector components: A tutorial worksheet to help students develop a conceptual framework

(Cálculo das componentes de um vetor: Um tutorial para ajudar os alunos a desenvolverem um quadro conceitual)

Pablo Barniol[1], Genaro Zavala[1,2]

[1] *Physics Education Research and Innovation Group, Department of Physics, Tecnólogico de Monterrey, Campus Monterrey, Monterrey, México*
[2] *Engineering School, Universidad Andrés Bello, Santiago, Chile*
Recebido em 26/1/2015; Aceito em 16/2/2015; Publicado em 30/9/2015

When we administered our Test of Understanding of Vectors (TUV) to students who were finishing a physics university remedial course (that covers subjects of a traditional high school physics course), we observed that they have considerable difficulties in calculating the x-component of a vector when the angle given is measured from the y-axis to the vector. As a result of this finding, we decided to design a tutorial worksheet that guides students through the development of a conceptual framework in this subject. The worksheet was implemented with 264 students of the same course in another semester. Upon using the TUV to evaluate the students' understanding, we confirmed that the tutorial worksheet had facilitated their learning. This tutorial worksheet is presented in the appendix and might be used by other physics instructors who teach this material in high schools, colleges or universities.
Keywords: vector components, vector concepts, tutorial worksheet, instructional material.

Quando aplicamos o nosso "Test of Understanding of Vectors" (TUV) para os alunos que terminaram um curso universitário de nivelamento de física (que abrange temas de um curso tradicional de física do Ensino Médio) observamos que eles têm dificuldades consideráveis para o cálculo da componente-x de um vetor quando o ângulo dado é medido a partir do eixo-y. Como resultado desta constatação, decidimos criar um tutorial para orientar o desenvolvimento de um quadro conceitual neste tópico. O tutorial foi implementado com 264 estudantes do mesmo curso em outro semestre. TUV foi utilizado para avaliar a compreensão do aluno e confirmou que o tutorial ajudou na sua aprendizagem. O tutorial é apresentado no apêndice e pode ser utilizado por outros professores de física para ensinar este material no pré-vestibular e cursos universitários.
Palavras-chave: componentes de um vetor, conceitos de vetor, guia tutorial, e materiais didáticos.

1. Introduction

When we administered our Test of Understanding of Vectors (TUV) [1] to students finishing a physics university remedial course, we observed that they have significant difficulties in calculating the x-component of a vector when the angle given is measured from the y-axis to the vector. This result confirms the need for new instructional material that fosters students' understanding of how to calculate vector components.

With this in mind, we referred to McDermott's study [2], which outlines a process for developing new instructional material. The process has three steps: 1) conduct research on student understanding, 2) use the findings to guide the development of instructional materials, and 3) carry out an effectiveness assessment of the new instructional materials, based on what the students have learned. "Tutorials in Introductory Physics" [3] are well-known instructional resources that were designed following this process [4].

Since the literature objectively reports that use of the Tutorials is one of the most effective teaching strategies for introductory physics [5], we decided to design a tutorial worksheet to increase students' understanding of the calculation of vector components. The three objectives of this study are:

(1) to present a tutorial worksheet whose design was based on the analysis of students' difficulties when calculating vector components;

(2) present evidence of the need for this tutorial;

(3) evaluate its effectiveness.

[1] E-mail: pablo.barniol@itesm.mx.

2. Previous research on students' difficulties in understanding vector components

Van Deventer [6] and Zavala and Barniol [7] studied students' difficulties in sketching the components of a vector. Van Deventer detected a frequent error in which students chose a component with a longer magnitude, and Zavala and Barniol found that this error is rooted in a misconception: students think that the magnitude of a component is equal to the magnitude of the vector.

Van Deventer [6] also studied students' difficulties in calculating vector components. He designed a multiple-choice problem in which students are asked to calculate the x-component of a vector when the given angle was measured from the y-axis. He determined that the most frequent error was to calculate it using the cosine function instead of the sine function, and concluded that this error was due to students memorizing the formula $A\cos\theta$ to calculate the x-component of a vector, a rule that is only valid when the angle is given from the x-axis.

Barniol and Zavala [1] designed the 20-item Test of Understanding of Vectors (TUV), demonstrating that it is a reliable assessment tool. The TUV evaluates ten vector concepts: direction, magnitude, components, unit vector, representation of a vector, addition, subtraction, scalar multiplication, dot product, and cross product. The authors modified the component calculation problem designed by Van Deventer and incorporated it in the TUV as item 14.

3. Theoretical framework of tutorial worksheets

McDermott [2] established that the tutorials provide experience in learning through guided inquiry and an emphasis on the constructing of concepts. The tutorial worksheets have two main aims: 1) to guide students to develop a conceptual framework of important topics that are known from the research to be difficult for students, and 2) to address persistent conceptual difficulties. The worksheets contain questions that break down the reasoning process into steps, in order to guide students to use scientific reasoning to build conceptual understanding. During the class session in which the tutorial worksheet is presented, students work collaboratively in groups of three or four.

McDermott mentions that instructors do not lecture or give answers but rather help students to arrive at their own answers by posing questions that guide them through the necessary reasoning. The tutorials require training for instructors in both the subject matter and instructional method. This training seminar is carried out using the same material and techniques that the instructors are expected to use in class. Finally, it is noteworthy that this journal has published studies in which the development of new instructional material based on McDermott's framework is presented [8].

4. Design of the tutorial worksheet

This section addresses the first objective. The worksheet is included in the Appendix. Following the theoretical framework, this tutorial worksheet has two main goals: 1) to guide students in developing a conceptual framework to calculate vector components, and 2) to address persistent conceptual difficulties in the calculation of these components. Both are described below.

The different sections of the tutorial worksheet permit a step by step description of how students develop a conceptual framework when using the tutorial. Section 1 leads students to recall two important aspects of the calculation of components: 1) the identification of right triangles, and 2) the use of sine and cosine functions in the right triangle. Next, section 2 guides students to draw the component of a vector placed on a grid in the two possible ways (i. e. the y-component can be drawn to the left or to the right of the vector) and to notice that the two triangles that result (one above and one below the vector) are right triangles. The identification of these aspects is crucial to the development of the conceptual framework. Note that we used a grid to allow students to sketch the components in the most accurate way, to observe that the components form right triangles, and to address the most frequent error when sketching vector components, i.e., sketching them with longer magnitudes.

Then, section 3 leads students to calculate both components when the angle is measured from the x-axis by using the triangle that is formed below the vector. Next, section 4 guides students to calculate both components when the angle is measured from the y-axis by using the triangle that is formed above the vector. Finally, section 5, "Integrated vision", leads students to first notice the trigonometric functions used in each of the two previous cases to calculate the x- and y-components of a vector, and to also be able to form their own conclusions with regard to the calculation of these components.

Questions e and f of section 5 explicitly address the most frequent error in the calculation of components: calculating the x-component of a vector using the cosine function when the given angle is measured from the y-axis. To address this, we decided to first present them with the phrase that they tend to memorize: "to calculate the x-component of a vector one has to use the cosine function". Then, we ask them if they believe that the statement is complete, and, if not, what should be added to complete it.

5. Methodology

This research study was conducted at a large private Mexican university. All participants in this study were enrolled in the *Introduction to Physics* course, which is a remedial course taken by those students who enter the university without having scored a passing grade on the Physics Selection Test (PST). This course covers subjects of a traditional high school physics course: dimensional analysis, significant figures, density, trigonometry, kinematics in one dimension, and vectors.

The methodology used in this study calls for both a control and an experimental group. For the control group, we analyzed students' results in the course from the fall semester of 2012. The textbook used by these students was "Introduction to University Physics" by Alarcon and Zavala [9], and in class they also worked on collaborative activities presented in the "Activities Manual" by the same authors [10]. (Note that these activities were not specifically designed based on research on students' difficulties.) Under the topic of vectors, students worked with three different activities from the manual. The first one dealt with magnitude, direction, addition, subtraction and scalar multiplication of a vector; the second mainly addressed unit vectors and unit vector notation; and the third activity dealt with vector products.

For the experimental group, we analyzed students' results in the course from the fall semester of 2013. These students had the same instructors and used the same textbooks as students from the previous year. They also worked with the first of the three collaborative activities. However, in this semester we implemented four new tutorial worksheets based on the research on students' most persistent difficulties. The four new tutorial worksheets were: 1) calculation of components, 2) unit vector, 3) dot product, and 4) cross product. The implementation of these tutorials was carried out following the recommendations by McDermott [2]. This article focuses only on the first tutorial worksheet. (The other three tutorial worksheets will be presented in future articles).

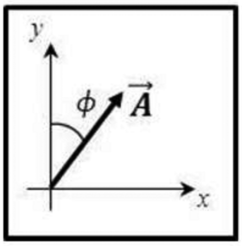
The PST was administered to all the participants before the beginning of the course. The PST is a multiple-choice test that evaluates students' knowledge of the main topics of a traditional high school mechanics course. We used the PST to validate that both groups were comparable. To evaluate the participants' understanding of vector concepts once the course had ended, we used the TUV. We had 313 students in the control group and 264 students in the experimental group.

6. Evidence of the need for a tutorial worksheet on calculation of vector components

In order to present the evidence (objective 2), we analyzed the scores from the TUV and the answers to item 14 of the students in the control group. The distribution of the scores was negatively skewed and the median was 13. It is noteworthy that the students who are on the median had difficulty answering 7 items correctly (out of 20).

Figure 1 presents item 14 of the TUV, the item that evaluates the calculation of a vector component. We observed that only 50% of students chose the correct answer (option C). Moreover, this item was one of the seven most difficult items in the test. This is evidence of the need for a tutorial worksheet on the calculation of vector components. Furthermore, we noted that 34% of the students incorrectly chose option D, which is the equation Acosø. This confirms the need to address this specific error in the tutorial.

14. The figure below shows vector $\vec{A}$ that forms an angle ϕ with the vertical axis. $|\vec{A}|$ is the magnitude of vector $\vec{A}$. Which option shows the magnitude of the *x*-component of vector $\vec{A}$, (i.e., $|\vec{A_x}|$)?



(A) $|\vec{A_x}| = |\vec{A}|\, tan\, \phi$

(B) $|\vec{A_x}| = \frac{|\vec{A}|}{cos\, \phi}$

(C) $|\vec{A_x}| = |\vec{A}|\, sin\, \phi$

(D) $|\vec{A_x}| = |\vec{A}|\, cos\, \phi$

(E) $|\vec{A_x}| = \frac{|\vec{A}|}{sin\, \phi}$

Figure 1 - Item 14 on the TUV.

7. Effectiveness of the four new tutorial worksheets used in the experimental group

To evaluate the effectiveness of these worksheets, we compared the TUV scores of the control and experimental groups. First, we evaluated whether both groups were comparable by analyzing the scores from the PST. Since both distributions of scores in the PST were not normal and their variances met the assumption of homogeneity, we used the Mann-Whitney test [11]. This test indicates that the PST scores obtained by students in the control group (Mdn = 44) did not differ significantly from those of students in the experimental group (Mdn = 38) before the course, U = 38560.5, z = -1.39, p = 0.164. Based on this result, we can state that both groups were comparable.

To evaluate the effectiveness of the new tutorials, we compared the scores obtained on the TUV by both groups. Similar to the control group, we found that the experimental group had a negatively skewed distribution of scores in the TUV with a median of 15. Since both distributions of scores in the TUV were not normal and their variances met the assumption of homogeneity, we used the Mann-Whitney test. This test indicates that the scores obtained by students in the experimental group (Mdn = 15) were significantly higher than those obtained by students in the control group (Mdn = 13), U = 51293.5, z = 5.015, $p < 0.01$, r = 0.2. This result shows that the newly designed tutorials help students to increase their understanding of vector concepts. It's interesting to note that the items for which we found more differences between students' performances in the two groups were mostly those related to the tutorials. One of these items is the component calculation item. We give a detailed analysis below.

8. Effectiveness of the tutorial worksheet on calculation of vector components

To evaluate the effectiveness of the tutorial worksheet (objective 3), we compared the control and the experimental groups' answers to item 14. To perform this comparison we first used the chi-square test ($p < 0.05$) to determine whether there was a significant difference in the distribution of answers for both the control and experimental groups. We found a significant difference in the distribution of both groups' answers, χ^2 (4, N = 577) = 21.5, $p < 0.01$. Then we determined which specific option was significantly different by using the chi-square test again following the procedure described by Sheskin [12]. We found significant differences in the selection of two options as shown in Table 1.

The most important difference found was in the selection of the correct answer (67% with the tutorial vs. 50% without it). The increase of the proportion of students selecting the correct answer is evidence that the tutorial helps students develop a conceptual framework to calculate vector components. Moreover, we also found a significant difference in the selection of the most common incorrect answer Acosø (23% with the tutorial, *vs.* 34% without it). This difference seems to be explained by the fact that this specific error is directly addressed in the tutorial worksheet. These differences present evidence of the effectiveness of the tutorial worksheet on calculation of vector components.

9. Evidence of increased student learning as a result of the tutorial worksheet

In order to further evaluate the effectiveness of the tutorial worksheet, we decided to also present evidence of improved student learning as a result of using this tutorial, by analyzing their answers when they completed it in the classroom. As mentioned previously, this worksheet has two main goals. The first is to foster the development of a conceptual framework to calculate vector components throughout the tutorial. The last section is "Integrated vision", in which students summarize the developed framework (questions c and d). The second goal is to address the most frequent error (questions e and f). Below, we present evidence of the students' learning upon completing these sections.

Table 1 - Proportion of students of the control and experimental groups who chose each option in the item 14 of the TUV. The arrows indicate which options have significantly different results.

Option	Answer	Control group		Experimental group
A	Atanø	6%		6%
B	A/cosø	6%		2%
C	Asinø (correct)	50%	↔	67%
D	Acosø	34%	↔	23%
E	A/sinø	4%		2%

In question c of section 5, the worksheet leads students to develop a conceptual framework to calculate the x-component of a vector. This question asks: *What can you conclude about the calculation of the x-component of a vector?* When we analyzed students' answers on the worksheet, we found two lines of thinking. In the first, students specified the trigonometric function that needed to be used in each possible case. For example, a student answered:

> If you want to obtain the x-component when the angle is measured from the x-axis, you use the cosine function, and if you want to obtain the x-component when the angle is measured from the y-axis, you use the sine function.

The second line of thinking is more general; an example of a student's statement is:

> the calculation depends on the axis from which the vector angle is measured.

We observed that these two lines of thinking are directly related and show evidence that the students have developed an adequate conceptual framework to calculate the x-component of a vector. Note finally that we found similar evidence with regard to the development of the framework to calculate the y-component of a vector (question d).

The last two questions on the tutorial worksheet address the most frequent error. These questions ask:

> e) In physics courses, students are accustomed to repeating the phrase: "To calculate the x-component of a vector, one has to use the cosine function". Is this statement complete? f) What should be added to this phrase in order for it to be complete?

In these two questions we observed similar answers. A frequent answer is:

> e) The phrase is not complete because it depends on the axis from which the vector is measured. f) This should be added: only when the angle is measured from the x-axis.

As we can see from this example, students discarded the most frequent error when they had completed the tutorial worksheet.

10. Conclusion

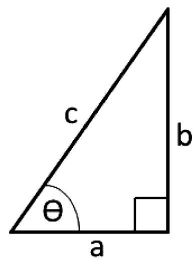
Many experienced physics teachers are aware of students' difficulties in calculating vector components, and attempt to address these issues in their instruction. An important aspect of this study is that we have carried out systematic research to design and evaluate a tutorial worksheet on this subject and we have presented evidence of the effectiveness of the tutorial. When using this tutorial, instructors do not teach to explicitly overcome this difficulty; rather, students come to understand it themselves through using the tutorial worksheet. This is part of what constitutes active learning. The tutorial worksheet is presented in the Appendix. Physics instructors who teach this material in high schools, colleges or universities are welcome to use it.

Appendix: Tutorial worksheet

Calculation of the x- and y-components of a vector

1. Trigonometry of the triangle

Consider the triangle shown in the figure.

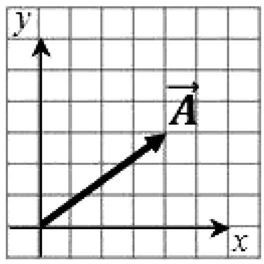


a) Is it a right triangle? _______ Explain.

b) Complete the next equations: $\cos(\theta)$ = _______, $\sin(\theta)$ =_______.

2. Sketch of the x- and y-components of a vector

The figure shows vector **A**.



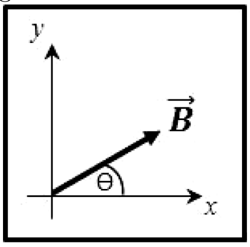
a) Sketch the x-component of the vector naming it A_x and sketch the y-component of the vector naming it A_y.

b) Notice that the x-component can be sketched *below* or *above* the vector. Also notice that the y-component can be sketched to *the left* or *to the right* of the vector. Identify and name these two ways of sketching A_x and A_y.

c) In the previous drawings, notice that a triangle is formed *above* the vector and another triangle is formed *below* the vector. Are these right triangles?

3. First calculation of the x- and y components of a vector

The figure shows vector **B** with a magnitude of 5 units that forms an angle $\theta = 30°$ measured from the x-axis.

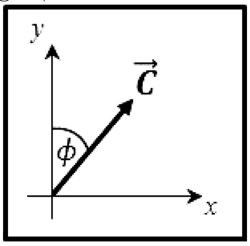


a) Calculate the x-component of the vector (Hint: use the triangle that is formed *below* the vector).

b) Using the same triangle, calculate the y-component of vector **B**.

4. Second calculation of the x- and y-components of a vector

The figure shows vector **C** with a magnitude of 5 units that forms an angle $\phi = 40°$ measured from the y-axis.



a) Given what you have learned in the previous sections, calculate the x-component of the vector (Hint: use the triangle that is formed *above* the vector).

b) Using the same triangle, calculate the y-component of the vector **C**.

5. Integrated vision

a) Consider section 3. The angle given was measured from which axis? Which trigonometric function did you use to find the x-component of the vector?

b) Consider section 4. The angle given was measured from which axis? Which trigonometric function did you use to find the x-component of the vector?

c) What can you conclude about the calculation of the x-component of a vector?

d) Following the same reasoning for the calculation of the y-component in sections 3 and 4, what can you conclude about the calculation of the y-component of a vector?

e) In physics courses students are accustomed to repeating the phrase: "To calculate the x-component of a vector, one has to use the cosine function". Is this statement complete?

f) What should be added to this phrase in order for it to be complete?

References

[1] P. Barniol and G. Zavala, Physical Review Special Topics - Physics Education Research **10**, 010121 (2014).

[2] L.C. McDermott, American Journal of Physics **69**, 1127 (2001).

[3] L.C. McDermott and P. Shaffer *Tutoriales para Física Introductoria* (Pearson Education, Argentina, 2001).

[4] P. Shaffer and L.C. McDermott, American Journal of Physics **60**, 1003 (1999).

[5] N. Finkelstein and S. Pollock, Physical Review Special Topics - Physics Education Research **1**, 010101-1 (2005).

[6] J. Van Deventer, *Comparing Student Performance on Isomorphic and Physics Vector Representations.* Master's Thesis, The University of Maine, United States (2008).

[7] G. Zavala and P. Barniol, AIP Conference Proceedings **1289**, 341 (2010).

[8] P.F.T. Dorneles, I.S. Araujo and E.A. Veit, Revista Brasileira de Ensino de Fisica **28**, 487 (2006).

[9] H. Alarcon and G. Zavala, *Introducción a la Física Universitaria* (Trillas, Mexico, 2005).

[10] H. Alarcon and G. Zavala, *Introducción a la Física Universitaria - Manual de Actividades* (Trillas, Mexico, 2006).

[11] A. Field, *Discovering Statistics Using IBM SPPS Statistics* (Sage Publications, United Kingdom, 2013).

[12] D. Sheskin, *Handbook of Parametric and Nonparametric Statistical Procedures* (Chapman and Hall, United States, 2007).