Revista Brasileira de Ensino de Física, vol. 45, e20230116 (2023)
www.scielo.br/rbef
DOI: https://doi.org/10.1590/1806-9126-RBEF-2023-0116

Chaotic dynamics in memristive circuits

Kelly C. Iarosz[*1,2], Enrique C. Gabrick[2], José Trobia[3], Rafael R. Borges[4], Paulo R. Protachevicz[5], Robson C. Bonetti[6], Silvio L.T. de Souza[7], Gabriel L. Batista[8], José D. Szezech Jr.[2,3], Iberê L. Caldas[5], Antonio M. Batista[2,3,5]

[1]Centro Universitário Telêmaco Borba, 84266-010, Telêmaco Borba, PR, Brasil.
[2]Universidade Estadual de Ponta Grossa, Programa de Pós-Graduação em Ciências, 84030-900, Ponta Grossa, PR, Brasil.
[3]Universidade Estadual de Ponta Grossa, Departamento de Matemática e Estatística, 84030-900, Ponta Grossa, PR, Brasil.
[4]Universidade Tecnológica Federal do Paraná, Departamento Acadêmico de Matemática, 84016-210, Ponta Grossa, PR, Brasil.
[5]Universidade de São Paulo, Instituto de Física, 05508-090, São Paulo, SP, Brasil.
[6]Faculdade Guairacá, 84400-000, Prudentópolis, PR, Brasil.
[7]Universidade Federal de São João-del-Rei, Campus Centro-Oeste "Dona Lindu", 35501-296, Divinópolis, MG, Brasil.
[8]Universidade Estadual de Ponta Grossa, Departamento de Física, 84030-900, Ponta Grossa, PR, Brasil.

Received on May 07, 2023. Revised on June 19, 2023. Accepted on July 17, 2023.

Memristors (abbreviation of memory and resistor), introduced as the fourth fundamental electrical circuit element, remember the electric charge that flowed through it. These nonlinear elements are considered as a class of two terminal resistive devices and offers a lot of possible applications in various areas. In this article, we review the dynamical behaviour of some electrical circuits with memristors. Initially, we show that a simple circuit with a capacitor and an inductor connected to a memristor exhibits periodic and chaotic attractors. After that, we show that the known Chua circuit, with a nonlinear resistance, can generate bifurcations and chaos. Substituting the nonlinear resistance by a memristor, the modified Chua circuit exhibits coexistence of attractors. Another known circuit, the Colpitts, is made of a combination of capacitors and inductors containing a bipolar junction transistor. We show that the modified Colpitts circuit, created by substituting the transistor by a memristor, presents multistability and coexistence of many attractors.
Keywords: Memristor, chaos, electric circuit, Chua, Colpitts.

1. Introduction

In 1971, Chua [1] proposed the fourth fundamental passive circuit element, called memristor. Memristor is a portmanteau of the words memory and resistor. Its valuable property is the possibility of changing the resistance as a function of the current, memorising the alterations. The first practical memristive device was based on a thin of titanium dioxide and fabricated by a group at HP Research Labs in 2008 [2]. It exhibits a pinched hysteresis between the current and the voltage. Nowadays other kinds of memristors have been developed [3].

Memristors offer a lot of potential applications in different areas, such as electrical engineering and computer science. They have been widely used in hardware security [4] and neural network [5]. Wang et al. [6] proposed a memristive chaotic system to achieve the encryption of image. Tan and Wang [7] demonstrated that a neural model composed of two neurons based on the memristor can show firing pattern transitions and different attractors. The memristors can also be used to generate digital modulate signals [8] and to implement programmable circuits [9].

A great deal of research has been devoted to analyse the dynamic behaviour of circuits with memristors [10]. Properties of basic electric circuits constructed from resistors, capacitors, inductors, and mainly memristors have been reported [11].

In this article, we introduce some chaotic electric circuits and how their dynamics is modified by including a memristor. Initially, we introduce a simple circuit, with a capacitor and an inductor coupled with a nonlinear active memristor [12], and show that this circuit presents periodic and chaotic attractors. After that, we present the common Chua and Colpitts nonlinear circuits that also exhibit chaotic attractors. The Chua circuit has been used as source of pseudo random signals and in secure communication [13]. The Colpitts circuit is able to generate high frequency sinusoidal signals and optical signals with application in optoelectronics [14]. However, when we replace the nonlinear resistance by a memristor in the Chua circuit, the modified circuit presents coexisting periodic and chaotic attractors [15]. Furthermore, for a sinusoidal voltage stimulus, this modified Chua circuit presents complex transient dynamics [16]. Finally,

*Correspondence email address: kiarosz@gmail.com

we introduce the Colpitts circuit composed of a combination of capacitors and inductors [17]. Furthermore, when we substitute the exponential nonlinear term in the Colpitts system by an ideal memristor, we find, depending on the initial condition, extreme multistability [18].

This paper is organised as follows. In Section 2, we introduce the memristor and a mathematical model, as well as some values of the parameters commonly used in the literature. A three circuit elements in series, a modified memristor Chua circuit, and a modified memristor Colpitts circuit are presented in Section 3. Depending on the parameter values, the three circuit elements in series can exhibit periodic and chaotic attractors, as shown in Subsection 3.1. Subsections 3.2 and 3.3 show that coexisting attractors can be find in modified memristor circuits based on Chua and Colpitts oscillators. Finally, we draw our conclusions in Section 4.

2. Memristor

Memristor is an abbreviation of memory resistor, namely a resistor with the capability to hold data in memory. It is composed of electrode and dielectric layers, as well as substrate. The materials play an important role in its performance [3]. The memristor model reported by Strukov et al. [2] is based on a two-layer film composed of undoped (TiO_2) and doped oxygen vacancies (TiO_{2-x}) with platinum electrodes (Pt), as shown in Fig. 1.

The memristor model is described by [2]

$$V = M \cdot I, \tag{1}$$

$$M = R_{ON}\frac{w}{D} + R_{OFF}\left(1 - \frac{w}{D}\right), \tag{2}$$

where V is the voltage potential, M is the generalised resistance, I is the electric current, and w is the state variable of the device [19]. R_{ON} and R_{OFF} correspond to the low and high resistance, respectively [11]. D represents the full length of the memristor.

Equation (1) can be rewritten as $I = G \cdot V$, where the memductance is $G = 1/M$ [10]. The time evolution of w is given by

$$\begin{aligned} \dot{w} &= \eta \cdot f(w,p) \cdot I = \frac{\eta \cdot f(w,p) \cdot V}{M} \\ &= \frac{\eta \cdot f(w,p) \cdot v_0 \cdot \sin\left(\frac{2\pi t}{T}\right)}{R_{on}\frac{w}{D} + (1 - \frac{w}{D}) \cdot R_{OFF}}, \end{aligned} \tag{3}$$

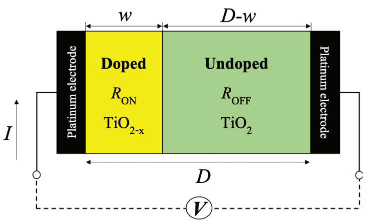

Figure 1: Schematic representation of the memristor composed of undoped (TiO_2) and doped oxygen vacancies (TiO_{2-x}) with platinum electrodes (Pt).

Table 1: Description and values of parameters [2].

Description	Parameter	Value
η	Polarity	1
D	Length of titanium diode memristor	1 nm
R_{OFF}	High resistance in low concentration dopant	70 Ω
R_{ON}	Low resistance in high concentration dopant	1 Ω
p	Integer value in the nonlinear function	10
v_0	Voltage Amplitude	1 V
T	Intrinsic period of oscillation	20 s
w_0	Initial device value	[0.0,1.0] nm

where η corresponds to the polarity, v_0 the voltage amplitude, T the period of potential oscillation, and t the time. In Eq. (3), $f(w,p) = 1 - (2w-1)^{2p}$ is the window function of the nonlinear dopant drift and p is a positive integer number [2, 20, 21]. Table 1 shows the description and some values of the parameters commonly used in the literature [2].

To illustrate the typical memristor temporal behaviour, we apply a periodic potential $V(t)$ to it. Thus, Fig. 2 displays the memresistance variable M, and current passing behind set I. The state variable of the device has the initial value $w(t=0) = w_0$ as the minimal, as well as the initial resistance as maximal $M(w_0)$. We see that oscillation in the applied potential (Fig. 2(a)) is able to change the state of the memristor device (Fig. 2(b)). As a consequence, the memresistance (Fig. 2(c)) generates oscillations in the current passing by the device (Fig. (2(d)).

Figure 3 exhibits I as a function of V for different values of w_0. For $w_0 = 0.0$ nm, I is directly proportional to V, as shown in Fig. 3(a). In Figs. 3(b) and 3(c) for $w_0 = 0.5$ nm and $w_0 = 0.6$ nm, respectively, we see a

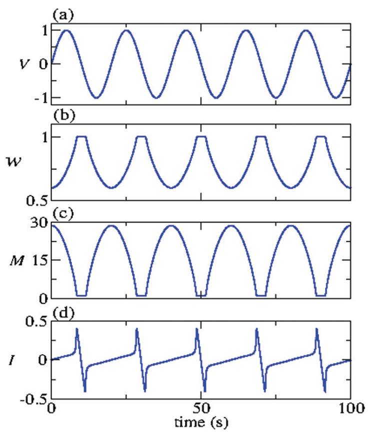

Figure 2: Temporal evolution of (a) V, (b) w, (c) M, and (d) I. We consider the parameters according to Table 1. We observe periodic oscillations.

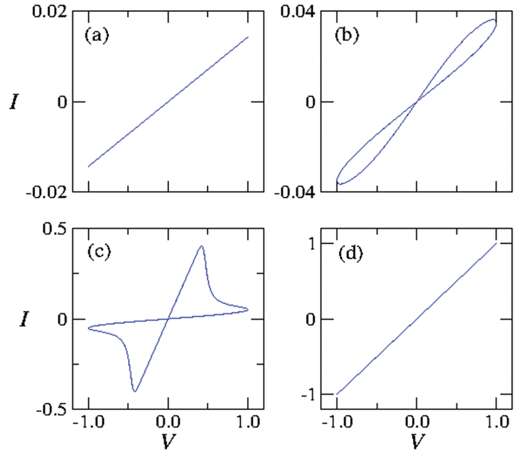

Figure 3: I as a function of V for (a) $w_0 = 0$ nm, (b) $w_0 = 0.5$ nm, (c) $w_0 = 0.6$ nm, and (d) $w_0 = 0.8$ nm. Depending on w_0, it is possible to observe the existence of a pinched hysteresis effect, as shown in the panels (b) and (c).

current-voltage characteristic of the memristor device, that is the pinched hysteresis effect. For $w_0 = 0.8$ nm, the $I - V$ curve is a straight line through the origin. To obtain the $I \times V$ curves, we consider $V = v_0 \cdot \sin(2\pi t/T)$.

3. Circuits

3.1. Three circuit elements in series

The nonlinear dynamic behaviour of memristors have been analysed in oscillatory and chaotic circuits [22]. Muthuswamy and Chua [12] proposed a circuit with a linear passive inductor, a linear passive capacitor, and a nonlinear active memristor. Figure 4 displays a schematic circuit diagram that has the three elements in series.

The dynamic equations of the circuit are given by

$$\begin{aligned} \dot{x} &= \frac{y}{C}, \\ \dot{y} &= -\frac{1}{L}[x + \beta(z^2 - 1)y], \\ \dot{z} &= -y - \alpha z + yz, \end{aligned} \tag{4}$$

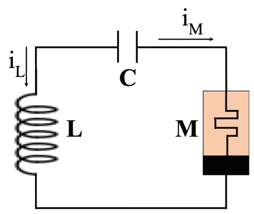

Figure 4: Schematic diagram for the three circuit elements in series [12]. The circuit is composed of a linear passive inductor (L), a linear passive capacitor (C), and a nonlinear active memristor (M).

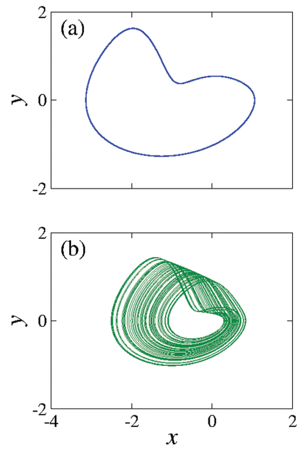

Figure 5: Phase portraits of $y(t)$ versus $x(t)$ for $C = 1.2$, $L = 3.3$, $\beta = 1.34$, (a) $\alpha = 1.15$, and (b) $\alpha = 0.85$. The panels (a) and (b) show periodic and chaotic attractors, respectively.

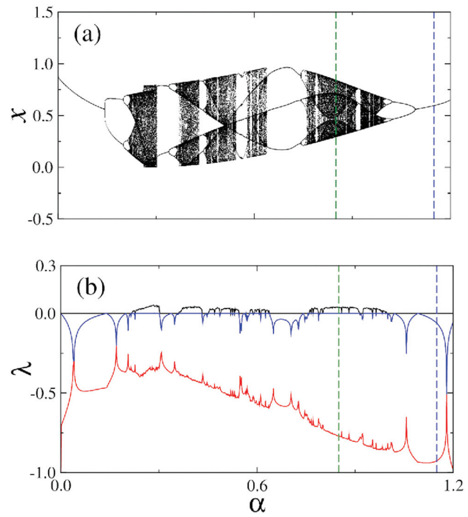

Figure 6: (a) Bifurcation diagram and (b) Lyapunov exponents as a function of the control parameter α for $C = 1.2$, $L = 3.3$, and $\beta = 1.34$. The blue ($\alpha = 1.15$) and green ($\alpha = 0.85$) dashed lines correspond to the α values considered in Figs. 5(a) and 5(b), respectively.

where x corresponds to the voltage across the capacitor C and y is related to the current through the inductor L. The variable z is the internal state of the memristive system. The parameters α and β are associated with the memristive device.

Figure 5 displays $y(t)$ versus $x(t)$ for $C = 1.2$, $L = 3.3$, and $\beta = 1.34$. The panels (a) and (b) exhibit a periodic attractor for $\alpha = 1.15$ and a chaotic attractor for $\alpha = 0.85$, respectively. The chaotic attractor shows sensitivity to initial conditions identified by the spectrum of Lyapunov exponents [23]. The system is chaotic when the maximal Lyapunov exponent is positive [24].

In Fig. 6(a), we present the bifurcation diagram of the maximum values of $x(t)$ as a function of α. The diagram shows period doubling as α increases and a parameter range with chaotic behaviour. By observing the maximal Lyapunov exponent in Fig. 6(b), the negative (periodic attractor) and positive (chaotic attractor) values can be compared with the bifurcation diagram to confirm the periodic and chaotic attractors.

3.2. Modified memristor Chua circuit

Around 1983, Chua invented an electronic circuit that produces irregular behaviour [25]. The circuit has a nonlinear negative resistance, known as Chua diode. In 1984, Matsumoto [26] demonstrated numerically the appearance of chaotic attractors in the Chua circuit. Experiments confirming the existence of periodicity and chaos from the Chua circuit were reported by Zhong and Ayrom [27] in 1985. After that, dynamical properties of this circuit have been theorethically investigated [28, 29].

A memristive circuit can be derived from the Chua circuit by replacing the Chua diode with the memristor [22], as shown in Fig. 7. Periodic and chaotic trajectories were observed in experimental setup of modified Chua circuit [30]. Xu et al. [31] found multiple attractors in a memristive Chua circuit.

The dimensionless model of the modified Chua circuit is described by [15]

$$\begin{aligned}\dot{x} &= a(y - x + xM(w)),\\ \dot{y} &= x - y - z,\\ \dot{z} &= by,\\ \dot{w} &= -cx - dw + x^2 w,\end{aligned} \tag{5}$$

where x and y are related to the voltages across the capacitors, z corresponds to the current through the inductor, and w is associated with the internal voltage of the memristor. The memresistance $M(w)$ is defined as $\alpha - \beta w^2$ and a, b, c, d, α, and β are the control parameters.

Figure 8(a) exhibits three periodic attractors, while Fig. 8(b) displays one periodic attractor (red line) and other chaotic (green line). We consider $b = 24$, $c = 37$, $d = 12$, $\alpha = 1.2$, $\beta = 0.5$, (a) $a = 9$, and (b) $a = 12$. In Fig. 9, we compute the bifurcation diagram of the maximum values of z as a function of the control parameter a. The diagram is plotted by forward (black) and backward (red) for $b = 24$, $c = 37$, $d = 12$, $\alpha = 1.2$, and $\beta = 0.5$. It is possible to observe period-doubling cascades to chaos and various narrow windows in which chaos goes to periodic behaviour. We find the existence

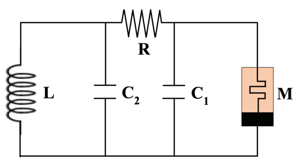

Figure 7: Schematic diagram for the Chua circuit with a non ideal voltage controlled memristor [15]. The memristor replaces the Chua diode.

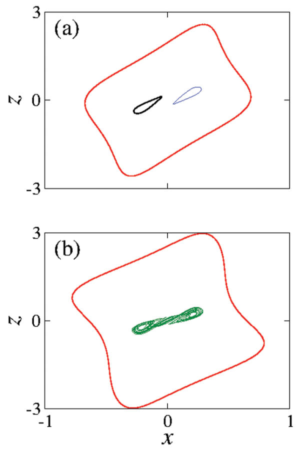

Figure 8: Phase portraits of $z(t)$ versus $x(t)$ showing the coexistence of attractors for $b = 24$, $c = 37$, $d = 12$, $\alpha = 1.2$, $\beta = 0.5$, (a) $a = 9$, and (b) $a = 12$. The panel (a) displays three periodic attractors, while the panel (b) exhibits a periodic (in red) and a chaotic attractor (in green).

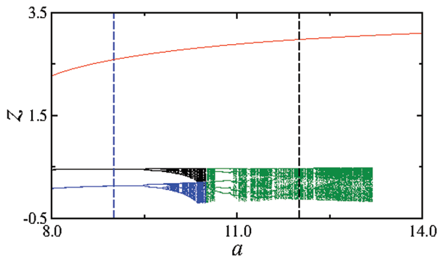

Figure 9: Bifurcation diagram of the local maximum of the dynamical variable z as a function of the control parameter a for $b = 24$, $c = 37$, $d = 12$, $\alpha = 1.2$, and $\beta = 0.5$. The blue and black lines mark the a values used in the phase portraits of Figs. 8(a) and 8(b), respectively.

of coexisting attractors, also called multistability. The diagrams show that periodic attractors coexist, as well as chaotic attractors with different complexities coexist. The blue and black lines mark the a values used in the phase portraits of Figs. 8(a) and 8(b), respectively.

Depending on the initial conditions, the trajectory can move toward different attractors. Figure 10(a) displays initial conditions $x(0)$ and $y(0)$ for orbits converging to three different attractors with period one (white, blue, and red). In Fig. 10(b), we observe trajectories going to a chaotic attractor (green region) and others going to a attractor with period one (red region).

3.3. Modified memristor Colpitts circuit

In 1918, Colpitts [32] invented a type of oscillatory circuit that utilises a combination of capacitors and inductors. The oscillating frequency can be adjusted from a few Hertz up to 10^9 Hertz (microwave region). For this circuit, there are numerical and experimental evidences of periodic and chaotic behaviours [33–35].

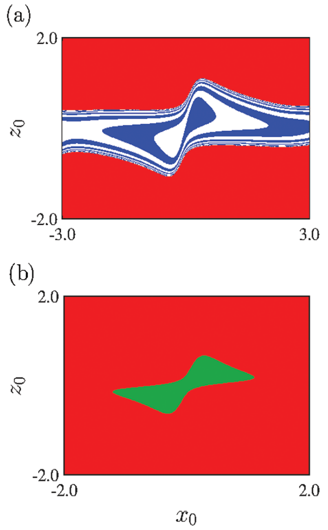

Figure 10: Basins of attraction $z(0)\times x(0)$ with $y(0)=w(0)=0.0$ for attractors depicted in Fig. 8(a) ($a=9$) and Fig. 8(b) ($a=12$). The white, blue, and red regions correspond to subsets of the initial conditions of different attractors with period one, while the green region corresponds to a chaotic attractor.

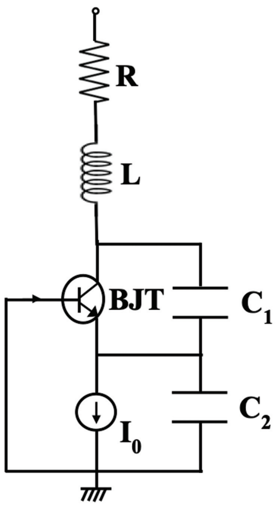

Figure 11: Schematic diagram for the Colpitts circuit where BJT is a bipolar junction transistor [36]. The circuit has a BJT as the gain element and a resonant network composed of a pair of capacitors (C_1 and C_2) and an inductor (L).

Figure 11 displays the classical configuration of the Colpitts oscillator with a bipolar junction transistor (BJT) [36].

Considering normalised parameters and dimensionless state variables, the mathematical model is given by [37]

$$\begin{aligned} \dot{x} &= \frac{g}{Q(1-k)}[z-n(y)], \\ \dot{y} &= \frac{g}{Qk}z, \\ \dot{z} &= -\frac{Qk(1-k)}{g}(x+y)-\frac{1}{Q}z, \end{aligned} \tag{6}$$

where x and y are associated with the voltages across the capacitors, z is related to the current through the inductor, while g, Q, and k are constants. $n(y)$ is an exponential nonlinear term defined as

$$n(y) = \mathrm{e}^{-y}-1, \tag{7}$$

that is related to the voltage-current relation of the BJT.

Memristors have been introduced into the Colpitts circuit by replacing the BJT element to study the dynamical behaviour [38]. Zhang et al. [18] reported multistability in a memristor based Colpitts system. The memristive circuit is created by substituting the exponential nonlinear term by a memristor. The model is formulated as

$$\begin{aligned} \dot{x} &= az-aW(w)y, \\ \dot{y} &= az, \qquad (8)\\ \dot{z} &= -\frac{0.5(x+y)}{a}-bz, \\ \dot{w} &= y, \qquad (9) \end{aligned}$$

where $W(w)=\alpha-\beta w^2$, $a=2g/Q$, and $b=1/Q$.

In Fig. 12, we plot w versus z for $a=5.2$, $b=0.9$, $\alpha=0.5$, and $\beta=0.1$. Considering the same values of the control parameters and different initial conditions,

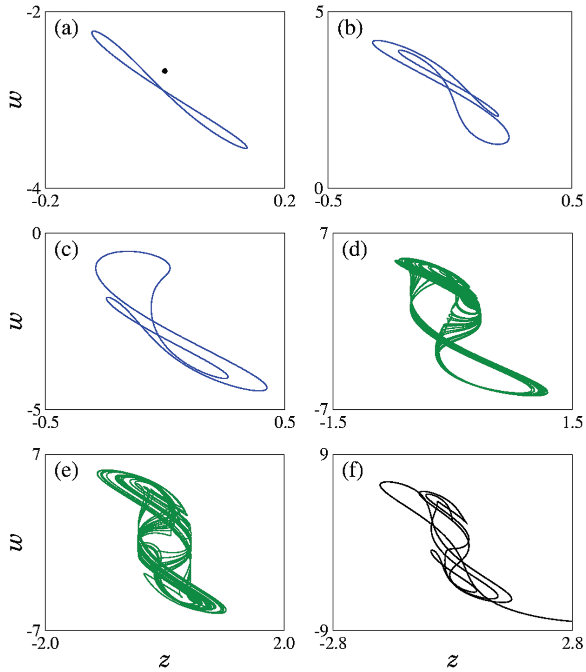

Figure 12: $w(t)$ versus $z(t)$ for $a=5.2$, $b=0.9$, $\alpha=0.5$, and $\beta=0.1$. The panels show the coexistence of many attractors, where we consider as initial conditions (a) $(-1,0,0,-3.2)$ (point attractor in black star) and $(-1,0,0,-3.4)$ (period 1 limit cycle), (b) $(10^{-9},0,0,3.3)$ (period 2 limit cycle), (c) $(10^{-9},0,0,-0.8)$ (period 3 limit cycle), (d) $(10^{-9},0,0,3.6)$ (asymmetric chaotic double-scroll attractor), (e) $(10^{-9},0,0,0)$ (symmetric chaotic double-scroll attractor), and (f) $(-1,2,0,2.2)$ (unbounded orbit).

the panels show that the modified memristor Colpitts circuit exhibits the coexistence of many attractors. For the initial conditions $(-1, 0, 0, -3.2)$ and $(-1, 0, 0, -3.4)$, we find a point attractor (black star) and a period 1 limit cycle (blue line), respectively, as shown in the panel (a). The panel (b) displays a period 2 limit cycle for $(10^{-9}, 0, 0, 3.3)$ and (c) shows a period 3 limit cycle for $(10^{-9}, 0, 0, -0.8)$. In the panels (d) and (e) for $(10^{-9}, 0, 0, 3.6)$ and $(10^{-9}, 0, 0, 0)$, respectively, we observe asymmetric and symmetric chaotic double-scroll attractors. The panel (f) exhibits an unbounded orbit for $(-1, 2, 0, 2.2)$.

4. Conclusions

The memristor is the fourth fundamental passive circuit element that exhibits a pinched hysteresis loop in the current versus voltage plane. It has attracted much attention of researchers due to applications in several areas of integrated circuit design and computing. Over the last few years, many experiments and mathematical models have been proposed to study the dynamic behaviour of memristive circuits.

In this work, we present some memristive circuits that can exhibit chaotic oscillations. We focus on the simplest, Chua, and Colpitts circuits. The simplest circuit constructed from a capacitor and an inductor connected to a memristor shows periodic and chaotic attractors. By replacing the Chua diode with the memristor, it is possible to observe chaotic attractors and multistability. With regard to the Colpitts circuit, it displays complex nonlinear phenomena by including a memristor. Depending on the parameter, the modified memristor Colpitts circuit exhibits extreme multistability.

Acknowledgments

This study was possible by partial financial support from the following agencies: Fundação Araucária, Brazilian National Council for Scientific and Technological Development (CNPq), and Coordination for the Improvement of Higher Education Personnel (CAPES). São Paulo Research Foundation (FAPESP 2018/03211-6, 2020/04624-2, 2022/05153-9, 2022/13761-9). We thank 105 Group Science (www.105groupscience.com).

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