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Playing with two rays of light

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The special theory of relativity is currently included in the curricula of high school in many countries. For this reason, we often organize in-depth lessons on this topic and, in this paper, we summarize a lecture that we have given in some extra activities addressed to last year high school students stimulating pupils "to play" with Lorentz transformations. We found a simple relation that allows us to deduce the rest length of a relativistic train without knowing the relative velocity and without using the lorentz factor and spacetime distortions.

Keywords: Special relativity, Lorentz transformations, Lorentz factor.

1. Introduction

The theory of relativity is currently part of the high school curricula in many countries [1–13]. Students are well aware that the Einstein's theory is based on the principle of relativity and on the fact that the speed of light in vacuum has the same value in all inertial frames of reference [14]. As a consequence of these two postulates, the Galilean transformations are replaced by the new relativistic transformations. Pupils know that the Galilean relations must be replaced and, although the mathematical derivation is not accessible to their knowledge, they know the final form of the so-called special Lorentz transformations which are reported in the textbooks. This is the simplest possible case where the relative velocity v is confined to the $x-x'$ direction obtaining

$$\begin{cases} t = \frac{t'+\frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \\ x = \frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}} \\ \quad y = y' \\ \quad z = z' \end{cases} \tag{1}$$

In the previous system, we have that (x, y, z, t) and (x', y', z', t') are the coordinates of an event in two frames, c is the speed of light and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is called Lorentz factor.

2. Tell me how long it takes for light to travel and I'll tell you how long is the train

Let us consider a train of length L that moves with velocity v with respect to the ground. Then we imagine a ray of light that starts from the rear of the train and reaches the front moving in the same direction as the motion. We now consider the coordinates of the "departure" event and the "arrival" event from two different frames. For the observer on the train, these two events have coordinates $(0, 0)$ and $(L, L/c)$ respectively. From the ground point of view, instead, thanks to special Lorentz transformations (1), we have $(0, 0)$ and $[\gamma(L + \frac{vL}{c}), \gamma(\frac{L}{c} + \frac{vL}{c^2})]$ respectively. Therefore, for the observer fixed to the ground, the light beam takes the following time interval

$$\Delta t_1 = \gamma \left(\frac{L}{c} + \frac{vL}{c^2} \right). \tag{2}$$

Now we consider a flash of light that starts from the front of the train and reaches the rear moving in the opposite direction to the motion. For the observer on the train, these two events have coordinates $(L, 0)$ and $(0, \frac{L}{c})$ respectively. Instead, thanks to (1), the observer on the ground measures $(\gamma L, \gamma \frac{vL}{c^2})$ and $(\gamma \frac{vL}{c}, \gamma \frac{L}{c})$ getting that light takes the following time interval

$$\Delta t_2 = \gamma \left(\frac{L}{c} - \frac{vL}{c^2} \right). \tag{3}$$

Since our goal is to be able to calculate the length of the train without knowing its velocity, we looked for a mathematical way to make the Lorentz factor vanish. We have observed that this occurs by multiplying relations (2) and (3). Indeed we get

$$\Delta t_1 \cdot \Delta t_2 = \gamma^2 \left(\frac{L^2}{c^2} - \frac{v^2 L^2}{c^4} \right) = \gamma^2 \frac{L^2}{c^2} \left(1 - \frac{v^2}{c^2} \right) = \frac{L^2}{c^2}. \tag{4}$$

Furthermore, we have

$$\sqrt{\Delta t_1 \cdot \Delta t_2} = \frac{L}{c} = \Delta t. \tag{5}$$

We can note that the Lorentz factor vanishes and we can find out the rest length of the train

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$c\sqrt{\Delta t_1 \cdot \Delta t_2} = c\Delta t$ without taking into account the relativistic time dilations. We just need to measure the travel time of the two light rays. It could be useful for didactic purposes to point out that to obtain the value of the length L without knowing the velocity v of the train, it is necessary to perform two different measures of time using two rays of light. Instead, if the observer at rest wants to determine the length of the train with only one measure of time, he must wait that the head of the train reaches his position and, from that instant, he measures the lapse of time Δt until also the end of the train reaches his position. In that case $L = v\Delta t$ and, to determine L, it is necessary to know also the velocity v of the train. It could certainly be interesting to understand if the result of the lesson has general validity, with a deeper physical meaning, or whether it is just a mathematical coincidence of this simple case. In other words, it would be useful to investigate the physical meaning of the multiplication between the two time intervals in the most general case. Such an in-depth study, of course, was not possible. Indeed, the young students do not know the more general Lorentz transformations or the Poincare group.

3. Conclusion

In this brief paper, starting from some in-depth lessons held at high schools, we present one of these, calculating the rest length of a relativistic train without knowing the relative velocity of the inertial observers.

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