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Articles

Interdisciplinary analysis of bank erosion and formation of river meanders: insights into the dynamics of non-inertial reference frames and implications for river management

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Erosion corresponds to the process of fragmentation, transportation, and deposition of soil and/or rock fragments. It is the main exogenous factor responsible for the alteration of the earth's surface, resulting in various geomorphological formations, many of which have significant economic interest due to their beauty. In this paper, we aim to discuss, from an interdisciplinary perspective, the general and specific elements that contribute to the erosion of river banks and the formation of meanders. We present qualitative and quantitative arguments that suggest that the Coriolis force, a non-inertial force, may play a fundamental role in understanding these phenomena. In addition, a detailed discussion of physics in the reference frame of the Earth (non-inertial) is carried out.
Keywords: Erosion, riverbank, Baer's law, meanders formation, interdisciplinarity.

1. Introduction

Erosions have always caught the attention of human beings, as they are often phenomena of great visual impact responsible for the transformation of the terrain. Appreciation of these events is a source of income in various tourist locations. After all, how not to be amazed by the canyons of Xingó in Sergipe, Capitólio in Minas Gerais, Itaimbezinho in Santa Catarina and Rio Grande do Sul, among several others in Brazil and around the world?

Unfortunately, not all erosion processes are beautiful, as they can impoverish soils, put homes and buildings at risk, damage access roads, among other harmful effects. Thus, the study and control of erosive processes on river banks are of paramount importance to society, especially for riverside populations, whose homes are built in the vicinity of riverbeds.

The field of knowledge competent to study erosive phenomena is geography and, in particular, geology. In this study, it is reasonable to imagine that the most relevant parameters are local, such as the structure, dimensions, and geometry of the banks; temporal-climatic conditions; mechanical properties of the composition material, such as the particle size of the soil; presence of riparian vegetation, among others [1, 2]. However, it is less intuitive to imagine global parameters that can influence erosions in all rivers of the world in a predictable manner.

Such a global phenomenon that influences erosion on river banks was suggested in 1859 by the French physicist Jacques Babinet and proposed more definitively by Karl Ernst von Baer in 1860 [3–5]. The proposal is that the rotation of the Earth significantly influences the erosion of river banks. This influence would be asymmetric in relation to the right and left banks of the river and would also be different in the northern and southern hemispheres. This proposal became known, more correctly, as the Babinet-Baer law or simply as Baer's law.

This phenomenon is so curious that it even drew the attention of Albert Einstein. He studied this topic in a brief article published in 1926 [6][1]. In this work, the author uses simple physics concepts to explain two empirical phenomena: (1) the course of river waters tends to run in sinuous lines, rather than following the line of steepest slope on the terrain; (2) rivers in the northern hemisphere preferentially erode their right banks, while those in the southern hemisphere erode their left banks predominantly. This latter phenomenon is Baer's law [3, 5].

In this work, we revisit Einstein's article. That is, we use basic physics arguments to understand some of the important factors for the formation of meanders

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[1] For interested readers, this article can be found translated into Portuguese in the reference [7].

in rivers, as well as the process of erosion of river banks. Einstein, however, focused on the general aspect common to all rivers. He did not bother to discuss the particular aspects that influence these phenomena. Therefore, in order to make the work more complete, we added a discussion of these phenomena from the point of view of geography, which encompasses the particular elements not explained by Einstein. In addition, we provide a detailed treatment regarding the influence of the Earth's rotation and water friction with the banks in the process of erosion and meander formation in rivers. This treatment allows us to discuss physics in non-inertial reference frames, a subject little discussed in basic physics courses. The dialogue between areas of knowledge is always important and desirable in the academic context. Moreover, it plays an important role in highlighting to students and teachers the global nature of knowledge.

The work is structured as follows: in Section 2, we present a discussion of the local aspects that most influence the process of erosion of river banks, while in Section 3, we conduct a qualitative discussion on the formation of river meanders in light of Einstein's arguments. Section 4 is dedicated to a quantitative derivation of the non-inertial forces that arise on the surface of the Earth, taking into account that it is a non-inertial reference frame due to its rotation. It is in this section that the equation for the calculation of the Coriolis force will be derived. The qualitative and quantitative arguments supporting the idea that the Coriolis force can have a significant influence in explaining the Babinet-Baer law are presented in Section 5. The conclusions of the work are presented in Section 6.

2. The erosion of riverbanks

In a broader sense, erosion corresponds to the fragmentation, transport, and deposition of sediment from rocks and/or soils. Depending on the causal agent, it can be classified as: wind erosion, fluvial erosion, glacial erosion, marine erosion, and rainfall erosion. Various human actions, such as inadequate soil management, deforestation, and burning, favor the intensification of erosive processes.

Regarding the subject, riverbanks are constantly subject to fluvial erosive processes since the water flow is continuous. Moreover, they have great relevance for fluvial morphology, as rivers become more straight or meandering due to the occurrence of marginal erosion. Studies on the erodibility of riverbank margins experienced notable development from the 1950s and 1960s onwards, among which the works of Wolman, Wolman and Miller, and Schumm and Lichty [8–10] can be highlighted.

There are different direct methods for measuring and monitoring marginal erosion, such as the pin method [8], the stake method [11], and successive profiling [12].

The erodibility of the margins depends on several factors, among which it is possible to highlight: structure, dimension, and geometry of the margins; temporal-climatic conditions; mechanical properties of the composition material, such as the granulometry of soil particles; presence of riparian vegetation; among others [1, 2].

In summary, the height and angle of the margin are inversely proportional to its stability. Thus, the greater the angle formed between the margin slope and the free surface of the flow and the higher the margin, the more unstable it will be, and consequently, the greater its susceptibility to erosive processes [13].

The climate to which the hydrographic basin belongs and the season of the year also interfere with the erodibility of riverbanks [2]. Climates with higher annual rainfall rates or with a predominance of more intense rainfall tend to present greater activity of rainfall and fluvial erosive processes under the margins, as occurs during the rainier seasons. On the other hand, drier climates suffer less from water erosion.

Soil granulometry has a significant influence on erosive processes that act on margins since the aggregation of grains and the permeability of soils vary according to the size of the predominant particles. Clay soils have greater cohesion between their grains, so they are less permeable and more resistant to erosion caused by water flow. Sandy soils have less aggregation between their particles, as sand has a larger diameter than other soil fractions, such as clay and silt. Therefore, they are characterized by high permeability and susceptibility to fluvial erosion [14].

Riparian vegetation contributes to marginal stabilization since it reduces the speed of water colliding against the margins. Furthermore, the connections established between tree roots and soil particles also contribute to the fixation of the margins. Therefore, their absence or incipiency causes the accentuation of fluvial erosion and the increase of sediment deposition in the riverbed. It should be noted that the presence of trees contributes to the formation of litter, which increases soil coverage and, consequently, its protection against erosion [15, 16].

Marginal erosion directly interferes with water flow, since sediments from the banks are deposited in the channel. Therefore, river siltation occurs, a phenomenon in which the volume of sediments deposited in the river exceeds its transport capacity. As a consequence, there is an increase in water turbidity, a decrease in its water volume, a reduction in available oxygen, and the death of aquatic animals. In more severe cases, it can result in the extinction of the body of water [17].

The erosion of river banks also causes the straightening or meandering of rivers, as well as the formation of abandoned meanders.

3. Formation of river meanders

Einstein proposes that small changes in the velocity vector of the flow between the river banks are responsible for producing secondary circular currents in the plane perpendicular to the direction of the flow. These small changes can be caused by the Coriolis force, curves in the river, or even characteristics of the banks. This phenomenon, according to Einstein, produces erosion on one of the banks and sediment deposition on the other, resulting in the formation of a winding watercourse, which we call a meander [7].

There are three main theories to describe the formation of meanders [19]. Depending on the characteristics of each river, one theory may be more suitable than the others for such a study, but all, in some way, are relevant to understanding the formation of meanders. They are:

- **Stochastic theory:** Suggests that random fluctuations in flow velocity are responsible for the formation of meanders;
- **Equilibrium theory:** Assumes that meanders result from the adjustment process between terrain erodibility and the erosive power of the flow;
- **Geomorphologic theory:** Attributes the cause of meanders to deviations caused by tectonic features acting as an obstacle.

We see that all three theories agree on the importance of the asymmetry of flow velocity at the banks proposed by Einstein to understand the formation of meanders.

In addition to these factors, Einstein proposes that even a perfectly symmetrical river with respect to the composition and geometry of its banks and depth would still exhibit meanders due to erosions caused by transverse circular currents to the flow. This would explain the spatial similarity of scale of meanders observed in rivers around the world.

To understand Einstein's proposal, we will analyze how transverse currents are formed in a straight segment and also in a circular segment of the river. In both analyses, the Coriolis force plays a fundamental role. We will evaluate this force quantitatively in the next section. Now, the important point for our purposes is that this force arises as a consequence of the rotation of the Earth. In other words, it is a non-Newtonian force that arises because the Earth is a non-inertial reference frame. Moreover, it is a force that acts perpendicular to the flow of the river and has opposite directions in the northern and southern hemispheres of the Earth (derivation in the next section).

In Figure 1, we schematically present the top view of the profile of a straight segment of a river. Let us suppose that in this case, the Coriolis force acts from point A to point B, as shown in Figure 1. Due to friction with the bottom, the resultant force on the fluid is smaller at the bottom. This difference in transverse force between the surface of the river and the bottom is responsible for generating the circular currents proposed by Einstein, presented in Figure 2 (a).

Figure 2 (b) schematically presents a circular section of a river from a top view. Let us take the Coriolis force acting in the direction from A to B again. However, in

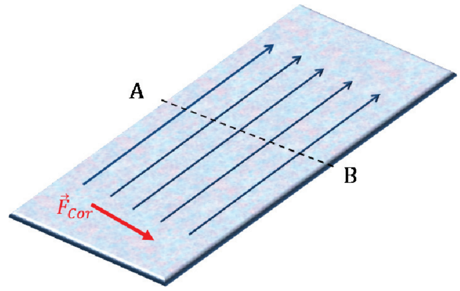

Figure 1: Schematic figure of the top view of the profile of a straight segment of a river.

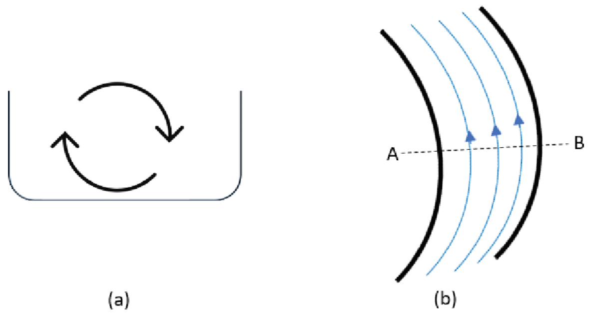

Figure 2: Schematic figure of the circular currents proposed by Einstein(a) and schematic figure of a circular section of a river from a top view (b).

this case, in addition to the Coriolis force, we will also have the action of a centrifugal force (non-inertial) in the direction from A to B. That is, in circular points of a river where Coriolis and centrifugal forces act in the same direction, we should observe a reinforcement of the circular currents suggested by Einstein in relation to those in a straight segment. It may happen, however, that the Coriolis and centrifugal forces are in opposite directions. In this case, obviously, we should expect attenuated circular currents compared to those observed in straight river sections.

4. Physics in the Earth's reference frame and the Coriolis force

A reference frame is said to be inertial if a particle observed in this frame moves in a uniform linear motion (ULM) whenever the resultant forces acting on it are zero. In other words, a reference frame is inertial when it satisfies Newton's first law.

The Earth performs a circular[2] motion around the Sun and a rotation around an imaginary axis passing through its center. Therefore, strictly speaking, a reference frame on the surface of the Earth is not inertial. Nevertheless, in the study of systems at low velocities (velocities less than 10^3 m/s) and for short times (less than a few

[2] To be more precise, an ellipsoidal motion.

hours), we can ignore the Earth's movements and treat reference frames on its surface as inertial, obtaining good results with this approximation. However, when dealing with meanders in rivers, this approximation can no longer be used. Although rivers are low-velocity systems, the time considered for the analyzed phenomenon is not.

In non-inertial reference frames, effects that resemble the action of forces appear. These effects are not forces in the Newtonian sense, since they do not result from the interaction between bodies and, consequently, they also do not satisfy Newton's third law. However, if we treat these effects as if they were forces, even of another nature, we can use the same tools already used for the analysis of dynamics in inertial systems. Therefore, we call these effects inertial forces, non-Newtonian forces, or fictitious forces[3].

Consider an inertial reference frame Σ_I at the center of the Earth and a non-inertial reference frame Σ at a point on the surface of the Earth (Figure 3). In the reference frame Σ, the z-axis points vertically upward relative to the Earth's surface, the x-axis is tangent to the meridian line in the north-south direction, and the y-axis points in the west-east direction tangent to the parallel line. As demonstrated in Appendix A, the acceleration of a particle at position $\vec{r}$ relative to the non-inertial reference frame $\vec{a}_\Sigma$ is given by:

$$\vec{a}_\Sigma = \vec{a}_{\Sigma_I} - \vec{a}_{0\Sigma_I} + 2m\vec{v}\times\vec{\omega} + \vec{\omega}\times(\vec{r}\times\vec{\omega}) + \vec{r}\times\frac{d\vec{\omega}}{dt}, \quad (1)$$

where $\vec{a}_{\Sigma_I}$ is the acceleration of the particle with respect to the inertial reference frame Σ_I, $\vec{a}_{0\Sigma_I} = \left(d^2\vec{R}/dt^2\right)_{\Sigma_I}$ is the acceleration of the origin of the non-inertial coordinate system with respect to the inertial coordinate system, $\vec{v}$ is the velocity vector of the particle with respect to the non-inertial reference frame, and $\vec{\omega}$ is the angular velocity vector of the Earth.

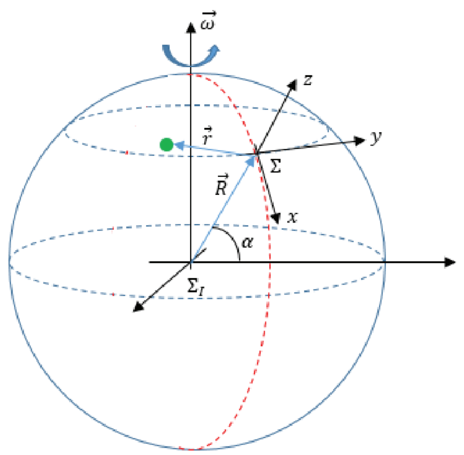

Figure 3: Schematic figure of a particle on the surface of the Earth and an inertial reference frame at the center of the Earth Σ_I and another non-inertial reference frame on the surface of the Earth Σ.

Keeping in mind Newton's second law, by multiplying equation (1) by mass, we obtain that:

$$m\vec{a}_\Sigma = \underbrace{m\vec{a}_{\Sigma_I}}_{\vec{F}} - m\vec{a}_{0\Sigma_I} + \underbrace{2m\vec{v}\times\vec{\omega}}_{Cor.} + \underbrace{m\vec{\omega}\times(\vec{r}\times\vec{\omega})}_{Centr.} + m\vec{r}\times\frac{d\vec{\omega}}{dt}. \quad (2)$$

For a system that moves only under the influence of gravity (as is the case for rivers), the first term on the right-hand side of equation (2) corresponds to the weight component analyzed from the point of view of the inertial reference frame Σ_I. Therefore, this term does not bring us any new information.

The second term on the right-hand side of equation (2) can be written in the form[4]

$$m\vec{a}_{0\Sigma_I} = m\left(\frac{d^2\vec{R}}{dt^2}\right)_\Sigma = -m\vec{\omega}\times(\vec{R}\times\vec{\omega}), \quad (3)$$

Note that this term has its modulus proportional to ω^2. Let us consider the angular velocity of the Earth to be constant with a modulus approximately equal to[5]

$$\omega = \frac{2\pi}{\underbrace{24\times 3.600}_{Seconds\,in\,a\,day}} \approx 7,3\times 10^{-5}\ \text{rad/s}. \quad (4)$$

Where we can conclude that this squared term is very small (of the order of 10^{-9}), which justifies neglecting its influence in relation to other larger terms. The last term on the right-hand side of equation (2) vanishes since we are considering $\vec{\omega}$ to be constant. We can observe that the fourth term on the right-hand side of equation (2), known as the centrifugal term or centrifugal force, has its magnitude proportional to the square of ω. Therefore, this term is much smaller than the third term on the right-hand side of equation (2), known as the Coriolis term or Coriolis force. For this reason, we will disregard the contribution of the centrifugal term. With this, we can observe that the most relevant term among the fictitious forces for the dynamics of the system is the Coriolis force.

4.1. The Coriolis Force

From equation (2), we have that the Coriolis force is given by:

$$\vec{F}_{cor} = 2m(\vec{v}\times\vec{\omega}). \quad (5)$$

[3] And thus we set aside any reservations about the ontological nature of forces, after all, "being" is not exactly the subject of study of science.

[4] The interested reader can find the proof of this result in the appendix.

[5] To be more precise, we should use the sidereal day, which is the time it takes for the Earth to complete a 360° rotation around the imaginary axis that passes through its center. This time is 23 hours and 56 minutes. However, this difference of 4 minutes is very small compared to the order of magnitude of ω. For this reason, we will consider the Earth's period to be 24 hours.

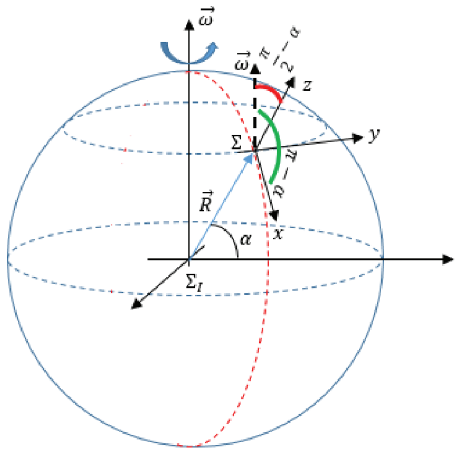

Figure 4: Schematic representation of the vector $\vec{\omega}$ with respect to the coordinate axes in the reference system (non-inertial) on the surface of the Earth.

To solve for the term $\vec{v} \times \vec{\omega}$, we decompose the vector $\vec{\omega}$ into its components along the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ of the x, y, and z axes, respectively. From Figure 4, we can see that $\vec{\omega}$ can be written as:

$$\vec{\omega} = -\omega\text{cos}(\alpha)\hat{\imath} + \omega\text{sen}(\alpha)\hat{k}. \tag{6}$$

To analyze how the Coriolis force influences river currents, we will analyze currents along the two coordinate axes in the reference system on the surface of the Earth (non-inertial). To define right and left, we will always take the direction of the river water flow as a reference.

4.1.1. Velocity along the meridians

If the river flows in the direction of the meridians, then in the adopted coordinate system (Figure 3), the velocity of the flow is given by:

$$\vec{v} = v\hat{\imath}. \tag{7}$$

In this case, the Coriolis force takes the following form:

$$\begin{aligned} \vec{F}_{cor} &= 2m\,(v\hat{\imath}) \times [-\omega\text{cos}(\alpha)\hat{\imath} + \omega\text{sen}(\alpha)\hat{k}] \\ &= -2m\omega v\text{sen}(\alpha)\hat{\jmath}. \end{aligned} \tag{8}$$

4.1.2. Velocity along parallels

If the river flows in the direction of the parallels, then the velocity of the flow is given by:

$$\vec{v} = v\hat{\jmath}. \tag{9}$$

In this case, the Coriolis force takes the following form:

$$\begin{aligned} \vec{F}_{cor} &= 2m\,(v\hat{\jmath}) \times [-\omega\text{cos}(\alpha)\hat{\imath} + \omega\text{sen}(\alpha)\hat{k}] \\ &= 2m\omega v[\text{sen}(\alpha)\hat{\imath} + \text{cos}(\alpha)\hat{k}]. \end{aligned} \tag{10}$$

Unlike the previous case, the Coriolis force now has a component in the vertical direction. Since cosine is an even function ($\cos(\alpha) = \cos(-\alpha)$), the direction of this component depends only on the sign of the velocity v. If $v > 0$, it acts in the opposite direction to weight, and if $v < 0$, it acts in the same direction as weight. For velocities of the order of 10^0 m/s and masses of the order of 10^0 kg, however, this term is about 10^{-5} times smaller than the weight force and can be neglected.

Regarding the term of the Coriolis force in the $\hat{\imath}$ direction, let us assume that the river is in the **northern hemisphere** ($\alpha > 0$). If $v > 0$, then the component points in the positive direction of x (north-south), and if $v < 0$, it points in the negative direction of x (south-north). However, note that from the point of view of an observer descending the river in a boat, in both cases, the component points to their right. The same reasoning made for the **southern hemisphere** ($\alpha < 0$) leads us to the conclusion that, in this case, the component always points to the left of the observer descending the river in a boat.

5. Babinet-Baer's Law

In a work from 1860, Karl Ernst von Baer discussed the asymmetric erosion on the banks of rivers [5]. According to what became known as Babinet-Baer's law, it is observed that in the northern hemisphere the right banks of rivers erode more than the left banks, while in the southern hemisphere the opposite occurs. As we saw earlier, this asymmetry is similar to that of the Coriolis force. Therefore, it is natural to try to attribute to the Coriolis force the responsibility for this asymmetric erosion of riverbanks.

A pertinent objection to establishing a cause-and-effect relationship between the Coriolis force and Baer's law is the fact that the ω term appearing in the Coriolis force is very small (of the order of 10^{-5}, as we can see from equation (4)), which suggests that the Coriolis force is also small. We will try to verify below whether this objection is really valid.

The ω term appearing in the Coriolis force, however, is very small. The Coriolis force will be larger if the mass is larger, therefore, it is expected that more voluminous rivers will suffer more erosion. In addition, the higher the flow velocity of the rivers, the greater the Coriolis force will be. Even with mass and velocities causing the Coriolis term to increase, the result will still be a small absolute value of this force. The question is: can a small absolute value term like the Coriolis term cause a noticeable influence on the process of erosion of riverbanks?

To try to answer this question, let us consider two situations where the Coriolis term will be significant: systems with high velocity and when the force can act for a long time[6]. As an example of high-speed

[6] The Coriolis term is also significant in systems with large masses; however, in this case, the weight force (considering systems on Earth) will be much larger and make it negligible.

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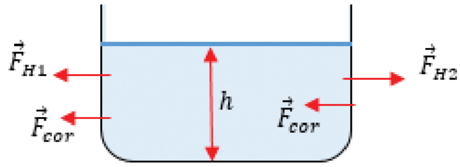

Figure 5: Schematic figure of the profile of a river in the form of a channel.

systems where the Coriolis term is important, we can cite supersonic projectiles. We can mention the Foucault pendulum as a system in which the action of the Coriolis force over long periods is relevant. Both effects can act together, as in the case of intercontinental ballistic missiles and more pronounced wear on the left (right) side of railway lines in the southern (northern) hemisphere. The Coriolis term also has a strong influence on meteorological phenomena, such as the trade winds and the sense of rotation of cyclones, which tends to be counterclockwise in the northern hemisphere and clockwise in the southern hemisphere.

Rivers are in the low velocity situation (so that the Coriolis term is not relevant), but with a movement that takes place over centuries. Thus, we can consider it reasonable to justify that the appearance of the Coriolis force in the reference frame of the Earth's surface, due to its rotation, is a relevant factor to explain Babinet-Baer's law.

We can use a simplified model, proposed by Unger et al., to quantitatively estimate the influence of Coriolis force on asymmetric erosion processes, as well as the influence of other river characteristics [18]. To do so, let us consider the profile of a river in the form of a channel, as shown in Figure 5. In the figure, F_H represents the hydrostatic forces[7] that act on the river banks, h represents the depth of the river, and F_{cor} represents the Coriolis force, which in this case, we are considering to act from left to right. Note that the Coriolis force is the same on both banks, but the same cannot be said for the hydrostatic force. Since the Coriolis force acts from right to left, it will cause the left bank of the river to have greater mass, density, and depth than the right bank. For these reasons, we expect the hydrostatic forces to be different on the two banks.

Note that on the left bank, the resultant normal force will be given by

$$F_{N1} = F_{H1} + F_{cor},$$

and on the right bank, the resultant normal force will be

$$F_{N2} = F_{H2} - F_{cor}.$$

For the sake of simplification, let us take[8] $F_{H1} = F_{H2} = F_H$. The erosion of the banks occurs due to the friction of the flow with the banks, which depends on the normal force and the characteristics of the surfaces in contact. Let us take the friction forces on the left and right banks as, respectively,

$$f_{at1} = \mu_1 F_{N1},$$

and

$$f_{at2} = \mu_2 F_{N2},$$

once again, for the sake of simplification, let us take[9] $\mu_1 = \mu_2 = \mu$.

The rate at which water flow erodes the bank can be associated with the rate of energy of the water that is dissipated by friction with the bank. This rate is the power developed by the frictional force. The power on the left and right banks is given, respectively, by

$$P_1 = f_{at1} v,$$

and

$$P_2 = f_{at2} v,$$

where v is the velocity of the river flow. As we are interested in the difference of erosions on the two banks, we will analyze the difference in power between the banks:

$$\Delta P = P_1 - P_2 = (f_{at1} - f_{at2})\, v$$
$$\Delta P = \mu\,(F_{N1} - F_{N2})\, v,$$

or

$$\Delta P = 2\mu F_{cor} v = 4\mu m \omega v^2 \sin(\alpha).$$

We can write this last equation in terms of water density (ρ) and channel dimensions ($A \times L$), where A is the wetted area[10] and L is the length of the considered channel. In this case, we can write the power difference between the banks per unit length of the river as

$$\frac{\Delta P}{L} = 4\mu A \rho \omega v^2 \sin(\alpha). \tag{11}$$

As the flow rate Q is given by vA, the equation can be rewritten as:

$$\frac{\Delta P}{L} = 4\mu \rho \omega Q v \sin(\alpha). \tag{12}$$

[7] Strictly speaking, we should perform a hydrodynamic treatment here, but since our goal is only to present a simplified model that provides an estimate of the influence of Coriolis force on the erosive processes of river banks, the hydrostatic treatment is sufficient.

[8] As discussed earlier, we expect $F_{H1} > F_{H2}$, so our assumption will make the Coriolis force have less influence than it actually does.

[9] That is, we are considering that the coefficient of friction between the banks and the water flow is the same on both banks of the river.

[10] The wetted area corresponds to the cross-sectional section of the river that is in contact with water at a given moment.

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Table 1: Average flux velocity v (m/s), flow rate Q (m^3/s), and P/L (W/m) for the Yangtze, Araguaia, and São Francisco rivers. We used the values $\rho = 1000kg/m^3$ and $\mu = 0.1$, following [18].

River	Latitude	v	Q	P/L
Yangtse [20]	30°	1.91	82957	2316
Araguaia [21]	−8.29°	1.361	25650	146
São Francisco [22]	−15.94°	1.58	9714	123

Note that this result shows us that the difference in erosions on the two banks due to the Coriolis force depends on ω, which is a small factor as we saw earlier, but also depends on the velocity of water flow in the river in a quadratic form.

In the Table 1, we can observe an estimate for the value of P/L for three large rivers[11]. Note that the high flow rate and velocity of the Yangtze River result in a high value for P/L. For the São Francisco and Araguaia rivers, this value is much lower, but not negligible. After all, we are talking about systems that operate on large time scales. This result is strong empirical evidence that the Coriolis force plays an important role in the asymmetric erosion of river banks.

6. Conclusion

In this paper, we presented an interdisciplinary discussion on the phenomena of riverbank erosion and meander formation. There are local and global factors that influence marginal erosion and, consequently, the emergence of meanders. The complexity of the topic requires several areas of knowledge to be used for a more complete understanding. The study of particular aspects is an obligation assigned to geographic science, while the evaluation of general elements is a function of physics. Thus, an interdisciplinary approach to the subject can increase the scope of the analysis.

Furthermore, we discussed how non-inertial forces arise in non-inertial reference frames, such as the surface of the Earth. Among these forces, the Coriolis force stands out, which plays a fundamental role in explaining the phenomena of asymmetric erosion (Babinet-Baer's law) and meander formation. Our arguments were based on the reflections presented by A. Einstein in 1926.

Supplementary material

The following online material is available for this article:
Appendix: Derivation of some equations

[11] Due to the absence of specific data for the rivers in question, we attempted to establish a lower limit as follows: According to [18], the values of μ for the Tisza River fluctuate between 0.1 and 0.3, and we adopted a value of 0.1. The values for the density of water in the river ρ range from $1000kg/m^3$ to $1300kg/m^3$ due to the amount of suspended material in certain situations, but we adopted $\rho = 1000kg/m^3$.

References

[1] D.L. Rosgen, in: *Proceedings of the Seventh Federal Interagency Sedimentation Conference* (Reno, Nevada, 2013).

[2] J.H.R. Andrade and C.E. Maia, Boletim de Geografia **37**, 71 (2019).

[3] J. Babinet, Compt. Rend. **49**, 638 (1859).

[4] R.W. Fairbridge, in: *Geomorphology* (Springer, Berlin, 1968).

[5] K.E.V. Baer, *Kaspische Studien* **8**, 1 (1860).

[6] A. Einstein, Die Naturwissenschaften **11**, 223 (1926).

[7] A. Einstein, *Como vejo o mundo* (Nova Fronteira, Rio de Janeiro, 2016).

[8] M.G. Wolman, American Journal of Science **257**, 204 (1959).

[9] M.G. Wolman and J.P. Miller, The Journal of Geology **68**, 54 (1960).

[10] S.A. Schumm and R.W. Lichty, Geological Survey Professional Paper **352-D**, 71 (1963).

[11] D.J. Hughes, in: *River Channel Changes*, edited by K.J. Gregory (John Wiley & Sons, Chichester, 1977).

[12] H.R. Hudson, Canadian Journal of Earth Sciences **19**, 381 (1982).

[13] M. Rinaldi and S.E. Darby, Developments in Earth Surface Processes **11**, 213 (2008).

[14] W. Teixeira, M.C.M. Toledo, T.R. Fairchild and F.Taioli, *Decifrando a Terra,* (Oficina de textos, São Paulo, 2000).

[15] J.C. Ritchie and G.W. Mccarty, Soil & Tillage Research, **69**, 45 (2003).

[16] J.P. Julian and R. Torres, Geomorphology **76**, 193 (2006).

[17] S.B. Cunha, in: *Geomorfologia: Uma atualização de bases e conceitos*, edited by A.J.T. Guerra and S.B. Cunha (Bertrand Brasil, Rio de Janeiro, 1996).

[18] Z. Unger and I. Németh, Carpathian Journal of Earth and Environmental Sciences **8**, 201 (2013).

[19] R.M. Khatsuria, *How and why rivers meander* (Central Water and Power Research Station, Pune, 2010).

[20] Z.N. Zhu, X.H. Zhu, C. Zhang, M. Chen, H. Zheng, Z. Zhang, J. Zhong, L. Wei, Q. Li, H. Wang et al., Frontiers in Earth Science **9**, 723123 (2021).

[21] AGÊNCIA NACIONAL DE ÁGUAS, *HidroWeb: Sistema de Informações Hidrológicas. Estação Conceição do Araguaia (Código 27500000)*, disponível em: https://www.snirh.gov.br/hidroweb/serieshistoricas, acessado em 14/5/2023.

[22] AGÊNCIA NACIONAL DE ÁGUAS, *HidroWeb: Sistema de Informações Hidrológicas. Estação São Francisco (Código 44200000)*, disponível em: https://www.snirh.gov.br/hidroweb/serieshistoricas, acessado em 14/5/2023.