

Developing computer use skills for problem solving in engineering students from the first year physics course

(Desarrollo de habilidades relacionadas con el uso de los ordenadores como herramienta para resolver problemas en el primer curso de física para ingeniería)

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Usually one can find three subjects in the first year of the syllabus of any technical engineering career, namely, calculus, general physics and programming. Being physics a matter lying on the grounds of technical engineering it becomes naturally appropriate to introduce the use of calculus and programming as useful tools in the context of a physics problem. This can be accomplished by moving some Practical Classes of Physics (problem solving) into the computer pool and by reformulating the physics problems in order to make them more appropriate for this kind of approach. In this environment, students put together, for instance, programming tools and numerical methods, along with the physical laws in order to address more realistic models, different from those which can usually be treated on the blackboard. This kind of computational physics problems increases the motivation of the engineering students by embedding them into sceneries whose models are closer to those real problems they will be facing later in their professional and scientific life. This is particularly relevant for the first year of the engineering careers when the development of this kind of professional skills is usually skipped. In the present work we will illustrate these ideas by means of the known problem of “The motion of a body subject to air drag force”. The basic ideas of this work have been experienced in the physics course of first year undergraduate students of telecommunication and electronics engineering of Pinar del Río University, Cuba in 2010.

Keywords: physics for engineers, computer-aided teaching.

Es usual encontrar tres asignaturas en el primer año de las carreras de ingenierías técnicas, es decir: cálculo, física general y programación. Como la física se encuentra en la base de conocimiento de las ingenierías técnicas, es naturalmente apropiada la introducción del Cálculo y la Programación como valiosas herramientas en el contexto de un problema de Física. Esto se puede lograr trasladando algunas Clases Prácticas de Física (dedicadas a la solución de problemas) hacia el laboratorio de ordenadores y por medio de una reformulación del problema, de modo que sea más adecuado para este tipo de situaciones computacionales. En este entorno, los estudiantes logran reunir, por ejemplo, herramientas de programación y métodos numéricos, junto con las leyes de la Física, con el objetivo de abordar modelos más realistas, diferentes de los que usualmente se tratan comúnmente en el pizarrón. Este tipo de problema computacional de Física incrementa la motivación de los estudiantes de ingeniería por medio de una imbibición en escenarios cuyos modelos son más cercanos a los problemas reales que ellos enfrentarán luego en el desempeño profesional y científico. Este hecho es particularmente relevante para el primer año de las carreras de ingeniería donde el desarrollo de habilidades profesionales es obviado y relegado para años superiores. En el presente trabajo ilustraremos estas ideas a través del conocido problema del “Movimiento del un cuerpo sujeto a la fuerza de arrastre del aire”. Las ideas básicas de este trabajo han sido experimentadas en el curso de física de primer año de la carrera de telecomunicaciones y electrónica de la Universidad de Pinar del Río, Cuba en el año 2010.

Palabras-clave: física para ingenieros, enseñanza asistida por computadoras.

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1. Introduction

Computers can be found to be an essential part in many spheres of the society and are commonly found in schools at all teaching levels [1]. The link between computers and teaching has been a continuous process during the last years. Diverse types of softwares as tools for teaching have been developed [2-4]. There has been such a big impact that the way teachers teach and the way students learn have undergone deep changes with the advent of computers. Some examples of this are the computer simulators, teleconferences and distance education courses on internet [5, 6]. Every new technology in general (interactive tutorials, internet forum, etc) gains more space in teaching process [7-9].

The use of computers makes teaching-learning process more optimal, interesting and meaningful. Students can get better prepared for the professional and scientific life that they will be facing later after graduation and also for integrating to the society in general [5].

Computer-related skills are becoming every time more important in the engineer profile [10-12]. It is common to see an increasing use of computational tools for the modelling, simulation and visualization of processes in the everyday scientific and professional life of engineers [12, 13]. This has to do with the fact that reasonable powerful computers are becoming commercially available at an affordable price parallel to the development of numerical methods. In this respect, computer based research constitutes a good alternative in front of costly experiments, particularly in the developing countries. The Scientific Computing deals with mathematical models and numerical techniques applied to scientific problems with the help of computers. This new field of modern science finds applicability in many other traditional fields, such as the natural sciences, human and social sciences. The numeric simulation allows the study of complex systems where analytic solutions are not possible or even impossible to be studied by direct experimentation, *e.g.* Refs. [14, 15].

In physics teaching, computers have been used in two main directions. One consists in using computers for the familiarization of students with styles of works and proceedings behind the technical and scientific activities nowadays [12, 13, 16]. The other has been more popular and it addresses the use of computers to make the teaching-learning process easier, becoming a valuable tool for the teachers [6, 10, 11, 17-19], usually in the introduction of more complex concepts [20-22]. This is the case for example of interactive tutorials, simulators, multimedia, animations, etc. Most of works in the literature focus on the second direction and a small part on its use as scientific tools to analyze and solve problems. Even when computers are used as scientific tools, students are demanded only to change some parameters of the model in a very interactive way since the model has been already prepared. This case mis-

ses an important part related to the construction of the physical and mathematical model, which results more meaningful for the professional formation of the students [23]. In the case of the Technical Engineering at the University level, this fact should be more taken into consideration since it allows students the development of essential professional skills from early physics courses.

Although there is a visible increment of computers and softwares integrated to the teaching at the different levels, a bigger impact would be expected in the next few years. Although a lot of effort is being devoted to the Introductory and first physics courses for engineers in the last years [18, 19, 24], usually the use of computers as scientific tools is still incipient, being left for higher years courses. It happens that most of times the integration of mathematical and computational tools is not properly reached in the first two years of the career. Students usually see these tools apart and have problems in applying them to solve a specific problem, which is a mayor goal in engineering teaching. Since physics, calculus and programming are three usual subjects in the first year of the syllabus of any technical engineering career; it comes naturally to look for the proper integration in the context of the physics problem whose solution might combine analytical, computational and experimental aspects. This is more relevant for those technical engineering where physics is more fundamental, for instance, in the telecommunications and electronics engineering of Cuban Universities.

In the present work we will illustrate how in the context of a physics problem, students can integrate programming tools and numerical methods, along with the physical laws in order to face a more realistic situation, different from those which can usually be treated in the traditional blackboard classes. This kind of computational physics problems increases the motivation of the engineering students by embedding them into sceneries which bring working schemes very similar to the nowadays engineers sceneries [24].

We introduce as a computational physics problem an example based on the "The motion of a body subject to air drag force" [26]. This is a typical problem in the first year physics course of engineering careers [23, 26, 28]. Besides commenting on how this computational approach enhances the didactic possibilities of a physics problem we comment on the way it is introduced in the course. One important view is that we focus more on following the professional styles while solving the problem than being consequent with the traditional didactic approach. Some usual professional schemes such as: comparison between analytical and numerical approaches, graphical analysis, integrated use of mathematical and computational tools are included in the problem solution. In order to show how this problem is applied to the teaching, we comment on an

experience with first year undergraduate students of Telecommunication and Electronics Engineering of Pinar del Río University, Cuba in 2010. Specifically, we aim at a Practical Class (problem solving) of the subject Physics I (mechanics and thermodynamics).

The outline of the paper is the following. In section 2, we introduce the computational problem and the context where it is introduced. In section 3, the solution is showed item by item and didactic comments are added in between in a comparative way with the traditional classes. Each comment is preceded by the expression “Comment # x”, where x is the ordering number. In section 4, a practical experience of the application of these ideas in a physics course is commented. In section 5, some conclusions are drawn and finally two appendixes have been included. The first appendix contains the numerical procedure that has been followed to solve the problem and the second one the code in Fortran 90 [29] used for the calculations.

2. The computational problem and the context

The problem of “The motion of a body subject to air drag force” is usually avoided in the Practical Classes (for problem solving on the blackboard) of General Physics courses, since it implies to face some mathematical work for the derivation of the trajectory equations, which also means class time. Although it would have been simpler to make an interactive program where students only would have to vary a group of parameters, we intentionally indicate the students to solve the problem from the beginning, even touching elements of the physical model construction.

We have used a simple approach for this problem, where the drag force of the air is represented by a linear dependence of the velocity $\mathbf{F}_d \propto \mathbf{v}$. It is true that it would have been a more realistic model if we had included parameters such as the shape, dimensions of the launched body and the density of the fluid [26], but we considered important to keep the analytical work behind the problem somehow simple. It is important to point out that this problem is inserted in a first year physics course, when simultaneously new concepts of Calculus, such as derivatives and integrals are introduced.

In the Cuban technical careers we use basically four types of classes in the physics courses: Lecture, Practical Class, Laboratory Class and Seminars (or workshops). All of them form a system in developing specific skills within a topic. The Practical Class is devoted to problem solving on the blackboard. Our proposal is to move some Practical Classes to the computers pool and become them in computational physics practical classes. In this sense, we have explored two variants for this kind of practical classes: 1) to include them as a preliminary to the laboratory class. It means

that the problem will find continuity in the experimental corroboration of the theoretical and simulated model and 2) as a class for the integration of different physics contents usually given as separated topics within the course. Although the integration of the different subjects of the career is intended to be reached via a good curriculum derivation of the professional model, much more should be done in this respect. Our proposal represents a contribution in that direction in order to develop professional skills by an integration of subjects. This purpose overcomes the boundaries of any particular subject within the academic semester.

2.1. Statement of the problem

A body of 2.5 kg is launched to the air with an initial velocity of $v_0 = 150$ m/s and with an initial launching angle of $\alpha = 40^\circ$ with respect to the horizontal direction. This body is subject to the Earth gravity force and to a drag force $\mathbf{F}_d = -30\mathbf{v}$ where \mathbf{F}_d is expressed in N (Newtons) and the velocity of the body \mathbf{v} in m/s (meters per second).

a) Solve numerically the Newton equations of motion, starting from $t = 0$ (moment of the launch) up to $t = 20$ s. For this purpose, make a code to implement and run the numerical procedure described in Appendix 1 [26], otherwise you may use the code in Fortran 90 listed in Appendix 2. Increase the time interval in $\Delta t = 0.01$ s. Update the output file every 0.5 s. We suggest the use of output text files with the following heading to register the data. These files can be uploaded using Microsoft Excels or whatever program using an electronic worksheet.

t	$x(t)$	$y(t)$	$x^{exact}(t)$	$y^{exact}(t)$

In the table, headings $x^{exact}(t)$ and $y^{exact}(t)$ represent the analytic exact values, given by the following expressions

$$x^{exact}(t) = \frac{m}{b} v_0 \cos \alpha (1 - e^{-\frac{m}{b}t}) \quad (1)$$

and,

$$y^{exact}(t) = -\left(\frac{mg}{b}\right)t + (1 - e^{-\frac{m}{b}t})\left(\frac{m^2g}{b^2} + \frac{m}{b}v_0 \text{sen}\alpha\right). \quad (2)$$

b) From the table above, make a figure showing the curves for $y(t)$ vs. $x(t)$ and $x^{exact}(t)$ vs. $y^{exact}(t)$. Based on this figure, make a comment qualitatively on the quality of the numerical approximation.

c) From the figure of item b, obtain the value of the maximum height of the body with respect to the ground (H_{max}) and the maximum horizontal distance reached (X_{max}). Using the values of the table, seek the position interval that contains the value of X_{max} and

report the mean value of the time for the corresponding row in the table. This value will be an estimate for the total flight time of the body (t_v).

d) Using the expressions 1 and 2, calculate the exact value of the maximum height H_{\max}^{exact} . By substituting this value and the value for the maximum distance (X_{max}) into Eq. (1), obtain the exact flight time (t_v^{exact}).

e) Obtain new values of $x(t)$ and $y(t)$ using the procedure described in Appendix 1 and make a graph of $y(t)$ vs. $x(t)$, but this time take different values for the launching angle, namely, $\alpha = 15^\circ, 30^\circ, 45^\circ, 60^\circ$ and 75° and keep the initial velocity constant ($v_0 = 150$ m/s). Extend the time interval up to 25 s and make a comment on what is observed in the figure.

f) Using the equations for the case when the air drag force is not present, (let us denote them as $x^{vac}(t)$ and $y^{vac}(t)$), make a comparative graph including both cases, the trajectory without the air drag and with air drag (numerical trajectory: $y(t)$ vs. $x(t)$). Please, use the data supplied in the problem statement. Also make a comment on the effect of the air on the features of the trajectory. Extend the time interval up to 30 s. Use a time step of $\Delta t = 0.01$ s, but show results every 0.5 s.

3. Solution and comments

• Comment # 1:

In order to solve the Newton equations numerically the students are asked to program a code where elements of the Programming course should be included. This is the case of variable declarations, loops, arithmetic and logic operators and data output to files. On the other hand, by means of a finite differences scheme, concepts of the subject Calculus are approached.

One important point in this is that the numerical algorithm is based on a physical approximation that uses concepts of preceding educational levels, such as the motion at constant velocity [26, 28]. As a preliminary to this exercise, students are asked to understand the details of the physical model on the grounds of the problem and to derive analytically Eqs. (1) and (2) aforementioned. Here, we focus more on the numerical and computational way to solve problem, the exploration of its solutions and the comparison to analytical solutions and to the case of the body launched in the vacuum (case traditionally treated).

3.1. Response to item a

In Table 1, the data for the numerical and exact trajectory are shown.

Tabela 1 - Data for the numerical and exact trajectories of the body.

t	$x(t)$	$y(t)$	$x^{exact}(t)$	$y^{exact}(t)$
0	0	0	0	0
0.5	55.76263	45.58899	55.76362	45.59052
1.0	108.276	86.14221	108.27981	86.14811
1.5	157.72945	121.95308	157.73777	121.96586
2.0	204.30128	153.29793	204.31539	153.3198
2.5	248.1594	180.43701	248.18061	180.46988
3.0	289.46193	203.61538	289.49131	203.66092
3.5	328.35779	223.06383	328.39627	223.12346
4.0	364.98721	238.99969	365.03557	239.07465
4.5	399.48225	251.62764	399.54117	251.71896
5.0	431.96729	261.14044	432.03732	261.24898
5.5	462.55945	267.7196	462.64105	267.84607
6.0	491.36902	271.53606	491.46255	271.68102
6.5	518.49987	272.75081	518.60561	272.91469
7.0	544.04983	271.51544	544.16799	271.69858
7.5	568.111	267.97272	568.24173	268.17533
8.0	590.77015	262.2571	590.91353	262.47932
8.5	612.10896	254.49519	612.26502	254.73706
9.0	632.20437	244.80623	632.3731	245.06774
9.5	651.12883	233.30251	651.31017	233.58356
10.0	668.95057	220.08976	669.14444	220.39022
10.5	685.73386	205.26759	685.94012	205.58725
11.0	701.53919	188.92975	701.75769	189.2684
11.5	716.42355	171.16458	716.65412	171.52193
12.0	730.44062	152.05525	730.68305	152.43098
12.5	743.64091	131.68006	743.895	132.07386
13.0	756.07203	110.1128	756.33755	110.5243
13.5	767.7788	87.42291	768.0555	87.85174
14.0	778.80343	63.67582	779.09104	64.12157
14.5	789.18565	38.93311	789.48393	39.3954
15.0	798.96291	13.25282	799.27158	13.73122
15.5	808.17045	-13.31043	808.48925	-12.81635
16.0	816.84148	-40.70519	817.17012	-40.19585
16.5	825.00725	-68.883	825.34545	-68.35884
17.0	832.6972	-97.79825	833.04469	-97.2597
17.5	839.93907	-127.40795	840.29556	-126.85545
18.0	846.75896	-157.67165	847.12418	-157.10563
18.5	853.18145	-188.55124	853.55512	-187.97212
19.0	859.22971	-220.01082	859.61156	-219.41903
19.5	864.92555	-252.01661	865.3153	-251.41257
20.0	870.28949	-284.53677	870.68688	-283.92089

• Comment # 2:

The numeric procedure that has been used here is very simple and it is based on considering small intervals where the velocity is assumed to have a constant value. This approximation is better the smaller are the time intervals. The constant velocity type of motion is studied in levels before the university so a numerical procedure based on this would help in the transition towards the study of more general motions.

3.2. Response to items b y c

In the Fig. 1 the curves for $y(t)$ vs. $x(t)$ and y^{exact} vs. $x^{exact}(t)$ are shown. The values of the maximum height (H_{max}) and maximum distance (X_{max}) were determined from the graph and shown on it. Also an estimation of the flight time is shown (t_v).

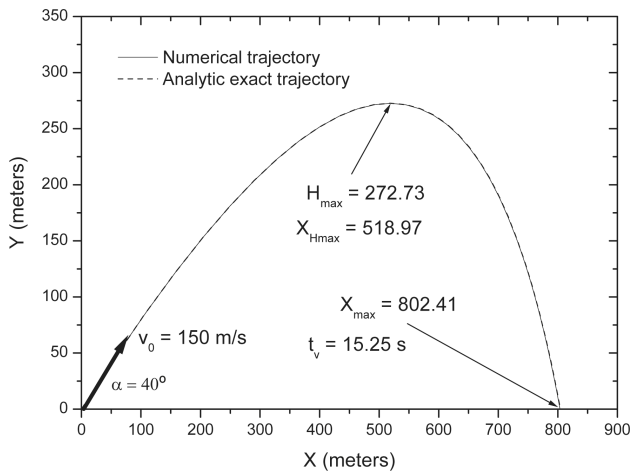


Figure 1 - Curves for $y(t)$ vs. $x(t)$ and y^{exact} vs. $x^{exact}(t)$.

It can be noticed that both curves overlap, which is a qualitative indicator of the good quality of the numerical integration.

• Comment # 3:

If the curve of $y = y(x)$ and the values of the maximum height (H_{max}) and maximum distance (X_{max}) were to be obtained without the help of the numerical approach, these steps should have been followed:

- Analytic determination of the expression for $x(t)$, $y(t)$ and $y = y(x)$. This step involves the use of integrals.

- In order to calculate the maximum height (H_{max}), the first derivative of $y = y(x)$ should be performed.

- For the calculation of the maximum distance (X_{max}) two variants include: (1) the calculation of the total flight time and substitute in the equation $x = x(t)$ and (2) solving $y = y(x)$ for $y = 0$.

All these steps take a lot of time for the mathematical work and little for the physical analysis of the situation. Maybe this is one of the reasons why teachers usually decide to solve the problem without air drag force on the blackboard. However the problem with air drag is very important since brings new situations concerning the type of motion differently from those uniform motions studied in previous levels.

3.3. Response to item d

Using the Eqs. (1) and (2), $y = y(x)$ can be obtained:

$$y(x) = \left(\frac{mg}{v_0 b \cos \alpha} + \tan \alpha \right) x + \frac{m^2 g}{b^2} \ln \left(1 - \frac{b}{mv_0 \cos \alpha} x \right). \quad (3)$$

By taking the first derivative of this expression and making it equal to zero, the value of the maximum height can be obtained H_{max} . Let the value for the corresponding x coordinate be x_{Hmax}

$$x_{Hmax} = \left(\frac{mv_0^2 \text{sen}(2\alpha)}{2(mg + v_0 b \text{sen} \alpha)} \right). \quad (4)$$

By evaluating $y = y(x = x_{Hmax})$, H_{max} is obtained

$$H_{max} = y(x_{Hmax}). \quad (5)$$

The flight time of the body can be obtained by substituting the value of X_{max} (maximum distance), in Eq. (1)

$$t_v^{exc} = -\frac{m}{b} \ln \left(1 - \frac{x_{max} b}{mv_0 \cos \alpha} \right). \quad (6)$$

By substituting the constant values supplied in the statement of the problem in the Eqs. (4), (5) and (6), the values for x_{Hmax} , H_{max} and t_v are obtained. These results are registered in the Table 2 comparatively with the numerical values.

Tabela 2 - Numeric and exact values for x_{Hmax} , H_{max} and t_v .

Variables	Exact value	Numeric value	Relative difference (%)*
x_{Hmax}	518.44	518.97	0.10
H_{max}	272.91	272.73	0.07
t_v	15.17	15.25	0.53

*(exact value - numeric value)*100/exact value.

• Comment # 4:

The analytical work and the code programming are done previously by the students so that the class time is used mostly to focus on physical aspects of the problem. This kind of semi-empirical and numerical work where theoretical and numerical results are combined and on the other hand, the comparison between pure theoretical work and experimental work, constitute a usual scheme of work in the research profile of engineers nowadays.

3.4. Response to item e

The Fig. 2 represent the several trajectories for different values of the launching angle, $\alpha = 15^\circ$, 30° , 45° , 60° and 75° . It can be seen that if the initial velocity is kept constant, the maximum distance reached by the body changes with the angle. When the air drag is present the maximum distance is reached for $\alpha = 30^\circ$, differently from the case when the air drag is not considered, when the value is $\alpha = 45^\circ$.

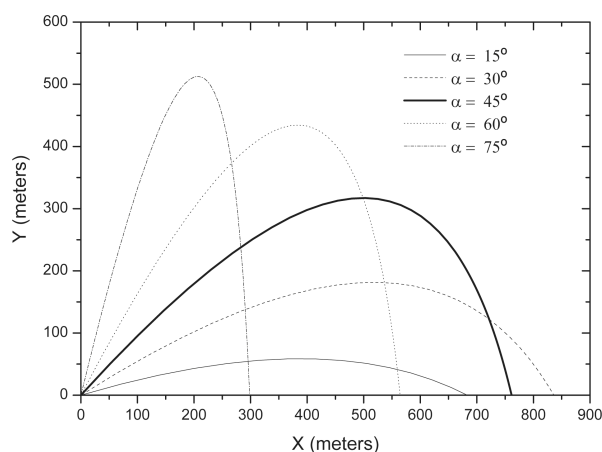


Figura 2 - Numerical trajectories for different values of the launching angle, $\alpha = 15^\circ$, 30° , 45° , 60° and 75° .

• Comment # 5:

It can be noticed that in the numerical approach, only the value of the launching angle has to be changed in the code in order to get the new trajectory. If the different trajectories for different launching angles are to be obtained in the classroom on the blackboard, some mathematical work should be done, extending the limited time of the class.

3.5. Response to item f

The Fig. 3 shows comparatively, the curves for the trajectory of the body with and without air drag force, being the launching velocity constant. It can be observed that the drag force makes the curve to be asymmetric with respect to a perpendicular line crossing over the point of maximum height. This is a difference with respect to the case when the air drag is not considered. In this case of the body moving through the air, the trajectory is not a simple parabola, but a more complex expression. For this case the maximum height and the maximum distance show smaller values than the case for the body in the vacuum. At the beginning, both trajectories are almost the same (up to 251.95 m in the horizontal line) but alter a while they separate due to the influence of the drag force of the air.

• Comment # 6:

In order to be short we have chosen some few items for this problem. In general, there are many other aspects can be addressed, for instance:

1. It is very interesting to study the behavior of the velocity and acceleration *vs.* time. In the x axis an acceleration different from zero appears and in the y axis the acceleration is not constant anymore when this case is compared to the case of not considering the air drag.
2. The drag force expression can be changed, for example to illustrate the case of a denser medium, such as water or oil. If this change should be considered in the traditional class, the equations of $x(t)$ and $y(t)$ should be obtained again however in the numerical case some minor changes in the code would be enough.

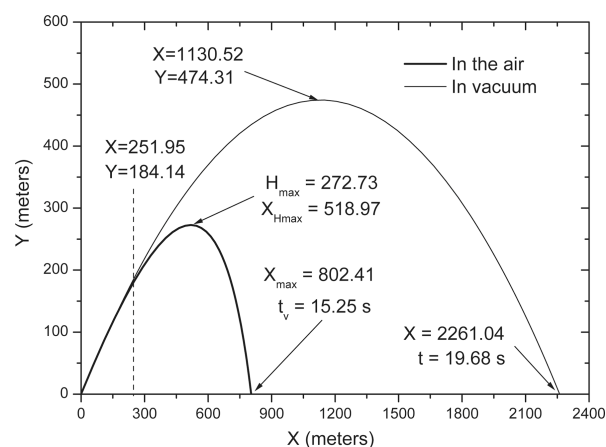


Figura 3 - Trajectories of the body with and without air drag force.

3. An interesting exercise could be the measurement of the viscosity parameter b of the drag force from experiments. For example, some experimental values for the position and time, for different launching angles and different initial velocities can be included in the model. The values obtained with this semi empirical model can be compared to other experimental quantities characterizing the trajectory.

On the other hand, the contribution to the following professional skills is addressed by means of this proposal, for instance:

1. Physical and mathematical modeling of the situation.
2. Implementation of finite differences schemes to numerically approximate exact solutions.
3. Comparison between the analytical and numerical solutions to the equation of motions.
4. The comparison to a limit model, which is the “falling body without air drag force”.
5. The analysis of the errors introduced by the numerical procedure.
6. The construction of the plots from the output data.
7. Graphical analysis to obtain relevant information from the curve plots.
8. If this problem were to be treated later in a Laboratory Class, the results discussed here would help in the design of the experiment, which is a common work style for engineers, being manifested the important relation between theory and experiment.

4. Practical experience in a physics course

In our university (Pinar del Río University, Cuba) the General Physics for Telecommunication and Electronics Engineering is organized in 3 subjects: Physics I (basically mechanics and thermodynamics), Physics II (electromagnetism and wave optics) and Physics III (quantum optics, atomic and nuclear physics). In each

course, in general, as mentioned before, four types of classes are combined to form a system within each topic. These types of classes are: The Lecture, the Practical Class, the Laboratory Class and the Seminars (or workshops). The problem presented here was inserted along with two other problems in a Practical Class on “kinematics of the particle”. In the Practical Classes the main objective is the problem solving and in this case the duration was of 1 h and 30 min. Our proposal consisted in moving this class to the computer classroom and become it a Computational Practical Class, where the problem solving is assisted by computers and numerical methods as valuable tools.

In the following we will comment shortly an experience of how this Computational Practical Class was organized in a group of 18 students of the first year course of telecommunication and electronics engineering in the academic year 2009-2010.

4.1. Previous preparation of the students for the class

- a) The students were asked to review all contains related to the topic of the class. The necessary bibliography was provided by the teacher.
- b) The students obtained the analytical solutions for the problem. For this purpose, certain level of help was provided by the teacher.
- c) The students implemented the programming code for the numerical algorithm that was to be used later in the class.

4.2. The preparation of the teacher and computer classroom

- a) Two students were placed at each workplace having a personal computer. In total, the group of 18 students was distributed in 9 workplaces.
- b) The programming software was installed at each computer.
- c) The teacher had its own PC connected to the video projector whose image was visible from all workplaces.
- d) The teacher prepared all aspects related to the solution of the problem, namely, the programming code, the output data, the figures, the answers, etc, in order to contrast his solution to the students' solutions.

4.3. The parts of the class

4.3.1. Introduction

In 15 min the teacher stated the objective of the class and recalled the main directions about the organization of the activity. Some key concepts related to the topic of the class were refreshed by means of a dialog of the teacher with the students. The teacher also checked the previous preparation of the students for the class.

4.3.2. Development

The development of the class was divided into two times. The first of about 25 min was devoted to the individual work of the students at the workplace. The teacher paid attention continuously to the inquiries of the students. In the second time, of 40 min, the teacher conducted a debate where students presented their results. All experiences were socialized by means of questions and answers on the problems of the class. This time was mostly used to discuss about the outputs of the models, mainly those, which are usually difficult to face analytically.

4.3.3. Closing

In 10 min, the teacher gave the students the conclusions of the class, focusing on the regularities behind the solution of the problems and the way mathematical and computational tools were combined to solve the problem. Also the advantages with respect to the traditional way of solving the problem, namely, using the blackboard, were commented.

Subsequently, the teacher linked this Computational Practical Class to the next Laboratory Class of the same topic, giving information to the students about the Orientation Guide for this activity. Finally, some homework was left for the reaffirmation of the objective of the class.

4.3.4. Opinion of the students about the experience

In order to have a general idea of the opinion of the students about this experience, a brief questionnaire of three questions about the computational problem has been applied to the students (18 in total were sampled).

a) Which of the following aspects resulted more difficult for you? Please, mention three of them.

1. ____ The understanding of the physical model, 2. ____The analytical derivation of the trajectories (math work), 3. ____ The understanding of the numerical algorithm based on physics approximations, 4. ____ The programming of a code for the algorithm, 5. ____ To make changes in the code to get the results for new initial conditions, 6. ____ The construction of the plots from the output data, 7. ____ The analysis of the results from the figures.

b) Which of the following aspects resulted easier to understand (to do)? Please, mention three of them.

1. ____ The understanding of the physical model, 2. ____The analytical derivation of the trajectories (math work), 3. ____ The understanding of the numerical algorithm based on physics approximations, 4. ____ The programming of a code for the algorithm, 5. ____ To make changes in the code to get the results for new initial conditions, 6. ____ The construction of the plots

from the output data, 7. ---- The analysis of the results from the figures.

c) Which of the following aspects resulted more meaningful for you as an engineering student (more motivating)? Please, mention three of them.

1. ---- The understanding of the physical model, 2. ----The analytical derivation of the trajectories (math work), 3. ---- The understanding of the numerical algorithm based on physics approximations, 4. ---- The programming of a code for implementing the algorithm, 5. ---- To make changes in the code to get the results for new initial conditions, 6. ---- The construction of the plots from the output data, 7. ---- The analysis of the results from the figures.

The results of the application of the questionnaire have been displayed in Fig. 4. Although the collected data does not allow a good statistical characterization of any aspect, some subtle tendencies can be noticed. For instance, from results of question a), we can read that the understanding of physics and math concepts is still a complicated issue for the students. This is normal for first year students since new mathematical concepts are still being introduced which allows the generalization of physics situations. From results of question b), we can read that there is a preference in the engineering students for practical things, such as, working on a code or maybe for more pragmatic things such as the interpretations of figures. In the case of question c), items 1, 4 y 7 have been mostly selected by the students. Students show to be conscious of the importance of the understanding of the physics model but at the same time they prefer something practical to deal with it as the implementation of a code. Also they seem to appreciate the benefits of getting information in a better presented form which is a figure.

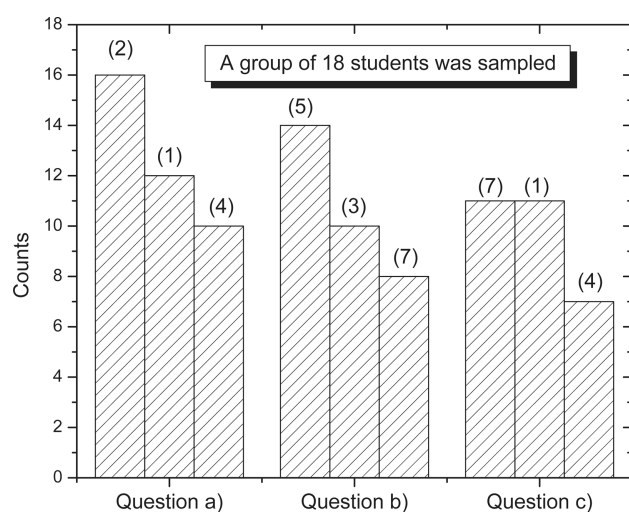


Figura 4 - Results of the application of the questionnaire about the impact of the computational class. For each question the item is indicated by a number above the column.

5. Conclusions

The computational physics problems allow the integration of Numerical Calculus and Programming as valuable tools in order to make the learning-teaching process more efficient in the first year physics course of engineering. This type of computational physics problem increases the motivation of the engineering students by embedding them into sceneries whose models are closer to those real problems which they will be facing later in their professional and scientific life. At the same time, the computational approach in Practical Classes helps in enhancing the didactic possibilities as a valuable tool for the teacher. Although this problem has been solved using a Fortran 90 code, any other softwares can be used, for example Matlab [30] and Derive [31]. With some more work, an Excel electronic sheet can also be used. In this sense, the computational physics problem can be adapted to the different engineering levels by properly choosing the level of the physics and computation concepts. The scenery where this problem applies has been commented along with an example of how to proceed in the daily practice. Also the general opinion of the students has been commented.

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Appendixes

1. Numerical procedure for the calculation of the trajectory of a particle of constant mass moving in a force field depending on the particle velocity [27]

Let v_0 and x_0 be the initial velocity and position of a particle of constant mass. The time interval is first divided into N sub-intervals of size Δt . Then, the cinematic quantities are calculated in each sub-interval by means of using the expressions for the uniform motion with constant acceleration, that is, the velocity varies uniformly.

For the first time interval Δt , the acceleration is assumed constant (a_{m1}), and its value is given by the direct evaluation of the acceleration in the initial velocity (v_{i1})

$$a_{m1} = a(v_{i1}).$$

The velocity, at the end of the interval, is calculated by the expression

$$v_{f1} = v_{i1} + a_{m1}\Delta t.$$

The mean value of the velocity in the first interval is

$$v_{m1} = \frac{(v_{i1} + v_{f1})}{2}.$$

If a uniform motion of constant velocity v_{m1} is assumed in the interval, the displacement Δx_1 can be calculated as follows

$$\Delta x_1 = v_{m1}\Delta t.$$

The final position of the particle after a period of time Δt is calculated as

$$x_{f1} = x_{i1} + \Delta x_1.$$

Following this procedure for the remaining sub-intervals, the numerical trajectory of the particle can be obtained.

2. Code lines in Fortran 90 language [29] used for the calculation of the numerical trajectory of the particle. This code uses the scheme of finite differences explained in the Appendix 1

```
PROGRAM Trajectory

!Variable declarations
!Exact values of the position in x and !y
Double precision xexac, yexac
!Time, mass, parameter b of the drag force, gravity constant, launching
!angle in degrees and radians
Double precision t, m, b, g, alphagrad, alphasad
!Time step
Double precision dt
!pi
Double precision pi
!Initial and final positions and !velocities in x and y for each time step
Double precision x0, y0, v0x, v0y, xf, yf, vfx, vfy
!Mean velocity in x and y
Double precision vmx, vmy
!Temporal variables to store the initial positions
Double precision x0t, y0t
!Acceleration in x and y
Double precision ax, ay
!Constants definition
!Number of subintervals
Parameter (N=2000)
!Time step
Parameter (dt=0.01)
!Parameter of air viscosity
Parameter (b=0.30d0)
!Mass of the particle in kilograms
Parameter (m=2.5d0)
!Initial position in x and y in meters.
Parameter (x0=0.0d0, y0=0.0d0)
!Absolute value of the initial velocity in meters per second
Parameter (v0=150.0d0)
!Launching angle in radians
Parameter (alphagrad=40.0d0)
!Gravity constant in meters per squared second
Parameter (g=9.8d0)
```

```

!pi
Parameter (pi=3.1416d0)
!Initializing the time
t=0.0d0
!Conversion from degrees to radians.
alphanad = alphagrad*pi/180.0d0
!Calculating the initial velocity in x and y
v0x = v0*dcos(alphanad)
v0y = v0*dsin(alphanad)
!Output file for the trajectory data
open(unit=1,file='Salida.txt')
!Using temporal variables
x0t=x0
y0t=y0
!Time loop
do i=0, N
  !One time step forward, dt
  t=i*dt
  !In x axes
  ax=(-b/m)*v0x
  vfx = v0x + ax*dt
  vmx = (vfx + v0x)/2.0d0
  xf = x0t+dt*vmx
  !In y axis
  ay=-g+(-b/m)*v0y
  vfy = v0y + ay*dt
  vmy = (vfy + v0y)/2.0d0
  yf = y0t+dt*vmy
!Checking if the time elapsed is
!0.5 seconds in order to update the
!output file
  if (mod(real(i),50.0).eq.0.0d0) then
!Evaluating the exact expressions for
  !'x(t) y y(t)
  xexac=v0*dcos(alphanad)*(m/b)*(1.0d0- dexp(-(b/m)*t))
  yexac=-(m*g/b)*t +
(1.0d0-dexp(-(b/m)*t))*((m**2)*g/(b**2)+v0*dsin(alphanad)*(m/b))
  !Writing data points for the positions in x and y to the output file
    write(1,'(5(1X,F20.5))') t,xf,yf,xexac,yexac
  endif
!Transferring the resulting values as !initial values for the
velocity and !positions in x and y
  v0x = vfx
  v0y = vfy
  x0t = xf
  y0t = yf
enddo !End of the time loop
!closing the output file
close(1)
END !End of program

```

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