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Maximizing tangential frictional forces effects in bidimensional collisions: a study of the tangent restitution coefficient through a video analysis approach

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In this work we used the video analysis technique to investigate the contribution of the friction forces to the dynamic of a series of oblique collisions of a basketball against the floor's surface. This contribution was evaluated through the tangent restitution coefficients associated to these collisions, which, in general, presented values distinct from the unit, proving, thus, its importance. Additionally, the appreciably values bigger than unity obtained for this quantity quantitatively confirms the physical scenario suggested in a previous work by some of the authors [1] which is governed by the torques transmitted to the ball during the successive collisions and characterized by variations on the magnitudes and even on the directions of the friction forces. To support this dynamic we considered a ball of relatively sizeable radius and mass, and launched with a certain spin velocity. Also, in order to increase the friction coefficient, the floor's surface was covered with a rough and rubberized material.

Keywords: video analysis, inelastic collisions, tangential restitution coefficient.

1. Introduction

The use of information and communication technologies (ICTs) [2] through computers, software and the most varied apps, brought to the physical classes the opportunity to perform teaching and learning experiences combining physics experiments with computational simulations and modelling [3]. The simulations are virtual experiments that allow the students to visualize the physical phenomenon outlined in a computer screen [4], being of extreme importance in dealing with phenomena that are hard and expensive to reproduce in laboratory. In addition, this technique emphasizes the mathematical framework underlying the physical phenomena concerned, enabling, thus, a better understanding by the students. Similarly, studies combining real experiments and modelling techniques have grown in teach science scenario due the popularization of smartphones, through which, by means of an app, as the Tracker [5], the video analysis technique [6] can be applied in large scale, permitting more rich teaching experiences.

The video analysis technique is a powerful tool which has been widely applied by the physics teachers in order to improve the teaching and learning processes during regular classes, enabling better and more didactic discussions about the basic physics concepts inherent to the physical system under study, as, for instance, the kinematics [7, 8] and the conservation laws of motion of mechanical systems [9, 10]. Furthermore, by means of an app and a simple smartphone camera [11], it is possible to deepen the discussion about some physical concepts by considering systems and situations that are apart from that usually considered in regular physics' books. Thus, the combination video analysis, experiment and modelling, allows the teachers to promote interesting discussions about the design and development of determined physical models, regarding their predictability, limitations, and others important aspects of these models to the development of physical science.

The study of collisions in basic high school and in the first years of university courses are usually presented into a theoretical scope which includes the possibility of a scenario in which two physical quantities are conserved: (1) the linear momentum of the system, when there are no external forces acting upon the system and (2) the kinetic energy of the system, when inelastic processes, such as permanent deformations, are nonexistent. In a system composed by a set of massive bodies, each of its components suffers variations on its individual linear momentum, even in the absence of external forces. These variations correspond to the mutual impulses transmitted between the particles due to the action of contact forces on the collisions. In this context, an important quantity is the restitution coefficient, which quantifies the amount of kinetic energy that is lost on the collisions and, consequently, the degree of elasticity associated to

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them. In the particular case of a massive body colliding against a flat and rigid surface, the variation of linear momentum corresponds to the impulse transmitted to the body by the external forces applied by the surface, namely the normal and the friction forces, with the first associated to the normal restitution coefficient, which is quite frequent in basic literature, in the context of collision models where the effects of friction force are neglected [12]. The friction forces acting on the collisions, on the other hand, are associated to the tangent restitution coefficient, a quantity incorporated to more sophisticated collisions' models and which consists in the main object of study of this paper.

In this work we investigate the role played by the friction forces during the collisions of a basketball launched against the horizontal surface of the floor. In order to strengthen the effects produced by these forces, specially to increase the variation of linear momentum in the tangential direction, i.e., the tangential impulse produced by this force, we prepared the experimental setup with the floor's surface covered with a rough rubberized material of a determined thickness. Additionally, the basketball was launched with a certain angular velocity. The analytic approach applied combines video analysis and mathematical modeling, and is based on the calculation of the tangential restitution coefficient [13], a quantity that carries information about the magnitude of the friction forces during the collisions and allows us to know, considering the set of its magnitudes, corresponding to successive collisions, the direction of the friction forces during each of them, an important information for the force's diagrams considered in collisions models.

This paper is organized with the second section presenting the experimental setup and the collision model considered, along with a brief discussion concerning the video analysis performed with the Tracker software. The third section presents the results, and, finally, the fourth section, presents the conclusions.

2. Experiment and Modelling

The experimental procedure consisted in horizontally launching a basketball with translation and angular velocities from a height of approximately 1.5 m above the floor and studying the physics concerning the series of subsequent inelastic collisions between this ball and the floor's surface. This surface was prepared with a rough rubberized material of thickness t, composed by a superposition of N horizontally overlaid layers, as indicated in Fig. 1(A). The incident velocity $\vec{v}_b$ immediately before a certain collision is altered during this event mainly due the normal and the friction forces, $\vec{N}$ and $\vec{f}$, respectively, acting on the ball during this collision. These forces produce an impulse $\vec{J}$, which is the quantity responsible for the mentioned variation of the incident velocity $\vec{v}_b$ and, in turn, is quantified by the variation $\Delta\vec{p}$ of the ball's linear momentum before and after the

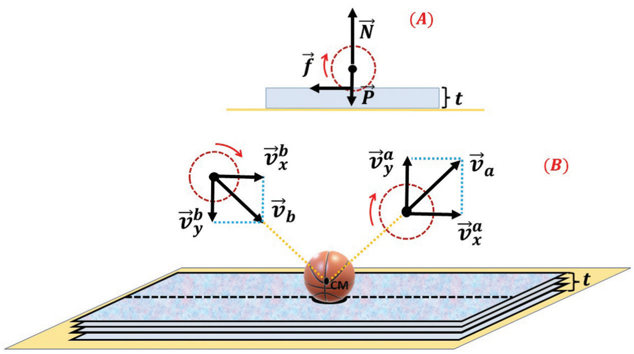

Figure 1: The figure indicates in (A) the forces acting on the ball during the collision against the rubberized surface and in (B) the ball center of mass (CM) velocities immediately before and after the collision.

collision, $\vec{p}_b = m\vec{v}_b$ and $\vec{p}_a = m\vec{v}_a$, respectively, being $\vec{v}_a$ the velocity after the collision and m the ball's mass ($m = 0,301$ kg). This process is depicted in Fig. 1(B). Considering the contributions of the forces acting on the ball during the collision to the impulse $\vec{J}$, we have that the normal force $\vec{N}$ is associated to the component J_y of this quantity, i.e., to the component Δp_y of the variation of the linear momentum vector along the y direction, which is perpendicular to the floor's surface. The action of the friction force, on the other hand, produces the components J_x and Δp_x, which are, respectively, the components of the $\vec{J}$ and $\Delta\vec{p}$ along the direction tangent to this surface. Now, considering the weight force $\vec{P}$, it's possible to show that the magnitude of this force is substantially smaller than the magnitude of the normal force acting on the ball during the collision, and, for this reason, its contribution to the component $J_y = \Delta p_y$ of the impulse can be neglected. The variation of the ball's linear momentum and, consequently, of the kinetic energy during the tiny time intervals of the collisions, in which the ball's and the floor's surfaces are in contact, are also evaluated in terms of the restitution coefficients. Particularly, we have the well-known normal restitution coefficient e, defined by

$$e = \frac{v_y^a}{v_y^b}, \tag{1}$$

with v_y^b and v_y^a being the components of the velocities immediately before and after the collisions along the direction perpendicular to the floor's surface. This coefficient is widely discussed in literature, specially in studies of inelastic collisions, in which the energies dissipation are evaluated [14, 15]. For this reason, it will not be explored in the present work. Conversely, we will put emphasis on the physics underlying the effect produced by the friction force acting on the ball during the collisions, which is quantified by the tangent restitution coefficients β [13], defined by the relation between the horizontal components of the centre of mass velocities immediately after, v_x^a, and before, v_x^b, these collisions,

$$\beta = \frac{v_x^a}{v_x^b}. \tag{2}$$

Through the magnitudes of this quantity, it is possible to evaluate some characteristics of the friction force during the collisions, as well as its influence on the dynamic of these events, which is typically governed by the angular and translation momenta of the ball. In fact, it's observed that the frictional force produces antagonistic effects on its angular velocity ω and on the tangential component v_x of its centre of mass velocity, as can be seen, for instance, in a physical scenario where this force is initially opposite to the v_x direction, reducing, thus, the magnitude of this component and, according to equation (4), resulting in a value of β which is less than the unity, $\beta < 1$. In this case, the torque produced by the friction force increases the angular velocity ω of the ball, diminishing then the relative tangential velocity between the floor and ball surfaces and the friction force in the subsequent collision. If the angular velocity, and, consequently, the torque transmitted to the ball, are sufficiently huge, the sign of the relative velocity and of the friction force can be inverted. In this scenario, the friction force enhances the tangent component v_x of the center of mass velocity and the tangent restitution coefficient reaches magnitudes greater then the unity, $\beta > 1$. In order to strength this effect, and evaluate the collisions into a regime where these values of β manifests, we launched the ball with a huge spin and a relatively small translation velocities. The results obtained actually corroborates with this context, suggesting a dynamic characterized by repeated inversions in the direction of the friction forces in the series of collisions studied.

The dynamics of the collisions were investigated through a video analysis approach realized with the free software Tracker and based on the tracking of the ball's trajectory, as presented on the main panel of Fig. 2, where one can identify, through the yellow circles, the trajectory described by the ball in instants before and after a certain collision. Through the tracking it was possible to obtain the horizontal component x of the ball's position as a function of time t (Fig. 2(C)) during the period of analysis. As there are no horizontal forces acting on the ball in the time intervals between the collisions, the horizontal components v_x of the velocities are constants in these intervals and can be extracted from the linear fitting of x, as indicated in the insets (A) and (B) of Fig. 2 for the collision illustrated in this figure. Specifically, the x components of these velocities are given by the parameter A of the linear function

$$f(x) = Ax + B, \tag{3}$$

indicated on the insets and describing the uniformity variation of the horizontal component of the position with time. The parameter B is a constant with no

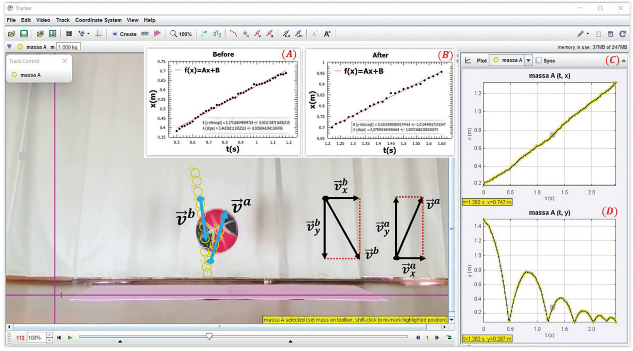

Figure 2: The figure shows in the main panel the image of the experiment indicating the tracking of the ball's trajectory during the time interval comprising the second collision and the quantities obtained through the video analysis realized with the Tracker, namely the velocities immediately before ($\vec{v}^b$) and after ($\vec{v}^a$) this collision. The insets (A) and (B) show, respectively, the horizontal coordinate $x(t)$ in the time intervals immediately before and after these collisions with the corresponding linear regressions through the horizontal components of $\vec{v}^b$ and $\vec{v}^a$ obtained. The insets (C) and (D), in turn, show, respectively, the horizontal and vertical component of the ball position.

relevance to the analysis performed. The errors associated to v_x^b and v_x^a are generated by the fitting and presented in Table 1, together with its related velocities. The Figs. 2(C) and 2(D) show, respectively, the x and y coordinates of the ball's position vector into an extended time interval, containing the complete set of studied collisions. The linear behavior observed in the first graph evidences an almost uniform motion, due to the absence of acceleration along the horizontal direction, except during the tiny time intervals of the collisions, in which the friction forces are present. The second graph, alternatively, shows a parabolic characteristic of the vertical coordinate as a function of time in the non-contact regions, between the successive collisions, indicating the exclusive action of a constant acceleration, the gravitational acceleration, on this direction. The $x(t)$ and $y(t)$ coordinates compose the vector position $\vec{r}(t) = (x(t), y(t))$ which describes the ball's position as a function of time and is characterized by a succession of oblique launches governed by the properties of the corresponding collisions. The gradual decrease in maximum height obtained after subsequent collisions reflects the inelastic nature of these collisions, associated to the loss of mechanical energy related, mostly, to the permanent deformations on the rubberized surface of the floor. An energy dissipation is usually evaluated in terms of the normal restitution coefficient.

3. Results and Discussion

The horizontal component of the ball's velocities immediately before and after the collisions, evaluated through linear fittings of $x(t)$ on the appropriated time intervals,

Table 1: Velocity v_x^b before, velocity v_x^a after and tangential restitution coefficients $\beta = v_x^a/v_x^b$ associated to the nth collision for different thickness $t = \alpha N$ of the rubberized material covering the surface, where $\alpha = 2$ mm is the thickness of one layer of this material.

		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$N=1$	$v_x^b(m/s)$	0.337 ± 0.006	0.447 ± 0.006	0.576 ± 0.007	0.452 ± 0.010	0.495 ± 0.019
	$v^a(m/s)$	0.446 ± 0.006	0.576 ± 0.007	0.452 ± 0.009	0.495 ± 0.019	0.606 ± 0.032
	β	1.325 ± 0.031	1.289 ± 0.023	0.784 ± 0.018	1.097 ± 0.048	1.223 ± 0.080
$N=2$	$v_x^b(m/s)$	0.346 ± 0.003	0.388 ± 0.003	0.353 ± 0.003	0.396 ± 0.005	0.427 ± 0.008
	$v^a(m/s)$	0.389 ± 0.003	0.353 ± 0.003	0.396 ± 0.005	0.427 ± 0.008	0.378 ± 0.056
	β	1.124 ± 0.015	0.908 ± 0.012	1.121 ± 0.017	1.079 ± 0.024	0.884 ± 0.131
$N=3$	$v_x^b(m/s)$	0.413 ± 0.005	0.470 ± 0.003	0.540 ± 0.004	0.383 ± 0.005	0.442 ± 0.020
	$v^a(m/s)$	0.470 ± 0.003	0.540 ± 0.004	0.383 ± 0.005	0.442 ± 0.020	0.242 ± 0.019
	β	1.136 ± 0.015	1.149 ± 0.011	0.710 ± 0.011	1.154 ± 0.055	0.546 ± 0.050
$N=4$	$v_x^b(m/s)$	0.373 ± 0.006	0.475 ± 0.002	0.566 ± 0.004	0.403 ± 0.009	0.503 ± 0.016
	$v^a(m/s)$	0.474 ± 0.002	0.566 ± 0.004	0.403 ± 0.009	0.503 ± 0.016	0.536 ± 0.001
	β	1.270 ± 0.021	1.194 ± 0.010	0.711 ± 0.016	1.249 ± 0.049	1.067 ± 0.035
$N=5$	$v_x^b(m/s)$	0.467 ± 0.006	0.506 ± 003	0.556 ± 0.003	0.419 ± 0.005	0.584 ± 0.007
	$v^a(m/s)$	0.506 ± 0.006	0.556 ± 0.003	0.419 ± 0.005	0.584 ± 0.007	0.430 ± 0.059
	β	1.083 ± 0.015	1.099 ± 0.008	0.755 ± 0.010	1.393 ± 0.025	0.737 ± 0.102

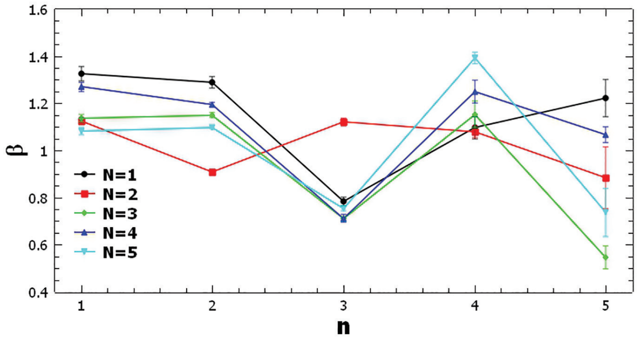

Figure 3: Tangential restitution coefficient of the nth collision and for different thickness of the rubberized material covering the surface.

as described on the preceding section, are presented in Table 1, together with its corresponding errors, for the first five collisions ($n = 1, 2, 3, 4, 5$) and considering the experimental setup prepared with the floor's surface covered with rubber layers of different thickness $t = \alpha N$, with $N = 1, 2, 3, 4$ and 5. The velocities' error bars were directly estimated from the fittings performed with the free software Scidavis [16] and propagated, from Equation (4), through the expression

$$\delta\beta = \beta\sqrt{\left(\frac{\delta v_x^a}{v_x^a}\right)^2 + \left(\frac{\delta v_x^b}{v_x^b}\right)^2} \qquad (4)$$

to give the error's bars associated to the tangential restitution coefficient β, also presented in the referred table. The results obtained for the restitution coefficients β as a function of the collision index n and for the experimental setup prepared with different numbers of layers N are presented in Fig. 3. During the first ($n = 1$) and the second ($n = 2$) collisions, the majority of points, for different numbers of layers, give restitution coefficients magnitudes greater than unity ($\beta > 1$), indicating, according to our model, that the friction force points along the forward direction of the movement. In this case, the friction force favors the horizontal component of the translational motion, at the same time that it reduces the angular velocity of the ball, allowing to explain the reason why the magnitudes of the horizontal components of velocities immediately after the collisions are greater than immediately before these events, as indicated on the Table 1. In this paper, we explore and maximize this effect, whose existence was suggested in a previous work [1], by horizontally launching the basketball with a relatively small center of mass velocity v_x, $0.34\,\text{m/s} < v_x < 0.47\,\text{m/s}$, while simultaneously imprinting a considerable initial angular velocity. In addition, the horizontal surface was covered with a rubberized material in order to increases its roughness and strengthen the friction force in the motion direction due to the considerable high rotation of the ball.

In the third collision ($n = 3$) the tangential restitution coefficients are smaller than unity for the collisions with the majority of the surfaces thickness, except for $N = 2$. This effect can be explained in terms of the successive inversions on the friction force direction due to the reduction of the angular velocity during the second collision ($n = 2$), pointing to the backward direction in relation to the translational movement for ($n = 3$). As a consequence, the magnitudes of the restitution coefficients measured, in this case, for surfaces prepared with different number of layers N, assume similar values, suggesting a weak dependence on the restitution coefficient with the rigidity of collisions for ($n = 3$). It's explained due to the fact that in the first two collisions the friction forces points on the forward direction of the movement, reducing the ball's angular velocity and, at the same time, increasing the horizontal component of its center of mass' velocity so that, during the third collision ($n = 3$), the influence of the angular velocity is negligible and this collision is governed by the translational motion. This physical scenario, governing the third collision, is indicated in Fig. 3 through the collapse of the β obtained for $N = 2, 3, 4$ e 5 into a value near $\beta = 0.7$.

The restitution coefficients associated to the fourth collision ($n = 4$), for all the thickness considered for the the floor's surfaces, assume, once more, magnitudes that are larger than the unity, indicating a second inversion in the friction force, occurred during the third collision. This force, then, switches back to pointing on the forward direction, favoring the center of mass' motion with the increasing of its velocity at the same time that its angular velocity decreases. For the fifth and last analysed collision ($n = 5$), it's observed a random spreading on the magnitudes of β obtained for collisions with surfaces of different thickness, with some of them assuming values below and others above unity, indicating a random dependence on thickness, an effect that can be explained by the substantial modification on the rotational axis of the ball, that is presumed to occur after the first four collisions.

Intuitively, we would expect a monotonic behaviour of the tangential restitution coefficients as a function of the thickness $t = \alpha N$ of the rubberized surface, related to the rigidity and the degree of deformation of this surface during the collisions, which tends to decrease with the increasing of N. Nonetheless, according to Fig. 3, the measured restitution coefficient β presented an irregular and non intuitive behavior as a function of t for each of the collision orders n, specially for $n = 2$. To a certain extent, this can be explained by the lack of control of the launches, which certainly presented different initial magnitudes for the angular and the center of mass' velocities for each of the experimental realizations (for different numbers of layers N), making β exhibit a non regular behaviour.

4. Conclusions

In this work, we investigated the influence of the friction forces on the dynamic of a series of collisions between an spherical object against a flat, rough and deformable surface, represented by the floor covered with a rubberized material. To enhance the effects of this force, we performed the experiment with an object of considerable dimension and high deformability, specifically, a basketball of radius $R = 0,75$ cm and mass $m = 0,301$ kg, which, additionally, was launched with a certain angular velocity. These characteristics were important to increase the magnitude of the torque generated on the ball due the action of friction forces during the collisions and provide the rotation dynamic, which is on the core of the emergent physical scenario, dominated by successive variations on the magnitude and inversions on direction of the friction forces. This scenario was identified through the magnitudes of the tangent restitution coefficient β presented on Fig. 3 and obtained for the experiment performed with the floor' surface covered with rubberized material of distinct thickness. The results indicated a similar behavior regardless of the rubber thickness, with β regularly oscillating between values above and below unity ($\beta = 1$), a behavior that we believe to be associated to the mentioned inversions in the direction of friction forces acting on the ball during the collisions. This effect is a consequence of the persistent antagonism between the centre of mass tangential and the angular velocities.

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References

[1] R.S. Franco, V.S. Miranda, R.S. Dutra and L.C. Ribeiro, Revista Brasileira de Ensino de Física **43**, e20200528 (2021).

[2] A. Albirini, Computers & Education **47**, 373 (2006).

[3] W.P.N. dos Reis, D.P. Rodrigues, I.L. Rocha, R.S. Dutra and P.V.S. Souza, Physics Education **56**, 013005 (2020).

[4] M.D.R. Uribe, A.J. Magana, J.H. Bahk and A. Shakouri, Computer Applications in Engineering Education **24**, 428 (2016).

[5] https://physlets.org/tracker/.

[6] D. Brown and A.J. Cox, The Physics Teacher **47**, 145 (2009).

[7] J. Poonyawatpornkul and P. Wattanakasiwich, Physics Education **48**, 782 (2013).

[8] P. Klein, S. Gröber, J. Kuhn and A. Müller, Physics Education **49**, 37 (2014).

[9] M. Erol, H. Yıldız and M. Tekingündüz, The Physics Educator **2**, 2050008 (2020).

[10] J.A. Bryan, Physics Education **45**, 50 (2010).

[11] E.A. Lima, R.S. Dutra and P.V.S. Souza, Physics Education **55**, 045021 (2020).

[12] D. Halliday, R. Resnick and J. Walker, *Fundamentals of physics* (John Wiley & Sons, New York, 2013).

[13] J.T. Jenkins, ASME. J. Appl. Mech. March **59**, 120 (1992).

[14] D.P.M. Filho, J.K.S. Kamassury and R.C.S. Meira, Revista Brasileira de Ensino de Física **39**, 4 (2017).

[15] A. Wadhwa, Physics Education **44**, 517 (2009).

[16] https://scidavis.sourceforge.net/