A very simple way to measure coaxial cable impedance
(Um modo simples para medir a impedância de um cabo coaxial)

P. Fonseca, A.C.F. Santos[1] and E.C. Montenegro

Instituto de Física, Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, Brazil

A very simple experiment is described to measure the impedance of a coaxial cable and the velocity of propagation of a pulse. The experiment uses a fast pulse, which has the advantage of avoiding long cables and it allows the observation of pulse dispersion.
Keywords: coaxial cable, impedance matching, signal transmission.

Descrevemos um experimento simples para medir a impedância de um cabo coaxial e a velocidade de propagação de um pulso. O experimento utiliza um pulso rápido, que possui a vantagem de evitar cabos muito longos e permite a observação da dispersão do pulso.
Palavras-chave: cabo coaxial, casamento de impedâncias, transmissão de sinais.

1. Introduction

Coaxial cables are the interconnections that transmit pulses from one end to another, preserving the information in the signal. Their basic geometry is that of two concentric cylindrical conductors separated by a dielectric material. Coaxial cables are standard transmission lines in nuclear, atomic and high-energy electronics.

Some papers [1-4] have been devoted for demonstrating the properties of transmission lines whether using expensive and sophisticated instruments or low-cost equipment. Most of them use slow pulses (rise time comparable to the transit time in the transmission line). This paper aims to describe a very simple experiment using fast signals that are mostly used in atomic, nuclear and high-energy physics. One advantage of this approach is that it allows ones to use relatively short cables as opposed to cables of hundreds of meters that are often used.

The equivalent circuit for a unit length of transmission line is shown in Fig. 1. For a loss less line $R = G = 0$ and the gradient of voltage and current are given by

$$\frac{\partial V}{\partial x} = L\frac{\partial I}{\partial t}, \tag{1}$$

and

$$\frac{\partial I}{\partial x} = C\frac{\partial V}{\partial t}. \tag{2}$$
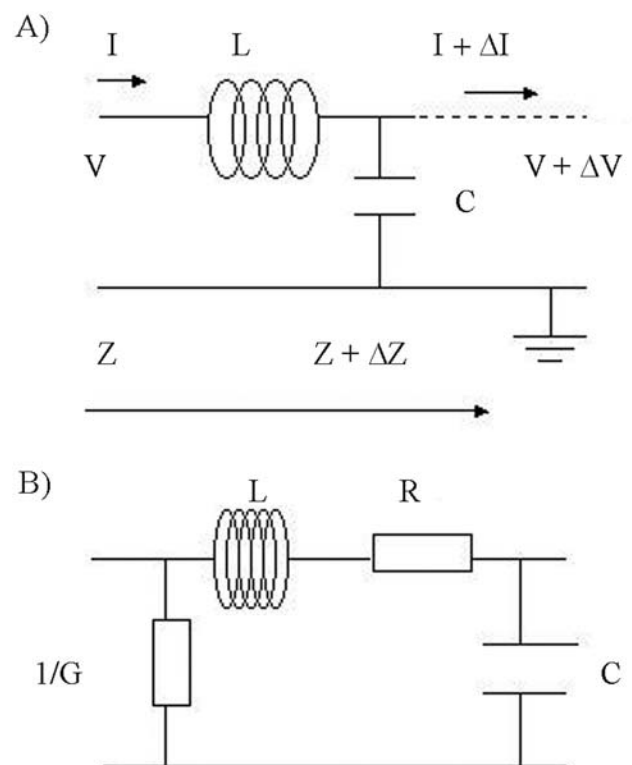

Figure 1 - A model for an A) ideal and B) lossy coaxial cable.

Differentiating Eqs. (1) and (2) with respect to distance and time, respectively, one obtains

[1]E-mail: toni@if.ufrj.br.

$$\frac{\partial^2 V}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 V}{\partial x^2}, \tag{3}$$

which is the transmission line wave equation. One can easily show by comparing Eq. (3) to the general wave equation that the velocity with which the electromagnetic energy propagates along this loss less line is given by

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}}. \tag{4}$$

The speed of signal propagation is more often expressed as its reciprocal, the time of propagation per unit length or delay of the cable, and its typically on the order of 5 ns/m. The characteristic resistance of line is given by

$$Z_{cable} = \sqrt{\frac{L}{C}}. \tag{5}$$

2.   Procedure and results

The signal in a coaxial cable is, in general, the sum of the incident signal and a reflected signal traveling in the opposite direction. In this section, we describe very simple experiments that make use of some of the transmission lines characteristics, namely, velocity of propagation, reflection and impedance matching.

Pulses from a commercially available generator OR-TEK are fed to a constant fraction discriminator (CFD). The timing output of the CFD produces fast logic pulses that are seen by a 100 MHz oscilloscope transmitted by means of a 50 Ω coaxial cable. Figure 2 shows the main and reflected pulses observed using the oscilloscope without impedance matching. Since oscilloscopes are high impedance devices ($\approx$ 1 MΩ), direct entry of a fast signal will lead in an impedance mismatch and a mistaken signal indication.

2.1.   Measuring the impedance of the coaxial cable

If the line is terminated with a purely resistive load of impedance R, pulse will be reflected or transmitted at the end of the line. The transmission coefficient T is a function of the load as

$$T = \frac{2R}{R + Z_{cable}}, \tag{6}$$

where T is defined as the ratio between the transmitted (V_t) and incident (V_o) voltages
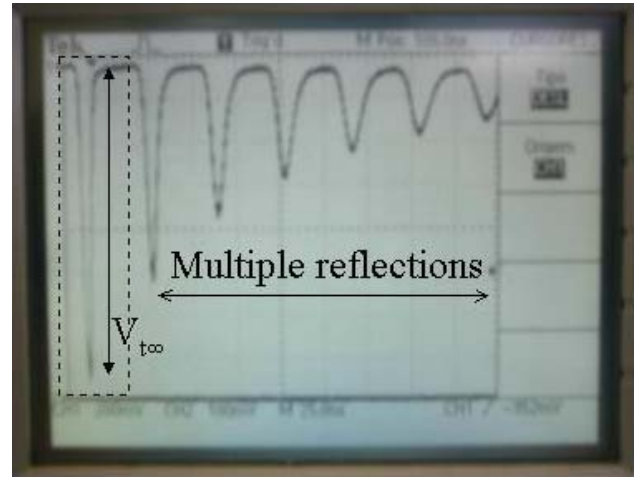
$$T = \frac{V_t}{V_o}. \tag{7}$$

Figure 2 - Fast pulses measured at the end of the coaxial cable using an oscilloscope without impedance matching. The cable length is 3.49 m. The distance in time between adjacent pulses is 35.8 ns. The parameter scale of the X and Y axis of the oscilloscope screen are 20 ns/div and 200 mV/div.

In the limiting case of infinite load impedance, the transmitted signal is twice the incident signal. On the other hand, if R equals to the impedance of the transmission line, Z_{cable}, the transmitted signal is equal to the incident one.

By adding a potentiometer in parallel (shunt), as shown in Fig. 3, one is able to measure the transmitted signal as a function of R. Figure 4 shows the plot of the transmission coefficient T as a function of R. Another manner of studying the dependence of T as a function of R is by linearizing Eq. (6) as follows

$$\frac{1}{T} = \frac{1}{2} + \frac{Z_{cable}}{2R}. \tag{8}$$

Figure 3 - A potentiometer of 1 kΩ used in shunt with the oscilloscope.

So, the plot of T^{-1} as a function of $2R^{-1}$ should give rise to a straight line with the angular coefficient equals to Z_{cable}. The transmission coefficient is determined as follows: Firstly, one measures the amplitude $V_{t\infty}$ of the fast pulse at the end of the coaxial cable using an oscilloscope without impedance matching as shown in Fig. 1. The pulse of interest is the very first

one (in time). The following pulses are related to multiple reflections at both ends of the coaxial cable. According to Eq. (6), the measured signal corresponds to twice the incident signal, *i.e.*, $V_{t\infty} = 2V_o$. Then, one adds the potentiometer in shunt, and measures the amplitude of the transmitted pulse, V_t (again the very first one in time) as a function of R. The transmission coefficient is determined as (see Eq. (7))

$$T = 2\frac{V_t}{V_{t\infty}}. \qquad (9)$$

Figure 4 shows this plot. The experimental value for Z_{cable} is $48 \pm 4\Omega$, in agreement with the nominal value from the coaxial cable impedance.
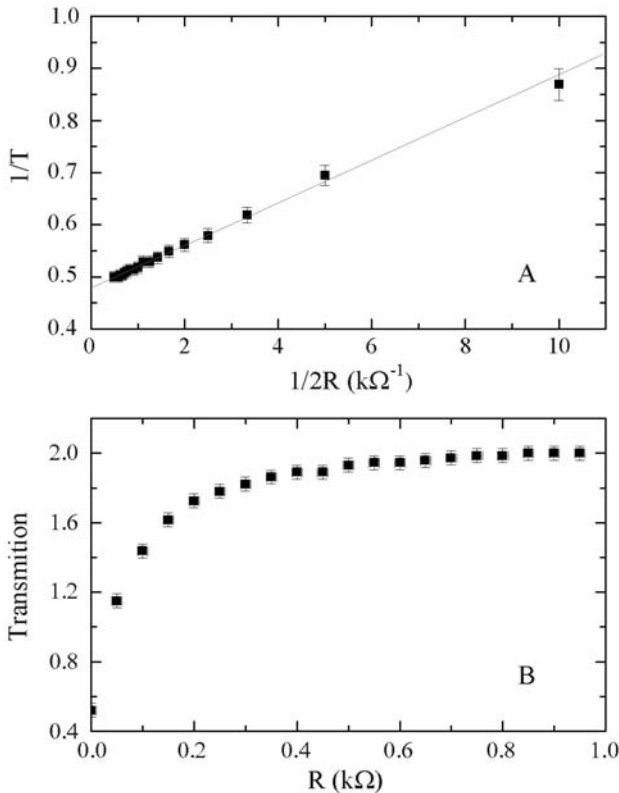

Figure 4 - A) Plot of the reciprocal of T *vs.* the reciprocal of $2R$. B) The transmission coefficient as a function of the equivalent impedance.

2.2. Velocity of propagation

One advantage of using fast signal is the fact that it is not necessary to use very long cables. From Fig. 2, one can measure the distance in time between adjacent pulses corresponding to a cable of 3.49 ± 0.01 m which allow us to obtain the velocity of propagation of the pulse as

$$v = \frac{2d}{\Delta t} = \frac{2 \times 3.49\text{m}}{35.8 \times 10^{-9}\text{s}} = 1.9 \times 10^8 \text{m/s}.$$

2.3. Pulse dispersion and the lossy cable

From Fig. 2 one can see that width of the reflected pulses increase with the number of reflections. This dispersion of the pulse packet is due to a differential attenuation of the frequency components as a consequence of the fact that the coaxial cable is not ideal, *i.e.*, the velocity depends on the frequency. The general solution of the wave equation for a lossy cable is

$$V(x,t) = V_1 e^{-\alpha x} e^{[i(\omega t - kx)]} + V_2 e^{+\alpha x} e^{[i(\omega t + kx)]},$$

where

$$\alpha^2 = \frac{\left\{ \left[RG - \omega^2 LC \right] + \sqrt{\left[R^2 + \omega^2 L^2 \right] \left[G^2 + \omega^2 C^2 \right]} \right\}}{2},$$

$$k^2 = \frac{\left\{ - \left[RG - \omega^2 LC \right] + \sqrt{\left[R^2 + \omega^2 L^2 \right] \left[G^2 + \omega^2 C^2 \right]} \right\}}{2},$$

which represents two traveling waves. The lossy cable leads to an exponential attenuation of the signal with the distance. The attenuation coefficient is often so small that it is significant only in very long cables of many ten's of meters. The dependence of the phase velocity $v = dk/d\omega$ with the frequency is responsible for the dispersion seen in Fig. 2. For coaxial cables, frequencies above 100 MHz are attenuated more efficiently.

3. Conclusions

We described a very simple experiment to measure the impedance and velocity of pulse propagation of a coaxial cable using a potentiometer and a non-expensive oscilloscope. Besides the low cost, one additional advantage is the use of not very long cables.

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