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Geometrical aspects of Venus transit

(Aspectos geométricos do trânsito de Vênus)

A.C. Bertuola[1], C. Frajuca[1], N.S. Magalhães[2], V. dos Santos Filho[3]

[1] *Instituto Federal de São Paulo, São Paulo, SP, Brasil*
[2] *Universidade Federal de São Paulo, São Paulo, SP, Brasil*
[3] *H4D Scientific Research Laboratory, 04674-225, São Paulo, SP, Brasil*

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We obtained two astronomical values, the Earth-Venus distance and Venus diameter, by means of a geometrical treatment of photos taken of Venus transit in June of 2012. Here we presented the static and translational models that were elaborated taking into account the Earth and Venus orbital movements. An additional correction was also added by considering the Earth rotation movement. The results obtained were compared with the values of reference from literature, showing very good concordance.
Keywords: Venus transit, geometrical methods, Earth-Venus distance, Venus diameter.

Nós obtivemos dois valores astronômicos, a distância Terra-Vênus e o diâmetro de Vênus, por meio de um tratamento geométrico de fotos tiradas do trânsito de Vênus em junho de 2012. Aqui nós apresentamos os modelos estático e translacional que foram elaborados levando-se em conta os movimentos orbitais da Terra e de Vênus. Uma correção adicional foi também acrescentada, considerando o movimento de rotação da Terra. Os resultados obtidos foram comparados com os valores de referência da literatura, mostrando uma concordância muito boa.
Palavras-chave: trânsito de Vênus, métodos geométricos, distância Terra-Vênus, diâmetro de Vênus.

1. Introdução

In June of 2012, in some places of the Earth some people had the privilege to witness Venus transit [1, 2], an astronomical event of singular beauty, observed and studied for a long time [3–6]. This event is very rare and it is possible that this generation of researchers does not again see the phenomenon because the next one will occur only in 2117. The phenomenum is very important in order to obtain some physical informations of the planet. For instance, spectrographic data can be combined with refraction measurements from Venus transits to give a scale height of the scattering centers in Venus haze [7]. Further in a recent work. [8], it is proposed that accurate astronomical distances may be determined from recent observations which were collected in the last event.

In the literature, investigations concerning to calculations of orbits and astronomical observables associated to them have been considered in many contexts with very sophisticated theoretical frameworks [9–12], including aspects like either the sensibility to disturbances in the orbit of satellites [9] or the study of equatorial circular orbits in static axially symmetric gravitating systems [10] and the calculation of radius of the orbits as well. In general, relativistic frameworks have been used [11, 12]. Here we intend to show that it is possible in a very simple case to calculate some astronomical observables with a simple geometrical method and elementary algebra, by solely considering the photos of Venus transit.

A lot of photos were taken of the phenomenon and published by several different authors. The illustration shown in the Fig. 1 displays a mosaic composed by a

[1] E-mail: acbertuola@gmail.com.

selection of photos that were chosen for our study. The mosaic in the Fig. 1 was constructed from several photos obtained in different places [2].

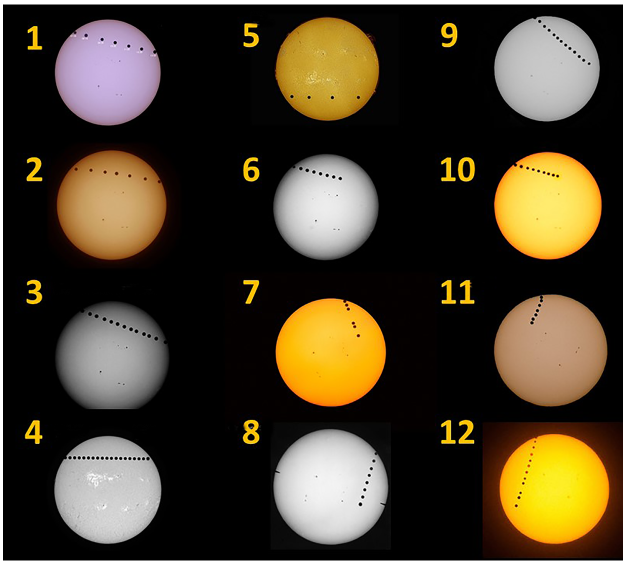

Figure 1 - Mosaic formed by photos in which it is shown the composition of the positions of Venus during the transit. From frames 1 up to 5, one observes the complete composition of the transit. For the others remaining photos, the composition of the transit is incomplete.

In the frames 1 up to 5, Venus transit can be seen in a complete way. The other photos (6 up to 12) are those that have the incomplete composition because the unset happened before the end of the transit. Working directly with the photos of Fig. 1, there is the possibility to obtain two ratios: the length of the trajectory of the spot by the Sun diameter and the diameter of Venus spot by the Sun diameter.

Our work was initially inspired in Ref. [13], in which different methods could be constructed to accurately calculate the Sun's distance from a discussion of observations of Venus transit, so that British expeditions could be organized. We follow an inverse way, that is, known the Earth-Sun distance, we obtained the Earth-Venus distance and Venus diameter analyzing the transit phenomenon.

We constructed two physical models in order to calculate Venus diameter and the Earth-Venus distance. The first model considered is the static case, in which the Earth and the Sun are in rest; the second one is the translation model, in which the translational movement of the Earth is taken into account, but it is not considered the known spin of the planets [14]. Additionally, the latter had a correction due to the rotation of the Earth. We used in our calculations some physical quantities known [15–18], as shown in the Table 1. In that table, d_{ES} is the maximum distance from the Earth to the Sun (The Earth is close to the apogee.) and D_S is the Sun diameter. Here we consider the diameter as the Sun equatorial diameter [17]. The other astronomical values are the Earth orbital period (T_E), Venus orbital period (T_V), the value of the time spent for Venus to complete the transit (t) and the radius of the Earth (R_E). The official values of the Earth-Venus distance and Venus diameter used for comparison in this work are respectively $d_{EV} = 4.31 \times 10^7$ km and $D_V = 1.2104 \times 10^4$ km [15–18].

The numerical values of the Earth-Venus distance and Venus diameter were calculated when Venus was also farthest away from the Sun. In the following, we describe the geometrical methods that we have elaborated.

2. Geometrical aspects

The direct geometric method is based on a technique of image treatment of photos of Venus transit. By means of this technique, two important ratios are obtained. The first ratio is the length (L') of the trajectory of Venus spot on the Sun by the Sun diameter. The second ratio is the value of the diameter of Venus spot (D') by the Sun diameter. The indirect geometric method corresponds to a mathematical triangulation technique of the relative positions of the Earth, the Sun and Venus during the transit phenomenon. The Venus diameter and its distance to the Earth were obtained by combining those two methods. In the following, we describe details of both geometrical methods.

Table 1 - The quantity d_{ES} is the Earth-Sun distance (in km), D_S is the Sun diameter, T_E is the Earth revolution period (in days), T_V is Venus orbital period (in days), t is the duration of transit (in hours) and R_E is the radius of the Earth (in km).

d_{ES} (km)	$\frac{d_{ES}}{D_S}$	T_E (day)	T_V (day)	t (h)	R_E (km)
1.521×10^8	109.3	365.26	224.7	6.67	6378

[2]The photos used in the construction of the mosaic in Fig. 1 were: Photo 1: Joetsu City (Niigata, Japain), 06 June 2012; Photo 2: Manila (Philippines). Credit: James Kevin, 06 June 2012; Photo 3: Quezon City (Philippines). Credit: Jett Aguilar, 06 June 2012; Photo 4: Mauna Loa (HI, USA). Credit: National Solar Observatory; Photo 5: Mauna Kea (HI, USA), 05 June 2012,Hα Composite Image; Photo 6: Landers (CA, USA), 05 June 2012; Photo 7: Credit: © Indranil Sinharoy; Photo 8: Mount Wilson Observatory. Credit: Jie Gu; Photo 9: Port Angeles (Wash, USA). Credit: Rick Klawitter, 05 June 2012; Photo 10: Scottsdale (Arizona, USA). Credit: © 2012 Brian Leckett; Photo 11: Abu Dhabi. Credit: Nik Syahron, 06 June 2012; Photo 12: Sta Rosa (Laguna, Philippines). Credit: Doun Dounel, 06 June 2012.

2.1. Direct geometrical method

We used the Photoshop software, so that the coordinates x and y of any point can be obtained. The distance between two points can also be calculated by considering elementar analytical geometry. The direct geometrical study of the photo 5 (Fig. 1) was performed according with the Fig. 2.

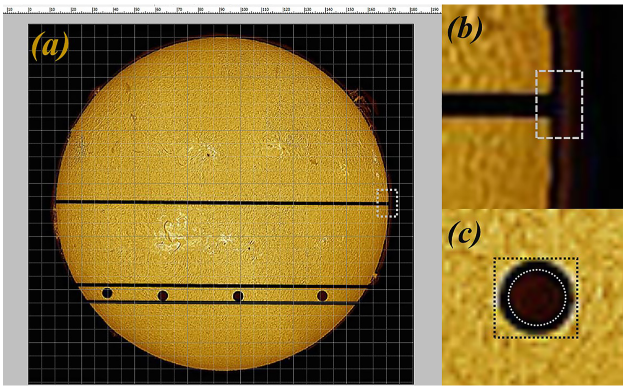

Figure 2 - Geometrical treatment of Venus transit of the photo 5 from Fig. 1. In the frame (a), we drew with Photoshop three straight lines, being one the diameter of the Sun and the others enclosing all the spots of Venus during the transit. In the frame (b), we zoom in the right extreme of the diameter, so that we could securely obtain one point in the illuminated region and the other one in the dark region. In the frame (c), we zoom in one of the spots, so that it were entirely contained inside a square drawn with the software. Inside the spot, a circle was drawn in order to outline the effective dark region.

That figure displays some steps of the procedure adopted to obtain the direct measurements, using the edition tools of the software. The Fig. 2(a) displays two parallel straight lines, drawn to enclose all the spots on the Sun. The length of the trajectory of Venus spot can be estimated by means of the measurements of the lengths of the chords drawn in the figure.

Above those two lines, a horizontal straight line was traced and the value of the Sun diameter was obtained. In fact, in agreement with the Fig. 2(b), two values for the Sun diameter were obtained in that same image. For example, we obtained the underestimated value of the Sun diameter considering two points in the extremes of the segment, that are entirely contained on the internal illuminated part of the Sun. The overestimated value of the diameter was obtained choosing the extremes of the segment on the dark area of the image. It was also measured the value of the diameter of the spots of Venus in a direct way according to Fig. 2(c), in which a square encloses the whole spot. A circumference was drawn in its internal dark region. By means of the square and the circumference, we obtained two values for the diameter of the spot. All the possible values obtained from Fig. 2 are showed in the Table 2 and with them we obtained four independent values for each ratio.

Now we define two geometrical parameters of the direct model as

Table 2 - Ratios obtained from the photo 5. Each numerical matrix element represents the ratio of the diameter or length in the column by the diameter in the row. We considered internal values for D' and L', respectively indicated by the first and third columns; and external values for both, respectively indicated in the second and fourth columns.

-	D'	D'	L'	L'
D_S	0.03191	0.02808	0.8596	0.8239
D_S	0.03197	0.02813	0.8587	0.8254

$$\alpha = \frac{D'}{D_S}, \quad \text{and} \quad \beta = \frac{\mathrm{L}'}{\mathrm{D_S}}. \tag{1}$$

These parameters serve to indicate the quality of the photos and to evaluate the ability of the researcher in the procedure of choosing and treating the images. So, we presented in the Table 3, their average values and their correspondent errors. We also presented in the Table 3 all the results for the other photos of the mosaic in Fig. 1, that is, we provided the average values of the ratios α and β, as well their respective standard deviations Δ and percentage error $\Delta(\%)$. This latter value was calculated only to evaluate the quality of the image. For instance, the parameters α and β were numerically obtained by the direct geometrical method applied to the best photos and the database were chosen, so that the photos 9 and 12 were discarded because they presented high percentage error for the ratio D'/D_S.

Such values are available for being used in the denominated indirect geometrical method.

2.2. Indirect geometrical method

The geometrical method is a flat triangulation among the relative positions of the Earth, the Sun and Venus allowed by transit phenomenon. We here present three geometrical models, beginning with the simplest one and increasing the complexity for the others. The results obtained from initial model have guided us to the proposal of a new complemental model.

A straight line can be imagined instantly connecting the Earth, Venus and its spot on the Sun in the beginning of the transit and other in the end of the transit to construct the three triangulations of the method according to the Fig. 3.

In that figure, the triangulation is accomplished in the apex of the transit, with Venus, its spot on the Sun and one observer in the Earth forming the referred triangle. This is a very special instantaneous situation that is always worth. In this triangle, the distance $\overline{Q'M}$ is the radius of the spot of Venus projected on the Sun, the distance $\overline{S'R'}$ is the radius of Venus and O' is the point in which one finds the observer in the Earth. By using geometry of triangles from that illustration, we found that $\Delta Q'MO' \sim \Delta R'S'O'$, so that it was obtained the equation

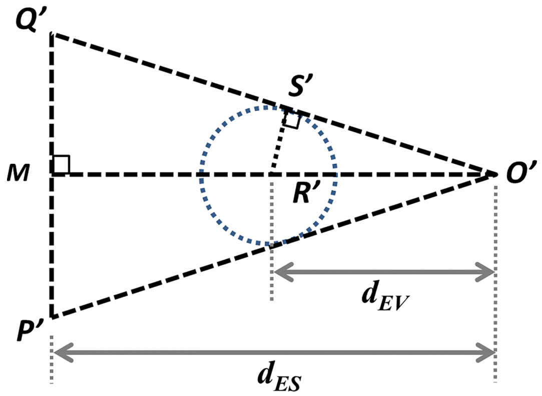

Figure 3 - Initial triangulation proposed from Venus transit. In the scheme, we show the basic triangulation, in which is drawn the entire planet and its spot on the Sun.

$$\frac{d_{EV}}{D_V} = \sqrt{\frac{1}{4} + \frac{1}{\alpha^2}\left(\frac{d_{ES}}{D_S}\right)^2}. \qquad (2)$$

The numerical value of the parameter α was obtained by the direct geometrical method defined by the Eqs. (1) and $\frac{d_{EV}}{D_V}$ can be obtained using the numerical value previously known of the ratio $\frac{D_{ES}}{D_S}$ given in the Table 1.

The Fig. 4 refers to the first model initially proposed, that corresponds to the static case. In that model, the translation and rotation movements of the Earth are not considered. The entire movement happens as if the Earth and the Sun were approximately in rest state in relation to the distant stars, during the event [19]. In that figure, the point P is the center of the spot in the beginning of the transit, when it is just to enter in the solar disk. Analogously, Q is the center of the spot in the end of transit. Besides, in the interval of time in which the transit happens, we conjectured Venus movement to be circular uniform one. The triangulation showed in that figure is mathematically described by

$$d_{EV} = \left[\frac{2\delta_V \frac{d_{ES}}{D_S}}{\alpha + \beta + 2\delta_V \frac{d_{ES}}{D_S}}\right] d_{ES}, \qquad (3)$$

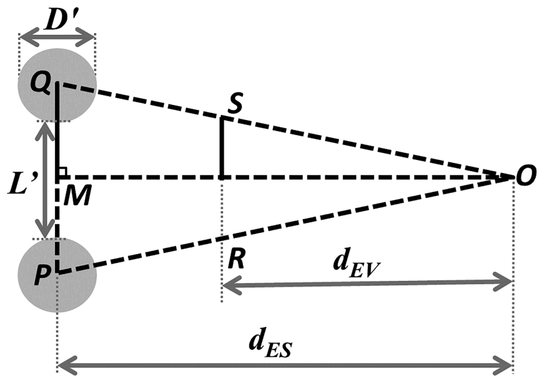

Figure 4 - Triangulation proposed from Venus transit in the case of the first model. In the scheme, we have a representation of the static model, in which it is not considered the Earth revolution movement.

in which δ_V is a physical parameter defined by $\delta_V = \frac{\pi t}{T_V}$, whose numerical value can be obtained from Table 1. The Eqs. (3) and (2) can already be enough to obtain a good value for the Earth-Venus distance and Venus diameter, but it is important to verify the influence of the Earth revolution movement on the results. So, we proceed with the translation model.

The Fig. 5 is a triangulation in which the movement of translation of the Earth around the Sun is added. In that figure, the distance $\overline{TU}$ is the displacement of the Earth during the transit. It is important to observe that the length d is the distance of the Earth up to the point O. Despite of being an unknown value for us, this does not represent any problem because it is eliminated in the calculations.

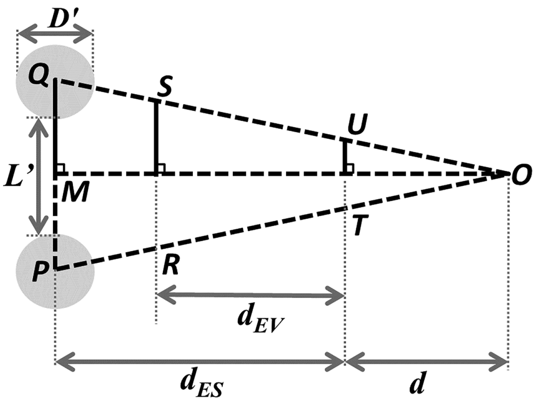

Figure 5 - Triangulation proposed from Venus transit in the case of the second model. In the scheme, we have a representation of the translational model, in which we consider the Earth revolution movement.

The scheme shown in the Fig. 5 is a complement to the Fig. 4, in which is added the translation movement of the Earth. The segment $\overline{RS}$ represents the real trajectory of Venus, while the segment $\overline{TU}$ is the trajectory of the Earth during the phenomenon. We propose a flat model in which the two support straight lines of the segments $\overline{SU}$ and $\overline{RT}$ cross each other in the point O. That means that the Earth, the Sun and Venus stand together on a plane. Inside the triangle ΔQMO, two similar triangles are identified and this lead us to the equations

$$\frac{\delta_E d_{ES}}{d} = \frac{\delta_V(d_{ES} - d_{EV})}{(d_{EV} + d)} = \frac{(\alpha+\beta)D_S}{2(d_{ES} + d)}. \qquad (4)$$

in which $\delta_E = \frac{\pi t}{T_E}$ and the numerical value of T_E can be obtained from the Table 1. The physical parameters δ_E and δ_V were obtained by surmising uniform circular movements for the planets. The unknown parameter d can be eliminated in Eqs. (4), then we obtained

$$d_{EV} = \left[\frac{2\left(\delta_V - \delta_E\right)\frac{d_{ES}}{D_S}}{\alpha + \beta + 2\left(\delta_V - \delta_E\right)\frac{d_{ES}}{D_S}}\right] d_{ES}. \qquad (5)$$

The numerical value of Venus diameter was quickly obtained, using the Eq. (2), after the results obtained from Eq. (5).

3. Results and analysis

All the values of the distances were calculated using the data from the Tables 1 and 3, respectively substituted into Eqs. (3), (5) and (2). Our results are showed in the Table 4.

By analyzing the data obtained, we can compare the results from the static model with the the corresponding ones from the translational model. We observe that the results calculated from static and translational models were very different and the best values of Venus diameter and the Earth-Venus distance were obtained by means of the translational model, when compared with the values of reference adopted. Both values are within of the respective error range provided by the translational model. Even so, in order to securely check if it was necessary to correct the results, we investigated the influence of the Earth intrinsic rotation movement on the values of Table 4. So, we considered the values of the translational model in the model with correction due to the Earth rotation.

Table 3 - Average values of the ratios and their respective errors: Δ (standard deviation) and $\Delta(\%)$ (percentage error). We used the percentage error to study the quality of the images.

Photo	α	Δ	$\Delta(\%)$	β	Δ	$\Delta(\%)$
1	0.0284	0.0028	9.7	0.8108	0.025	3.1
2	0.0297	0.0024	8.0	0.8250	0.031	3.8
3	0.0311	0.0009	3.0	0.8039	0.028	3.5
4	0.0301	0.0017	5.7	0.8118	0.020	2.4
5	0.0305	0.0020	6.7	0.8408	0.020	2.3
6	0.0291	0.0038	13	0.8018	0.020	2.5
7	0.0326	0.0044	13	0.8220	0.012	1.5
8	0.0370	0.0041	11	0.8116	0.029	3.6
9	0.0232	0.0038	17	0.8210	0.030	3.6
10	0.0239	0.0030	12	0.8187	0.023	2.8
11	0.0291	0.0018	6.2	0.8637	0.030	3.5
12	0.0211	0.0045	21	0.8157	0.031	3.8

Table 4 - Numerical results calculated by means of the indirect geometrical method for the static and translational cases. Note that the best results for the Earth-Venus distance and Venus diameter were obtained from the translational method. The first line of the numerical results refers to the complete transit and the last one corresponds to the incomplete one.

Static		Translational	
$d_{EV}(10^7$ km)	$D_V(10^4$ km)	$d_{EV}(10^7$ km	$D_V(10^4$ km)
7.610 ± 0.060	2.086 ± 0.065	4.231 ± 0.048	1.160 ± 0.037
7.586 ± 0.097	2.11 ± 0.30	4.187 ± 0.077	1.16 ± 0.17

We decomposed the movement of the Earth in two sequential movements in order to obtain the correction. The first one was the translation of the Earth and afterwards the rotation motion. The Fig. 6 shows the triangulation of the transit when it is considered that additional movement. The first circle to the left represents the beginning of the transit and the right one corresponds to its end. The radius of the Earth R_E appears in a natural way as a parameter of correction, with value given in Table 1. In that figure, we can analyze a scheme of a particular symmetrical configuration for the correction in the translational model due to the rotation of the Earth. In the circles representing the Earth, the point T indicates the position of the observer in the beginning of Venus transit and V is the position of the observer in the end of transit, taking into account both movements. We recognize the translation distance $\overline{TU}$, earlier used in the translational model. The angle of rotation of the Earth that corresponds to the time of transit is 100.05° and $\overline{TV}$ is a corrected distance due to the new position of the observer after the rotation of the Earth, so that the difference between those positions of the observer corresponds to $\overline{UV} = \Delta l$. We also see in both circles the angle δ of observation of the phenomenon in relation to the horizon. The correction $\overline{UV}$ can be obtained from the equation $\overline{TV} = 2\delta_E d_{ES} - \Delta l$.

In such a symmetrical configuration, one can in a straightforward way obtain the correction $\Delta l = 2R_E \sin 50.025°$, so that

$$d_{EV} = \left\{ \frac{\left[2\left(\delta_V - \delta_E\right) + \frac{\Delta l}{d_{ES}}\right] \frac{d_{ES}}{D_S}}{\alpha + \beta + \left[2\left(\delta_V - \delta_E\right) + \frac{\Delta l}{d_{ES}}\right] \frac{d_{ES}}{D_S}} \right\} d_{ES}. \tag{6}$$

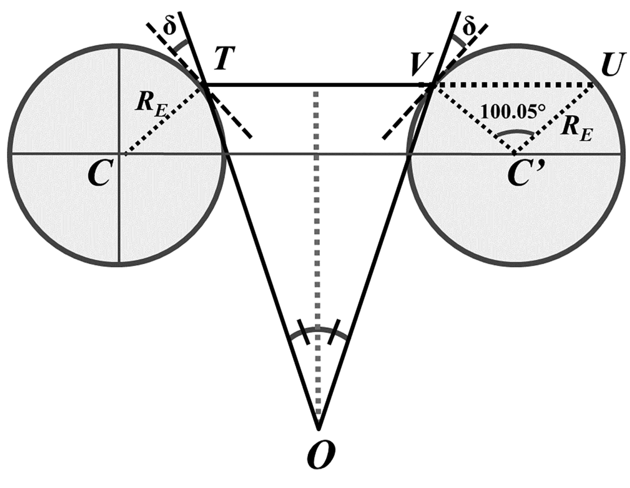

Figure 6 - Scheme for visualization of the correction in the translational model due to the rotation of the Earth. In the calculation, we first consider a translation of the center of the Earth from C up to C′, characterizing the translation $\overline{TU}$. In the end of the translation, one realizes a rotation of 100.05° and the observer in the Earth is dislocated from U to V, characterizing the effective distance $\overline{TV}$. So, the variation in the positions of the observer corresponds to $\overline{UV} = \Delta l$. In both circles, the parameter δ is the angle of observation in relation to the horizon.

So, we calculated the corrected values by considering the symmetrical configuration and our final values are shown in the Table 5.

Table 5 - Final results for the Earth-Venus distance and Venus diameter by means of the translational model with correction.

d_{EV} (km)	D_V (km)
4.296 ± 0.048	1.178 ± 0.038

The corrections had a small influence on the values of the Table 4, mainly on the average values, but these corrected values are closer to the values of reference adopted.

4. Concluding remarks

This work achieved the objective of obtaining the numerical values of Venus diameter and the Earth-Venus distance by means of models based on image treatment and triangulations allowed by Venus transit. The proposal highlights the efforts from several researchers and observers that obtained the photos used in the geometrical treatment of the images and presents as advantage to consider simple flat geometrical models in the description. In fact, we completely reached our aims by means of the translational model with corrections, within a relatively good precision. In the calculations, we adopted as input data two geometrical parameters obtained from a set of photos with the composition of the transit. In the case of the rotational correction, the Earth radius appeared in a natural way as a correction parameter. The final results took into account the systematic error due to the equipments, such as the type of cameras with its respective coupled filters, and errors in the composition of images. The geometrical methods here proposed depend on the researcher and his choices, which can present small variations from one to the other (as evaluations of spots diameter or positions of extreme points in diameters), but the method is itself robust, with all the values laid within the error bar. It is important to mention that the average values of the Earth-Venus distance and their respective percentage errors were more precise, but in the case of Venus diameter the error propagation significantly increased the percentage error in the incomplete transit. Despite of that problem, we did not discard them because the average values were very close to the measurements of reference, so that one could identify the differences between the results of the complete and incomplete transit. We believe that the new technique outlined here can be applied to other planets and other Moons, as in the case of the transits of Jupiter Moons. In the case of an exoplanet, we could determine the distance between it and its respective star if we knew the apparent diameter of the star, the distance between the star and the Earth and the composition of the transit, so that the method here described should work well. However, to analyze distances or orbital parameters of those orbs we need additional investigations that are now in progress.

Acknowledgments

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