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Estimating the parameters of a monthly hydrological model using hydrological signatures

Estimativa dos parâmetros de um modelo hidrológico mensal utilizando assinaturas hidrológicas

Ana Clara de Sousa Matos[1], Francisco Eustáquio Oliveira e Silva[1] & Gustavo de Oliveira Corrêa[1]

[1]Universidade Federal de Minas Gerais, Belo Horizonte, MG, Brasil
E-mails: aclaram@ufmg.br (ACSM), fsilva@ehr.ufmg.br (FEOS), gustavoliveira@ufmg.br (GOC)

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ABSTRACT

In the most common Bayesian framework for estimating the parameters of a hydrological model (time domain), the specification of the likelihood function can be challenging. In addition, scarcely gauged regions might be hard to model, due to the lack of sufficient timeseries to calibrate the model. To circumvent these problems, the present study seeks to evaluate the applicability of hydrological signatures and Approximate Bayesian Computation methods to estimating the parameters and analyzing the uncertainty of a hydrological model (signature domain). We used the GR2M monthly model, aiming to approximate the signatures estimated from the simulated timeseries to those calculated from the monitoring data. As a result, we found KGEs of over 0.91 and 0.83 for most signatures in the calibration and validation periods, respectively (0.95 and 0.90 in the time domain). The uncertainty intervals varied from signature to signature, with the tendency of being smaller for the signature-domain than for the time-domain.

Keywords: Hydrological modeling; Hydrological signatures; Approximate Bayesian Computation; GR2M, DREAM.

RESUMO

Na abordagem Bayesiana mais comum para estimativa de parâmetros de um modelo hidrológico (no domínio do tempo), a especificação da função de verossimilhança pode ser um desafio. Além disso, regiões com monitoramento escasso podem ser de difícil modelagem, dada a ausência de séries temporais suficientes para calibração do modelo. A fim de contornar esses problemas, este estudo busca avaliar a aplicabilidade de assinaturas hidrológicas e métodos de aproximação computacional Bayesiana para fins de estimativa de parâmetros e análise de incerteza de modelos hidrológicos (domínio das assinaturas). Foi adotado o modelo mensal GR2M, buscando aproximar as assinaturas estimadas a partir das séries temporais simuladas àquelas calculadas usando os registros de monitoramento. Como resultado, foram encontrados valores de KGE acima de 0.91 e 0.93 para a maioria das assinaturas durante os períodos de calibração e validação, respectivamente (0.95 e 0.90 no domínio do tempo). Os intervalos de incerteza variaram de assinatura para assinatura, tendendo ser menores para o domínio das assinaturas que para o domínio do tempo.

Palavras-chave: Modelagem hidrológica; Assinaturas hidrológicas; Aproximação Computacional Bayesiana, GR2M, DREAM.

INTRODUCTION

In Brazil, the 8 biggest metropolitan regions are supplied by mixed systems composed by at least one reservoir. Some examples are the metropolitan region of São Paulo (RMSP, in Portuguese), that encompasses over 20 million people, the Cantareira system is responsible for over 50% of the water supply; the Metropolitan Region of Rio de Janeiro (>12 million people), with some small reservoirs such as Registro; for Belo Horizonte (> 5 million people), three reservoirs integrate the Paraopeba system, providing water to roughly 50% of the population; in the country's capital's (Brasilia) metropolitan region, Paranoá and Descoberto lakes are used for supplying part of the over 4 million population's demand. In the northeast, Castanhão and Pirapama are some of the reservoirs used for proving water for the metropolitan regions of Fortaleza and Recife, respectively, both with almost 4 million inhabitants.

In southeast and central Brazil, there was a severe water crisis during the mid-2010s, resulting in water scarcity and even rationing in various cities. The above-mentioned systems of São Paulo, Rio de Janeiro, Belo Horizonte and Brasília were deeply affected. As a consequence, the operation of these reservoirs, as an instrument to improve water supply reliability, as well as the expected impacts of the climate change on it, has become a topic of interest. To do so, it is fundamental to quantify the inflows and outflows to the reservoirs.

Hydrological models are mathematical tools for quantitative analysis, extrapolation and prediction of events (Beven, 2012), allowing for estimating variables in scenarios not yet observed, such as severe droughts or floods. In this sense, they make it possible to simulate the inflows to reservoirs, create operational rules and help decision-making.

Hydrological models are described by parameters that seek to synthesize the hydrological behavior of the basin through equations and assumptions. The main approach to estimating the parameters of a hydrological model is based on optimization and calibration processes, in which one aims to find the parameter set that best relates the model outputs to the real system. The final result is a single set of parameters and, consequently, a single simulated timeseries. Therefore, it is not possible to evaluate the many uncertainties related to these estimates: the assumptions and equations used in the model's structure, the natural variability of the hydrological variables, data errors, etc. (Beven, 2009). Another point to be addressed is the data availability to calibrate these models and make their predictions minimally trustworthy.

Regarding the uncertainty analysis, the most common approach is based on a Bayesian framework using Monte Carlo Markov Chain (MCMC) sampling methods. This methodology was vastly used in the last decades (Diao et al., 2021; Hopp et al., 2020; Sheng et al., 2020). However, they depend on the definition of the likelihood function, which may not have an explicit form or may have a high computational cost. Consequently, it is crucial to take the results of these assessments cautiously, particularly when the model errors are correlated, non-stationary and non-Gaussian (Bennett, 2019).

An alternative approach is the Generalized Likelihood Uncertainty Estimation – GLUE (Beven & Binley, 1992), which aims to find regions within the parameter space, up to an acceptable limit, whose predictions are similar to the observed data, without specifying a likelihood function. While this method provides an assessment of the parametric uncertainty (Ragab et al., 2020; Yan et al., 2020), the scientific community argues that the subjective choice of a likelihood function can hinder the posterior validation of the assumptions made a priori, potentially leading to statistically incoherent and/or debatable predictions and parameters distributions (Vrugt et al., 2009).

The so-called "likelihood-free" methods rise as an alternative in these cases. They propose sampling from a posterior distribution, with no need to evaluate the likelihood function. Approximate Bayesian Computation (ABC) algorithms are one of the likelihood-free methods: similar to MCMC, they require a formal probabilistic model, but do so by sampling realizations of the model outputs, rather than computing the likelihood function (Kavetski et al., 2018). As a consequence, they relax some of the assumptions regarding the likelihood function, and the previously-mentioned problems related to MCMC or GLUE applications. ABC applications evaluate the model performance using summary statistics: assuming sufficient summary statistics are chosen, it is possible to empirically estimate the variables' posterior distribution (Beaumont, 2019). In this sense, the need for extensive data to calibration is reduced. Fenicia et al. (2018) and Kavetski et al. (2018) proposed using hydrological signatures as summary statistics.

Although widely adopted in studies on hydrological processes for decades, the concept of hydrological signatures was formalized by Gupta et al. (2008), who described them as the minimum relevant representation of the hydrological information contained in a data set. They are characteristics derived from monitoring data or modeled series of hydrological data, such as rainfall, flow or soil moisture, and can range from simple statistics, such as the average or quantiles of a time series, to more complex metrics, such as those that describe recession and are related to storage in the basin.

Addor et al. (2018) point out that hydrological signatures are particularly useful for characterizing and comparing the dynamics of basins in which there is a predominance of flow gauges and a scarcity of data such as evapotranspiration and water table level. In other words, the signatures can be an important source of indirect information about the basin's hydrological processes, when these processes cannot be isolated due to the absence of monitoring data. They can also be regionalized and used for model calibration, since the attributes of the basin are generally more related to the signatures than to the model parameters, and because the regionalization of the signatures is independent of the choice of prediction model or error model (McMillan, 2021).

Previous studies used hydrological signatures and ABC algorithms to estimate the parameters of a daily hydrological model (Fenicia et al., 2018; Kavetski et al., 2018) or a monthly model using synthetic data (Fenicia et al., 2018). Brazil has a particular socioeconomic and territorial context, with continental dimensions, several different biomas and a scarce gauging network of hydrological variables, especially in small to medium-sized catchments. In addition, the long-term operation of the previously-mentioned reservoirs used for water supply in the major urban agglomerations usually adopts a monthly step, as the concern is more on the volume inflow and impact on storage than on singular events and hydrograms' peaks.

Given this context, this work proposes the first application of hydrological signatures to estimate the parameters of a monthly hydrological model, using ABC methods and real data from a tropical catchment. Our goal is to evaluate the goodness of fit and the parametric uncertainty estimated using streamflow time series modeled in the signature domain, compared to the time domain. We aim to test if it is possible to obtain similar performance in both domains and if the parametric uncertainty is affected by the loss of information due to the consideration of signatures instead of the full time series.

CASE STUDY

We conducted a case study using monitoring data from the Serra Azul creek basin, located in Minas Gerais state, in Southeast Brazil. This catchment plays an important role in the social-economical dynamics of the Belo Horizonte Metropolitan Region (RMBH, in Portuguese), being responsible for the water supply of roughly 20% of the RMBH population (Santos & Silva, 2015). The main economic activities are related to agriculture, livestock, mining and industries (Fernandes, 2012). Moreover, this basin is the main contributor for a homonymous reservoir, which is used for, other than water supply, environmental conservation: the region is declared Special Protection Area (APE, in Portuguese) by the Decree MG nº 20.792/1980. In this area, there are programs that focus on recovering rivers' sources and protecting native fauna and flora.

The selected catchment is also part of the Juatuba catchment, considered a representative basin of the hydrological behavior of the surrounding catchments and of the Brazilian savannah (Franz, 1977). Besides the importance of this catchment to water supply and economic activities, it also has a higher gauging density than other relatively small catchments. At the Jardim monitoring gauge (Figure 1), the Serra Azul creek basin has approximately 113 km^2. There are two distinct seasons: a warm and wet period from October and March, and the dry and low-temperature season between April and September. The average air temperature varies between 22 ºC and 15 ºC, the mean relative humidity is approximately 70%, the mean annual precipitation is 1476 mm and the annual evapotranspiration is around 1033 mm (Neves & Rodrigues, 2007).

In this study, we used the monthly averages of the hourly data available between January 1997 and May 2008, for Jardim streamflow gauge (Code 40811100), and of the daily data available for Alto da Boa Vista (Code 2044021), Fazenda Laranjeiras – Jusante (Code 2044041), Jardim (Code 2044052) and Serra Azul (Code 2044054) rainfall gauges. The hourly data was gently provided by the Geological Survey of Brazil during previous works; the daily data was obtained from the HidroWeb Portal (www.snirh.gov.br/hidroweb), which integrates the National Water Resources Information System in Brazil. The evapotranspiration timeseries was obtained from the INMET Florestal station (Code 83581). Figure 1 shows the location of the study area and the monitoring gauges considered.

The period from January1997 to November 1997 was used to warm up the model. The simulations were carried out using the hydrological years from 1997/1998 to 2007/2008, with the period from December 1997 to February 2003 selected for calibration and the remaining period used for validation. Exceptionally, the last hydrological year was considered only until May 2008 due to lack of information for the remaining days of the year.

METHODS

Multiple realizations of a hydrological model, in both time and signature domains, are used in this study, aiming to evaluate the differences and similarities in the modeling results when a "likelihood-free" approach is used, compared to the Bayesian approach in the time domain. In the signature domain, the approximation technique focused in reproducing, for the simulated time series, the signatures estimated for the monitored time series.

Models and sampling algorithms

Hydrological model

In this study, we applied the GR2M – Génie Rural à 2 paramètres Mensuel (Mouelhi et al., 2006) hydrological model in

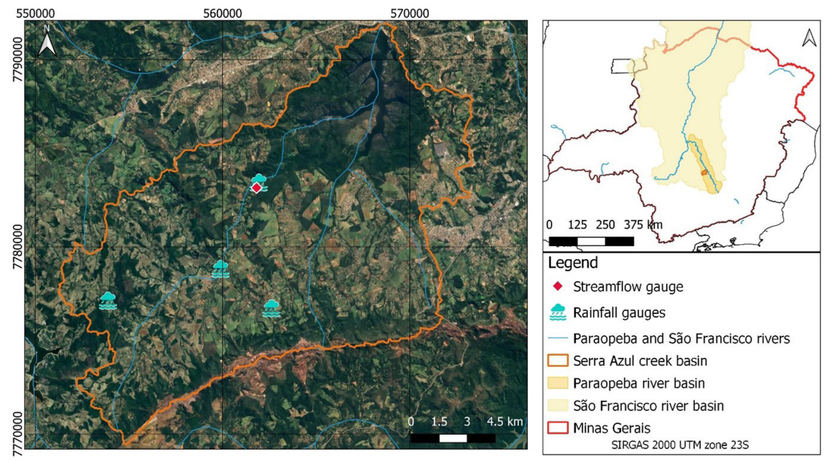

Figure 1. Study area and the selected gauging stations.

both the time and the signature domains. The GR2M model is a variant of the GR4J model (Perrin et al., 2003) with a monthly scale and only two parameters to be estimated: θ_1, the maximum capacity of the production store; and θ_2, the groundwater exchange coefficient. The first parameter controls the function of production that revolves around a reservoir-ground of a maximum capacity. Parameter θ_2 modifies a transfer function represented by a quadratic draining reservoir with its capacity limited to 60 mm. The equations considered in this model and a more detailed description can be found in Mouelhi et al. (2006) and in Mouelhi et al. (2013).

Okkan & Fistikoglu (2014) point out that the θ_1 parameter controls the basin's response to rainfall events and, to a certain degree, the variability of the modelled flow. High values of θ_1 tend to generate significant storage in the basin, making runoff less dependent on instantaneous rainfall, but more dependent on preceding events. On the other hand, for lower θ_1 values, storage is reduced and direct runoff is increased.

Table 1 presents the values and intervals assumed for the GR2M model used in this work.

Likelihood function

We considered the time domain as the paradigm solution against to which the results from the signature domain would be tested. Seeking for flexibility and a better representation of the modeling errors, we adopted the generalized likelihood function – GL (Schoups & Vrugt, 2010), which allows for the characterization of heteroscedastic, correlated, non-normal errors through a skew exponential power density (SEP) function. It is derived from a non-linear regression model given by Equation 1:

$$Q = Z + \delta \tag{1}$$

Where Q corresponds to the N flow observations, Z is a vector of average flows and δ, a vector of random residuals with zero mean.

The average flows in each time interval, Z_t, are calculated from the modelled flows, q_t, as shown in Equation 2:

$$Z_t = q_t\left(x \mid \theta\right) \cdot \mu_t \tag{2}$$

Where q_t is a function of the inputs x and the model parameters θ, and μ_t corresponds to a multiplicative factor that seeks to characterise the bias introduced into the model's outputs due to errors in the observations and in the model structure. Since μ_M is a parameter that represents the bias estimated from the input data, the value of μ_t is calculated using Equation 3:

$$\mu_t = \exp\left(\mu_M \cdot q_t\right) \tag{3}$$

To take autocorrelation and dependence into account, the residuals δ - Equation 1 - are characterized by the set of parameters θ_δ and a probability density function, modeled according to Equation 4:

$$\begin{aligned} &\Phi_p\left(B\right)\delta_t = \sigma_t \cdot a_t \\ &a_t \sim SEP\left(0,1,\xi,\beta\right) \end{aligned} \tag{4}$$

In Equation 4, $\Phi_p\left(B\right) = 1 - \sum_{j=1}^{p}\phi_j B^j$ is an autoregressive polynomial with p parameters ϕ_j, B is the lag operator $\left(B^j\delta_t = \delta_{t-1}\right)$, σ_t is the standard deviation at time t, a_t expresses independent and equally distributed random errors, with mean equal to zero and standard deviation equals to one.

Heteroscedasticity is explicitly considered using Equation 5, which admits linear variation for the standard deviation as a function of the flow :

$$\sigma_t = \sigma_0 + \sigma_1 \cdot Z_t \tag{5}$$

The values of the linear σ_0 and angular σ_1 coefficients are estimated from the monitoring records. This formulation seeks to represent the uncertainties associated with the upper branches of the rating curve (Schoups & Vrugt, 2010).

A more detailed explanation of the GL can be found in Schoups & Vrugt (2010). In Table 2, we show the fixed values and uniform prior uncertainty ranges considered in this study. Due to the high computational cost to convergence, the values of the coefficients ξ (skewness), β (kurtosis) and ϕ_j (autocorrelation) were fixed one by one, after a first round of initial simulations. In these simulations, the complexity of the model, represented

Table 1. Parameters intervals considered *a priori* for the GR2M model.

Parameter	Description	Min	Max
θ_1	Maximum capacity of the production store (mm)	500	6000
θ_2	Groundwater exchange coefficient (mm)	0.01	1

Table 2. Parameters intervals and fixed values considered for the generalized likelihood function.

Parameter	Description	Interval	Fixed value
σ_0	Heteroscedasticity: intercept	-3 mm/ month to 3 mm/month	-
σ_1	Heteroscedasticity: slope	0 a 1	-
ξ	Skewness	-	1.0
β	Kurtosis	-	0.2
ϕ_j	Autocorrelation coefficient	-	0.55
μ_M	Bias parameter	-	0 mm/month

by the number of parameters considered in the analysis, was gradually increased, looking for convergence trends for a given parameter around a small uncertainty interval. The uncertainty intervals considered for the other parameters were defined based on the data and the numerical validity limits of the formulations.

Sampling algorithms

As a sampling algorithm in the time-domain, we used the Diffential Evolution Adaptive Metropolis – DREAM (Vrugt et al., 2008) method. The DREAM algorithm allows the estimation of posterior parameter distributions and the likelihood function. This MCMC algorithm assumes uniform prior distributions for model parameters and allows the simulation of multiple chains simultaneously. In addition, the scale and shape of the distribution models are continuously updated throughout the simulation, resulting in greater efficiency when simulating complex, non-linear or multimodal target distributions (Vrugt et al., 2009). Table 3 lists the main parameters used to run the algorithm and their assumed values.

In the signature domain, we adopted the SABC algorithm (Albert et al., 2014) for the approximation step, as it combines principles of the Simulated Annealing and the Approximation Bayesian Computation methods. We set the algorithm to return 5,000 parameters sets, after 1,000,000 iterations.

When $g\left(q^{(j)}\right)\{g_1, g_2, ..., g_N\}$ is the vector of N signatures calculated from observed data Q and $\tilde{g}\left(\tilde{q}^{(j)}\right)=\{\tilde{g}_1, \tilde{g}_2, ..., \tilde{g}_N\}$ the one estimated from the simulated series $\tilde{Q}$, the steps used to evaluate the posterior distribution are summarized below:

Pseudo-Algorithm to signature evaluation (Adapted from Kavetski et al., 2018)

1. Sample $\theta^{(j)}$ from the priori $\pi(\theta)$
2. Compute the simulated series $\tilde{q}^{(j)} \leftarrow \tilde{Q}\left(\ ^{(j)}, x\right)$ from parameters $\theta^{(j)}$ and from the model with x characteristics
3. Compute $\tilde{g}\left(\tilde{q}^{(j)}\right)$
4. Accept $\theta^{(j)}$ if $\rho\left(\tilde{g}, g\left(q^{(j)}\right)\right) < \varepsilon$, ρ being a distance metric and ε an accepted tolerance
5. Repeat steps 1 to 5 for $j=1,2,...,N$ parameters sets.

The distance ρ considered for posterior approximation is the relative difference between observed and simulated values, as described by Equation 6 for case of vector-valued signatures:

$$\rho(\tilde{g}, g)=\frac{1}{n_k}\sum_{l=1}^{n_k}\left|\frac{g_{k,l}-\tilde{g}_{k,l}}{g_{k,l}}\right| \tag{6}$$

3.2 Hydrological signatures

Consider t as the temporal dimension, the hydrological year starts in October, $P=(p_1, p_2, \ldots, p_N)$ is the set of N rainfall observations and $Q=(q_1, q_2, \ldots, q_N)$ represents the streamflow observations. The subsequent signatures are considered:

(1) Average monthly flow (q_{mean}): this metric is frequently employed in water resources management studies, especially for the reservoir operation. Its relevance has grown in works that seek to evaluate the environmental flows related to the sustenance of aquatic ecosystems (Zhang et al., 2020). The calculation takes into account the mean hourly observed flows for each year, and considers their average.

(2) Percentiles of the flow duration curve (P_{FDC}): the flow duration curve (FDC) allows a graphic and statistical analysis of the flow variability and its empirical distribution. The shape is influenced by factors such as rainfall patterns, land use and physiographic characteristics of the basin (Chiles, 2019). Compared to the previous signature (q_{mean}), which describes the average behavior of the hydrograph, we considered P_{FDC} to evaluate the extremes of the flow duration curve, specifically flows that were equal to or exceeded 1% (Q_1) and 99% (Q_{99}) of the time. The Weibull plot position was utilized, and a unified FDC was applied to all entries within the observed series.

(3) Slope of the flow duration curve (S_{FDC}): this metric aids in evaluating the water storage within the catchment and its vertical redistribution (McMillan, 2020). It is calculated using Equation 7 (Sawicz et al., 2011):

$$S_{FDC} = \frac{\ln(Q_{33}) - \ln(Q_{66})}{0.66 - 0.33} \tag{7}$$

Where Q_{33} and Q_{66} represent the flows that were equal to or exceeded 33% and 66% of the time, respectively.

(4) Annual runoff coefficient (c_a): this coefficient is used as an indicator of the general water loss to the deeper groundwater layers (McMillan, 2020). The mean value of the coefficients calculated for each year in the observed series was taken into account. The defining equation is as follows (Equation 8):

$$c_a = \frac{\sum_{t=1}^{12} q_t \Delta t}{\sum_{t=1}^{12} p_t} \tag{8}$$

Table 3. Parameters values considered for the DREAM algorithm.

Variable	Description	Value
nseq	Number of evaluated chains	4
ndraw	Maximum number of iterations	300,000
burn-in	Number of iterations discarded after the begging of the simulation	30%
thin.t	MCMC chain thinning interval	10
Rthres	Value of Gelman & Rubin's convergence diagnostic R value below which the sequences are considered to have converged	1.01

Evaluation and comparison criteria

To evaluate model performance, we considered the streamflow time series composed for the median, for each time interval, of the flows simulated with the selected parameter set. We evaluated the results using the KGE index and its components, as well as the simulated hydrographs. The KGE index expresses the distance from the point of ideal model performance in a re-scaled criteria space (Gupta et al., 2009) and is calculated from 3 components: Pearson's correlation coefficient (r), the ratio between the mean of simulated values (in this context, streamflow values) and the mean of observed values (γ), and the ratio between standard deviations of simulated and observed values (α). In an optimal scenario, both the KGE and its the three components would equal to 1. According to Knoben et al. (2019), KGE addresses some shortcomings in the Nash-Sutcliffe Efficiency metric and has an increasingly use for model calibration and evaluation.

In addition, we evaluated the root mean square error (RMSE), which is sensitive to outliers. It has the same unit as the simulated variable and can be interpreted as a measure of the average deviation between observed and simulated variables. Ideally, its value is equal to zero.

RESULTS AND DISCUSSIONS

Table 4 and Table 5 show the performance indices for the calibration and validation periods, respectively.

In Table 6 we present the percentages of the observations that are within the 95% credibility intervals.

From the tables, we observe that the timeseries simulated using q_{mean}, P_{FDC} and c_a reached similar performances to the one for the time domain, showed a good correlation between simulated and observed timeseries and were capable of reproducing the average catchment response. However, we found a completely different result for S_{FDC}: the simulated streamflows were meaningly higher than the observed ones, leading to γ and α values much higher than 1. It is worth noting that, considering the timeseries simulated using c_a, which showed the best performance among the proposed signatures in the monthly timestep, the slope of the flow duration curve is equal to 2.37. For the Jardim streamflow gauge, S_{FDC} = 1.95. Therefore, we conclude that the poor result we found for the slope of the flow duration curve in a monthly timestep is not due to the computational approximation; instead, it is possibly related to the incapacity of this signature in predicting the catchment's response in this timescale.

Regarding the RMSE, we found similar values for both the signature and the time domains. In addition, they were less than the standard deviation of the observed timeseries (22 mm/month).

Figure 2 shows the relation between simulated and observed streamflow for all tested signatures during the validation period, and Figure 3 disregards the poor result found for S_{FDC}.

Figure 4 to Figure 7 present the timeseries modeled according to the parameters estimated from each one of the signatures.

Table 4. Performance indices: calibration period.

Statistic	Time-domain	Signature-domain			
		q_{mean}	P_{FDC}	S_{FDC}	c_a
r	0.97	0.97	0.97	0.96	0.97
γ	1.01	1.07	0.93	21.38	0.99
α	1.04	1.03	0.95	20.15	1.04
KGE	0.95	0.91	0.91	-26.96	0.95
RMSE (mm/month)	4.96	5.56	5.27	737.02	4.89

Table 5. Performance indices: validation period.

Statistic	Time-domain	Signature-domain			
		q_{mean}	P_{FDC}	S_{FDC}	c_a
r	0.96	0.96	0.96	0.93	0.96
γ	1.08	1.15	0.99	21.62	1.05
α	0.94	0.94	0.87	18.42	0.95
KGE	0.90	0.83	0.86	-25.99	0.92
RMSE (mm/month)	5.60	6.80	5.49	663.54	5.34

Table 6. Percentage of the observations within the credibility interval.

Period	Time-domain	Signature-domain			
		q_{mean}	P_{FDC}	S_{FDC}	c_a
Calibration	39%	8%	9%	0%	85%
Validation	23%	13%	11%	0%	86%

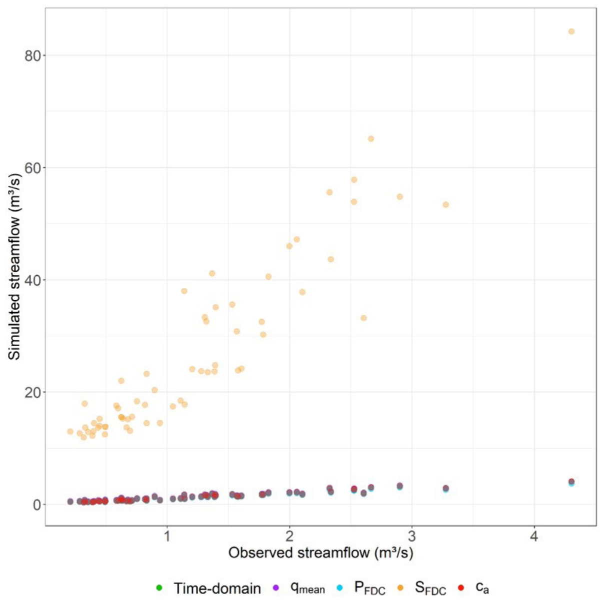

Figure 2. Simulated *versus* observed streamflow – All the signatures (validation period).

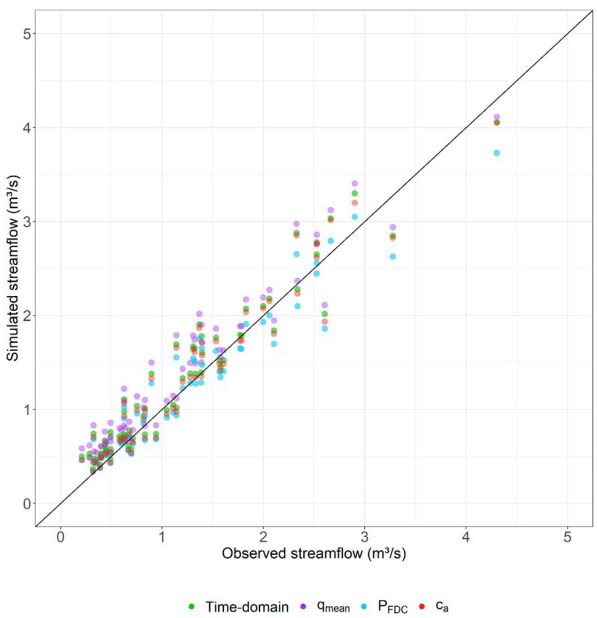

Figure 3. Simulated *versus* observed streamflow – Except S_{FDC} (validation period).

From the figures above, we notice that q_{mean}, P_{FDC} and c_a lead to very similar hydrographs to the one found in the time-domain, with a significantly wider uncertainty interval for the later signature. Along with the good performances discussed before, this result indicates the potential of this approach in estimating the parameters of a monthly hydrological model, corroborating the founds of Fenicia et al. (2018) for a catchment with different meteorological conditions and a hydrological model with different equations and timestep. The consideration of monthly averages reduces the influence of isolated events, such as intense rainfall events. Furthermore, it is closer to the Brazilian gauging context, in which hourly data is very scarce; for small to medium-sized catchments, even daily information can be hard to find.

A consequent possible application of this methodology is predicting in ungauged or poorly gauged basins (Dal Molin et al., 2023), as some hydrological signatures might be regionalized (Addor et al., 2018). Moreover, it can improve water management by allowing for reservoir operation and water allocation, for example. Even though this study did not intend to test regional data in the monthly step, we believe this would be a natural consequence of the results presented hereby and the next aspect to be addressed.

Regarding the parametric uncertainty, we noticed that, in general, the results were "excessively confident", with a majority of the observations outside the credibility interval. In the time domain, this finding might be related to the parameters tuning, in particular to fixing the skewness ξ. Ideally, all the parameters should have been sampled, but the chains would not converge when some of the parameters were not fixed. The ξ (skewness), β (kurtosis) and ϕ_j (autocorrelation) parameters were fixed after initial tests in an hourly timestep and based on the found of previous works (Ammann et al., 2019; Evin et al., 2014; Schoups & Vrugt, 2010). Narrower credibility intervals were also found by Sadegh & Vrugt (2013) when comparing DREAM to ABC.

In the signature domain, c_a was the only signature to show plausible credibility intervals, as q_{mean} and P_{FDC} also failed in capturing the observations, despite the good performances, in a similar situation to the time domain. It is worth noting that this signature was the only one where we considered the average of the M annual runoff coefficients – all the other signatures were estimated using the complete time series at once.

Finally, the opposite results found for P_{FDC} and S_{FDC} show that signatures derived from the flow duration curve may or may not lead to good performances – thus, a careful selection of the signatures is essential. Even though the performances may be equivalent to the ones for the time domain, the signatures derived from the FDC might not capture the variability of the process and cause overly confident uncertainty intervals, as discussed before.

Carefully selecting the hydrological signatures to be used is a challenge. Aspects such as data availability, predominant processes in the catchment and hydrological model's characteristics must be taken into account. McMillan (2020) groups several signatures by hydrologic processes to be represented, which can be a reference for further studies and provide some insights about possible signatures to be considered. Moreover, seasonality and time discretization are other very important points to be addressed, as they can have an impact in the analysis and even invalidate algorithms, especially for transition signatures (McMillan et al., 2023). Some more criterion to help selecting the signatures can be found in McMillan et al. (2017).

In general, the results presented hereby corroborate the founds of Fenicia et al. (2018) and Dal Molin et al. (2023), in different hydroclimatic conditions and using different hydrological models and timesteps. Unfortunately, the computational cost related to the simulations in the signature-domain was prohibitive to replicating the experiments in a broader range of catchments, this being the main limitation of this work. On the other hand, the consideration of a monthly timestep in this study is more coherent with the most common situation we find in

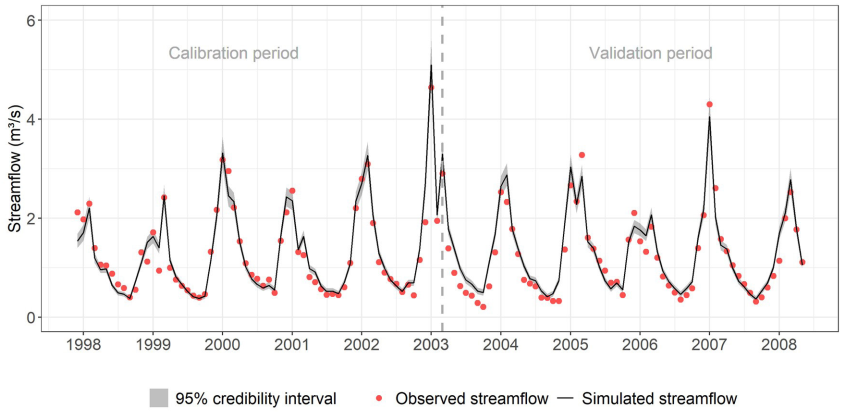

Figure 4. Simulated and observed streamflow timeseries: time-domain.

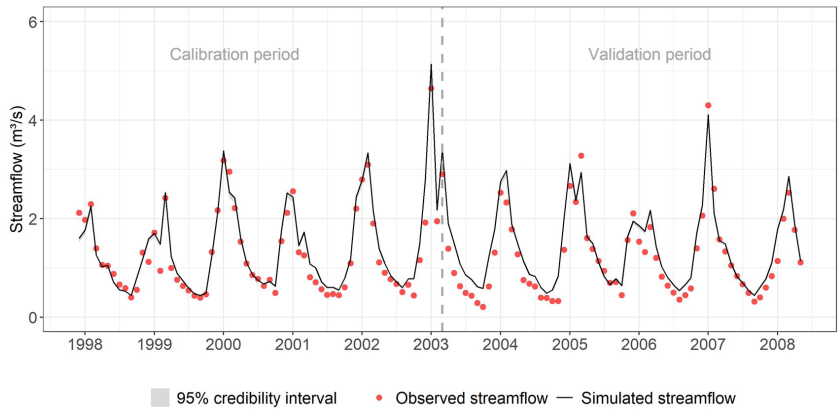

Figure 5. Simulated and observed streamflow timeseries: q_{mean}.

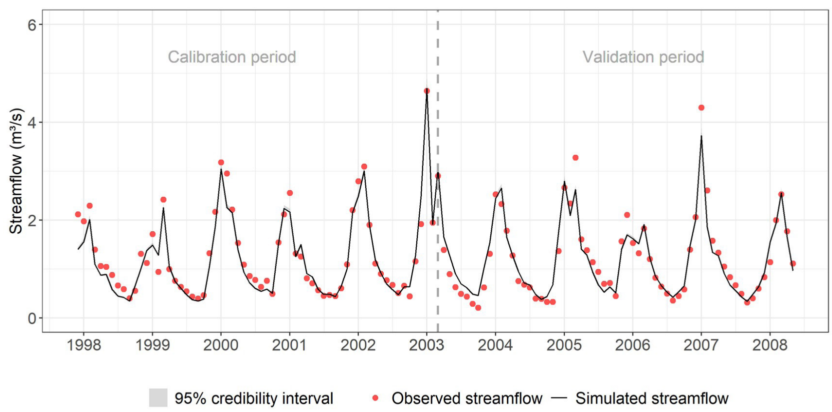

Figure 6. Simulated and observed streamflow timeseries: P_{FDC}.

Brazil, where the monitoring frequency (usually daily) is often inadequate to represent the catchment's hydrological processes, especially in small to medium-sized catchments with sub daily processes. In the specific case of the Serra Azul catchment, the consideration of a monthly step is also justified by the existing reservoir downstream, which water balance is fundamental to water supply in the RMBH and is usually taken in this timescale, for management purposes.

Other experiments, using different timesteps and hydrological models, as well as regional data, were presented in Matos (2021), and led to similar results, reinforcing that the methodology can be an interesting tool to predicting in ungauged basins.

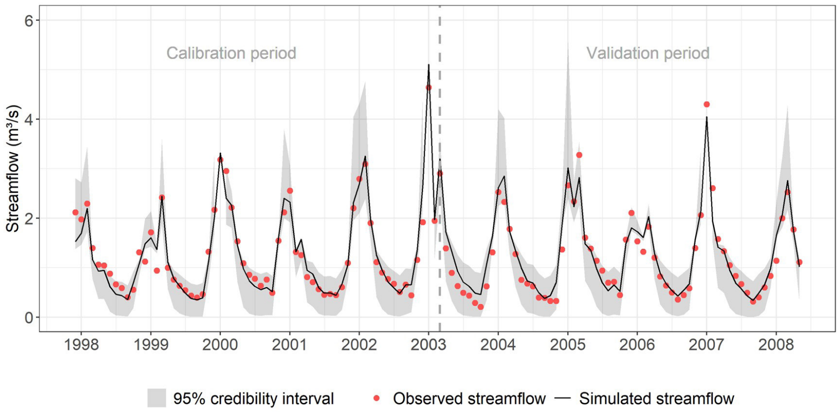

Figure 7. Simulated and observed streamflow timeseries: c_a.

CONCLUSIONS

This paper investigated the applicability of hydrological signatures to estimate parameters of hydrological models in a monthly step, using Approximate Bayesian Computation methods. We tested four different signatures – average monthly flow (q_{mean}), 1% and 99% percentiles of the flow duration curve (P_{FDC}), slop of the flow duration curve (S_{FDC}) and annual runoff coefficient (c_a) – and considered a single value (or vector, in the case of q_{mean} and P_{FDC}) for the signatures, estimated from the complete monitored timeseries. Except for S_{FDC}, all the signatures showed performances close to the paradigm solution, estimated in the time domain. However, the 95% credibility intervals for q_{mean} and P_{FDC} were extremely narrow and unable to encompass the observations. For c_a, the goodness of fit and the percentage of the observations within the 95% credibility interval demonstrate the applicability of the methodology.

Despite the good performances found for the majority of the tested signatures, the major limiting factor of this methodology is the computational cost, as hundreds of thousands or millions of iterations are needed in order for the Markov chains to converge, depending on the signature. Even on a monthly scale and with a parsimonious model, this high number of iterations can lead to running the model for over an hour.

Given that hydrological signatures may be regionalized, the main advantage of this approach is the possibility of predicting in poorly gauged or ungauged basins. In countries such as Brazil, with continental dimensions and a monitoring network mostly focused in bigger catchments, this approach may represent an alternative to improve water management or reservoir operation, for example.

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Author's contributions

Ana Clara de Sousa Matos: Conceptualization, Methodology, Validation, Formal analysis, Writing - original draft, Writing - review & editing.

Francisco Eustáquio Oliveira e Silva: Conceptualization, Methodology, Validation, Formal analysis, Writing - review & editing, Supervision.

Gustavo de Oliveira Corrêa: Writing - review & editing.

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