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Universal equations for rainfall distributions from SCS and Huff methods

Equações universais para as distribuições de chuva dos métodos SCS e Huff

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ABSTRACT

The temporal distribution of design rainfall hyetographs can be obtained through various procedures. One such method involves using temporal distribution curves to disaggregate a rainfall value of longer duration into chronological values within shorter time intervals. This is demonstrated by SCS distribution curves for durations of 6 and 24 hours and the well-known Huff curves. Another approach involves deriving temporal distribution using design hyetograph calculation methods, such as the Chicago and alternating block methods, based on Intensity-Duration-Frequency (IDF) equations. This paper aims to combine these approaches and develop universal equations capable of accurately fitting smooth rainfall time distribution curves. The advantage is the replacement of tables, like SCS and Huff, with analytical equations, enhancing usability in spreadsheets. Furthermore, these universal equations can determine the temporal rainfall distribution using an IDF equation or fitting a specified design hyetograph like Euler type II. Four numerical examples are provided.

Keywords: Rainfall distribution; SCS curves; Huff curves; IDF; Rainfall disaggregation.

RESUMO

A distribuição temporal dos hietogramas de precipitação de projeto pode ser obtida através de diversos procedimentos. Um desses métodos envolve a utilização de curvas de distribuição temporal para desagregar um valor de precipitação de maior duração em valores cronológicos dentro de intervalos de tempo mais curtos. Isto é demonstrado pelas curvas de distribuição SCS para durações de 6 e 24 horas e pelas conhecidas curvas de Huff. Outra abordagem envolve derivar a distribuição temporal usando métodos de cálculo de hietograma de projeto, como os métodos de Chicago e de blocos alternados, com base em equações de Intensidade-Duração-Frequência (IDF). Este artigo visa combinar essas abordagens e desenvolver equações universais capazes de ajustar com precisão curvas suaves de distribuição temporal da chuva. A vantagem é a substituição de tabelas, como as do SCS e Huff, por equações analíticas, melhorando a usabilidade em planilhas. Além disso, essas equações universais podem determinar uma distribuição temporal da precipitação usando uma equação IDF ou ajustando um hietograma de projeto específico como o Euler tipo II. São fornecidos quatro exemplos numéricos.

Palavras-chave: Distribuição temporal das chuvas; Curvas SCS; Curvas Huff; IDF; Desagregação de chuvas.

INTRODUCTION

The availability of a standardized rainfall time distribution curve simplifies the process of obtaining rainfall heights at shorter chronological intervals from a rainfall accumulation over an extended period. Classic examples of such curves include the SCS rainfall percentage distribution curves for 24-hour rainfall, which were established for four regions in the United States (United States Department of Agriculture, 1986), as well as the dimensionless Huff curves (Huff, 1990), which apply to both rainfall height and duration.

Typically, curves like those of SCS and Huff are presented in tabular format or integrated within more complex computer applications like TR-55 (United States Department of Agriculture, 1986). Working with tabular data might be difficult for people who prefer spreadsheets. This problem can be mitigated by introducing equations that replicates such tables or distributions.

As a result, by making it compatible with handheld calculators or spreadsheets, the suggested equations intend to simplify the process of getting design hyetographs. At the same time, these universal equations connect time distribution curves, IDF equations, and coefficients for the disaggregation of daily rainfall episodes.

While examples showcasing the compatibility of the universal equations with SCS and Huff curves are provided, this article does not delve into the practical merits of these methodologies in real-world scenarios. The primary objective of the article is to illustrate the adaptability (universality) of a well-fitted model to any rainfall distribution curve. For a more in-depth understanding of the application of SCS and Huff methods, it is recommended to refer to the works of Chin (2023), Fontoura (2019), Yin et al. (2016), and Back (2011).

METHODOLOGY

The methodology begins with the temporal cumulative equations of a design hyetograph, as outlined by Silveira (2016), derived from the Chicago method (Keifer & Chu, 1957). These equations consist of one for the period before the peak time and another for the period after the peak time.

Before the peak

$$P_t = \gamma P_{TOT} - \frac{a\left(t_p - t\right)}{\left(b + \frac{t_p - t}{\gamma}\right)^n} \quad (1)$$

After the peak

$$P_t = \gamma P_{TOT} + \frac{a\left(t - t_p\right)}{\left(b + \frac{t - t_p}{1-\gamma}\right)^n} \quad (2)$$

"Pt" represents the cumulative rainfall height up to the time instant "t" while "γ" is a parameter that varies between 0 and 1, determining the position of the intensity peak within the total duration. "P_{TOT}" stands for the accumulated rainfall of the entire hyetograph, and "t_p" signifies the time instant at which the rain intensity peak occurs. The parameters "a," "b," and "n" are derived from the Sherman-type Intensity-Duration-Frequency (IDF) equation, expressed as:

$$i_t = \frac{a}{\left(t+b\right)^n} \quad (3)$$

The rain intensity with duration "t" is denoted as "i_t", with "a," "b," and "n" being parameters that define the location and probability of occurrence. The parameter "a" is commonly determined using an expression such as:

$$a = k\,T^m \quad (4)$$

In this case, "T" indicates the return period in years, while "k" and "m" are constants.

Given that Equations 1 and 2 reflect chronological cumulated rainfall heights through time in a continuous way with no pauses, in order to standardize the rainfall variable (expressed as a percentage of total rainfall), it suffices to divide these equations by "P_{TOT}". The IDF equation can be used to achieve this normalization:

$$P_{TOT} = i_{t=t_{TOT}} \cdot t_{TOT} = \frac{a \cdot t_{TOT}}{\left(t_{TOT}+b\right)^n} \quad (5)$$

Performing this normalization yields the following equations:

Before the peak

$$\frac{P_t}{P_{TOT}} = \gamma - \frac{a\left(t_p - t\right)}{\left(b + \frac{t_p - t}{\gamma}\right)^n} \cdot \frac{\left(t_{TOT}+b\right)^n}{a \cdot t_{TOT}} \quad (6)$$

After the peak

$$\frac{P_t}{P_{TOT}} = \gamma + \frac{a\left(t - t_p\right)}{\left(b + \frac{t - t_p}{1-\gamma}\right)^n} \cdot \frac{\left(t_{TOT}+b\right)^n}{a \cdot t_{TOT}} \quad (7)$$

Now, we can get the following equations by canceling out the "a" in the numerator with the "a" in the denominator and using an algebraic manipulation of multiplying the equation by (t_{TOT}/t_{TOT})^n, where "t_{TOT}" is the total duration of the hyetograph, keeping in mind that $t_p = \gamma . t_{TOT,}$ we reach the following equations:

Before the Peak

$$\frac{P_t}{P_{TOT}} = \gamma - \frac{\left(\gamma - \frac{t}{t_{TOT}}\right) \cdot \left(1 + \frac{b}{t_{TOT}}\right)^n}{\left(1 + \frac{b}{t_{TOT}} - \frac{1}{\gamma}\frac{t}{t_{TOT}}\right)^n} \quad (8)$$

After the Peak

$$\frac{P_t}{P_{TOT}} = \gamma + \frac{\left(\frac{t}{t_{TOT}} - \gamma\right) \cdot \left(1 + \frac{b}{t_{TOT}}\right)^n}{\left(\frac{b}{t_{TOT}} + \frac{\frac{t}{t_{TOT}} - \gamma}{1-\gamma}\right)^n} \quad (9)$$

This equation preserves continuity throughout time, which means it stays valid regardless of time interval.

The ratios P_t/P_{TOT}, t/t_{TOT} and b/t_{TOT} are dimensionless; thus the dimensionless expressions for the continuous time distribution curve of rainfall heights over time are given by :

Before the Peak

$$P_{t'} = \gamma - \frac{\left(\gamma - t'\right)\cdot\left(1+b'\right)^{n}}{\left(1+b' - \frac{t'}{\gamma}\right)^{n}} \tag{10}$$

After the Peak

$$P_{t'} = \gamma + \frac{\left(t' - \gamma\right)\cdot\left(1+b'\right)^{n}}{\left(b' + \frac{t'-\gamma}{1-\gamma}\right)^{n}} \tag{11}$$

Here, $P_{t'}$ ($=P_t/P_{TOT}$) represents the dimensionless accumulated precipitation, ranging between 0 and 1. Similarly t´ ($=t/t_{TOT}$) denotes the dimensionless time, also varying between 0 and 1. Additionally, $b' = b/t_{TOT}$, signifies the normalized parameter "b´".

The resulting equations were validated by fitting them to the SCS Type I and IA, SCS Type II and Type III distributions (United States Department of Agriculture, 1986), as well as the Huff curves proposed by Huff (1990). For this purpose, the Solver™ module within Microsoft Excel™ spreadsheet software was utilized. The objective functions chosen for minimization were the Mean Squared Error (MSE) and the Mean Squared Percentage Error (MSPE). The parameters subjected to fitting were b´, n and γ.

RESULTS

Fitting the SCS distributions

The Type I and IA, Type II, and Type III curves account for the geographical variations in rainfall. Types I and IA reflect the maritime climate of the Pacific coast of the USA, characterized by wet winters and dry summers. The Type II distribution is applicable to almost the entire continental United States, excluding the Gulf of Mexico and Atlantic coast regions. These regions are represented by the Type III curve, which considers tropical storms and their associated high 24-hour rainfall values. Despite being initially developed for the United States, SCS curves are frequently adopted for research in other countries. While the original curves were formulated for 24-hour durations, variations for shorter durations exist, focusing on the most intense precipitation periods.

After minimizing the Mean Squared Error (MSE) objective function, the achieved results, for both SCS 24-hour and 6-hour durations, are presented in Table 1.

The outcomes depicted in Table 1 indicate reduced Mean Squared Percentage Error (MSPE) values, which accounts for the good visual alignments seen in Figures 1 to 4. The alignment for the Type IA 24-hour curve, on the other hand, may be slightly less marked.

The parameter γ, which signifies the degree of symmetry in the hyetograph (where γ = 0.5 implies perfect symmetry), is approximately 0.5 or very close to it in the Type II and Type III curves for both 6 hours and 24 hours. The conventional graphical and tabular representations of the original Type II and Type III curves also suggest this symmetry. In contrast, for the Type I and IA curves, there is an evident shift of the peak of the hyetograph. This shift is more pronounced in the 24-hour Type IA fit.

Overall, the use of the universal equation effectively quantified the differences between the curves. In terms of the γ

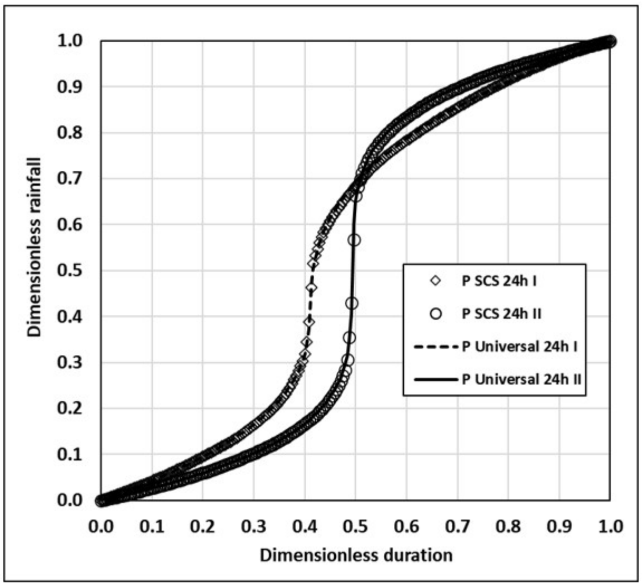

Figure 1. Fitting the 24-hour SCS I and II curves.

Table 1. Parameters b´, n and γ of the SCS curves.

Type	b´	n	γ	MSPE
I (24h)	0.001466	0.608	0.410	0.017195
IA (24h)	0.129108	0.546	0.293	0.219439
II (24h)	0.001957	0.755	0.493	0.010405
III (24h)	0.022281	0.794	0.500	0.040088
I (6h)	0.025795	0.629	0.383	0.048977
IA (6h)	0.001577	0.415	0.465	0.020974
II (6h)	0.007717	0.762	0.488	0.078862
III (6h)	0.014864	0.694	0.500	0.025629

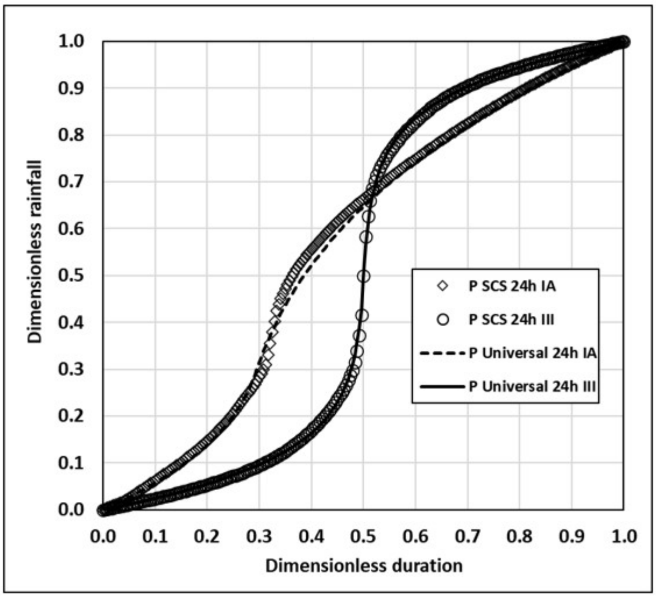

Figure 2. Fitting the 24-hour SCS IA and III curves.

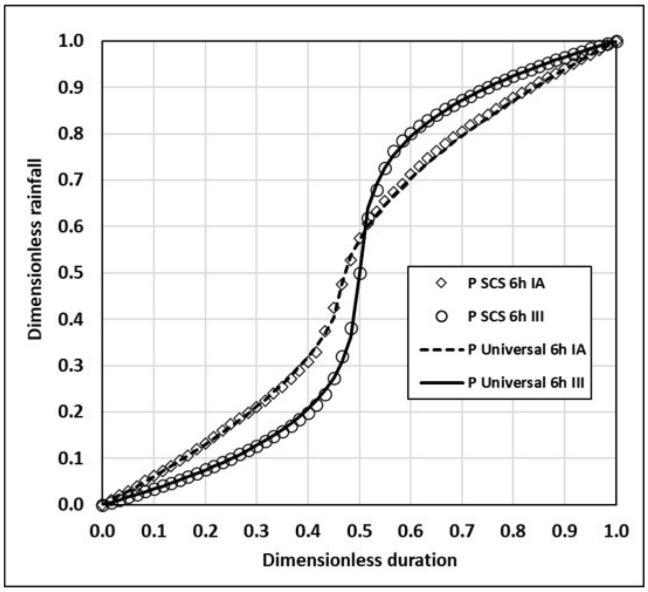

Figure 4. Fitting the 6-hour SCS IA and III curves.

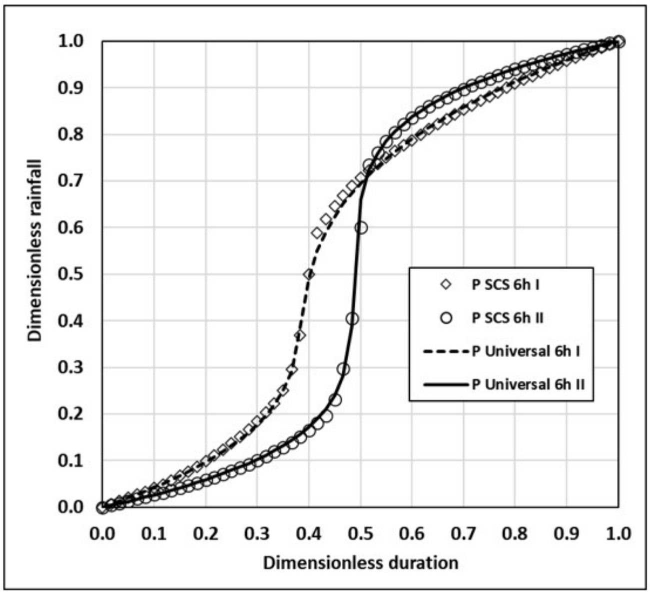

Figure 3. Fitting the 6-hour SCS I and II curves.

parameter, there were clear distinctions between SCS curves I, IA, and II (for both 6 and 24-hour durations), with a resemblance between curves II and III. The parameters b' and n play a role in shaping the curve and encompass the dynamics of rainfall processes aggregated over time (Van de Vyver and Demarée, 2010). It was expected that these parameters would exhibit variability.

Fitting the HUFF distributions

Huff (1967) conducted an analysis involving 261 hourly rainfall events recorded by 49 rain gauges positioned in a triangular arrangement within a 7 x 7 grid, covering a square area of 1,036 km^2 (400 mi^2) in Illinois, USA, over a period of 12 years. In this study, Huff (1967) organized the rainfall data into graphs that depict dimensionless cumulative precipitation (expressed as a percentage) as a function of dimensionless cumulative duration (also expressed as a percentage). Subsequently, he grouped these graphs into four major quartiles.

The first quartile pertains to rainfall events, with their peak occurring within the initial 25% of the total duration. The second quartile involves events with peaks between the 25% and 50% duration marks. The third and fourth quartiles consist of events where the peaks fall within the ranges of 50-75% and 75-100% of the total duration, respectively. Within each of these quartiles, Huff introduced the concept of "Huff curves," which are presented in probability bundles ranging from 10% to 90%. In essence, there are four sets of graphs, each comprising nine dimensionless curves depicting temporal distributions of rainfall.

Huff (1990) defines probabilities as the historical frequencies of accumulated rainfall values surpassing a certain threshold. With this perspective, the 90% curve signifies the boundary where most historical data points from accumulated rainfall curves for actual events lie above it. This interpretation similarly applies to the other curves representing probabilities of 10%, 20%, 30%, 40%, 50%, 60%, 70%, and 80%. In this conceptual framework, a 90% curve characterizes the realm of more common rainfall occurrences, while a 10% curve corresponds to rarer and less frequent events.

When referring studies that use the Huff concept, caution is advised because the interpretation can differ. For instance, in the study conducted by Bonta (2004), there's a reversal of the concept. In his work, a 10% "Huff" curve actually corresponds to the 90% Huff curve, based on the original concept presented by Huff (1990). Such discrepancies highlight the significance of clarifying the interpretation and context when discussing or using the Huff curves in different studies to ensure accurate communication and understanding.

Clearly, the original Huff curves pertain to Illinois (USA). However, Huff's (1967) contribution was to establish an analytically replicable method that could be applied to other locations. Bonta (2004) remarks on the challenge of selecting from the 36 available Huff curves and proposes utilizing all of them within a Monte Carlo process that is employed in rainfall-runoff models. On the other hand, Huff (1990) recommends employing the median curves (50% curves) for design purposes and reserves the suggestion of the 10% curves for situations involving unusual conditions in runoff generation.

The current paper refrains from delving into the specifics of how applicable the original Huff curves are to different locations, mirroring the approach taken with the SCS curves. Ideally, the collection of all 36 Huff ("Huff-like") curves should be derived using data from the particular site of interest. The proposed adaptation outlined in this paper is focused on the median Huff curves, drawing from the data explained in Huff's (1990) work.

Huff (1990) observed a predominance of first quartile curves for rainfall durations of up to 6 hours, second quartile curves for durations ranging from 6 to 12 hours, third quartile curves for durations of 12 to 24 hours, and fourth quartile curves for rainfall lasting longer than 24 hours. The outcomes achieved through the minimization of the Mean Squared Error (MSE) objective function, considering the 15 tabulated curves presented in Huff (1990), are shown in Table 2.

The outcomes displayed in Table 2 demonstrate reduced Mean Squared Percentage Error (MSPE) values, which account for the good visual alignments in Figures 5, 6, 7 and 8.

Fitting any dimensionless rainfall distribution

It is expected that the proposed analytical equations can fit any dimensionless rainfall distribution because this is S-shaped curve, a smooth function. Soldevila et al (2019) presents the S-shaped rainfall curves of some classical methods like Alternating Block, Triangular and Sifalda for Valencia, Spain. Another method like these is Euler's Type II hyetograph, which is more recent (DWA Rules and Standards, 2006). It is recommended for modelling sewer systems in Germany, being a hyetograph with the highest precipitation intensity located at 0.3 times the total duration of precipitation.

There are no generalized dimensionless precipitation curves for these methods, so it is not possible to fit representative equations in advance as was done for SCS and Huff. However, the proposed equations can be fit to dimensionless hyetographs already calculated by these methods.

Following are four examples of fitting the equations proposed for the synthesis of hyetographs based on SCS and Huff curves, and Chicago and Euler II distributions.

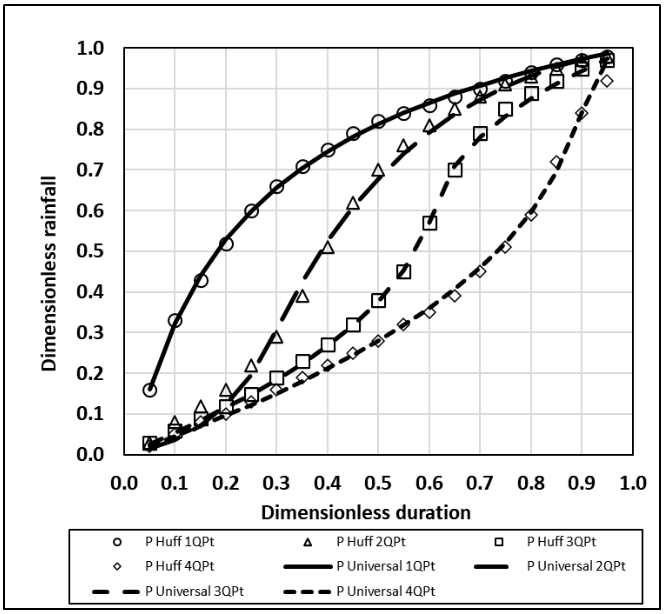

Figure 5. Fitting the Huff´s point median curves of dimensionless precipitation from 1st to 4th quartile.

Table 2. Parameters b´, n and γ of the Huff curves.

Huff curve 50%	b´	n	γ	MSPE
1QPt	0.192882	0.898	0.018	0.009850
2QPt	2.058141	0.486	0.295	4.222665
3QPt	0.077070	0.519	0.612	0.170782
4QPt	0.063740	0.590	0.916	0.310243
1Q10-50	0.379444	1.164	0.035	0.286376
2Q10-50	2.866894	3.339	0.295	4.539326
3Q10-50	0.124159	0.653	0.626	0.521046
4Q10-50	0.120152	0.743	0.902	0.771908
1Q50-400	0.233444	1.069	0.087	3.588326
2Q50-400	2.787747	3.530	0.293	3.804585
3Q50-400	0.213556	0.843	0.634	1.354016
4Q50-400	0.054689	0.776	0.864	0.960277
1Q10%	0.118290	1.137	0.032	0.082248
1Q50%	0.233444	1.069	0.087	3.588326
1Q90%	0.000000	0.207	0.085	7.375398

Meaning of the acronyms: iQPt = point curve (Pt) ith quartile (iQ), iQa-b = areal (a-b, range a to b in mi^2) curve ith quartile (iQ), iQj% = median areal probability (j) ith quartile (iQ), j=10, 50 or 90% probability.

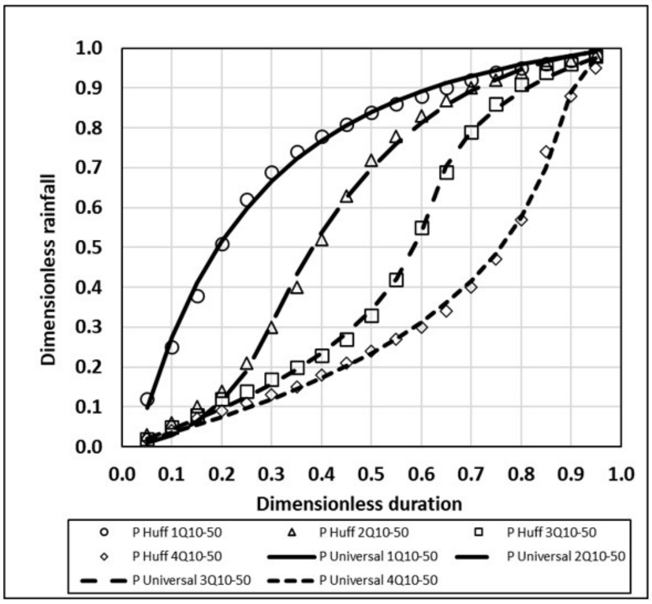

Figure 6. Fitting the Huff's median areal curves of dimensionless precipitation from the 1st to the 4th quartile in the range 10 to 50 mi².

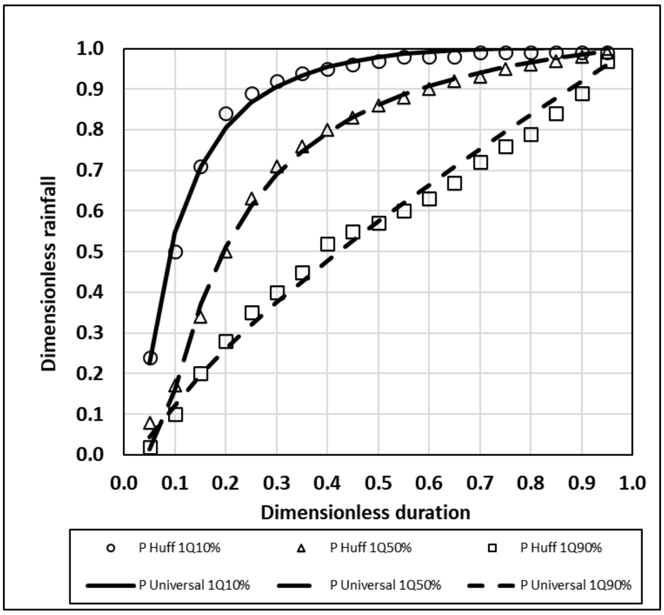

Figure 8. Fitting the Huff's median areal curves of dimensionless precipitation from the 1st quartile 10%, 50% and 90% probabilities.

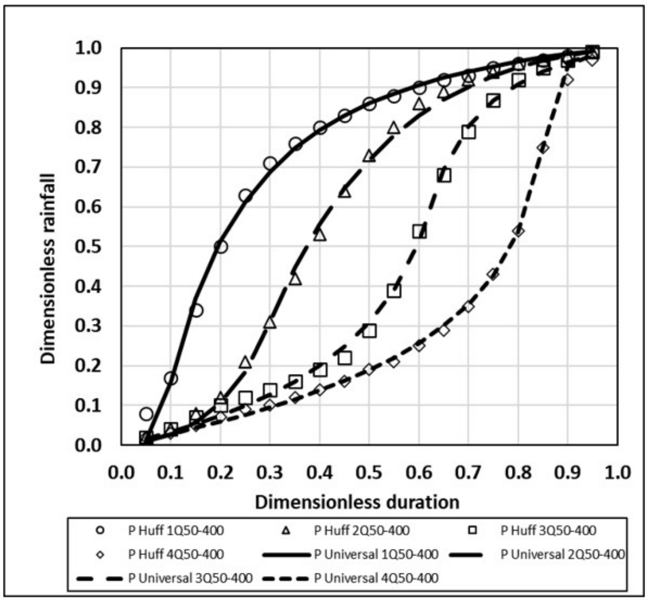

Figure 7. Fitting the Huff's median areal curves of dimensionless precipitation from the 1st to the 4th quartile in the range 50 to 400 mi².

EXAMPLES

Example 1: Calculating the cumulative time distribution curve with a 60-minute time step for a 250 mm rainfall in 24 hours, utilizing the proposed analytical equations for 24-hour SCS I curve:

Solution:

The necessary parameters come from Table 1:

b´= 0.001466

n = 0.608

γ = 0.410

With the given parameters, the two equations for calculating the dimensionless cumulated rainfall height using Equations 10 and 11 are as follows :

Before Peak

$$P_{t'} = 0.410 - \frac{\left(0.410 - t'\right)\cdot\left(1 + 0.001466\right)^{0.608}}{\left(1 + 0.001466 - \dfrac{t'}{0.410}\right)^{0.608}} \quad (12)$$

After Peak

$$P_{t'} = 0.410 + \frac{\left(t' - 0.410\right)\cdot\left(1 + 0.001466\right)^{0.608}}{\left(0.001466 + \dfrac{t' - 0.410}{0.590}\right)^{0.608}} \quad (13)$$

It should be remembered that $P_{t'}$ represents the cumulative dimensionless rainfall height and t´ represents the cumulative dimensionless duration. Table 3 presents the results, where the fourth column displays the disaggregated precipitation values (in mm), denoted as Pcalc = $P_{t'}$ * Ptot (where Ptot = 250 mm). For the sake of comparison, the sixth column showcases the discrepancy between the calculated precipitation at each duration and the values from the original tables of the 24-hour SCS Type I distribution (fifth column). This comparison demonstrates a strong agreement between the calculated values and the original data.

Example 2: Calculating the cumulative time distribution curve using 10-minute time steps for a 120 mm rainfall over 100 minutes, using analytical equations of the first quartile point median Huff curve (1QPt parameters from Table 2):

Solution:

The necessary parameters come from Table 2:

b´= 0.000116

n = 0.651

γ = 0.048

Using these parameters, we can formulate the two equations for calculating the dimensionless rainfall height by combining Equations 10 and 11:

Before Peak

$$P_{t'} = 0.048 - \frac{\left(0.048 - t'\right)\cdot\left(1+0.000116\right)^{0.651}}{\left(1+0.000116-\frac{t'}{0.048}\right)^{0.651}} \quad (14)$$

After Peak

$$P_{t'} = 0.048 + \frac{\left(t' - 0.048\right)\cdot\left(1+0.000116\right)^{0.651}}{\left(0.000116+\frac{t'-0.048}{0.952}\right)^{0.651}} \quad (15)$$

Table 4 presents the outcomes, where the fourth column displays the values of accumulated precipitation over the time intervals, denoted as Pcalc = Pt´*Ptot (where Ptot = 120 mm). For purposes of comparison, the fifth column exhibits the

Table 3. Solution of Example 1.

t (min)	t´	$P_{t'}$	P calc (mm)	P SCS I (mm)	Error (%)
60	0.042	0.017	4.2	4.4	-4.5
120	0.083	0.035	8.8	8.8	0.0
180	0.125	0.054	13.7	13.5	1.5
240	0.167	0.076	19.0	19.0	0.0
300	0.208	0.100	25.0	25.0	0.0
360	0.250	0.125	31.7	31.3	1.3
420	0.292	0.156	39.7	39.0	1.8
480	0.333	0.194	49.6	48.5	2.3
540	0.375	0.254	63.8	63.5	0.5
600	0.417	0.515	126.1	128.8	-2.1
660	0.458	0.623	157.3	155.8	1.0
720	0.500	0.684	172.7	171.0	1.0
780	0.542	0.732	184.2	183.0	0.7
840	0.583	0.770	193.6	192.5	0.6
900	0.625	0.802	201.6	200.5	0.5
960	0.667	0.832	208.8	208.0	0.4
1020	0.708	0.860	215.3	215.0	0.1
1080	0.750	0.886	221.3	221.5	-0.1
1140	0.792	0.910	226.8	227.5	-0.3
1200	0.833	0.932	232.0	233.0	-0.4
1260	0.875	0.952	236.8	238.0	-0.5
1320	0.917	0.970	241.4	242.5	-0.5
1380	0.958	0.986	245.8	246.5	-0.3
1440	1.000	1.000	250.0	250.0	0.0

Table 4. Solution of Example 2.

t (min)	t´	$P_{t'}$	P calc (mm)	P Huff (mm)	Error (%)
10	0.100	0.330	47.1	39.6	19.0
20	0.200	0.600	66.0	72.0	-8.4
30	0.300	0.660	77.6	79.2	-2.0
40	0.400	0.750	86.5	90.0	-3.9
50	0.500	0.820	93.8	98.4	-4.6
60	0.600	0.860	100.2	103.2	-2.9
70	0.700	0.900	105.9	108.0	-2.0
80	0.800	0.940	111.0	112.8	-1.6
90	0.900	0.970	115.7	116.4	-0.6
100	1.000	1.000	120.0	120.0	0.0

corresponding precipitation values using Huff's point median curve table for the first quartile (Huff, 1990). The final column in Table 4 illustrates the discrepancy between the calculated cumulative precipitation at each duration and the actual values, demonstrating the robust agreement achieved through the utilization of Equations 10 and 11.

Example 3: Calculating a cumulative design hyetograph from a Sherman-Type IDF.

Solution:

As an illustrative instance, the following Intensity-Duration-Frequency (IDF) relationship is employed, with a total duration of 90 minutes, a return period of 50 years and a time step of 10 minutes. The parameter γ is set to 0.35.

$$i_t = \frac{1100 T_R^{0.15}}{(t+30)^{0.75}} \tag{16}$$

To obtain the equations we then have:

$b' = b/t_{TOT} = 30/90 = 0.3333$

$n = 0.75$

$\gamma = 0.35$

Equations 10 and 11 then become:

Before Peak

$$P_{t'} = 0.35 - \frac{\left(0.35 - t'\right)\cdot\left(1+0.3333\right)^{0.75}}{\left(1+0.3333-\frac{t'}{0.35}\right)^{0.75}} \tag{17}$$

Table 5. Solution to Example 3.

t (min)	t′ (=t/t$_{TOT}$)	$P_{t'}$ (=P_t/P_{TOT})	P_t (mm) (=$P_{t'}*P_{TOT}$)
10	0.1111	0.0571	3.67
20	0.2222	0.1425	9.16
30	0.3333	0.3073	19.76
40	0.4444	0.5537	35.59
50	0.5556	0.7025	45.16
60	0.6667	0.8058	51.80
70	0.7778	0.8842	56.85
80	0.8889	0.9473	60.90
90	1.0000	1.0000	64.29

After Peak

$$P_{t'} = 0.35 + \frac{\left(t' - 0.35\right)\cdot\left(1+0.3333\right)^{0.75}}{\left(0.3333+\frac{t'-0.35}{0.65}\right)^{0.75}} \tag{18}$$

Recalling that $Pt' = Pt/P_{TOT}$. With the provided data, the aforementioned IDF yields a value of $P_{TOT} = 64.29$ mm.

Table 5 shows the results of applying the equations.

The same example, addressed differently but yielding the same results, was presented in Silveira's work (2016).

Example 4: Fitting cumulative rainfall equations for a Euler Type II hyetograph.

Solution:

Table 6 shows in third column a five-minute cumulative hyetograph presented in DWA (DWA Rules and Standards, 2006). In fourth column, the same hyetograph is presented in a dimensionless form.

Equations 10 and 11 that fit this Euler type II hyetograph had the following parameters:

$b' = 0$

$n = 0.721$

$\gamma = 0.221$

Equations 10 and 11 then become:

Before Peak

$$P_{t'} = 0.221 - \frac{\left(0.221 - t'\right)}{\left(1-\frac{t'}{0.221}\right)^{0.721}} \tag{19}$$

After Peak

$$P_{t'} = 0.221 + \frac{\left(t' - 0.221\right)}{\left(\frac{t'-0.221}{0.779}\right)^{0.721}} \tag{20}$$

$P_{t'}$ in in the fifth column shows the good results provided by the fitted universal equations.

Table 6. Solution to Example 4.

t (min)	t′ (=t/t$_{TOT}$)	P_t Euler II (mm)	$P_{t'}$ Euler II (=P_t Euler II/P_{TOT})	$P_{t'}$calc
5	0.1111	1.9	0.114	0.039
10	0.2222	5.3	0.319	0.319
15	0.3333	11.4	0.687	0.667
20	0.4444	12.8	0.771	0.772
25	0.5556	13.8	0.831	0.840
30	0.6667	14.7	0.886	0.891
35	0.7778	15.3	0.922	0.933
40	0.8889	16.0	0.964	0.969
45	1.0000	16.6	1.000	1.000

CONCLUSION

The equations here called universal, due to their versatility in fitting dimensionless accumulated precipitation curves, demonstrated excellent performance in the adjustment of the traditional SCS Type I, IA, II and III curves, as well as the historic Huff curves for Illinois (median curves have been adjusted). In addition to these methods, other adjustment possibilities have been shown.

The advantage of the presented analytical equations lies in their simplicity of use within spreadsheets, along with the capability to generate dimensionless accumulated precipitation curves for any dimensionless time interval ranging from zero to one. As a result, in cases where fitting is applied to SCS Type I, IA, II and III curves, as well as Huff curves, these equations can effectively replace existing tables. This substitution not only minimizes interpolation requirements for various time subdivision scenarios but also ensures minimal error introduction.

The suggested equations demonstrated remarkable adaptability in fitting cumulative rainfall curves, allowing them to be utilized for representing pre-existing curves in specific locations, established by any method (like Euler type II, for example).

Furthermore, the universal curves introduced here can be employed without modifications to construct cumulative hyetographs of varying durations. given a Sherman-type IDF, the return period, and the peak position (in cases where the peak position is absent, it can be assumed that $\gamma = 0.5$).

The potential of these universal equations is illustrated through four solved numerical examples.

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