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Applying the Chicago hyetograph for intense rainfall equations of the LnLn type

Aplicação do hietograma de Chicago para equações de chuvas intensas do tipo LnLn

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ABSTRACT

The Chicago Method is one of the most common methods to determine design rainfall hyetographs for urban drainage infrastructures. Among the advantages of the method is that it is easy to apply and is based on traditional IDF equations. Nevertheless, applications to other IDF equation models presents some limitations. The aim of the present study was to adapt and apply the Chicago hyetograph method with intense rainfall equations of the LnLn type. Equations to estimate rainfall intensity before and after peak duration were presented. In addition, equations to obtain accumulated rainfall before and after peak rainfall volumes were adapted. With this information, we were able to obtain rainfall blocks for each interval of the hyetograph. The method was applied to determine the hyetograph based on the intense rainfall equation for the town of Piracicaba, São Paulo, Brazil. The equations presented here can be implemented on electronic spreadsheets or in programming routines, hence allowing Engineering professionals to apply the most adequate methods to local data.

Keywords: Design rainfall; Urban drainage; Intense rainfall.

RESUMO

O método de Chicago é um dos métodos mais utilizados na determinação do hietograma da chuva de projeto para obra de drenagem urbana. Entre as vantagens do método está a facilidade de aplicação e ser baseado nas equações IDF tradicionais. No entanto, a aplicação para outros modelos de equação IDF apresenta limitação. Este trabalho teve como objetivo adaptar e aplicar o hietograma do método Chicago com as equações de chuvas intensas do tipo LnLn. Foram apresentadas as equações para estimativa das intensidades de chuva para duração anterior e posterior ao pico. Também foram adaptadas as equações para a obtenção das laminas acumuladas ou volumes da chuva anterior e posterior ao pico. Com essas informações pode-se obter os blocos de chuva de cada intervalo do hietograma. O método foi aplicado para a determinação do hietograma com base na equação de chuvas intensas da cidade de Piracicaba-SP. As equações aqui apresentadas podem ser podem ser implementadas em planilhas eletrônicas ou rotinas de programação permitindo aos profissionais de Engenharia aplicação de métodos mais adequados aos dados locais.

Palavras-chave: Chuva de projeto; Drenagem urbana; Chuvas intensas.

INTRODUCTION

In several civil and rural engineering projects, such as urban and agrarian drainage, installation must be assessed according to estimated maximum flows. In most cases, there is no flow data that is measured in loco, hence the need to estimate the flow or flood volume by using a model that transforms rainfall into flow (Bemfica et al., 2000).

When applying hydrological models, design rainfall must be defined taking into account the relation between height, intensity and duration distribution, which is the hyetograph. The temporal distribution of rainfall volumes will influence the infiltrated volume and the form of the direct surface runoff hydrograph resulting from excess rain. There are several assessment methods for temporal distribution, such as Alternating Block methods (Chow et al., 1988), the Triangular Method (Griffiths & Pearson, 1993), the Chicago Method (Keifer & Chu, 1957), The Sifalda Method (Sifalda, 1973), and temporal variation curves (Huff, 1990). Several studies (Cen, 1993; Canholi, 2005; Abreu et al., 2017; Back & Nurnberg, 2022) demonstrate that peak flood flows obtained by the different hyetograph models are significantly different.

The Chicago Hyetograph, or Chicago Method, was proposed by Keifer & Chu (1957) to estimate design rainfall for urban drainage of up to 3 hours (Chen et al., 2023). Several studies indicate that the Chicago Method was the simplest and most efficient (Balbastre-Soldevila et al., 2019; Yang et al., 2022) and was widely used and recommended for urban drainage (Su et al., 2019; Liao et al., 2021).

Among the advantages of the Chicago Method is that it is easy to apply and demands a low number of parameters, such as rainfall duration and the intense rainfall equation. Added to that, the inclusion of the delay factor allows one to alter the hyetograph format. Familiarity of usage and the straightforwardness with which the IDF equations are obtained have facilitated acceptance of the Chicago Method in Engineering, where it is one of the most used methods (Krvavica & Rubinić, 2020; Wittmanová et al., 2021).

However, dependence on the IDF equation can limit the use of the Chicago method. Although the IDF model equations for intense rainfall are the most commonly used in Brazil and in many other countries (Back & Cadorin, 2021), there are other intense rainfall equation models, or even other methods, to obtain intense rainfall through daily rainfall disaggregation (Aragão et al., 2013; Rangel & Hartwig, 2016; Back, 2020).

In the state of São Paulo the use of an intense rainfall equation model called LnLn is very common (Magni, 1984; Martinez Júnior & Magni, 1999, 2014: Martins et al., 2017). The Departamento de Águas e Energia Elétrica (1982)[1] established that the intense rainfall equations for the state of São Paulo would be the LnLn type. More recently, the Departamento de Águas e Energia Elétrica (2018) presented an update of intense rainfall equations for 75 pluviographic stations in the state of São Paulo with the use of the LnLn model.

In locations where only this equation model is available there are some complications in applying the Chicago hydrograph. Therefore, the aim of this study was to adapt and apply the Chicago hydrograph method to the LnLN-type intense rainfall equations.

[1] The DAEE is the Department of Water and Power in the state of São Paulo, Brazil.

DEVELOPMENT

Chicago method

The Chicago Method is based on two analytical equations to obtain rainfall intensity. One equation rates duration before the peak and the other rates time after the peak. These equations were deduced from the IDF equation presented by Sherman (1931), given by:

$$i_m = \frac{a}{\left(t+b\right)^n} \tag{1}$$

where i_m = average maximum rainfall intensity in mm/h; t = rainfall duration in minutes; a, b, n are parameters of the equation determined for each location.

In Brazil, the IDF equation is very common, considering:

$$a = K\,T^m \tag{2}$$

where T = return period (years); K and m are coefficients determined for each location.

Keifer & Chu (1957) presented the equations to determine rainfall intensity in which the asymmetry is defined by the parameter r (0< r < 1), also called rainfall advancement coefficient or peak factor. Hence, for rain with t_d duration, the peak takes place in time t_p = r t_d. The equations to estimate rainfall intensity in the time before the peak (t_b) and the time after the peak (t_a) are given by Equations 3 and 4 respectively:

$$\mathbf{i_b} = \frac{\mathbf{a}\left[\left(1-\mathbf{c}\right)\frac{\mathbf{tb}}{\mathbf{r}} + \mathbf{b}\right]}{\left[\frac{\mathbf{tb}}{\mathbf{r}} + \mathbf{b}\right]^{1+\mathbf{c}}} \tag{3}$$

$$\mathbf{i_a} = \frac{\mathbf{a}\left[\left(1-\mathbf{c}\right)\frac{\mathbf{ta}}{\left(1-\mathbf{r}\right)} + \mathbf{b}\right]}{\left[\left(\frac{\mathbf{ta}}{1-\mathbf{r}}\right) + \mathbf{b}\right]^{1+\mathbf{c}}} \tag{4}$$

where i_b = rainfall intensity before the peak (mm/h)/ ia = rainfall intensity after the peak (mm/h)/ t_b = time before the peak (min); ta = time after the peak (min).

Silveira (2016) describes the Chicago method and presents applications with the IDF equations. Chicago hyetograph requires the rainfall duration, usually assumed to be the concentration time for the basin, and the position of rainfall peak. The concentration time can be estimated based on empirical formulas considering the basin characteristics such as length and slope. In the literature there are dozens of empirical formulas, some more suitable for urban areas, others for rural areas. Silveira (2005) presented an excellent comparative study on these equations. The position of the peak of the hyetogram can be obtained by calculating the relationship between the time of peak intensity and the duration of the storm of equal duration. There are several studies showing that this value is generally less than 0.5, indicating that the peak intensity occurs in the middle of the storm (Chow et al., 1988; Canholi, 2005).

LnLn equation

In adapting the method, the format of the equation was considered for estimating the average maximum rainfall intensity in the LnLn model, given by:

$$Im = A(t+B)^C + D(t+E)^F \left[G + Hln\left(\ln\left(\frac{T}{T-1}\right)\right)\right] \quad (5)$$

where Im = average maximum intensity (mm/min); T = rainfall duration (min); T = return period (years); A, B, C, D, E, F, G, H = parameters of the equation adjusted to each location.

Rainfall volume can be calculated by:

$$V = im\,td \quad (6)$$

It can also be calculated as the integral of the intensity function, given by:

$$V = td\int_0^{td} i\,dt \quad (7)$$

Making:

$$K = G + H\ln\left(ln\left(\frac{T}{T-1}\right)\right) \quad (8)$$

Rainfall volume in time t can be expressed by:

$$V_t = \left[A(t+B)^C + D(t+E)^F K \right] t \quad (9)$$

Rainfall intensity distribution in relation to time is shown by the blue curve in Figure 1, with maximum intensity (i_{max}) at the beginning of the curve (t = 0) and exponentially decreasing with time according to a function f(t) which so far is unknown.

For rainfall duration t_d, rainfall volume is represented by the area under the curve of t = 0 to t = td. The average rainfall intensity (i_m) can be estimated by the relation between volume and duration, or:

$$i_m = \frac{V}{t_d} \quad (10)$$

The method considers a function of average rainfall intensity I_m that is different from instant intensity (i_t), whose integral with time corresponds to rainfall height of Equation 9. Deriving equation dv_t/dt, we have:

$$I_i = AtC(t+B)^{C-1} + A(t+B)^C + kD(t+E)^F + DKtF(t+E)^{F-1} \quad (11)$$

This equation presents maximum intensity for t = 0, like the intense rainfall equation. However, when t > 0, one can observe

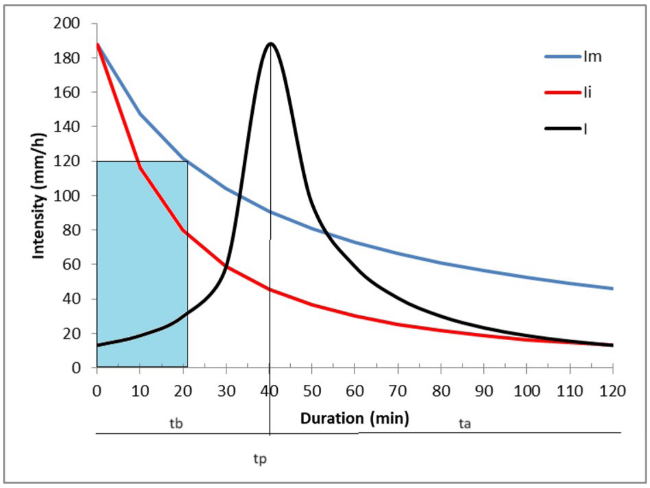

Figure 1. Representation of intensities for the Chicago Method.

(Figure 1) that instantaneous intensity (Ii) is lower than average intensity (lm), in other words:

$$I_i < I_m \quad (12)$$

Rainfalls with different durations (t_d), but the same intensity distribution, will produce i_m values that decrease while t_d increases.

Hence, we can express this as:

$$i_m = \frac{V}{t_d} = \frac{1}{t_d}\int_0^{t_d} f(t)\,dt \quad (13)$$

The average intensity i_m over time t can be described by an empirical function such as equation:

$$i = A(t+B)^C + D(t+E)^F K \quad (14)$$

Combined with Equation 13, the function (ft) can be obtained by differentiation:

$$f(t) = \frac{d}{dt}\left\{ \left[A(t+B)^C + D(t+E)^F K \right] t \right\} \quad (15)$$

Which results in:

$$i_m = f(t) = AtC(t+B)^{C-1} + A(t+B)^C + kD(t+E)^F + DKtF(t+E)^{F-1} \quad (16)$$

Considering the delay factor r (0 < r <1), peak time intensity for specific duration t_d is given by $t_p = r\, t_d$.

Rainfall distribution in relation to time before the peak ($0 < t_b < r\, t_d$) is given by:

$$i_b = A\frac{t_b}{r}C\left(\frac{t_b}{r}+B\right)^{C-1} + A\left(\frac{t_b}{r}+B\right)^C + kD\left(\frac{t_b}{r}+E\right)^F + DK\frac{t_b}{r}F\left(\frac{t_b}{r}+E\right)^{F-1} \quad (17)$$

Rainfall distribution in relation to time after the peak t_a ($(1-r)t < t_a < t_d$) is given by:

$$i_a = A\frac{t_a}{(1-r)}C\left(\frac{t_a}{(1-r)}+B\right)^{C-1} + A\left(\frac{t_a}{(1-r)}+B\right)^{C} + kD\left(\frac{t_a}{(1-r)}+E\right)^{F} + D\,K\frac{t_a}{(1-r)}F\left(\frac{t_a}{(1-r)}+E\right)^{F-1} \qquad (18)$$

The calculation of the rainfall hyetograph blocks is done by integrating these equation to obtain a curve of accumulated volume. For the sake of convenience, this curve is calculated so volume V is zero in $t = t_p$ and is defined in terms of the time elapsed before and after tp. The expressions for volumes before and after t_p are given by Equations 19 and 20, respectively:

$$V_a(t_a) = \left\{\left[A\left(\frac{t_a}{1-r}+B\right)^{C} + D\left(\frac{t_a}{1-r}+E\right)^{F} K\right]t_a\right\} \qquad (19)$$

$$V_b(t_b) = \left\{\left[A\left(\frac{t_b}{r}+B\right)^{C} + D\left(\frac{t_b}{r}+E\right)^{F} K\right]t_b\right\} \qquad (20)$$

Adapting the model indicated by Silveira (2016), we can write the equations to obtain the accumulated rainfall heights from the beginning of the hyetograph, where for time before the peak:

$$P_t = rP_{tot} - \left\{\left[A\left(\frac{(t_p-t)}{r}+B\right)^{C} + D\left(\frac{(t_p-t)}{r}+E\right)^{F} K\right]\left(t_p-t\right)\right\} \qquad (21)$$

For time after the peak, the equation is given by:

$$P_t = rP_{tot} + \left\{\left[A\left(\frac{(t-t_p)}{1-r}+B\right)^{C} + D\left(\frac{(t-t_p)}{1-r}+E\right)^{F} K\right]\left(t-t_p\right)\right\} \qquad (22)$$

In which P_t is the accumulated rainfall in time t from the beginning of the hyetograph; P_{Tot} is the total rainfall height of the hyetograph.

Applying the method

As an example of application, the hyetograph for the town of Piracicaba, SP will be determined. Design rainfall has a duration of 120 minutes, a return period of 25 years, considering an advancement coefficient of r = 0.333. The hyetograph will be established with 10-minute intervals.

The intense rainfall equation (Departamento de Águas e Energia Elétrica, 2018) is given by:

$$Im = 44.52(t+30)^{-0.8972} + 23.53(t+40)^{-0.9506}\left[-0.4847 - 0.6062 ln\left(\ln(\frac{T}{T-1}\right)\right]$$

Thus, we can define:

- Peak time: $t_p = t_d\, r$ = 120 x 0.333 =40 min
- $\mathbf{K} = -0.4847 - 0.6062\ln\left(\boldsymbol{ln}\left(\frac{25}{25-1}\right)\right) = 1.4542$
- $\mathbf{I_m} = 44.52(120+30)^{-0.8972} + 23.53(120+40)^{-0.9506}$ $1.4542 = 0.7716 mm/h$
- Rainfall height = 0.77156 x 120 = 92. 6 mm.
- Rainfall intensity: with Equations 17 and 18, the i_m of Table 1 was obtained.
- Rainfall volume before peak (V_b):
- $$V_b = \left\{\left[44.52\left(\frac{40}{0.333}+30\right)^{-0.8972} + 23.53\left(\frac{80}{0.333}+40\right)^{-0.9506} 1.4542\right]40\right\} = 30.9\,mm$$
- Rainfall volume after peak (V_a)
- $$V_a = \left\{\left[44.52\left(\frac{80}{1-333}+30\right)^{-0.8972} + 23.53\left(\frac{80}{1-0.333}+40\right)^{-0.9506} 1.4542\right]80\right\} = 61.7\text{mm}$$
- Total volume: ($V_b + V_a$)= 61.7 + 30.9 =92.6 mm
- Hyetograph blocks: with Equations 19 and 20, we have the accumulated volumes before and after the peak,
- Hyetrogram blocks: With Equations 21 and 22 we have the rain heights before and after the peak. Thus, for times of 10 minutes, Equation 21 can be applied and determined:
- $$\mathbf{P_t} = 0.33\,\mathbf{x}92.6 - \left\{\left[44.52\left(\frac{40-10)}{0.33}+30\right)^{-0.8972} + 23.53\left(\frac{40-10)}{0.33}+40\right)^{-0.9506} 1.4542\right](40-10)\right\} = 2.61$$
- For the time of 60 min, applying Equation 22 we have:

$$\mathbf{P_t} = 0.33\mathbf{x}92.6 + \left\{\left[44.52\left(\frac{60-40}{1-0.33}+30\right)^{-0.8972} + 23.53\left(\frac{60-40}{1-0.33}+40\right)^{-0.9506} 1.4542\right](60-40)\right\} = 65.53$$

- From the difference of the Pt values we obtain the blocks (Table 1) represented in Figure 2.

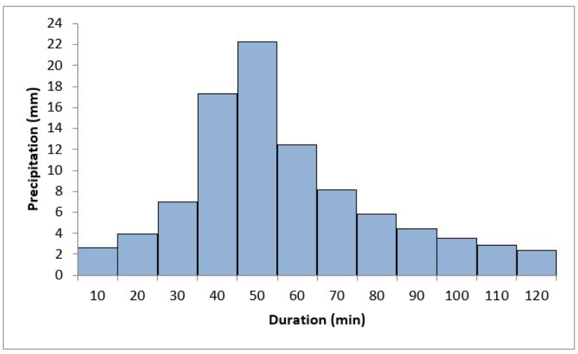

Figure 2. Chicago Hyetograph Method for Piracicaba, SP.

Table 1. Determination of Chicago hyetorgraph for Piracicaba, SP.

Interval	Duration (min)	t_b (min)	t_a (min)	I_m (mm/h)	Block (mm)
0	0	40	-	13.15	-
1	10	30	-	18.78	2.61
2	20	20	-	30.02	3.95
3	30	10	-	58.84	6.97
4	40	-	-	187.90	17.33
5	50	-	10	95.24	22.22
6	60	-	20	58.84	12.45
7	70	-	30	40.57	8.13
8	80	-	40	30.02	5.81
9	90	-	50	23.32	4,41
10	100	-	60	18.78	3.49
11	110	-	70	15.54	2.85
12	120	-	80	13.15	2.38

CONCLUSIONS

With the equations presented here, it is possible to determine the Chicago hyetograph method for locations using IDF equations of the LnLn type. These equations can be implemented in electronic spreadsheets or in programming routines hence allowing Engineering professionals to apply the most adequate methods to local data.

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Authors contributions

Álvaro José Back: Corresponding author, study design, methodology, data processing, data analysis, writing - original draft.

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