Proposition of a modified formulation to predict the lateral-torsional buckling resistance of steel beams: meta-analysis of test data

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Abstract

Lateral-Torsional Buckling (LTB) is an important limit state that must be considered in the design of steel members subjected to bending. During the process of revising of the Brazilian Standard ABNT NBR 8800:2008, it was questioned whether the present formulation would really lead to good results vis-à-vis the experimental tests. To this end, a comprehensive review of the technical literature was carried out, looking for test results that could be compared with the ones using the equations provided in the Brazilian Standard. Then, a meta-analysis was performed by comparing the test results with the ones of the ABNT NBR 8800:2008 methodology. It was observed that, for beams subjected to uniform bending moments and simple supports, the formulation proved to be adequate. However, for the case of beams with variable bending moments, for which the factor C_b (modification factor for non-uniform bending moment diagram) is used, it was realized that the formulation could lead to unsafe results, especially for high values of C_b. A modified formulation was then proposed, with the introduction of C_b only in the elastic range of the equations (only in the buckling moment equation) and in the equation for the slenderness parameter corresponding to the beginning of yielding, λ_r. The proposed modified formulation, as well as the EN 1993-1-1:2022 methodology, were subsequently included in the meta-analysis. Structural reliability analyses were also carried out for both the ABNT NBR 8800:2008 and the modified methodologies. The analyses showed that the reliability indexes of the modified formulation were higher than those of the ABNT NBR 8800:2008 one and closer to the recommended "target value".

Keywords: lateral torsional buckling, experimental analysis, steel beams, meta-data analysis, LTB .

1. Introduction

An open-section steel member (e.g., I-section), subjected to bending around its major axis, can suddenly lose stability and move laterally and rotate about its longitudinal axis. This phenomenon, called Lateral Torsional Buckling (LTB), is an important limit state in the design of steel structures. It is often critical in the design of beams without lateral restraint, especially during the erection process.

In Brazil, the design of steel beams with an I or H shape cross-section, subject to LTB, is done using the equations provided by the Brazilian Standard ABNT NBR 8800:2008 – *Design of Steel and Composite Structures for Buildings*, depending on different variables. Among them, the most important are the cross-section geometric characteristics, the beam boundary conditions, the steel properties, the point of load application in relation to the shear center and the presence and arrangement of lateral restraints.

The ABNT NBR 8800:2008 methodology is based on a simple supported beam subjected only to moments at its ends so that it bends in a simple curvature, causing a constant bending moment along its length. For cases of a non-uniform moment, the Standard uses a modification factor for non-uniform bending moment diagrams, the well-known factor C_b. It can be shown, using the classical buckling theory, that the moment which causes the onset of elastic instability, known as the critical moment, in I or H sections with double symmetry, is given by the equation:

$$M_{cr} = \frac{C_b \pi^2 E I_y}{L_b^2} \sqrt{\frac{C_w}{I_y}\left(1 + 0.039 \frac{J L_b^2}{C_w}\right)} \quad (1)$$

in which L_b is the unbraced beam length, i.e., the distance between lateral braces; J is the torsional constant; C_w is the warping constant; E is the modulus of elasticity; and I_y is the minor axis moment of inertia.

From the elastic stability theory (determination of the critical moment), ABNT NBR 8800:2008, based on the North American Specification for steel structural members ANSI/AISC 360–05, specifies expressions for determining the resistance moment, indirectly including the effects of geometric non-linearity, material non-linearity and geometric imperfections, determining three distinct ranges in the nominal resistant moment versus the slenderness parameter λ relationship:

- plastic range, in which the beam can reach the plastic bending moment, $M_{p\ell}$;
- inelastic range, the transition between the plastic and elastic ranges;
- elastic range, represented by the critical moment.

Therefore, the nominal resistant moment is given by:

$$\lambda \leq \lambda_p \rightarrow M_{Rk} = M_{p\ell} \quad \text{(plastic range)} \quad (2)$$

$$\lambda_p < \lambda \leq \lambda_r \rightarrow M_{RK} = C_b \left[C_b - (M_{p\ell} - M_r) \frac{\lambda - \lambda_p}{\lambda_r - \lambda_p} \right] \leq M_{p\ell} \quad \text{(inelastic range)} \quad (3)$$

$$\lambda > \lambda_r \rightarrow M_{Rk} = M_{cr} \quad \text{(plastic range)} \quad (4)$$

in which:

$$\lambda = M_b / r_y \quad (5)$$

$$\lambda_p = 1.76 \sqrt{E/f_y} \quad (6)$$

$$\lambda_r = \frac{1.38\sqrt{I_y J}}{r_y J \beta_1} \sqrt{1 + \sqrt{1 + \frac{27\, C_w\, \beta_1^2}{I_y}}} \quad (7)$$

$$M_{p\ell} = Z_x f_y \quad (8)$$

$$M_r = (f_y - \sigma_r) W_x \quad (9)$$

$$\beta_1 = \frac{(f_y - \sigma_r) W_x}{E J} \quad (10)$$

In these equations, r_y is the radius of gyration about the minor axis; *fy* is the steel yielding strength; Z_x is the plastic section modulus about the axis of bending; W_x is the elastic section modulus about the axis of bending; and σ_r is the section residual stress.

Analyzing these equations, one can see some inconsistencies regarding the use of the factor C_b. Although this modification factor was developed to adapt the elastic critical moment equation for the case of non-uniform moment diagrams, it was also used in the inelastic range. Furthermore, by using it to multiply the inelastic range equation, another inconsistency was created. The moment M_r, which is a limit at which the section ceases to be totally elastic, and which depends only on the section, the steel yield strength, and the cross-section residual stresses, in practice, became a C_b dependable variable, since the slenderness λ_r remains invariable.

Figure 1 shows a graphic representation of the variation of lateral-torsional buckling moment resistance as a function of slenderness parameter λ for different values of C_b, according to the current Brazilian Standard. It is noted that the inelastic range decreases as the value of C_b increases and even disappears at a value of C_b around 1.6. In other words, the regime abruptly changes from elastic to plastic, which is obviously inconsistent.

It is important to emphasize that the ABNT NBR 8800:2008 procedure for obtaining the nominal resistant moment is the same as ANSI/AISC 360–05 one. This procedure was still maintained in the latest edition of this North American Standard (ANSI/AISC 360–22).

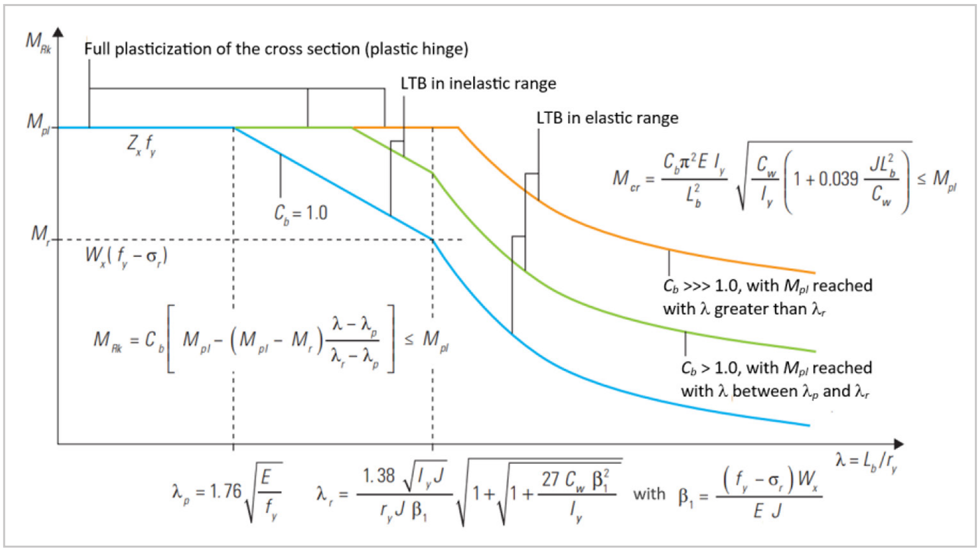

Figure 1 – Moment versus slenderness diagram as a function of C_b (Fakury *et al.*, 2016).

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2. Experimental bending tests

A total of 313 experimental tests were selected from literature (206 with rolled sections and 107 with welded sections), with slenderness parameters whose values could represent a wide range comprising the three ranges, plastic, inelastic and elastic. Table 1 shows the tests selected from different works available in literature.

The selected tests show a comprehensive range of boundary conditions (fork-type support, simple support with cantilever end), steel strength (mild and high strength steel, with nominal yield strength from 235 MPa up to 460 MPa), lateral restraints (no restraint, restraint on the load application and between loads), load application (on the middle of cross-section, on the upper flange and above the upper flange, cross section class (plastic and compact sections), fabrication type (hot-rolled and welded shapes), fabrication condition (as-delivered and stress-relieved shapes) and different bending moment shape (different C_b value). For illustration purpose, the bending moment shapes between effective lateral-torsional restraints are depicted in Table 1.

To facilitate the analyses and the interpretation of the results, as well as the comparison with those obtained using the equations in the Standard, the values of the maximum moment obtained in the tests were normalized by the cross-section plastic moment ($M_{p\ell}$). They were presented as a function of the reduced slenderness λ_{LT}, whose expression is shown below:

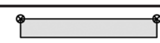
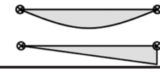
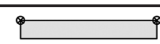
$$\lambda_{LT} = \sqrt{M_{p\ell} / M_{cr}} \qquad (11)$$

The relationship between λ and λ_{LT} depends on the cross-section characteristics and the steel strength as well. It can be shown that they are related by the following expression:

$$\lambda_{LT} = \sqrt{\frac{Z_x f_y \lambda^2 r_y^2}{C_b \pi^2 E \sqrt{I_y \left(C_w + 0.039 J \lambda^2 r_y^2 \right)}}} \qquad (12)$$

It can also be shown that the reduced slenderness limits between the plastic and inelastic ranges and between the inelastic and elastic ranges vary between 0.32 and 0.51 and between 1.22 and 1.46, respectively, for the cross-sections commonly used in design. For the tested sections, the reduced slenderness values ranged from 0.246 to 1.575. Therefore, it can be seen that the selected tests cover all three ranges of the diagram.

Table 1 – Selected tests.

Reference	Shape		C_b	Diagram Shape
	Rolled	Welded		
Lay *et al*. (1965)	7	-	1.00	
Janss and Massonnet (1967)	14	-	1.00	
			1.67	
Dibley (1969)	30	-	1.00	
Kitipornchai and Trahair (1975)	6	-	1.32	
Fukumoto et al. (1980)	75	-	1.32	
Dux and Kitipornchai (1983)	9	-	1.00	
			1.14	
			1.67	
Kubo and Fukumoto (1986)	44	-	1.32	
Wong-Chung and Kitipornchai (1987)	11	-	1.00	
Foster and Gardner (2015)	10	-	1.00	
			1.67	
Suzuki and Ono (1970)	-	12	1.32	
Suzuki and Ono (1973)	-	8	1.00	
Fukumoto and Itoh (1981)	-	68	1.32	
Xiong *et al*. (2016)	-	8	1.67	
Twizel (2021)	-	11	1.14	
Total	206	107		

3. Meta-analysis of test data

For each group of tests, Table 2 shows the mean and coefficient of variation (CoV) of the ratio between the moments calculated using the ABNT NBR 8800:2008 equations and the maximum moments obtained from the tests. It should be advised that the moments calculated using the Brazilian Standard equations were obtained using the actual properties of the cross-section and the steel strengths obtained in the tests, as well as the boundary conditions and load application. Table 2 also shows the C_b values for the test moment diagrams, calculated with the ABNT NBR 8800:2008 equation, as below:

$$C_b = \frac{12.5\, M_{max}}{2.5\, M_{max} + 3\, M_A + 4\, M_B + 3\, M_C} \qquad (13)$$

in which M_{max} is the absolute value of maximum moment in the unbraced segment; M_A is the absolute value of moment at quarter point of the unbraced segment; M_B is the absolute value of moment at centerline of the unbraced segment; and M_C is the absolute value of moment at the three-quarter point of the unbraced segment.

As one can see, for many tests, the mean was higher than 1.0, denoting unsafe results. Figure 2 shows the graphs of the test results ($\chi = M_{test}/M_{p\ell}$) and the resistance curves according to the Brazilian Standard equations as a function of reduced slenderness ($\chi = M_{Rk}/M_{p\ell}$) for values of C_b equal to 1.00, 1.14, 1.32 and 1.67, respectively. Again, several test results were below the nominal strength curve, meaning that the ABNT NBR 8800:2008 equations present many unsafe results, especially for C_b equal to or greater than 1.32.

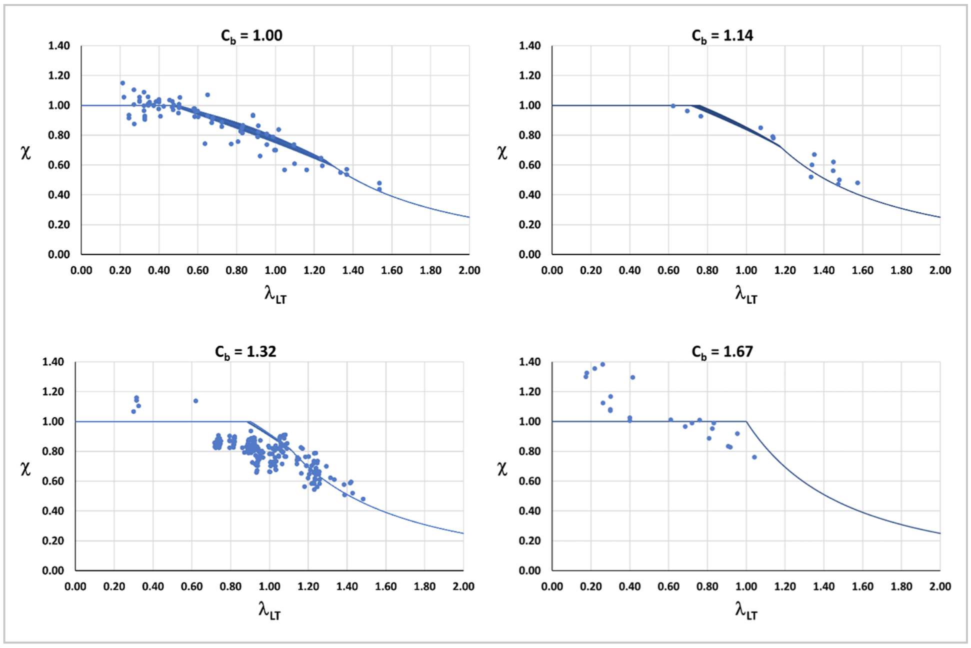


Figure 2 –Test results and ABNT NBR 8800:2008 resistance curves.

4. Proposed changes to the formulation

As explained in the Section 1 about the inconsistency of using C_b multiplying the inelastic range equation and the fact that some results are unsafe, it is proposed to change the formulation to consider the C_b only in the elastic range, keeping the moment M_r invariable. As a result, the slenderness λ_r is now variable with C_b. This modified formulation is the same as the 1986 version of ABNT NBR 8800, and as will be seen later, leads to better results than the 2008 version. Therefore, the equations for the nominal resistant moment are now as follows:

$$M_{Rk} = M_{p\ell}, \text{ for } \lambda \leq \lambda_p \quad (14)$$

$$M_{RK} = \left[M_{p\ell} - (M_{p\ell} - M_r) \frac{\lambda - \lambda_p}{\lambda_r - \lambda_p} \right], \text{ for } \lambda_p < \lambda \leq \lambda_r \quad (15)$$

$$M_{Rk} = M_{cr}, \text{ for } \lambda \leq \lambda_r \quad (16)$$

in which

$$\lambda_r = \frac{1.38 C_b \sqrt{I_y J}}{r_y J \beta_1} \sqrt{1 + \sqrt{1 + \frac{27\, C_w\, \beta_1^2}{C_b^2 I_y}}} \quad (17)$$

This proposed modified formulation was then included in the meta-analysis, the results of which are presented in Table 2. Just for comparison, the results from the EN 1993-1-1:2022 methodology were also presented. According to EN 1993-1-1:2022, the nominal resistant bending moment under LTB is obtained as follows:

$$M_b = \chi_{LT} M_{Rk} \quad (18)$$

$$\chi_{LT} = \frac{f_M}{\phi_{LT} + \sqrt{\phi_{LT}^2 - f_M \bar{\lambda}_{LT}^2}} \leq 1.0 \quad (19)$$

$$\phi_{LT} = 0.5\left[1 + f_M\left(\left(\frac{\bar{\lambda}_{LT}}{\bar{\lambda}_z}\right)^2 \alpha_{LT}\,(\bar{\lambda}_z - 0.2) + \bar{\lambda}_{LT}^2\right)\right] \quad (20)$$

$$\bar{\lambda}_{LT} = \sqrt{M_{Rk} / M_{cr}} \quad (21)$$

in which M_b is the nominal resistant bending moment under LTB; M_{Rk} is the characteristic value of the bending moment resistance; $\bar{\lambda}_{LT}$ is relative slenderness for lateral torsional buckling; χ_{LT} is the reduction factor for lateral torsional buckling; α_{LT} is the imperfection factor for lateral torsional buckling, given by Table 8.5 from EN 1993-1-1:2022; $\bar{\lambda}_z$ is relative slenderness related to weak axis flexural buckling, considering the buckling length $L_{cr,z}$ equal to the distance between discrete lateral restraints; and f_M is a factor that take into account the effect of the bending moment distribution between discrete lateral restraints.

Table 2 - Statistics of the ratio between the results of ABNT NBR 8800:2008, proposed modified formulations and EN 1993-1-1:2022 equations and experimental tests.

Reference	Section Type	C_b	ABNT NBR 8800:2008		Modified Formulation		EN 1993-1-1:2022	
			Mean	CoV	Mean	CoV	Mean	CoV
Lay *et al.* (1965)	hot-rolled	1.00	1.00	0.02	1.00	0.02	0.95	0.01
Janss and Massonnet (1967)	hot-rolled	1.00	0.98	0.09	0.98	0.09	0.93	0.09
		1.67	0.79	0.06	0.79	0.06	0.80	0.07
Dibley (1969)	hot-rolled	1.00	0.98	0.06	0.98	0.06	0.89	0.07
Kitipornchai and Trahair (1975)	hot-rolled	1.32	1.10	0.11	0.98	0.04	0.95	0.16
Fukumoto *et al.* (1980)	hot-rolled	1.32	1.14	0.09	0.98	0.08	0.87	0.11
Dux and Kitipornchai (1983)	hot-rolled	1.00	1.04	0.03	1.04	0.03	0.95	0.02
		1.14	1.04	0.03	0.95	0.00	0.92	0.00
		1.67	1.03	0.05	0.86	0.01	0.89	0.04
Kubo and Fukumoto (1986)	hot-rolled	1.32	1.17	011	1.01	0.07	0.88	0.07
Wong-Chung and Kitipornchai (1987)	hot-rolled	1.00	1.02	0.02	1.02	0.02	0.86	0.07
Foster and Gardner (2015)	hot-rolled	1.00	0.97	0.01	0.97	0.01	0.92	0.01
		1.67	0.96	0.03	0.95	0.03	0.96	0.03
Suzuki and Ono (1970)	welded	1.32	1.04	0.12	0.95	0.06	0.72	0.07
Suzuki and Ono (1973)	welded	1.00	1.09	0.01	1.09	0.01	1.06	0.01
Fukumoto and Itoh (1981)	welded	1.32	1.12	0.15	0.99	0.09	0.88	0.12
Xiong *et al.* (2016)	welded	1.67	1.10	0.09	0.92	0.03	0.86	0.06
Twizel (2021)	welded	1.14	0.91	0.09	0.90	0.08	0.61	0.05

Figure 3 shows the graphs of the test results and the resistance curves according to the equations from the modified formulation as a function of the reduced slenderness. The results for C_b equal to 1.00 are the same for both formulations. It is evident the significant improvement associated to the modified formulation by comparing the results in Table 2, as well as Figures 2 and 3. It is important to point out the conservativeness of EN 1993-1-1:2022 for welded shapes, especially for the heavier ones, such as those in Twizel tests.

Figure 4 shows a comparison between the experimental results and resistant bending moment predictions from the present Brazilian Standard ABNT NBR 8800:2008 – the same as ANSI/AISC 360–22 – (Figure 4-a), the proposed modified formulation (Figure 4-b) and EN 1993-1-1:2022 (Figure 4-c). The experimental data above the reference lines related to nominal flexural strength (M_{Rk}/M_{test} = 1.00) indicate non-conservative prediction. One can observe in Figure 4-b a decrease in the number of test results above the reference line, in comparison to Figure 4-a. Figure 4 also shows the statistics (mean and CoV – coefficient of variation) from all analyses, in which one can observe the non-conservativeness of the present Brazilian formulation, i.e., mean greater than 1.0 (1.08) and larger CoV (12.4%), and the better results from the modified formulation, evidenced by mean value close to 1.0 (0.98) and smaller CoV (8.7%). Analyzing the results from EN 1993-1-1:2022, one can observe an even greater decrease in the test results that lie above the reference line, with mean value smaller than 1.0 (0.87) but the largest CoV (13.1%). Finally, the results from the modified formulation show the smallest dispersion and the mean closest to 1.0, still on the safe side, as one can see by comparing the mean and CoV from all analyses.

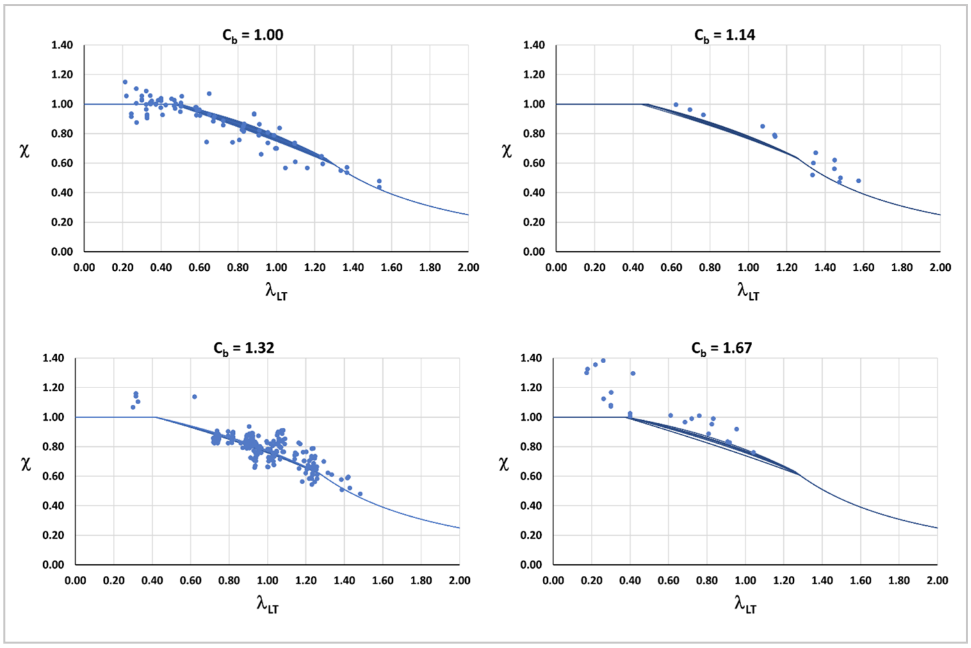


Figure 3 –Test results and resistance curves (modified formulation).

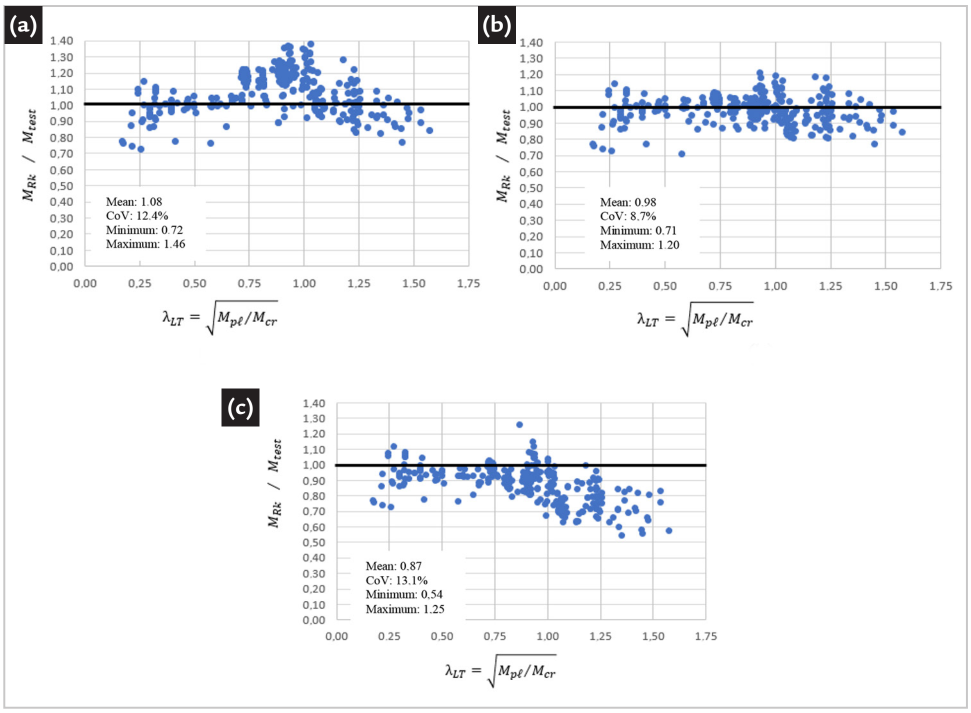


Figure 4 – Ratio between nominal resistant bending moment and ultimate experimental bending moment, related to: (a) ABNT NBR 8800:2008, (b) proposed modified formulation and (c) EN 1993-1-1:2022.

5. Structural reliability analysis

Based on the ABNT NBR 8800:2008 and on the modified formulation, the basic random variables for reliability analysis are defined, considered statistically independent (uncorrelated). The functional relationship between them, the so-called performance function, can be described generically by the following expression:

$$g(\boldsymbol{X}) = R(\cdot) - S(\cdot) \quad (22)$$

in which $\boldsymbol{X}$ is a vector containing the basic random variables. $R(\cdot)$ e $S(\cdot)$ are the functions that define the resistance and demand of the structural element, respectively. The functions $R(\cdot)$ e $S(\cdot)$ are defined for each limit state analyzed, in this case, the LTB.

According to the Brazilian Standard format, the nominal resistance R_n is related to the nominal demand S_n by the inequation below:

$$R_n \gamma_a \geq \gamma S_n = c\,(\gamma_D D_n + \gamma_L L_n) \tag{23}$$

in which γ_a is the safety factor given in the Brazilian Standard for the LTB limit state, equal to 1.10; γ_D e γ_L are dead and the live loads' partial safety factors, taken equal to 1.35 (average value) and 1.5, respectively, according to ABNT NBR 8800:2008; D_n e L_n are the nominal values of the dead and live loads, respectively, and c is a deterministic parameter for transforming loads into effects on the structure (e.g., $c=(sl^2)/8c$, in which s is the beam spacing and l is the span length).

This leads to the following performance functions for beams subjected to LTB:

- plastic range:

$$g(\cdot) = PZ_x F - c(D + L) \tag{24}$$

- inelastic range:

$$g(\cdot) = PZ_x (F - \alpha\Sigma_r) - c(D + L) \tag{25}$$

- elastic range:

$$g(\cdot) = PK_t E - c(D + L) \tag{26}$$

in which the random variables P is the model error, also known as the professional coefficient; Z_x is the cross-section resistance modulus; F is the yield strength of steel; D e L are dead and live loads, respectively; Σ_r is the residual stress; E is the steel modulus of elasticity; K_t is geometric parameter taken as follows:

$$K_t = \frac{\pi^2 I_y}{l^2}\sqrt{\frac{C_w}{I_y}\left(1 + 0.039\,\frac{J}{C_w}\,l^2\right)} \tag{27}$$

and α is a deterministic variable, related to the relative position of the beam length or the slenderness λ and the limits λ_p e λ_r, given by:

$$\alpha = \frac{\lambda - \lambda_p}{\lambda_r - \lambda_p} \tag{28}$$

Taking q as the ratio between the nominal live load and the nominal dead load, the performance functions in the limit state, i.e., in the condition in which $g(\cdot)=0$, evaluated at the design point, which will be used in the FORM analyses (First Order Reliability Method), can be described for each range as:

- plastic range:

$$p^* \frac{Z_x^*}{Z_{xn}}\frac{f^*}{f_{yn}} - \frac{1}{\gamma_a(\gamma_D/q + \gamma_L)}\left(\frac{1}{q}\frac{d^*}{D_n} + \frac{l^*}{L_n}\right) = 0 \tag{29}$$

- inelastic range:

$$p^* \frac{Z_x^*}{Z_{xn}}\left(\frac{f^*}{f_{yn}} - 0.3\alpha\frac{\sigma_r^*}{\sigma_{rn}}\right)\frac{1}{\gamma_a(\gamma_D/q + \gamma_L)}\left(\frac{1}{q}\frac{d^*}{D_n} + \frac{l^*}{L_n}\right) = 0 \tag{30}$$

in which the factor 0.3 is the ratio between the residual stress nominal values (σ_{rn}) and yield strength (f_{yn}).

- elastic range:

$$p^* \frac{K_t^*}{K_{tn}}\frac{e^*}{e_n} - \frac{1}{\gamma_a(\gamma_D/q + \gamma_L)}\left(\frac{1}{q}\frac{d^*}{D_n} + \frac{l^*}{L_n}\right) = 0 \tag{31}$$

In these equations, lowercase letters mean specific values of random variables; asterisks mean values at the design point and the subscript n means the nominal value of the variable - see Pimenta, 2008. It should be noted that only gravitational loads were considered in this study.

The statistical parameters of the basic random variables are given in Table 3, where δ is the bias coefficient. The statistical parameters of the professional coefficient were obtained by meta-analysis of the ratio between the test results and the values obtained from the ABNT NBR 8800:2008 equations and from the modified methodology.

Table 3 - Statistical parameters of the random variables.

Parameters					
Variables		δ	CoV	Distribution	Reference
Load	Dead *D*	1.06	0.120	Normal	Santiago *et al*. (2019)
	Live *L*	0.93	0.250	Gumbel	Costa *et al*. (2023)
Resistance	Plastic Modulus Z_x (RS)	1.00	0.040	Normal	JCSS (2001)
	Geometric K_t (RS)	1.00	0.040	Normal	
	Plastic Modulus Z_x (WS)	1.03	0.023	Normal	Pimenta (2008)
	Geometric K_t (RS)	1.05	0.030	Normal	
	Yield Strength *F* (RS)	1.11	0.055	Lognormal	This work
	Yield Strength *F* (WS)	1.17	0.065	Lognormal	Pimenta (2008)
	Residual Stress Σ_r	1.00	0.300	Normal	
	Modulus of Elasticity *E*	1.03	0.022	Lognormal	

The statistics for the plastic modulus, the geometric parameter and the yield strength variables were different for rolled shapes (RS) and welded shapes (WS). However, when analyzed together, the values for the rolled shapes were considered, as they were in the majority. The statistics for the rolled shape yield strength were obtained by analyzing 7872 certificates supplied by Gerdau, the main Brazilian steel mill that manufactures this type of profile, in 2022.

The professional coefficient statistics, obtained from the meta-analysis, are presented in Tables 4 through 7, together with the results of the reliability analyses via FORM, using the computational tools developed in Pimenta (2008).

Table 4 - Results for all shapes in the plastic and inelastic ranges.

All - Plastic		
q	β	
	ABNT NBR 8800:2008	Modified formulation
0.5	3.1	3.1
1.0	3.0	3.0
1.5	2.9	2.9
2.0	2.8	2.8
2.5	2.8	2.8
3.0	2.8	2.8
3.5	2.7	2.7
4.0	2.7	2.7

All - Inelastic		
q	β	
	ABNT NBR 8800:2008	Modified formulation
0.5	2.3	3.2
1.0	2.3	3.0
1.5	2.3	2.9
2.0	2.3	2.9
2.5	2.2	2.8
3.0	2.2	2.8
3.5	2.2	2.7
4.0	2.2	2.7

Professional coefficient *P*		
Parameters	ABNT NBR 8800:2008	Modified formulation
δ	1.047	1.047
CoV	0.112	0.112

Professional coefficient *P*		
Parameters	ABNT NBR 8800:2008	Modified formulation
δ	0.896	1.020
CoV	0.111	0.091

Table 5 - Results for rolled and welded shapes in the inelastic range.

Rolled - Inelastic		
q	β	
	NBR 8800:2008	Modified formulation
0.5	2.4	3.3
1.0	2.4	3.1
1.5	2.4	3.0
2.0	2.3	2.9
2.5	2.3	2.8
3.0	2.3	2.8
3.5	2.3	2.8
4.0	2.3	2.7

Welded - Inelastic		
q	β	
	NBR 8800:2008	Modified formulation
0.5	2.7	3.5
1.0	2.6	3.3
1.5	2.5	3.2
2.0	2.5	3.1
2.5	2.4	3.0
3.0	2.4	3.0
3.5	2.4	3.0
4.0	2.4	3.0

Professional coefficient *P*		
Parameters	NBR 8800:2008	Modified formulation
δ	0.913	1.024
CoV	0.109	0.084

Professional coefficient *P*		
Parameters	NBR 8800:2008	Modified formulation
δ	0.852	1.012
CoV	0.099	0.104

Table 6 - Results for all shapes in the inelastic range for C_b=1.0 and C_b>1.0.

All - Inelastic - C_b = 1.0		
q	β	
	ABNT NBR 8800:2008	Modified formulation
0.5	3.0	3.0
1.0	2.9	2.9
1.5	2.8	2.8
2.0	2.7	2.7
2.5	2.6	2.6
3.0	2.6	2.6
3.5	2.6	2.6
4.0	2.6	2.6

All - Inelastic - C_b > 1.0		
q	β	
	ABNT NBR 8800:2008	Modified formulation
0.5	2.2	3.3
1.0	2.3	3.1
1.5	2.2	3.0
2.0	2.2	2.9
2.5	2.2	2.8
3.0	2.2	2.8
3.5	2.1	2.8
4.0	2.1	2.7

Professional coefficient *P*		
Parameters	NBR 8800:2008	Modified formulation
δ	0.973	0.973
CoV	0.087	0.087

Professional coefficient *P*		
Parameters	NBR 8800:2008	Modified formulation
δ	0.877	1.029
CoV	0.107	0.089

Table 7 - Results for all shapes in the elastic range.

All - Elastic		
q	β	
	ABNT NBR 8800:2008	Modified formulation
0.5	2.9	3.5
1.0	2.8	3.2
1.5	2.7	3.0
2.0	2.6	2.9
2.5	2.6	2.9
3.0	2.5	2.8
3.5	2.5	2.8
4.0	2.5	2.8

Professional coefficient P		
Parameters	ABNT NBR 8800:2008	Modified formulation
δ	1.027	1.089
CoV	0.106	0.085

Figures 5 through 7 illustrate the comparison between the reliability indexes, β, from the ABNT NBR 8800:2008 formulations (in blue) and those from the modified formulation (in orange). When both results are the same, no graphs are shown. There is a significant improvement achieved with the modified formulation, as can be seen by comparing the reliability indexes (β), that are higher than those obtained from the current formulation and closer to the recommended target value of β=3.0.

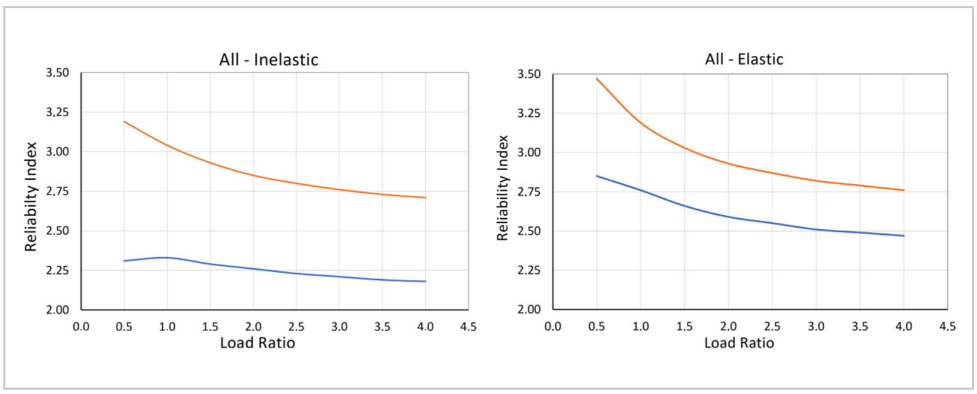


Figure 5 - Results of all shapes in the inelastic and elastic ranges.

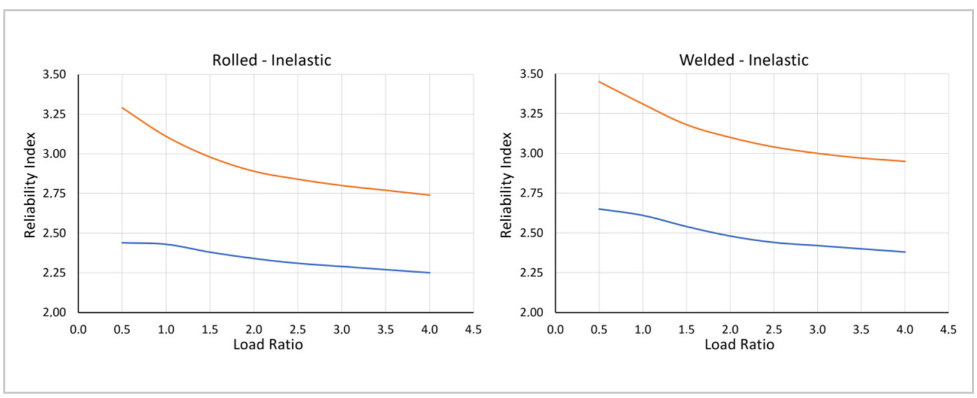


Figure 6 - Result for rolled and welded shapes in the inelastic range.

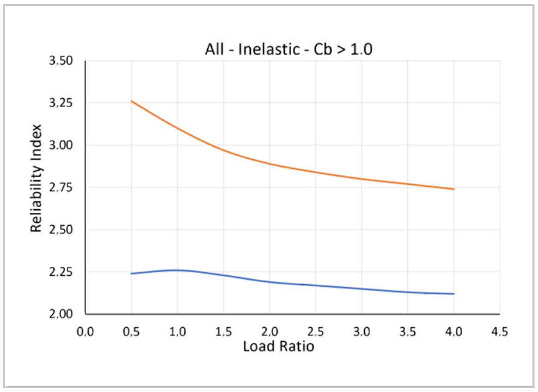


Figure 7 - Results for all shapes in the inelastic range for C_b>1.0.

6. Conclusion

In this study, a comprehensive literature review was carried out to gather test data that could be used to verify whether the lateral-torsional buckling (LTB) equations from ABNT NBR 8800:2008 are adequate to represent this important limit state. Fourteen articles, theses and reports were selected to obtain sufficient data to carry out a comparative meta-analysis with the equations of the Brazilian Standard.

It was observed that some tests, particularly those in which the critical moment modification factor for the non-uniform moment diagram, C_b, was equal to or greater than 1.32, presented lower values than those calculated using the equations of the Brazilian Standard. A modified formulation was then proposed, the same as the 1986 version of ABNT NBR 8800, in which the factor C_b is used only in the elastic range of the nominal moment versus the slenderness ratio diagram and no longer multiplies the equations in the inelastic range. This solved the inconsistencies found in the current version of the Standard. The limit moment between the elastic and inelastic regimes, M_r, remains now invariant with C_b. As a result, the limit slenderness λ_r is now variable with C_b. It was observed that the modified formulation showed better results than the current formulation, leading to values much closer to those found in the tests.

Structural reliability analyses were also carried out for both formulations using the First Order Reliability Method (FORM), using the computational tools developed in Pimenta, 2008. It was observed that the reliability indexes (β) of the results obtained with the modified formulation were higher than those obtained from the current formulation and closer to the recommended target value of β=3.0.

Therefore, it is proposed that this modified formulation be incorporated into the revision of ABNT NBR 8800:2008 to better represent the LTB limit state.

As future research, the authors will provide further comprehensive parametric analyses, considering different boundary conditions, load application positions, other bending moment diagram shapes and so on.

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