

Finite strip method computer application for buckling analysis of thin-walled structures with arbitrary cross-sections

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Joao Alfredo de Lazzari^{1,2}

<https://orcid.org/0000-0002-3176-3006>

Eduardo de Miranda Batista^{1,3}

<https://orcid.org/0000-0002-6317-6789>

¹Universidade Federal do Rio de Janeiro - UFRJ,
COPPE - Programa de Engenharia Civil,
Rio de Janeiro - Rio de Janeiro - Brasil.

E-mails: joaoadelazzari@outlook.com.br,

batista@coc.ufrj.br

Abstract

The Direct Strength Method (DSM) is a well-known formulation presented in the Brazilian standard ABNT NBR 14762:2010, that estimates the strength capacity of cold-formed steel (CFS) members. However, this formulation requires the elastic critical buckling loads as point of departure, regarding local (L), distortional (D) and global (G) modes, which can be obtained by (i) elastic buckling analysis or (ii) available direct equations. The present study is dedicated to the development of a computer program, FStr Computer Application, with graphical user interface (GUI), in order to make the buckling analysis easier and approachable for both research activities and engineering design of thin-walled structures with arbitrary cross-sections (sections combining closed cells with open branches). The proposed application uses the Finite Strip Method (FSM), mainly focused on a simple and accessible GUI, which was implemented in the MATLAB App Designer. Validation of the FStr was performed with the help of examples of open and closed cross-sections and comparison with the results of acknowledged computer programs, such as CUFSM and GBTul, based respectively on FSM and GBT (Generalized Beam Theory), as well as analytical procedures for the case of the global buckling modes. Both, the critical buckling loads expressed by the computed signature curve and the correspondent critical buckling modes are compared, confirming the adequate performance of the proposed computational tool. As future research, the authors plan updates for the FStr Computer Application, including the computation of the buckling modal participation and the automatic strength predictions, based on the DSM-based design prescriptions.

Keywords: Finite Strip Method, computer application, thin-walled structures, elastic buckling analysis, cold-formed steel members.

1. Introduction

Applying thin-walled members may be a frequent option due to light structural systems, less material consumption, engineering design and architectural concepts. However, steel thin-walled constructional systems are mostly slender structures, which present additional stability problems, (Batista, 2005).

Cold-formed steel (CFS) members are composed of thin-walled sections, usually conducive to slender structural systems, prone to buckling, obliging designers to deal with the complexity of the phenomenon and requiring as simple as possible design procedures in order to allow safe structural performance. The

current CFS structural design codes, Brazilian standard NBR 14762 (ABNT, 2010), Australian/New Zealand code 4600 (AS/NZS, 2018) and North-American standard S100-16 (AISI, 2016), have been improving their design approaches over the past decades. Revisions of the design procedures, usually based on semi-empirical procedures, obliges laboratory experimental campaigns combined with accurate numerical analysis in order to calibrate and improve the proposed equations and procedures. The usual formulations for the design of thin-walled CFS members require a previous elastic critical buckling analysis as the point of

departure, regarding the identification of local (L), distortional (D) and global (G) buckling modes, which can be obtained by (i) an elastic buckling analysis or (ii) available analytical equations.

The current study is dedicated to the development of a computer program for an elastic buckling analysis with graphical user interface (GUI), in order to make it easier and approachable for both research activities and engineering design of thin-walled structures with arbitrary cross-sections (e.g. symmetric and asymmetric open section CFS members, hollow sections, multi-cell box girder and monosymmetric I-shape beams).

1.1 The finite strip method

The present article provides the finite strip method (FSM) for an elastic buckling analysis. The FSM was originally formulated by Yau Kai Cheung, honorary professor of The University of Hong Kong (Cheung, 1976). On the other hand, it was Gregory J. Hancock, emeritus professor of The University of Sydney, who began using the FSM in structural members, such as hot-rolled sections and CFS sections (Hancock, 1978; Hancock, 1981; Hancock *et al.*, 1980).

The Finite Strip Method is a particular case of the Finite Element Method (FEM). Briefly, the FEM uses polynomial shape functions in all directions, while the FSM uses polynomials shape functions in a transverse direction and trigonometric shape functions in a longitudinal direction, which satisfies the boundary conditions (Figure 4.) for the case of small displacements

of the structural system. The main advantage in using FSM, as compared to FEM, is to reduce the structure's degrees of freedom, in order to acquire performance and time consumption in the elastic buckling analysis. In addition, the choice of the longitudinal deformation function as a trigonometric shape allows FSM to solve the buckling analysis (first order small displacements solution) with highly accurate results. However, the method is not eligible to perform structural analysis addressed to obtain displacements and stresses in structural systems.

The FSM formulation is based on classical plate theory assumptions, which are described in detail by Timoshenko and Woinowsky-Krieger (1959). In the present study, the computational matrix formulation is based on the main reference by Cheung (1976). Additionally, the

following sources are also used in the present study – *i.e.* (Bradford & Azhari, 1995; Schafer, 1997; Li & Schafer, 2009; Li, 2009; Lazzari, 2020).

The strip element is a lower order rectangular strip with two nodal lines (LO2) as shown in Figure 1. For each strip, the membrane strain is examined, considering plane stress assumptions and the bending strain, in accordance with Kirchoff thin plate theory assumptions (Cheung, 1976). Due to these assumptions, each strip has 8 degrees of freedom and 4 degrees per nodal line.

First, the displacement field inside de strip can be approximated by Eq. (1), using the nodal displacements $\{d\}$, shown in Figure 1-b, and the shape function matrix $[N]$. The displacements field for each strip, $\{u \ v \ w\}^T$, is determined as a summation of all longitudinal terms (p), from 1 to $m \in \mathbf{N}$.

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = [N]\{d\} = \sum_{p=1}^m [N]_p \{d\}_p = \sum_{p=1}^m \begin{bmatrix} [N_{uv}]_p & [0]_{2 \times 4} \\ [0]_{1 \times 4} & [N_w]_p \end{bmatrix} \{u_1 \ v_1 \ u_2 \ v_2 \ w_1 \ \theta_1 \ w_2 \ \theta_2\}_p^T \quad (1)$$

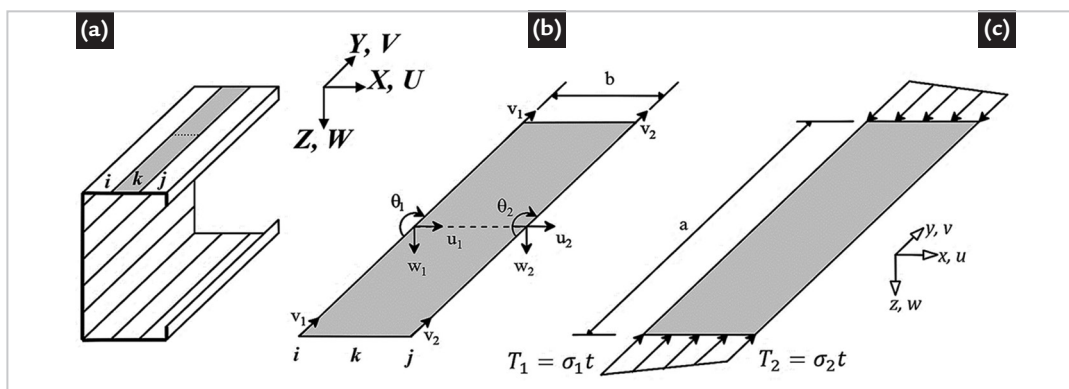


Figure 1 - Lower order rectangular strip with two nodal lines (LO2). (a) Strip discretization in a Lipped channel section. (b) Degrees of freedom on nodal lines. (c) External end stress distribution applied to the strip (Li & Schafer, 2009).

The shape function matrix can be found in Lazzari (2020) and is composed of polynomial functions times Y_p , which is a trigonometric function.

The formulation of the finite strip

now can be defined using the principle of minimum total energy. According to Cheung (1976), the principles states that “of all compatible displacements satisfying given boundary conditions,

those which satisfy the equilibrium conditions make the total potential energy assume a stationary value”. In other words, Eq. (2) appears in the variational form:

$$\left\{ \frac{\partial \Pi}{\partial \{d\}} \right\} = \left\{ \frac{\partial \Pi}{\partial \{d\}_1} \quad \frac{\partial \Pi}{\partial \{d\}_2} \quad \dots \quad \frac{\partial \Pi}{\partial \{d\}_m} \right\}^T = \{0\}, \quad \text{where } \Pi = U + W \quad (2)$$

in which Π is the total potential energy, U is the strain energy and W is the po-

tential energy of external forces. By definition, the strain energy of

a three dimensional solid is defined by Eq. (3).

$$U = \frac{1}{2} \iiint \{\epsilon\}^T \{\sigma\} dV = \frac{1}{2} \iiint \{d\}^T [B]^T [D] [B] \{d\} dV \quad (3)$$

In Eq. (3) $\{\epsilon\}$ is the strain, compounded with the sum of the bending

and twisting curvature strain, $\{\epsilon_b\}$, with the normal and shear strain, $\{\epsilon_M\}$. Also,

$\{\sigma\}$ is the stress, related to the strains, $[B]$ is the strain-displacement matrix

and $[D]$ is the elasticity matrix.

The stiffness matrix can now be computed by substituting Eq. (3) in the Eq. (2). Doing the appropriate differentiation and organizing in the form $[k]\{d\} - \{F\} = \{0\}$, the stiffness matrices can be determined. Solving for the membrane strain - considering plane stress assumption - it leads to the elastic

stiffness matrix for the membrane case, and solving for the bending strain - considering Kirchoff thin plate theory assumptions - it results in the elastic stiffness matrix for the bending case. Both matrices can be found in Schafer (1997), Li and Schafer (2009), Li (2009) and Lazzari (2020).

For the stability problem, it is

$$V = \frac{1}{2} \iiint_V \left\{ \sigma_1 - \left(\sigma_1 - \sigma_2 \right) \frac{x}{b} \right\} \left\{ \left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right\} dV \quad (4)$$

Working on the Eq. (4) using matrix formulation, and considering the principle of minimization of the total potential energy due to the initial stress, the geometric stiffness matrix or the initial stress matrix can be reached. Again, the geometric matrices are given from the membrane

and bending assumption separately. Both matrices can also be found in Schafer (1997), Li and Schafer (2009), Li (2009) and Lazzari (2020).

For the assumed flat shell strip (LO2), there is no interaction between the bending and the membrane, be-

cause the displacements are small enough. Due to that the elastic stiffness matrix, Eq. (5), and the geometric stiffness matrix, Eq. (6), are obtained by assembling the membrane and bending matrices through a simple combination, as described in Eq. (5) and Eq. (6).

$$[K] = \sum_{k=1}^s \sum_{p=1}^m \sum_{q=1}^m [R]_{8 \times 8} \begin{bmatrix} [K_M^{ij}]_{pq} & [0]_{4 \times 4} \\ [0]_{4 \times 4} & [K_B^{ij}]_{pq} \end{bmatrix}_{8 \times 8} [R]_{8 \times 8}^T \quad (5)$$

$$[KG] = \sum_{k=1}^s \sum_{p=1}^m \sum_{q=1}^m [R]_{8 \times 8} \begin{bmatrix} [K_G^M]_{pq} & [0]_{4 \times 4} \\ [0]_{4 \times 4} & [K_G^B]_{pq} \end{bmatrix}_{8 \times 8} [R]_{8 \times 8}^T \quad (6)$$

The matrices in Eq. (5) and Eq. (6) are the global matrices, which are obtained by assembling all the half-wave terms in each corresponding degree of freedom. For the assembling, it is necessary to transform

the local into global coordinates. In this case, there is considered a common y axis for the local and global coordinates. More details about the assembly of the global stiffness matrices can be found in Ádany

and Schafer (2006) and Lazzari (2020).

After the assembling, the general stability solution is obtained solving the classic generalized eigenvalue problem described in the Eq. (7).

$$([K] - [\Lambda][KG])[\Phi] = [0] \quad \text{or} \quad [K][\Phi] = [\Lambda][KG][\Phi] \quad (7)$$

Using the global elastic stiffness matrix $[K]$, the global geometric stiffness matrix $[KG]$

and a proper eigenvalue problem solver, it is possible to obtain the eigenvalues $[\Lambda]$, which

are the critical stresses, and the eigenvectors $[\Phi]$, which are the critical modal shapes.

1.2 Computer programs and methods for elastic buckling analysis

So far, the finite strip method is well consolidated in well-known computer programs. The two most famous computer programs that perform a finite strip method are: the Constrained and Unconstrained Finite Strip Method (CUFSM), by Ádany and Schafer (2006), Ádany and Schafer (2006a), Ádany and Schafer (2006b), Schafer (1997), Schafer (2020), and the THIN-WALL by Papangelis and Hancock (1995) and Nguyen *et al.* (2015).

The CUFSM (Schafer, 2020) program is a finite strip elastic buckling analysis application, which performs analyses for thin-walled sections. CUFSM is an open free source

program created by professor Ben Schafer's thin-walled structures research group at Johns Hopkins University (Baltimore, MD, United States of America) and it was developed in the MATLAB platform (Matworks, 2000).

The THIN-WALL is a Semi-Analytical Finite Strip Method (SAFSM), which has been recently updated to the THIN-WALL 2 (Nguyen *et al.*, 2015). The new updated version was developed at The University of Sydney (Sydney, NSW, Australia), with the help of a graphical user interface (Mathworks, 2000) and Visual Studio C++ computational engines.

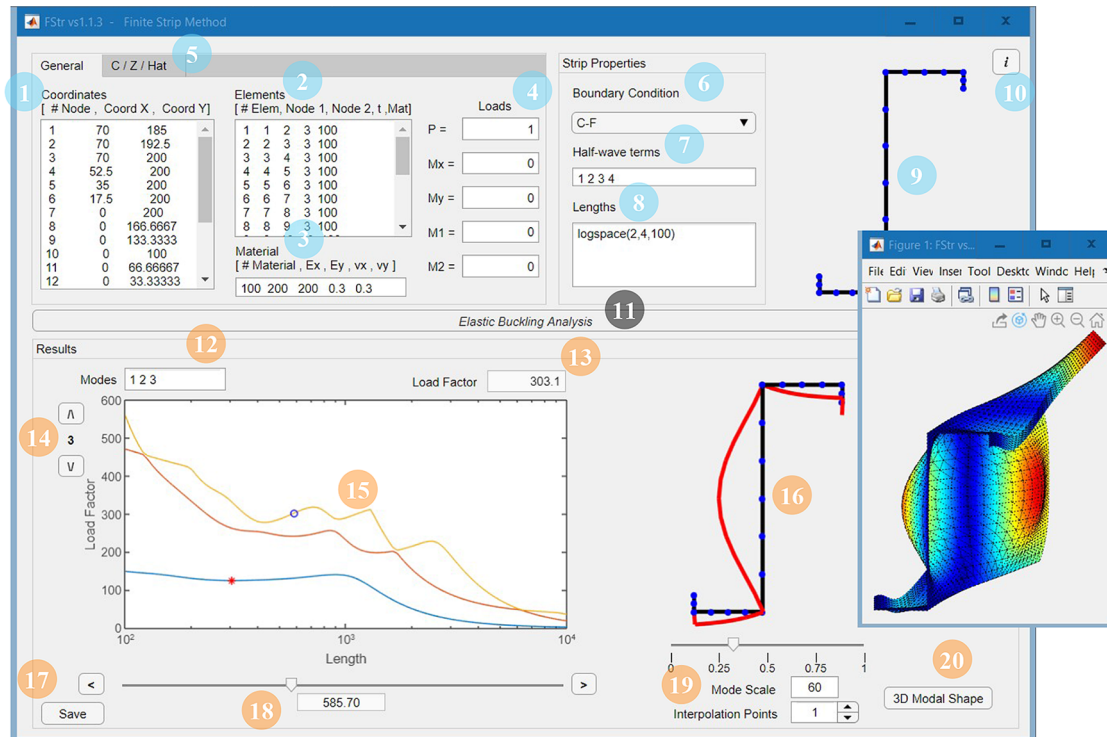
Besides the finite strip computer programs, there are other methods for performing the elastic buckling analysis. The Generalized-Beam-Theory (GBT) is a well consolidated method, originally proposed by Schardt (1989), that has been updated in the last decade by the IST research group (Instituto Superior Técnico – IST – University of Lisbon, Portugal) - e.g. Silvestre (2005), Bebiano (2010) and Camotim *et al.* (2010). The best performance computer program that uses this method is the GBTul 2.0, from the Generalized Beam Theory Research Group at the IST, Lisbon (Bebiano *et al.*, 2018).

2. FStr computer application

FStr Computer Application (Figure 2 and Figure 3.) is a software developed on the basis of the Finite Strip Method formulation, as described in section 1.1 (The Finite Strip Method). The GUI is implemented in

the MATLAB App Designer (Mathworks, 2000). The purpose of the GUI is to make it easier for the user to set up the data input and to analyze the data output. Figure 2 shows the FStr GUI with the data input and

output displayed in one single panel. The data input is marked from (1-10), the data preprocessing and the finite strip analysis are marked as (11) and the data output are indicated from (12-20).



- 1 Coordinates Panel [Node Number; Coordinate in x direction; Coordinate in y direction];
- 2 Elements Panel [Element Number; First Node; Second Node; Thickness; Material Name]
- 3 Orthotropic Material Panel [Material Name; Elastic Modulus for x direction; Elastic Modulus for y direction; Poisson's ratio for x direction; Poisson's ratio for y direction]
- 4 [P: Compression] [Mx: Moment about geometric x axis] [My: Moment about geometric y axis] [M1: Moment about major principal axis] [M2: Moment about minor principal axis]
- 5 Automatic C, Z and Hat Cross-section generation (Figure 3)
- 6 End Boundary Condition (Figure 4)
- 7 Half-wave terms for trigonometric series [1 2 3 4 ... m] $\{m \in \mathbf{N}\}$
- 8 Lengths of strips (e.g. $\text{logspace}(1,4,200)$, (10:100:10000))
- 9 Dynamic 2D cross-section geometry
- 10 About the FStr Computer Application
- 11 Elastic Buckling Analysis Button
- 12 Number of superior modes displayed [1 2 3 ... n] $\{n \in \mathbf{N} \mid n \leq 20\}$
- 13 Load Factor
- 14 Selection of Superior Modes
- 15 Signature Curve and Superior Modes (Load Factor *versus* Length)
- 16 Dynamic 2D Modal Shape, for each Length and Transversal Position Ratio ($\bar{y} = y/L$)
- 17 Save in *.txt* or *.mat* format
- 18 Selection of Length
- 19 Mode Scale and Interpolation points changer
- 20 3D Modal Shape for the selected Length

Figure 2 - FStr Graphical User Interface index description.

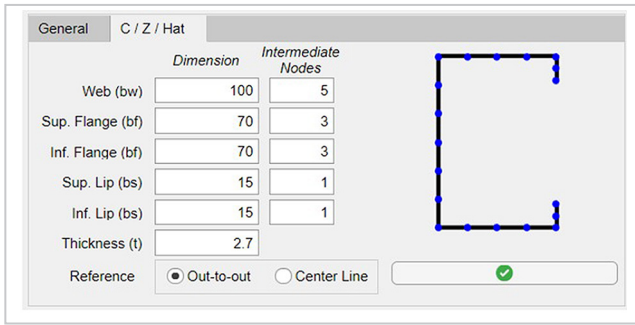


Figure 3 - Cross-section generation.

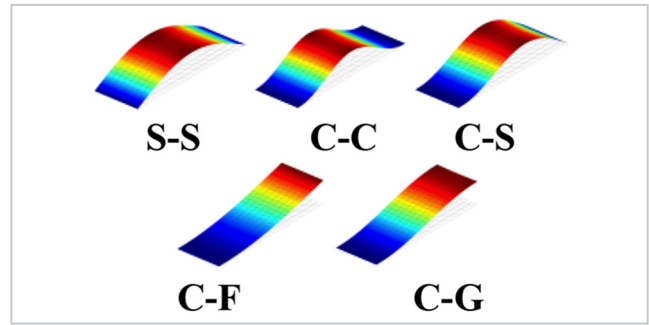


Figure 4 - End boundary condition illustration.

One must know that even though FStr, CUFSM and THIN-WALL are based on the same elastic buckling analysis method, they have their differences. First, the FStr source code has a more optimized code structure, which

makes it: (i) faster to assemble the global stiffness matrices; (ii) faster to generate the 3D buckling mode; (iii) faster elastic buckling analysis for axial compressive loading. However, the FStr do not perform any type of pure buckling mode

analysis yet, as well as not include the constrained buckling analysis method performed by CUFSM. Additionally, FStr is an ongoing development and limitations of the last version (FStr 1.3.0) are shown in Lazzari (2020).

3. Elastic buckling analysis validation

The validation is performed for different cross-section models. First, a lipped channel section with simply supported end boundary conditions is performed, in order to show the classical signature curve. Secondly, a

complex I-shaped cross-section is analyzed, under uniform bending and axial compression, and compared with the CUFSM and GBTul results. Additionally, more validation models are shown in Lazzari (2020).

The material employed to validate all the models is taken as an isotropic CFS, with elastic modulus of 200 GPa (200 kN/mm²), Poisson's ratio of 0.3 and transversal modulus of 76.92 GPa (76.92 kN/mm²).

3.1 Lipped channel column

The main goal of this validation is to compare a signature curve with analytical procedures found in literature. For this model, a CFS lipped channel column with a simply supported end boundary condition and only one term of buckling shape half-wave is analyzed. The geometry of the cross-section has $b_w=100$ mm, $b_f=70$ mm, $b_s=15$ mm and $t=2.70$ mm, web, flange and edge stiffener width and

thickness, respectively. The finite strip model is composed of 19 nodal lines, 18 strips and 76 degrees of freedom, with a total of 200 columns in a length range of 10 mm to 100000 mm, in logarithmic scale. The results are shown in Figure 5.

The critical buckling load from FStr is compared to: (i) the approximated semi-analytical expression for local buckling (Eq. 8), given by Batista (2010)

$$P_{crl} = K_l \frac{\pi^2 E}{12(1-\nu^2)} A \left(\frac{t}{b_w} \right)^2 \quad (8)$$

and presented in the Brazilian standard NBR 14762 (ABNT, 2010); (ii) distortional buckling equation, from Cardoso et al.(2017); (iii) global buckling equations given by Timoshenko and Gere (1961) and mostly into the codes, e.g. (ABNT, 2010; AS/NZS, 2018; AISI, 2016). Also, the results from the proposed program FStr are compared with those from CUFSM and GBTul.

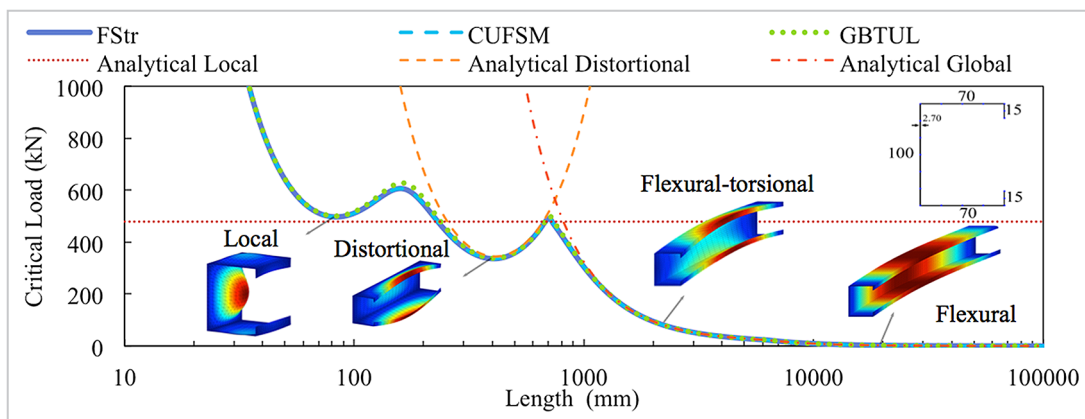


Figure 5 - Signature Curve comparison between FStr, CUFSM, GBTul and analytical procedures of LC 100x70x15x2.70 column with S-S end boundary condition with one term of half-wave.

Notice in Figure 5 that the FStr program obtained practically the

same signature curve as the CUFSM, with an average relative difference of

0.0012% and a standard deviation of 0.27%. The GBTul program also

provided a solution close to the finite strip method, with an average relative difference of 0.2377% and a standard deviation of 0.91%.

Further, the analytical procedures offered a great precision for the critical

loads at critical lengths. The critical buckling loads at half-wavelength for local and distortional buckling, as highlighted in Figure 5, has demonstrated that the matrix formulation from FStr is following the analytical formulations.

With a relative difference of 3.8% for the local buckling equation, Eq. 8, and 2.5% for the distortional buckling procedure (Cardoso *et al.*, 2017), the FStr accuracy against these analytical equations are validated.

3.2 I-shaped cross-section Beam and Column

For this validation, the purpose is to analyze the critical buckling loads and modes for a section with closed cells, in bending and axial compression. Given by Gonçalves

et al., (2009), the cross-section is I-shaped with two triangular closed cells separated by the web and with unequal flanges, described in Figure 6. The finite strip model cross-section is

composed of 21 nodal lines, 22 strips and 84 degrees of freedom, with a total of 100 lengths in a length range of 10 mm to 10000 mm, in logarithmic scale.

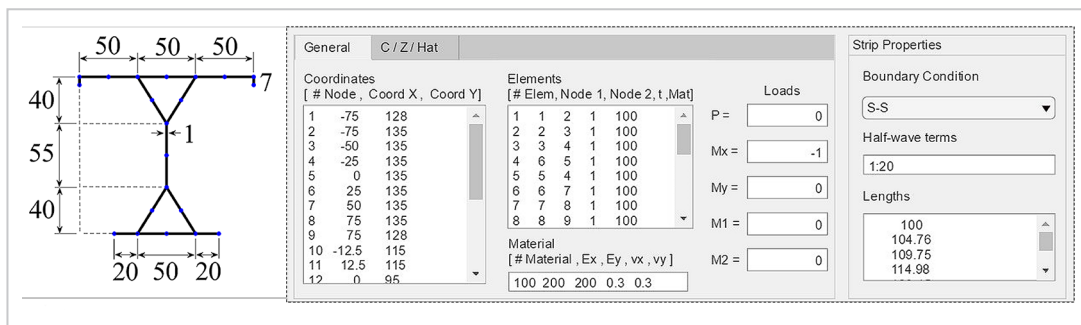


Figure 6 - I-shaped cross-section geometry (in mm) and initial parameters set up for the analysis in pure bending.

Figure 7 and Figure 8 show the signature curve for the FStr, CUFSM and GBTul programs. Also, in the same graphs, are displayed the relative difference between the proposed program with

CUFSM and GBTul. All the results given are with one term of half-wave. Additionally, with the purpose of comparing the single half-wave term, a solution from FStr with 10 half-waves (column) and

with 20 half-waves (beam) are illustrated. Moreover, in Figure 9, displayed are the critical buckling modes for the I-shaped column and beam, comparing the FStr with CUFSM and GBTul graphical results.

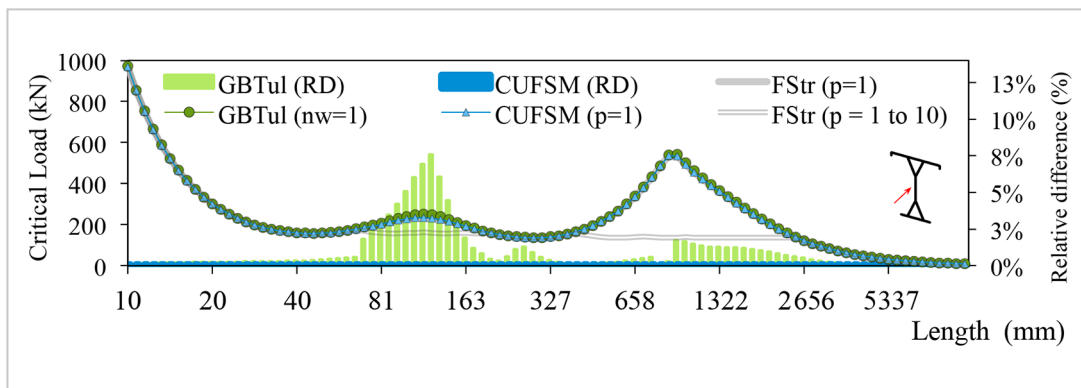


Figure 7 - Signature curve comparison and relative difference (RD) between FStr, CUFSM and GBTul of the I-shaped cross-section column with S-S end boundary condition with one term of half-wave ($p = 1$ and $n_w = 1$).

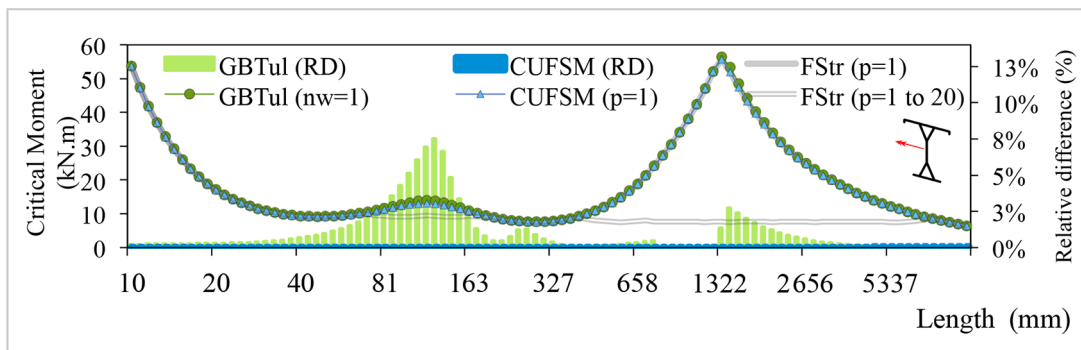


Figure 8 - Signature curve comparison and relative difference (RD) between FStr, CUFSM and GBTul of the I-shaped cross-section beam with S-S end boundary condition with one term of half-wave ($p = 1$ and $n_w = 1$).

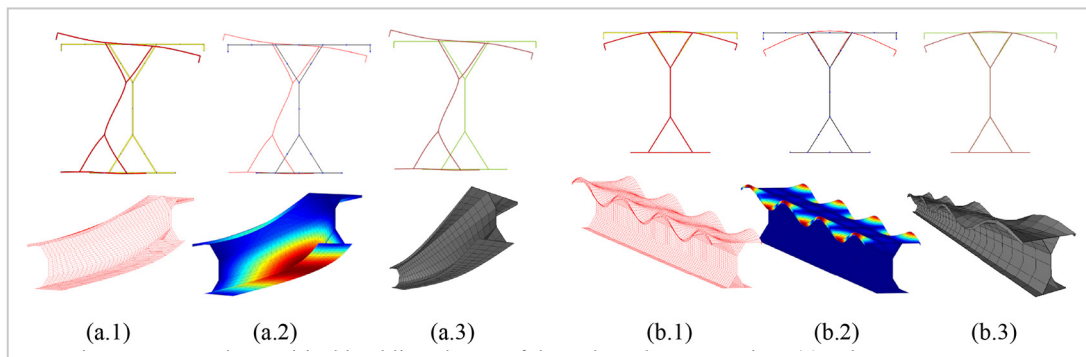


Figure 9 - 2D and 3D critical buckling shapes of the I-shaped cross-section: (a) column at $L=1000$ mm, $\bar{\gamma} = 0.5L$ and $p=1$, (a.1) CUFSM, (a.2) FStr and (a.3) GBTul; (b) beam at $L=2009$ mm, $\bar{\gamma} = 0.5L$ and $p = 1:20$, (b.1) CUFSM, (b.2) FStr and (b.3) GBTul.

As can be seen, the FStr gives the same solution as CUFSM, and compared to GBTul, the results are very close. At length of 123 mm, the GBTul gives the higher relative difference, of 7.5%, for the column and beam, while the CUFSM gives a maximum relative difference of 0.001% and 0.08%, for the column and

beam, respectively. The GBTul solution, gives an average relative difference of 0.94% and 1.01% with a standard deviation of 1.5% and 1.6%, for the column and beam analyzed with one term of half-wave ($p = 1$). When the half wave-terms are increased to 10 and 20 (column and beam), the average relative

difference decreases to 0.4% and 0.7% (column and beam) and the standard deviation also decreases to 0.3% and 0.4% (column and beam). On the other hand, when the FStr is compared to CUFSM, the average and standard deviation of the relative difference is not higher than 0.1% in any case.

4. Final remarks

An FSM computer application entitled FStr was developed, in order to assist the elastic buckling analysis. The program implemented in MATLAB, has an accessible and easy graphical user interface, and is conceived to attend research activities as well as engineering design of thinwalled structures with arbitrary cross-sections.

FStr is validated, comparing with results from analytical procedures

and other computer applications, i.e. CUFSM and GBTul. Additionally, in Lazzari, (2020) are given more validation models, including a boundary condition validation and a comparison with a finite element analysis solution. In summary, with all these validations, the FStr Computer Application is certified as a reliable source for an elastic buckling analysis, which can be applied in a numerous types of structural stability problems.

The FStr is a free computer application, however, it is not an open source i.e. the users do not have access to the computational routines. FStr can be accessed in the GitHub release repository <https://github.com/joaoadelazzari/FStr/releases>, in the file exchange from MathWorks website <https://www.mathworks.com/matlabcentral/fileexchange/74306> or in the google web site <https://sites.google.com/coc.ufrj.br/fstr/>.

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