A general method for designing non-symmetrical composite steel and concrete columns

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Abstract

The study of non-symmetrical composite columns is complex and European and North American standards only approach this issue superficially. In this text, a new proposal is presented for non-symmetrical composite column design subject to biaxial bending and axial compression, based on the compatibility of the deformation method, which considers the nonlinearity of the material with constitutive models according to Brazilian standards and geometric nonlinearity, as well as the equilibrium of the column in the displaced position. As a strategy for solving the general method, use is made of the "fiber element method", and for solving problems involving nonlinear equilibrium equations, the Newton-Raphson method.

Keywords: composite columns, design, general method, Strain Compatibility Method, fiber element method, Newton-Raphson.

1. Introduction

Eurocode 4 (2004) presents two methods for designing composite columns. There is a simplified one, which is valid for columns with uniform and bi-symmetrical section along the length and another, which generally can be used in columns that do not follow this regularity, considering the effect of the geometric and material nonlinearity, local buckling, cracking, creeping and shrinkage of concrete (if the latter two significantly reduce the structural stability). The influence of the stress between cracks on the increase in bending stiffness of concrete can be considered. The cross section can be assumed to remain plane after deformation. According to AISC (2010), the general method, called compatibility of deformations method, assumes a linear distribution of the cross section deformations. Stress x strain ratios for steel and concrete must be obtained experimentally, even for similar materials. The maximum allowed strain for concrete is 3‰.

Neither of the two foreign standards details the general method, only providing the recommendations transcribed herein. With some simplifications, a sequence of calculations is proposed here for a general method of composite column design based on the fiber element method and inspired in the procedures of Griffis (2003), Tawil *et al.* (1995), França (1984 and 1991) for calculating reinforced concrete members subjected to composite oblique bending and in the numerical model presented by Charalampakis and Koumousis (2008). For convergence analysis, the Newton-Raphson method will be used. Here, the term "general" refers to the fact that the method can be used in any type of cross section, with any variation of section along the longitudinal axis, any type of load, any boundary conditions and any stress x strain ratio of the material.

With the application of the general method, described in the sequence, as well as the simplified methods from Brazilian, European and North-American standards, the CalcPM code was developed in programming language C #, which is the main Microsoft's effort in programming language, being created in Visual Studio 2010, which is also of Microsoft.

2. Cross section analysis

The general solution of the cross section is to find the equilibrium between internal and external stresses applied to the cross section. The solution is presented in equations 1 to 3, wherein: N_{Sd} , $M_{Sd,x}$, $M_{Sd,y}$ are the design value of the efforts and N_{Rd}, $M_{Rd,x}$, $M_{Rd,y}$ are the design resistances, in relation to the cross section centroid.

$$N_{Sd} = N_{Rd} \tag{1}$$

$$M_{Sd,x} = M_{Rd,x} \tag{2}$$

$$M_{Sd,y} = M_{Rd,y} \tag{3}$$

The cross section resistances should be calculated from the imposed distribution of strains. The cross section studied consists of steel sections, reinforcement and concrete. In this text, the following hypotheses are adopted: there are no torsional stresses, there is no local buckling of the elements; there is complete solidarity between the materials (resulting in the fact that continuous points of two different materials, belonging to the contact surface, have the same specific deformation), the plane cross-section remains plane after deformation, there are neither initial nor residual strains or stresses.

Starting from these assumptions, in order to obtain the stress distribution in the cross section from the strain, three parameters of the plane equation are needed. The formulation of the plane equation used in this text is the same adopted by França (1984) and is shown in equation 4, wherein: ε_0 is the strain of the section centroid, positive in case of shortening, κ_x is the curvature of the plane in relation to the XX axis, positive as shown in Figure 2 and κ_y is the curvature of the plane relative to the YY axis, positive as shown in Figure 1.

$$\varepsilon_z(x,y) = \varepsilon_0 + \kappa_x X + \kappa_y Y \tag{4}$$

From the strains imposed to the cross section, resistances can be found by integrating the resultant stresses from each fiber, according to equations 5 to 7.

$$N_{Rd} = \int_{A_a} \sigma_a(x,y)dA + \int_{A_s} \sigma_s(x,y)dA + \int_{A_c} \sigma_c(x,y)dA \tag{5}$$

$$M_{Rd,x} = \int_{A_a} X\sigma_a(x,y)dA + \int_{A_s} X\sigma_s(x,y)dA + \int_{A_c} X\sigma_c(x,y)dA \tag{6}$$

$$M_{Rd,y} = \int_{A_a} Y\sigma_a(x,y)dA + \int_{A_s} Y\sigma_s(x,y)dA + \int_{A_c} Y\sigma_c(x,y)dA \tag{7}$$

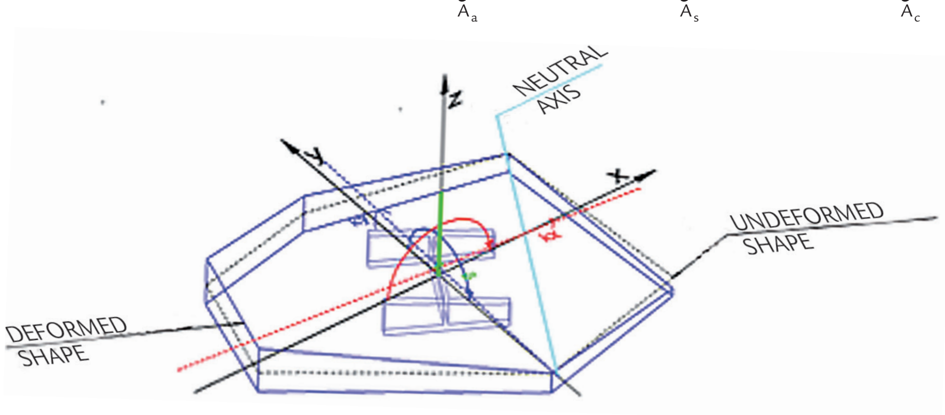


Figure 1
Strain of one dz thread of the element.

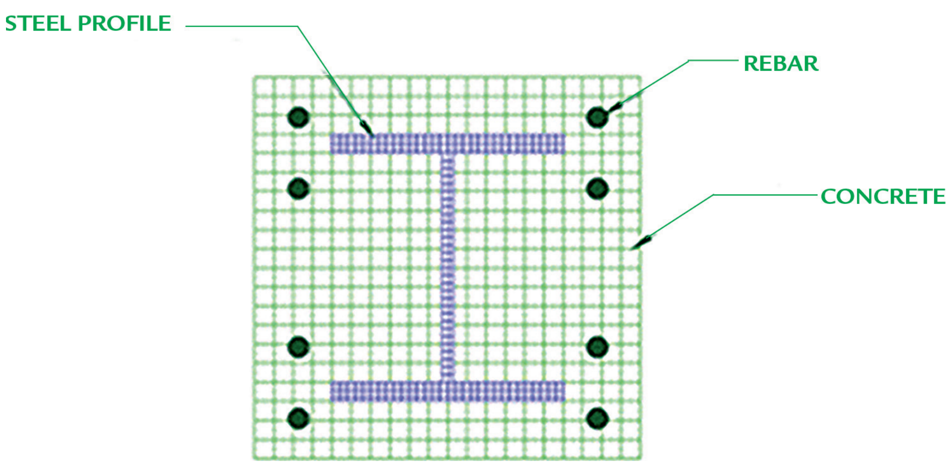


Figure 2
Discretization of the I section completely lined with concrete.

In order to obtain the equilibrium of the bending moments and the axial force, one should use the Newton-Raphson convergence method, by finding the equation of the plane that balances the stresses. The calculation method used in solving the general method is the "fiber element method", which, broadly speaking, is the discretization of the cross section into smaller elements (fibers) and each one of these fibers takes one of the three constitutive models: considering the stress-strain properties of concrete, of the steel reinforcement or of the steel section, as exemplified in Figure 2.

3. Analysis along the lenght of the column

The general solution along the bar is to find the equilibrium between action effects and resistances applied to all cross sections along the bar.

The conditions of equilibrium and strain compatibility must be satisfied in a finite number of cross sections along the longitudinal axis of the column, being the displaced longitudinal position of the column calculated by double integration of the curvature with variation of the displacement between the cross sections, according to the linear equations, parabolic equations, constants or any other function that represents this variation.

The method used herein for solving the composite column is the numerical integration of the column, the stiffness of which is obtained by the bending moment-curvature diagram and oblique biaxial composed flexion. Thus, analysis is performed considering the application of the actions in the deformed configuration, which is obtained by successive iterations until it reaches the equilibrium configuration. The column is simplified into a longitudinal bar formed by a finite number of elements and the equilibrium calculations of the cross section are performed at each node, see Figure 3.

In the case of initial eccentricity applied to the column, it is verified to be enough to introduce it linearly along the column, so that the procedure starts from an initial displaced position of the column. Despite the significant change in the final result, it in no way affects the sequence for the design.

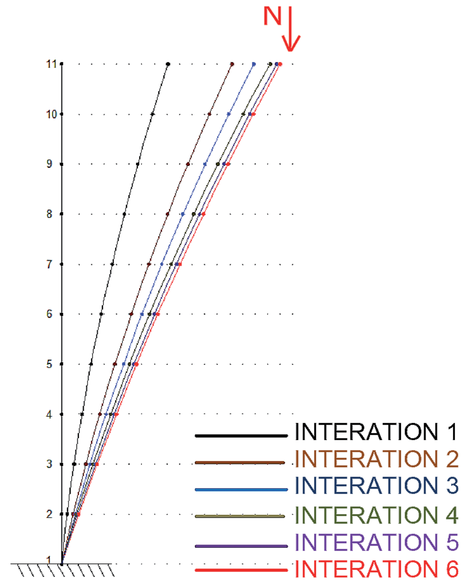


Figure 3
Load applied to the deformed configuration of the fixed-free composite column.

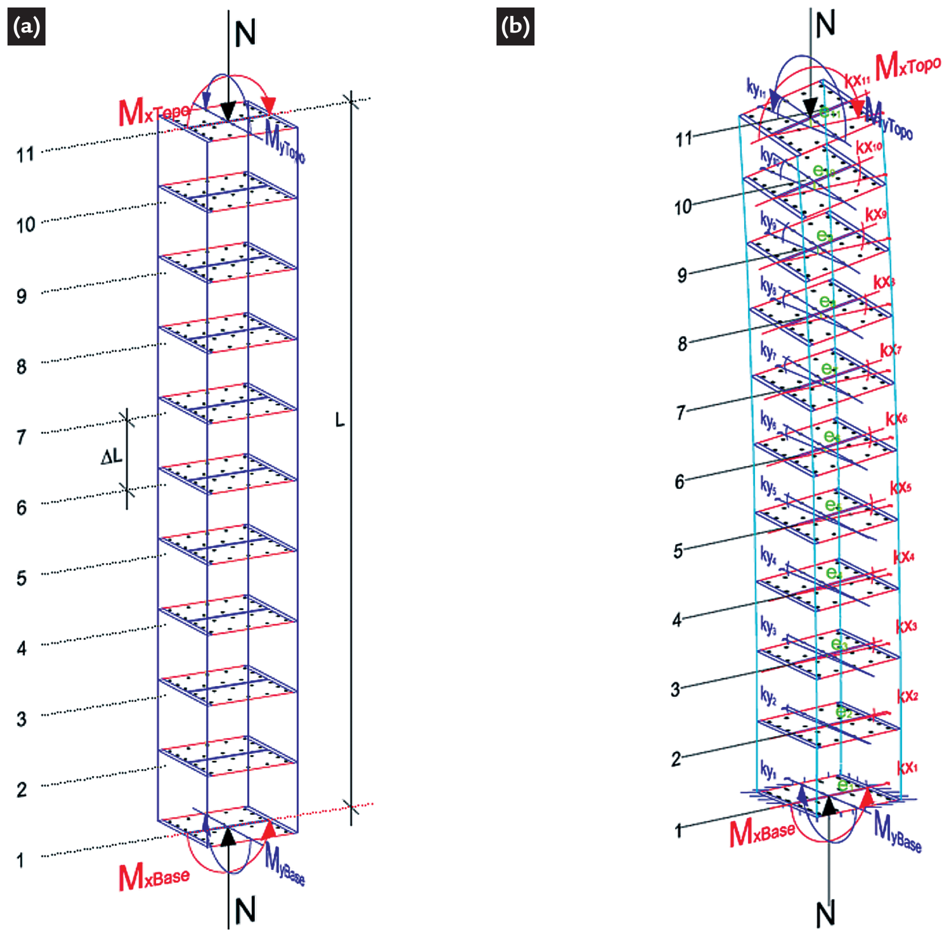


Figure 4
Discretized composite column in undeformed (a) and deformed (b) position.

3.1 Application to a fixed-free bar

One can divide the solution of the general method into steps, presented here for the fixed-free bar:

a) Introduction of the external loads and subdivision of the longitudinal bar.

b) Determination of the deformed position of the cross sections at the nodes, from the non-deformed position (1st Iteration).

The determination of the deformed position of the cross sections is performed according to section 2 in the cross section at each node along the length of the discretized composite column, obtaining the equations of the plane that balances them, equation 4, in relation to the loads acting on each node.

To exemplify, the composite column shown in Figures 4 (a) and (b) is used, with symmetrical cross section on XX and YY and loaded by biaxial composed bending.

c) Determination of displacements from the non-deformed position (1st iteration).

The equation of the plane provided in 4 defines coefficients κ_x and κ_y , which represent the curvature of the section relative to axis XX and YY, respectively.

The first integration of the curvatures provides the rotation of the cross section at any position of the longitudinal axis of the composite column, according to equations 8 and 9, wherein: $\varphi_x(Z)$ is the rotation of the cross section relative to axis XX at position 'z' and $\varphi_y(Z)$ is the rotation of the cross section relative to axis YY at position 'z'. Boundary conditions $\varphi_x(0) = 0$ and $\varphi_y(0) = 0$ imply that $C_{x1} = 0$ and $C_{y1} = 0$.

$$\varphi_x(z) = \int_0^z \kappa_x(z).dz + C_{x1} \quad \text{and} \tag{8}$$

$$\varphi_y(z) = \int_0^z \kappa_y(z).dz + C_{y1} \tag{9}$$

The displacement of the horizontal axis of the columns is achieved by double integration of the curvatures of the cross sections, as shown by equations 10 and 11 to axes XX and YY, wherein:

d_x is the displacement of the cross section relative to axis xx at position 'z' and d_y is the displacement of the cross section relative to axis yy at position "z". The boundary conditions $d_x(0)=0$ and $d_y(0)=0$ imply that $C_{x2} = 0$ and $C_{y2} = 0$.

$$d_x(z) = \int_0^z \varphi_x(z).dz \quad \text{and} \tag{10}$$

$$d_y(z) = \int_0^z \varphi_y(z).dz + C_{y2} \tag{11}$$

Therefore, the rotations of the cross sections and the displacements with respect to axes x-x and y-y for the fixed-free composite column are presented below in equations 12 through 15.

$$\varphi_x(z) = \int_0^z \kappa_x(z).dz \quad \text{and} \tag{12}$$

$$\varphi_y(z) = \int_0^z \kappa_y(z).dz \tag{13}$$

$$d_x(z) = \int_0^z \varphi_x(z).dz \quad \text{and} \tag{14}$$

$$d_y(z) = \int_0^z \varphi_y(z).dz \tag{15}$$

d) Determination of the stresses and strains of the 2nd iteration from the deformed position. The consideration of the application of the loads from the deformed position comprises applying the geometric non-linearity itself to the process, together with the material non-linearity, which has already been considered for obtaining the equation of the plane that balances the stresses.

In the case of columns subjected to loads at the top and bottom, this consideration leads to increased first iteration bending moments which are generated by the eccentric application of the normal force, as shown by equations Equations 16 and 17, wherein: e_x is the displacement at the top of the composite column in the X-X direction, e_y is the displacement at the top of the composite column in the Y-Y direction, d_{xi} is the displacement of section 'i' considered in the X-X direction and d_{yi} is the displacement of section 'i' considered in the Y-Y direction.

$$M_{xi(z)} = M_{xBase} + \left(M_{xBase} - M_{xTopo}\right)\frac{z}{L} + N\left(e_x - d_{xi}\right) \tag{16}$$

$$M_{yi(z)} = M_{yBase} + \left(M_{yBase} - M_{yTopo}\right)\frac{z}{L} + N\left(e_y - d_{yi}\right) \tag{17}$$

e) Successive iterations until either convergence or buckling. The additions of loads made in item (d) result in a new deformed shape of each cross section, which in turn generates a new displaced position of the bar along the longitudinal axis and the respective increase of stresses. Then, the process is repeated successively until one of the following three situations occurs: buckling of the bar by excessive displacement, rupture of materials due to tensions above the standardized limits or process convergence considering a certain criterion.

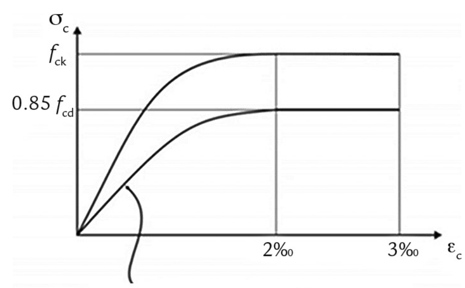
For solving the iterative process, the Newton-Raphson method was used.

4. Materials

The procedure hitherto described applies to any relationship of constitutive materials. For application purposes and as a suggestion, the following material properties were adopted. The general method is a powerful tool for designing columns, being often used in the design of reinforced concrete columns under the provisions of the ABNT NBR 6118:2007 standard. Thus, our aim is to adjust the general method using the constitutive models of the materials presented by the Brazilian standard for resolving composite columns. In principle, the main restrictions for the design of composite columns that distinguish them from concrete columns are the maximum allowed strain of concrete, which should be considered equal to 0.003 mm / mm, and the non-use of the method for composite columns subjected to tension or bending-tension stresses. Another important recommendation is that, in design, one should not use sections filled with concrete, classified as semi-compact or thin, since concrete plasticizing does not occur and steel plasticizing is either partial or nonexistent.

4.1 Stress x Strain Ratio of concrete

The stress x strain ratio of concrete is given by the ABNT NBR 6118:2007 standard, which allows the use of the concrete diagram shown in Figure 5, adapted to meet the strain requirement of 0.003 mm / mm, wherein: σ_c is the compressive stress in the concrete, ε_c is the strain of the concrete and f_{cd} is the compression strength for concrete calculation.



$$\sigma_c = 0.85\, f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{2‰}\right)^2\right]$$

Figure 5
Ideal Stress x Strain Diagram.
Source: adapted from the ABNT NBR 6118:2007.

4.2 Stress x Strain Ratio of steel, bars and section

The stress x strain ratio of the steel bars follows that proposed by the ABNT NBR 6118:2007 standard for passive steel reinforcement shown in Figure 6, wherein: fyd is the calculation yield strength of the reinforcement, f_{yk} is the characteristic yield strength of the reinforcement, and Ecs is the secant modulus of the elasticity of concrete.

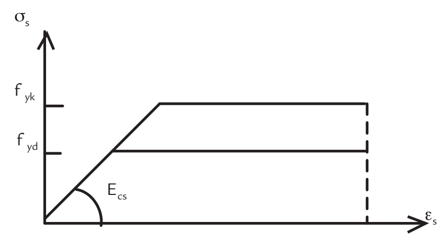


Figure 6
Stress x Strain Diagram for reinforcement steel.

According to the ABNT NBR 6118:2007 standard, this diagram is valid for temperatures between -20°C and 150°C and can be applied to tension and compression. The stress x strain ratio of the steel section is also shown in Figure 6, following the limits recommended by the ABNT NBR 8800:2008 standard, which allows the use of steels with maximum strength of 450 MPa and a ratio between ultimate stress (f_u) and the yielding stress (f_y) of not less than 1.18.

5. Results

The CalcPM code (see item 1 of this paper) allows designing composite columns according to the Brazilian standard ABNT NBR 8800:2008 (two models), AISC (2005 and 2010) and Eurocode 4 (2004), for four types of standardized columns. It is not the aim of this paper, but we inform that in Silva, Silva; Munaiar (2012) and Silva (2012), comparisons among the simplified methods are shown. Dozens of comparisons were done (for curiosity two examples are shown in Figure 7). In general, Model II of the Brazilian standard and Eurocode, its origin, is more economical than the Model I and its origin, the North-American standard. More comments in Silva (2012).

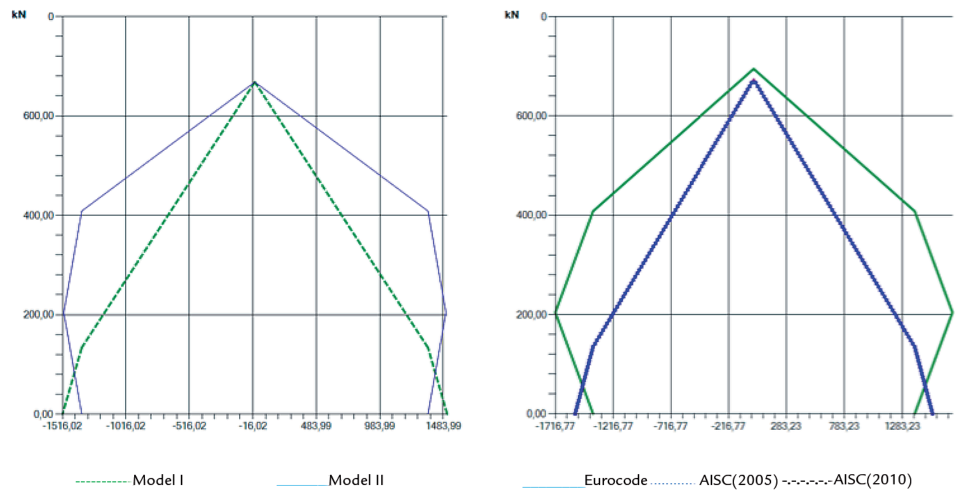


Figure 7
Combined compression and bending (kNxkN cm) to concrete filled circular hollow section columns (D=15.24 cm, t=0.225 cm). Left - Brazilian standard; right - Eurocode and AISC standards.
Source: Silva (2012)

Roik and Bergmann (1990)					CalcPM	Comparison
Column tested	Dimension (cm x cm)	Lenght (cm)	Excentricity (cm)	Test results (kN)	general method	gen.met./Test
1	28 x 28	346	0	3617.00	4258.99	1.18
2	=	=	4	2825.00	2855.81	1.01
3	=	=	10	1800.00	1622.53	0.90
4	30 x 30	546	1	3938.00	2864.61	0.73
5	=	=	3	2640.00	2093.50	0.79
6	=	=	11	1608.00	1023.24	0.64
7	28x28	346	0	2654.00	3428.71	1.29
8	=	=	4	1998.00	2509.51	1.26
9	=	=	10	1706.00	1440.56	0.84
10	30x30	546	1	2538.00	2128.07	0.84
11	=	=	3	1664.00	1694.29	1.02
12	=	=	11	1328.00	803.01	0.60
					Average	0.93

Table 1
Comparison between tests and general method (CalcPM).
Source: Silva (2012)

Within CalcPM, the general method was implemented as shown in this text and compared to the few experimental tests available, 12 tests by Roik and Bergmann (1990) for non-symmetrical totally encased columns. In Table 1 we present a summary of the types of columns tested and comparison of the results. Columns tested #1 to #6 are square concrete with profile, with section I eccentrically placed and columns tested #7 to #12 also, but section T instead I. The general method is about 10% of safety side.

6. Conclusions

The study of non-symmetrical composite columns is complex. Few standards address this issue and, when they approach it, do it so superficially. This text was a proposal for a general calculation method using section strains compatibility, fiber element method and Newton-Raphson method. The results, when compared to the few test results available, are favorable to safety. More experimental testing is necessary.

7. Ackowledgements

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