

## Structural reliability analysis of steel plane frames with semi-rigid connections

<http://dx.doi.org/10.1590/0370-44672017710044>

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### Abstract

The design codes for structural design traditionally establish the adoption of partial safety factors to deal with uncertainties. These standards do not allow one to know the real probabilities of failure of a structure. What is needed, from a practical and scientific point of view, is a more consistent approach, based on reliability theory. This article addresses an initial attempt at determining the safety levels of structures based on advanced analysis and design with structural reliability theory. In this context, the authors consider the effects of the geometric nonlinearity and flexibility of connections in the reliability analysis, which aims at setting a certain displacement as the service limit state. For this purpose, a computer program was written called Structural Reliability Module. This program uses a First Order Reliability Method to analyse reliability. To carry out the structural analysis of steel structures, this study uses the program Computational System for Advanced Structural Analysis. Validating the Structural Reliability Module involved comparing the results from other authors with those generated by the program. Results for the structures under study indicate the efficiency of the implemented measures. Results suggest that, when a certain displacement is set as a service limit state, the reliability of the structures under analysis are only slightly influenced by geometric nonlinearity but considerably so by semi-rigid connections.

**Keywords:** reliability analysis, steel plane frames, geometric nonlinearity, semi-rigid connections.

## 1. Introduction

This article presents the joint computational implementation of theories related to structural reliability and the advanced analysis of structures used by Agostini (2014). This research is the first step in an attempt to evaluate the structural safety of steel portal frames designed with ad-

vanced analysis.

Agostini (2014) aimed at developing a computational program that could provide an analysis of the structural reliability integrated into the structural analysis of steel portal frames. Reliability analysis was carried out with the First Order Re-

liability Method (FORM) method. The structural advanced analysis was carried out with the program Computational System for Advanced Structural Analysis (CS-ASA) (Silva, 2009) considering the nonlinear geometric behavior and the flexibility of beam-to-column connections.

## 2. Computational development and implementation

To create the Structural Reliability Module (SRM) computational program, researchers used the Fortran 95 programming language in the Microsoft Visual Studio 2010 environment (Microsoft Cor-

poration, 2010). SRM was designed to be integrated with the mathematical and statistical routines of the numeric library for Fortran, the IMSL™ version 6.0 (Visual Numerics®, 2006), as well as the CS-ASA

structural analysis program version developed by Silva (2009).

SRM was developed to work on an implicit performance function (limit state equation) allowing the reliability analysis of

different types of portal frames. The condition for not fulfilling a certain service limit state is linked to a specific displacement.

The FORM method, present in SRM, requires the calculation of gradients of the performance function for all random variables. SRM uses a central finite difference process for calculating gradients of the performance function. This process consists of changing the value of each random variable by a small increase and decrease. For each increase and decrease, a call to the CS-ASA finite element program for the displacement supply occurs. With the calculated displacement (cd) for the node and direction under analysis as well as the fixed displacement limit (dl), the performance function  $G(u)=1-(cd/dl)$  is evaluated. In the SRM, the performance function was allocated in a specific subroutine. The two results of the evaluation of the performance function, generated from the increase and decrease in the value of the random variable, will enable the calculation of the performance function gradient for the random variable in question. This entire process is performed for all the random variables present in the reliability analysis to compose the gradient vector of each iteration of the FORM method.

The proposed implementation allows one to consider, as random variables, one elastic modulus, two transverse cross-section areas, two moments of inertia, three dead loads, and five live loads. To model these variables, one implements the following probability distributions: normal, log-normal, and type I extreme (largest value). For the latter two, two subroutines were created with a specific link to the reliability module and aim at calculating the mean and the standard deviation equivalent to a normal distribution.

To transform the variables, a Nataf transformation (Melchers, 1999) was used and, to determine the design point in reduced space, the HLRF algorithm was used (Hasofer and Lind, 1974; Rackwitz and Fissler, 1978). Equations to determine the sensitivity indices and the importance

factors, as shown by Sagrilo (2004), were also implemented in the SRM. To integrate the SRM with the IMSL™ library (Visual Numerics®, 2006), it was necessary to make adjustments in the project developed on the Microsoft Visual Studio 2010 environment (Microsoft Corporation, 2010), following the orientations available on the Intel® website (Intel® Developer Zone, 2011).

The process of reliability analysis follows the original structure from the CS-ASA program. This program works with three input files. The first input file contains general information such as the type of analysis, coordinates, loads, properties of materials and sections. The second contains the parameters related to the nonlinear solution strategy. And, the third, refers to the dynamic analysis, which is not approached herein. A fourth input file was created to carry out the reliability analysis. Readings of this file can only be carried out by SRM. The first parameters that make up the fourth input file are as follows: reliability analysis method, number of iterations for FORM, tolerance for interruption of the iterative process involving the FORM method, displacement limit for the node in analysis, numerical parameter of finite differences for carrying out an increase and a decrease on the value of each random variable, and the indicator of correlated variables. In explanation, for the actual version of SRM, it is not possible to consider the correlation between variables. After those parameters and in the same line for each random variable: type of random variable, mean, and standard deviation values. Integrating the CS-ASA output files, there was created a specific output file that shows both the input file parameters related to the reliability analysis and the results of this analysis.

The reliability analysis starts, from the CS-ASA program, with readings on the first input file. The SRM is activated as soon as a reliability analysis option is found in this file. If the reliability analysis is not defined, reading of file with general information goes on normally to carry out one of the

types of analysis that make up CS-ASA. Managing the analysis process, SRM calls upon two subroutines which belong to the CS-ASA program. Firstly, the subroutine responsible for assembling the vector of nodal loads. Secondly, the subroutine that is responsible for producing the linear or nonlinear solution of the structure under analysis, providing the structural response in terms of displacements.

Besides taking the geometric nonlinearity into account, researchers enhanced the SRM with a functionality that enables it to carry out reliability analysis in structures with semi-rigid connections that display linear or nonlinear moment-rotation behavior. To do this, this study adjusted the Richard and Abbott, (1975) model, already implemented in the CS-ASA program, to carry out reliability analyses of structures that show a nonlinear moment-rotation behavior. The choice to adjust this model is due to the availability of results by Haldar and Mahadevan, (2000) related to the Richard and Abbott, (1975) model. Also, as affirmed by Pinheiro (2003), this model always offers a positive rigidity, is computationally effective, and is one of the most used to represent semi-rigid connections. To carry out the reliability analysis considering the connections present in the structure as semi-rigid, but with linear moment-rotation behavior, the input file with general information is edited so as to indicate the initial rigidity ( $Sc$ ) of the connection.

The flow chart in Figure 1 indicates CS-ASA with new functionality in the dotted box, reliability analysis. Interaction between static reliability analyses, denoted in Figure 1 by a double-headed arrow, occurs with a command from the SRM and aims at providing a structural response in terms of displacement. The dotted boxes in Figure 1 - labeled “geometric nonlinearity” and “rigidity of connections” - indicate the possibility of considering these effects when calculating displacement. Figure 2 presents the flow chart for SRM.

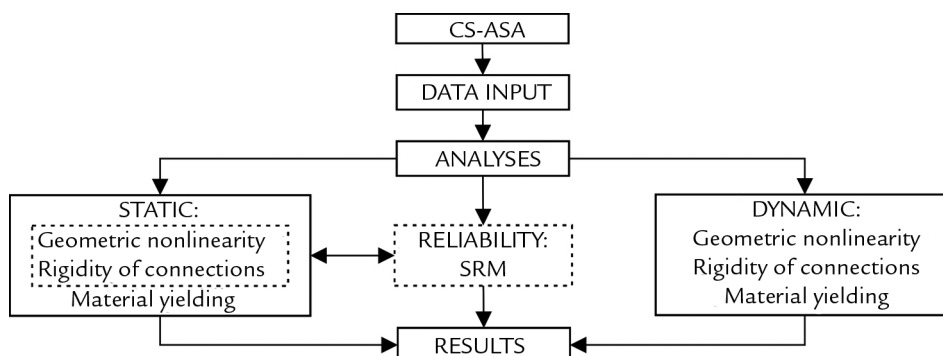


Figure 1  
New functionality  
of the CS-ASA: reliability analysis.

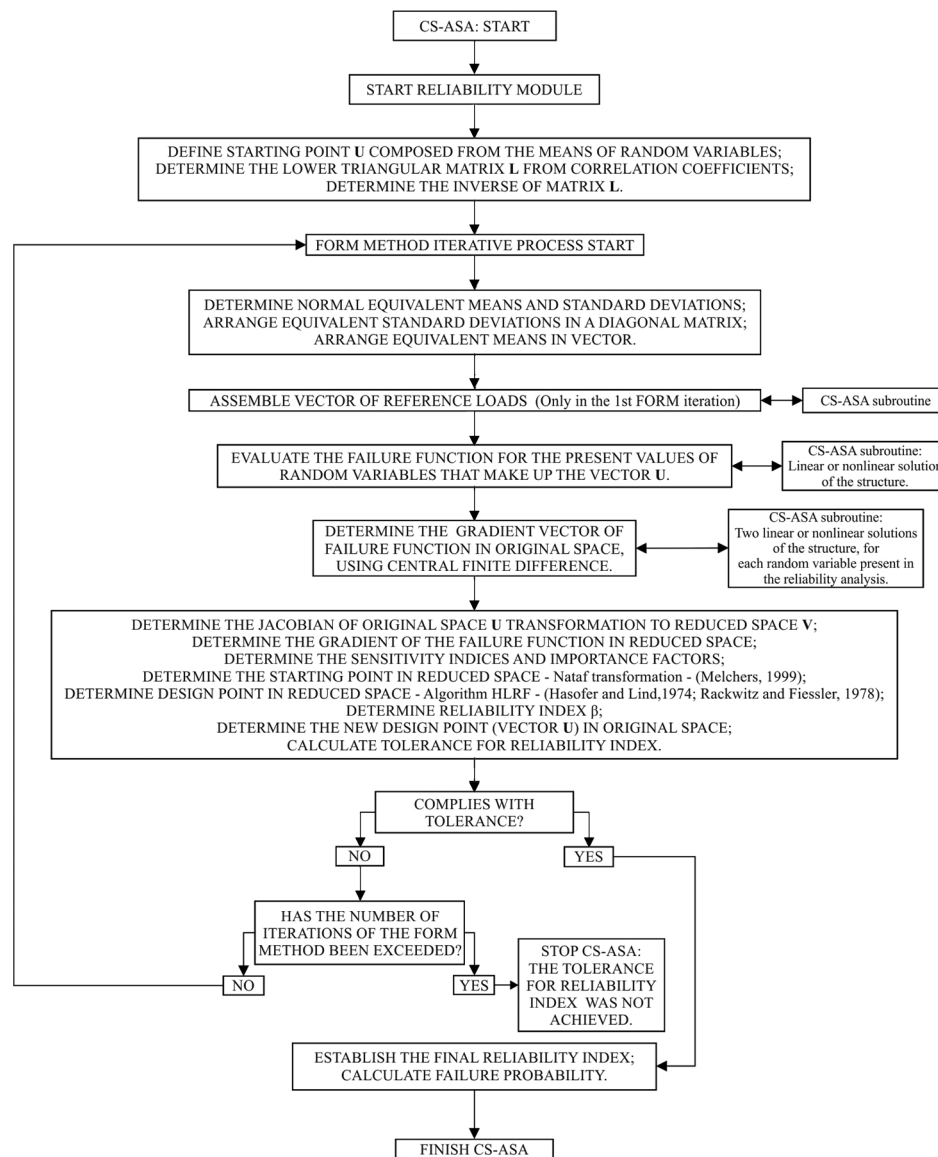


Figure 2  
Flow chart for SRM.

### 3. Results

Two examples analyzed by Haldar and Mahadevan, (2000) were used to compare with the methodology developed in the previous section. Among the main considerations regarding the geometric nonlinear analysis are the following: adoption of second order formulation

SOF-2 (Silva, 2009) defined in an updated Lagrangian referential and based on the Yang and Kuo, (1994) formulation and Euler-Bernoulli theory, use of a constant load increment strategy, use of constant load iteration strategy, determination of the initial increment in load parameter at

10% of the total load, division of loading into 10 increments, and adoption of Newton-Raphson standard method. It should be noted that the validation of the SRM was done by comparing it with data provided by Agostini and Freitas, (2011) and Haldar and Mahadevan, (2000).

#### Example 1: Plane frame with three elements

The first structure that was analyzed is represented in Figure 3. Statistical properties of random variables considered in the analysis of that structure are described

in Table 1. As regards the present example, structural reliability analysis was carried out considering the geometric nonlinearity and the presence of semi-rigid connections

in the joints between columns and beam. The limit state in question refers to the horizontal displacement of Node 2, limited to 0.0114m in service.

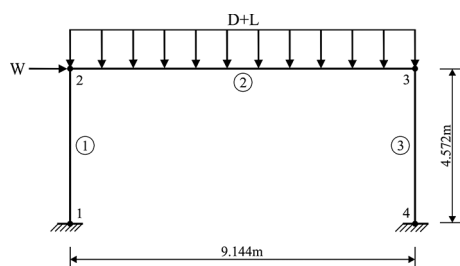


Figure 3  
Plane frame with three  
elements: geometry, load and discretization.

Variables	Units	Nominal values	Mean /Nominal value	Coefficient of variation	Type of distribution
<i>E</i>	MPa	199,948.04	1.00	0.06	Lognormal
<i>A</i>	m <sup>2</sup>	0.01271	1.00	0.05	Lognormal
<i>I</i>	m <sup>4</sup>	0.0003970848	1.00	0.05	Lognormal
<i>D</i> (Dead load)	kN/m	43.78	1.05	0.10	Lognormal
<i>L</i> (Live load)	kN/m	16.05	1.00	0.25	Extreme Type I
<i>W</i> (Wind load)	kN	28.91	0.78	0.37	Extreme Type I

Table 1  
Statistical properties of random variables.

As for the portal frame in Example 1, represented by the structure shown in Figure 3, three different connecting behaviors were simultaneously analyzed for Nodes 2 and 3. These analyses took into account the nonlinear behavior of the connection using the Richard and Abbott, (1975) mathematical model. The model

comprises the following parameters: initial stiffness (*k*), strain-hardening stiffness (*k<sub>p</sub>*), reference moment (*M<sub>0</sub>*), and a parameter defining the shape of the curve (*n*). Figure 4 shows the values of the four parameters for all three analyzed cases; the values were inserted into the analysis as non-random. Haldar and Mahadevan, (2000) analyzed

the reliability considering also the random behavior of the parameters that make up the Richard and Abbott, (1975) model. In order to better visualize the behavior of the connections under analysis, moment (*M*)-rotation (*φ<sub>c</sub>*) curves 1, 2, and 3 - indicated in Figure 4 - were generated using the equation presented in the figure.

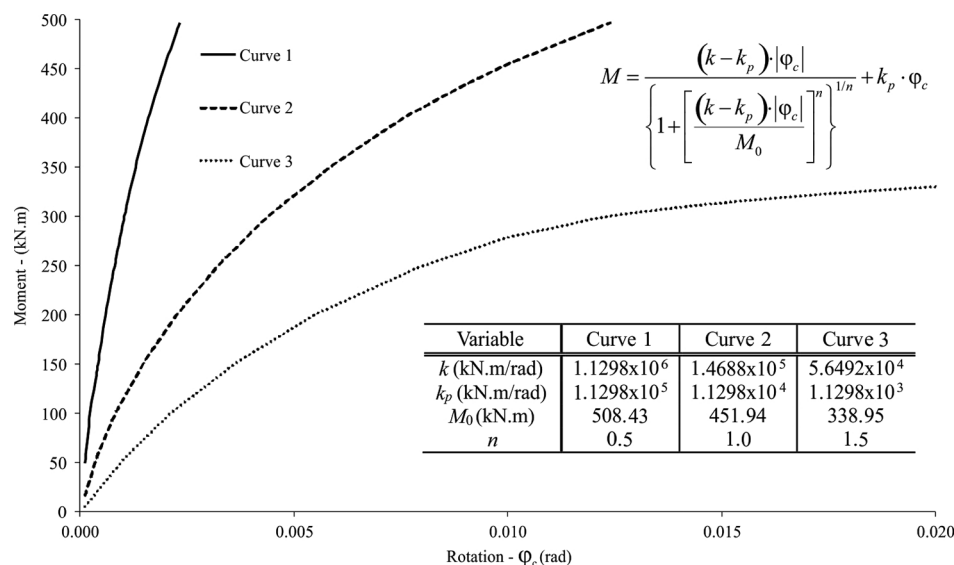


Figure 4  
Moment-rotation curves of semi-rigid connections.

Table 2 presents, for curves 1 to 3, the results from Agostini (2014) on the reliability analysis carried out considering the presence of semi-rigid

connections in the structure. Table 3 presents the comparison between the results of the reliability index *β* given by Agostini (2014) and those given

by Haldar and Mahadevan, (2000) considering the presence of semi-rigid connections.

Table 2  
Results considering the presence of semi-rigid connections.

Variable	Curve 1			Curve 2			Curve 3		
	Sensitivity index	Initial value of variable	Final value of variable	Sensitivity index	Initial value of variable	Final value of variable	Sensitivity index	Initial value of variable	Final value of variable
<i>E</i>	-0.1960	199,948.04	188,354.23	-0.1792	199,948.04	189,840.12	-0.1630	199,948.04	191,314.86
<i>A</i>	-0.0047417	0.01271	0.01268	-0.0042829	0.01271	0.01268	-0.003646	0.01271	0.01268
<i>I</i>	-0.1587	0.0003970848	0.0003813867	-0.1451	0.0003970848	0.0003834094	-0.1322	0.0003970848	0.0003853952
<i>D</i>	0.009346	45.97	45.95	0.012188	45.97	46.00	0.011894	45.97	45.98
<i>L</i>	0.00814	16.05	15.54	0.010107	16.05	15.56	0.009421	16.05	15.54
<i>W</i>	0.9676	22.55	109.21	0.9729	22.55	101.81	0.9776	22.55	92.84
Performance function	-	0.7987	-0.000115	-	0.7829	-0.0000456	-	0.7618	-0.0000155
Reliability index ( <i>β</i> )	-	12.14	4.93	-	10.94	4.66	-	9.60	4.33
Probability of failure	-	-	0.4114e-6	-	-	1.5657e-6	-	-	7.3658e-6
Number of iterations	-	-	4	-	-	4	-	-	4

Table 3  
Values for  $\beta$  according to Haldar and Mahadevan, (2000) and present work.

	$\beta$ Rigid connection (nonlinear analysis)		$\beta_1$ (Curve 1)		$\beta_2$ (Curve 2)		$\beta_3$ (Curve 3)	
	Haldar and Mahadevan, (2000)	Present work	Haldar and Mahadevan, (2000)	Present work	Haldar and Mahadevan, (2000)	Present work	Haldar and Mahadevan, (2000)	Present work
$\beta$ (Node 2 horizontal displacement)	5.47	5.03	5.17	4.93	4.47	4.66	4.06	4.33

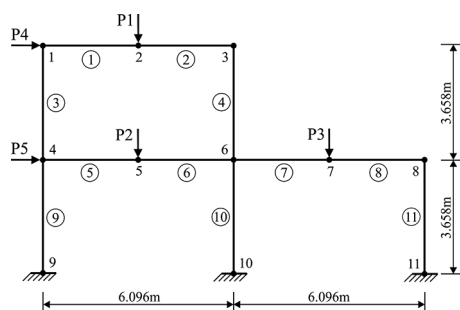
### Example 2: Plane frame with eleven elements

The second example, shown in Figure 5, consists of a frame with an asymmetric arrangement of its elements. For the actual example, the presence of semi-rigid connections was not taken into account. Statistical properties of the variables related to the structure in question are in-

dicated in Table 4, in which the sub-indices “b” and “c”, alongside the area and moment of inertia variables, stand for beam and column. The horizontal displacement of Node 1 was limited to 0.0254m, thus representing the service limit state. Table 5 presents the results of the reliability analy-

sis carried out for the structure in Example 2, based both on linear and nonlinear analysis. Table 6 presents a summary of the reliability indexes  $\beta$  determined by linear and geometric nonlinear analyses, including values obtained by Haldar and Mahadevan, (2000).

Figure 5  
Plane frame with eleven elements:  
geometry, loads and discretization.



Variable	Unit	Mean	Coefficient of variation	Type of distribution
$E$	MPa	199,948.04	0.06	Lognormal
$Ab$	m <sup>2</sup>	0.007613	0.05	Lognormal
$Ac$	m <sup>2</sup>	0.011419	0.05	Lognormal
$Ib$	m <sup>4</sup>	0.0002151916	0.05	Lognormal
$Ic$	m <sup>4</sup>	0.0001431836	0.05	Lognormal
$P1$ (Dead load)	kN	44.48	0.10	Lognormal
$P2$ (Dead load)	kN	88.96	0.10	Lognormal
$P3$ (Dead load)	kN	88.96	0.10	Lognormal
$P4$ (Wind load)	kN	44.48	0.37	Extreme Type I
$P5$ (Wind load)	kN	22.24	0.37	Extreme Type I

Table 4  
Statistical properties of random variables.

Table 5  
Results considering the horizontal displacement of Node 1.

Variable	Sensitivity index		Initial value of the variable		Final value of the variable	
	Linear	Nonlinear	Linear	Nonlinear	Linear	Nonlinear
$E$	-0.182986	-0.183625	199,948.04	199,948.04	194,716.96	194,731.43
$Ab$	-0.001573	-0.001566	0.007613	0.007613	0.007602	0.007602
$Ac$	-0.001099	-0.000923	0.011419	0.011419	0.011403	0.011404
$Ib$	-0.065100	-0.065437	0.0002151916	0.0002151916	0.0002133527	0.0002133548
$Ic$	-0.084845	-0.085225	0.0001431836	0.0001431836	0.0001416446	0.0001416474
$P1$	-0.001382	-0.000862	44.48	44.48	44.25	44.25
$P2$	0.008176	0.008469	88.96	88.96	88.68	88.69
$P3$	0.003952	0.004246	88.96	88.96	88.60	88.60
$P4$	0.973712	0.973498	44.48	44.48	91.69	91.21
$P5$	0.082917	0.083330	22.24	22.24	22.32	22.31
Performance function	-	-	0.4763	0.4743	-0.0000002	-0.0000002
Reliability index ( $\beta$ )	-	-	3.289	3.262	2.254	2.240
Probability of failure	-	-	-	-	1.208e-2	1.255e-2
Number of iterations	-	-	-	-	4	4



Table 6  
Values for  $\beta$  obtained by Haldar and Mahadevan, (2000) and by present work.

	$\beta$ (linear analysis)		$\beta$ (geometric nonlinear analysis)	
	Haldar and Mahadevan, (2000)	Present work	Haldar and Mahadevan, (2000)	Present work
$\beta$ (Node 1 horizontal displacement)	2.283	2.254	2.274	2.240

#### 4. Discussion

Comparing the results of Example 1 - presented in Tables 2 and 3 - it can be seen that the presence of semi-rigid connections resulted in a decrease in the reliability index, when the behavior of semi-rigid connections of nodes 2 and 3 goes from more stiff to less stiff. This decrease in the value of the reliability index  $\beta$  were also detected by Haldar and Mahadevan, (2000). These authors considered the idealization of connections as perfectly pinned or stiff as inadequate and suggested a careful investigation, even for the serviceability limit state, in frames modelled with

semi-rigid connections. Hadianfard and Razani's, (2003) reliability analysis also showed results that indicated substantial differences in behavior of structures modelled with semi-rigid connections compared to those modelled with ideally pinned or rigid connections. Hadianfard and Razani, (2003) stated that to achieve more reliable results, it is necessary to consider the semi-rigid behavior of connections in the reliability analysis.

By comparing the results in Example 2, shown in Table 6, with those determined by Haldar and Mahadevan,

(2000), one is able to notice an agreement between values. Results shown in Tables 5 and 6 indicate mainly that the consideration of the geometric nonlinearity did not result in great differences in the reliability index, sensitivity indices or final values of the variables. However, a small decrease in the reliability index  $\beta$  can be noticed. One noted that, for the incident loading, the properties of the material and the geometric characteristics of the structure in Example 2 show only small manifestations of the effects of geometric nonlinearity in the limit state under study.

#### 5. Conclusions

This study was carried out to understand the influence of the effects of geometric nonlinearity and of semi-rigid connections on the reliability of steel plane frames. The results indicate the efficiency of the proposed implementation. The

reliability analyses involved the service limit state and provided evidence of a slight decrease in reliability index  $\beta$  when including the effects of geometric nonlinearity. When semi-rigid connections were considered, the result was a considerable

decrease in the value of reliability index  $\beta$  as the stiffness of connections decreased. These results suggest the need to take the semi-rigid behavior of connections into account when carrying out reliability analyses of steel plane frames.

#### Acknowledgements

This work was supported by the Federal University of Ouro Preto, the IF

Sudeste MG at São João del-Rei, CNPq and FAPEMIG.

#### References

- AGOSTINI, B.M. *Análise de confiabilidade estrutural de pórticos metálicos planos com ligações semirrígidas*. Ouro Preto: Universidade Federal de Ouro Preto, 2014. 121 p. (Tese de Doutorado em Engenharia Civil).
- AGOSTINI, B.M., FREITAS, M.S.R. Procedimento para avaliação da confiabilidade estrutural de pórticos planos. In: IBERIAN LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGINEERING, 32. Ouro Preto. Abstracts... Ouro Preto: ABMEC, 2011. p. 210-210.
- HADIANFARD, M.A., RAZANI, R. Effects of semi-rigid behavior of connections in the reliability of steel frames. *Structural Safety*, v. 25, p. 123-138, 2003.
- HALDAR, A., MAHADEVAN, S. *Reliability assessment using stochastic finite element analysis*. England: John Wiley & Sons, 2000.
- HASOFER, A.M., LIND, N.C. An exact and invariant first-order reliability format. *Journal of Engineering Mechanics*, v. 100, p. 111-121, feb. 1974.
- INTEL® DEVELOPER ZONE. Santa Clara, c2011. Installing and using the IMSL libraries. Available in: <https://software.intel.com/en-us/articles/installing-and-using-the-imsl-libraries>. Access in: 6 sep. 2014.
- MELCHERS, R.E. *Structural reliability analysis and prediction*. (2ed.). England: John Wiley & Sons, 1999. 431p.
- MICROSOFT CORPORATION. Microsoft Visual Studio 2010. Version 10.0.30319.1

- RTMRrel. [S.l.]: Microsoft Corporation, 2010.
- PINHEIRO, L. *Análises não-lineares de sistemas estruturais metálicos rotulados e semirrígidos*. Ouro Preto: Universidade Federal de Ouro Preto, 2003. 245 p. (Dissertação de Mestrado em Engenharia Civil).
- RACKWITZ, R., FIESSLER, B. Structural reliability under combined random load sequences. *Computers & Structures*, v. 9, n. 5, p. 489-494, nov. 1978.
- RICHARD, R.M., ABBOTT, B.J. Versatile elastic-plastic stress-strain formula. *Journal of the Engineering Mechanics Division*, v. 101, n. 4, p. 511-515, 1975.
- SAGRILO, L.V.S. *Confiabilidade estrutural*. Rio de Janeiro: COPPE-UFRJ. 2004. (Notas de Aula).
- SILVA, A.R.D. *Sistema computacional para análise avançada estática e dinâmica de estruturas metálicas*. Ouro Preto: Universidade Federal de Ouro Preto, 2009. 322 p. (Tese de Doutorado em Engenharia Civil).
- VISUAL NUMERICS®. IMSL™ Fortran Numerical Library. Version 6.0. [S.l.]: Visual Numerics®, 2006.
- YANG, Y.B., KUO, S.R. *Theory & analysis of nonlinear framed structures*. Singapore: Prentice Hall, 1994.

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Received: 28 March 2017 - Accepted: 23 August 2017.