Optimization of structures subjected to dynamic load: deterministic and probabilistic methods

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Abstract

This paper deals with the deterministic and probabilistic optimization of structures against bending when submitted to dynamic loads. The deterministic optimization problem considers the plate submitted to a time varying load while the probabilistic one takes into account a random loading defined by a power spectral density function. The correlation between the two problems is made by one Fourier Transformed. The finite element method is used to model the structures. The sensitivity analysis is performed through the analytical method and the optimization problem is dealt with by the method of interior points. A comparison between the deterministic optimisation and the probabilistic one with a power spectral density function compatible with the time varying load shows very good results.

Keywords: optimization; sensitivity analysis; deterministic analysis; probabilistic analysis.

1. Introduction

A deterministic dynamic load is one whose value is determined in any instant and, consequently, the response value is determined in that instant by a deterministic analysis. This type of analysis is well known and a variety of methods, such as modal analysis, finite differences, and Newmark can be used to obtain the solution of the problem. The structural optimization problem for this type of load is presented in the work of Falco (Falco, 2000). In this work, a study is presented for the shape optimization of shells submitted to a deterministic load.

A random loading is one that cannot be foreseen in any instant of time and can only be defined through statistical properties. Therefore the analysis for this type of load can only be tackled by statistical or probabilistic methods. Random loads stem mainly from phenomena like earthquakes, wind, traffic and the vibration of aerospace structures. The analysis of structures submitted to random loads is very well established, whereas optimization studies are scarce. Some applications for this type of problem are: reduction of mass structures under the effect of earthquake without loss of stiffness; and mass reduction of satellite structures during launch so that their frequencies are often disengaged from the launch vehicle. Alves *et al* (2000) present a thorough study of the sensitivity analysis of structures submitted to random loading. In this work the equations of the analytical method are validated by the finite difference method. Alves and Vaz (2013) present the formulation and the results of the structural optimization of homogeneous and sandwich plates under random vibration.

Search is in this work presents a comparative study between the deterministic optimization problem, ie in the time domain, and the probabilistic optimization problem in the frequency domain. The equivalent formulation in the frequency domain was obtained from the application of Fourier Transforms in the formulation in the time domain. A comparative analysis of structural optimization for plates bending with random loading and the equivalent deterministic loading is presented to validate the formulation.

2. The finite element model

For the structural modeling of the plate, a 6-node triangular AST6 is used, and for the other examples, the beam element is used. This triangular element was developed by Sze (1997) for homogeneous plates and Goto (2000) extended it for laminated plates and shells. The motivation for the use of this element stems from the fact that its stiffness and mass matrices are explicitly given, permitting therefore the sensitivity analysis by the analytical method.

3. Deterministic optimization

According to Falco (2000), the deterministic optimization problem consists in the minimization of the structural mass subjected to a condition whereby the displacement in a given point and in a given instant of time is equal or less than a given threshold. Falco (2000) applies this formulation for the plate problem. This problem is formulated as follows:

$$\text{Minimize } \Sigma\, \rho_i A_i h_i$$

$$\text{Subject to: } u(t) \leq u_{max}$$

$$h_{lw} \leq hi \leq h_{up} \tag{1}$$

In Equation 1, ρ_i, A_i and h_i are, respectively, the mass density, the area, and thickness of the i^{th} element; $u(t)$ is the displacement in a given point; and h_{lw} and h_{up} are, respectively, the lower and upper bounds of the design variable. The displacement $u(t)$ is obtained by the Newmark method as follows.

$$^{t+\Delta t}\dot{\mathbf{u}} = {}^{t}\dot{\mathbf{u}} + (1-\delta)\,{}^{t}\ddot{\mathbf{u}}\Delta t + \delta\,{}^{t+\Delta t}\ddot{\mathbf{u}}\Delta t \tag{2}$$

$$^{t+\Delta t}\mathbf{u} = {}^{t}\mathbf{u} + {}^{t}\dot{\mathbf{u}}\Delta t + \left(\frac{1}{2}-\alpha\right){}^{t}\ddot{\mathbf{u}}\Delta t^2 + \alpha\,{}^{t+\Delta t}\ddot{\mathbf{u}}\Delta t^2 \tag{3}$$

$$\mathbf{M}\,{}^{t+\Delta t}\ddot{\mathbf{u}} + \mathbf{C}\,{}^{t+\Delta t}\dot{\mathbf{u}} + \mathbf{K}\,{}^{t+\Delta t}\mathbf{u} = {}^{t+\Delta t}\mathbf{R} \tag{4}$$

where ${}^{t}\boldsymbol{u}$, ${}^{t}\dot{\boldsymbol{u}}$, ${}^{t}\ddot{\boldsymbol{u}}$ are displacement, velocity and acceleration with the time t and ${}^{t+\Delta t}\boldsymbol{u}$, ${}^{t+\Delta t}\dot{\boldsymbol{u}}$ e ${}^{t+\Delta t}\ddot{\boldsymbol{u}}$ are displacement, velocity and acceleration with the time t $t+\Delta t$. Herein, δ equal to 0.5 and Δt equal to 0.1s were adopted.

4. Probabilistic optimization

The equivalent probabilistic optimization problem is the minimization of the structural mass with the restriction that the probability of the displacement in a given point be greater than a given value that must be less than a given probability. This problem is formulated as (Alves, 2013).

$$\text{Minimize } \Sigma\, \rho_i A_i h_i$$

$$\text{Subjected to: } Pr(u_i > u_{max}) \leq Pd_{max}$$

$$Pr(u_i > u_{max}) < Pa_{max}$$

$$h_l < h_i < h_u \tag{5}$$

In Equation 5 ρ_i, A_i and h_i are the same as in Equation 1; $Pr(u_i > u_{max})$ is the probability that the displacement in a given point be greater than a maximum displacement; Pd_{max} is the given probability; and h_{lw} and h_{up} are, respectively, the lower and upper bounds of the design variables.

The probability that the displacement be greater than the maximum displacement is

$$Pd\,(u_i > u_{j\,max}) = \int_{u_{max}}^{\infty} p(u_i)\,du \tag{6}$$

Where $p(u_i)$ is the probability density function of u_i. The probability density function considered here is the Gaussian distribution:

$$p(u_i) = \frac{1}{\sqrt{2\pi}\sigma_{u_i}} e^{\left[-\frac{\left(u_i - \bar{u}_i\right)^2}{2\sigma_{x_i}^{\,2}}\right]} \tag{7}$$

As the processes considered in this work are ergodics with zero mean, Equation 7 transforms into.

$$p(u_i) = \frac{1}{\sqrt{2\pi}\sigma_{u_i}} e^{\left[-\frac{(u_i)^2}{2\sigma_{u_i}^{\,2}}\right]} \tag{8}$$

In Equations 4 and 5 σ_{ui} is the standard deviation of u_i, from the theory of the stochastic response of a multi-degreeof-freedom system, and u_i is obtained by following equation:

(9)
$$\sigma_{u_i}^2 = \frac{1}{2\pi} \mathbf{B} \int_{-\infty}^{+\infty} S_y(\Omega)\, d\Omega\, \mathbf{B}^T$$

B is a matrix whose entries are the n_j modal components and $\mathbf{S_y}$ (Ω) is a spectral density matrix of the modal coordinates. This matrix is defined as;

(10)
$$\mathbf{S_y}(\Omega) = \mathbf{H}(\Omega)\, \mathbf{S_P}(\Omega)\, \mathbf{H}(\Omega)$$

where $\mathbf{S_p}$ (Ω) is the spectral matrix of the nodal loads and H (Ω) is the diagonal matrix whose entries are the complex frequency response functions associated to each normal mode. For the n generic mode:

(11)
$$H_n = [\,(K_n - M_n\Omega_n^2) + 2i\,\xi_n M_n \Omega_n]$$

K_n, M_n, Ω_n and ξ_n being, respectively, the generalized stiffness and mass and the damping ration of the n normal mode.

5. The solution of the optimization problem

The solution of the optimization problem presented through Equations 1 and 5 is performed by the Interior Points Method, Herskovits (1995).

The method works specifically with the feasible region of the problem. Where the problem is bounded by the objective function and the constraint functions, which may be equal or unequal. It basically consists in determining some points within this feasible region and from these, continue to search for the sweet spot that belongs equally to this region. Hence all points earned in sequence always possess decreasing values. So, even though the convergence to the optimum is not guaranteed, the last point found will always be less than or equal to the other, so it will be viable. Consider the minimization problem:

(12)
$$\text{Minimize } f(x)$$
$$\text{Subject to } c(x) \le 0 \quad i = 1.....m$$

And the Kuhn-Tucker conditions for this problem are:

(13)
$$g + \sum_{i=1}^{m} \lambda_i a_i = 0$$
$$\lambda^*_i c_i(x^*) = 0$$
$$c_i(x^*) \le 0$$
$$\lambda^*_i \ge 0$$

Where **A** a matrix containing the gradients of the constraints, and **C** a diagonal matrix that contains the values of these restrictions. Thus, the first two equations can be rewritten as follows:

(14)
$$g + A^t\lambda = 0$$
$$C\lambda = 0$$

Using Newton's method to solve this problem, we have:

(15)
$$\begin{bmatrix} W^k & A^t \\ \Lambda A & C \end{bmatrix} \begin{pmatrix} d_0 \\ \lambda_0 \end{pmatrix} = -\begin{pmatrix} g \\ 0 \end{pmatrix}$$

whereby Λ is a diagonal matrix where $\Lambda_{ii}=\lambda_i$, and d_0 is the search direction which is the estimate of the Lagrange multipliers. It can be shown that the search direction will always decrease, except in the case where the point x does not change the value. In this case the search direction $d_0 = 0$.

6. Examples

The application example is the optimization of the isotropic plate depicted in Figure 1. The plate properties are: Young`s modulus= 2.6 x $10^{10}$ N/m², shear modulus $1.0x10^{10}$ N/m²; Poison`s ratio ν=0.3 and mass density 25000N/m³. The imposed restriction is that the center displacement of the plate, w_c, must be less or equal than 2,9cm. The optimisation problem is considered with 1, 2 and 4 design variables. Figure 2 shows the distribution of the design variables in the finite element mesh.

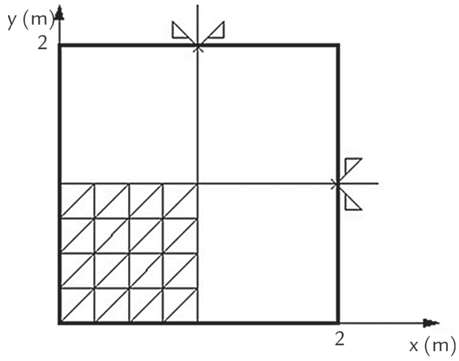

Figure 1
Symmetrical plate.
4x4 mesh.Clamped in 4 sides.

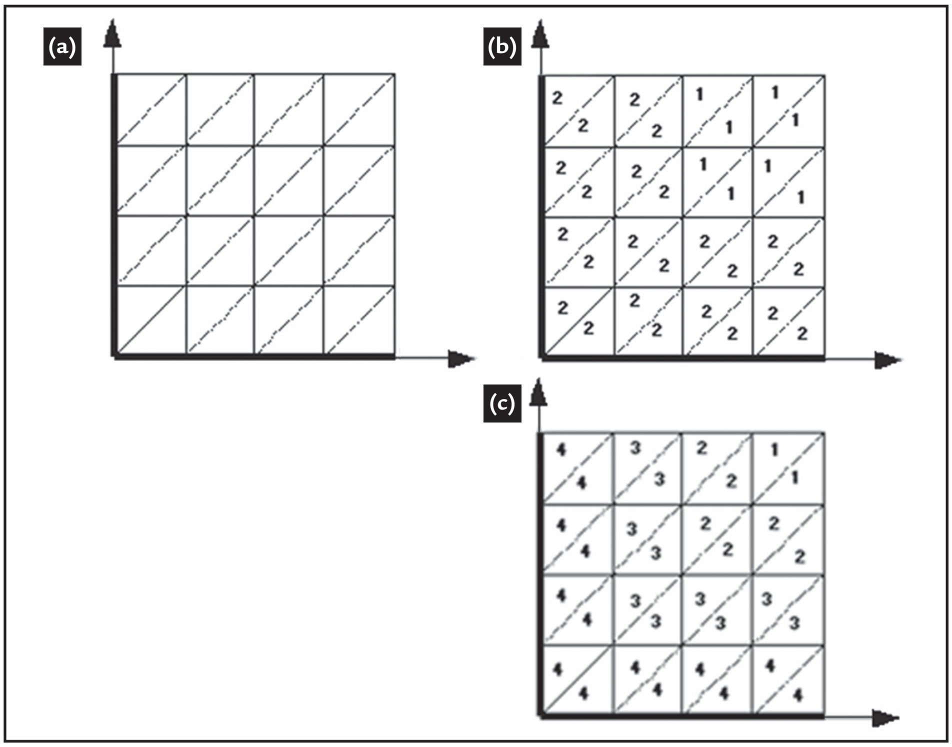

Figure 2
a) Optimization with 1 design variable.
b) 2 design variables.
c) 3 design variables.

The deterministic problem is formulated in the following way:

$$\text{Minimize } V = \sum_{i=1}^{n} \rho A_i h_i$$

$$\text{Subject to: } w_c(t) \leq 0.029$$

$$h_{lw} \leq h_i \leq h_{up} \quad (16)$$

The load time history is presented in Figure 3.

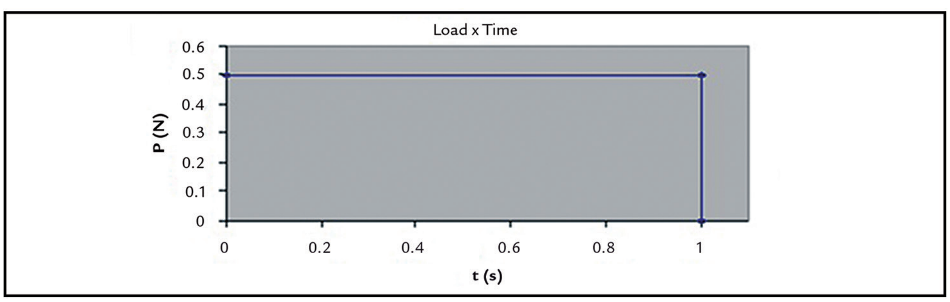

Figure 3
Time variation of concentrated
load for deterministic optimization.

The results of the optimization are shown in Table 1 and Figure 4.

Table 1
Results of the deterministic optimization.

Design Variables	Initial	Final		
		1 D. V.	2 D. V.	4 D. V.
h_1(m)	$1x10^{-2}$	$9.615x10^{-2}$	$1.605x10^{-2}$	$2.104x10^{-2}$
h_2(m)	$1x10^{-2}$	--	$3.657x10^{-3}$	$1.004x10^{-2}$
h_3(m)	$1x10^{-2}$	--	--	$5.243x10^{-3}$
h_4(m)	$1x10^{-2}$	--	--	$3.007x10^{-3}$
Objective. Function(kg)	25.0	24.04	16.89	15.38
Const.(m)	$2.58x10^{-2}$	$2.9x10^{-3}$	$2.9x10^{-3}$	$2.9 x10^{-3}$
Decrease Objective. Function(%)		3.85	29.74	8.9

Table 1 presents the results of the optimization problem with 1, 2 and 4 variables and reduction of the objective function when there is an increase in the number of project variables. With the increase in design variables above, 4 had no significant reduction in the objective function.

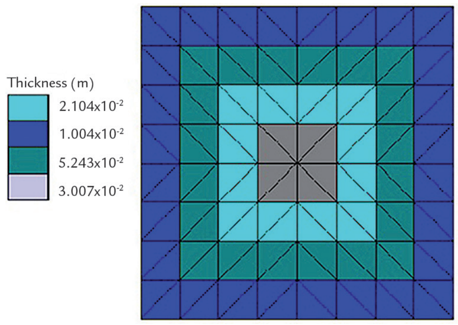

Figure 4
Thickness Distribution throughout the plate considering 4 design variables.

The observation of these results indicates that the increase in the number of design variables leads to a diminishing value of the objective function, which is to be expected.

In the sequel, the equivalent probabilistic optimization is performed. Now the problem is defined as

$$\text{Minimize } V = \sum_{i=1}^{n} \rho A_i h_i$$

$$\text{Subject to: } Prw_c(t) \leq 0.029 Pd_{max}$$

$$h_{lw} \leq h_i \leq h_{up}$$

To attain consistence, the value of the constraint is calculated from an analysis of the results obtained with the final values of the design variables in the deterministic optimization and the load is given from the Fourier transform (Figure 5) of the load considered in the deterministic optimization.

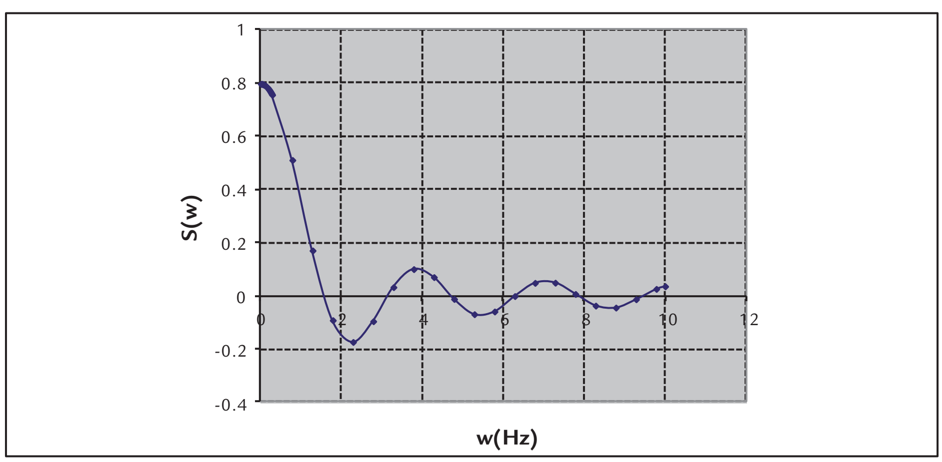

Figure 5
Equivalent Load for Probabilistic Optimization.

The results of the optimization are tabulated in Table 2 and Figure 6.

Design Variables	Initial	Final		
		1 D.V.	2 D.V.	4 D.V.
h_1(m)	$1x10^{-2}$	$7.036x10^{-2}$	$1.026x10^{-2}$	$1.155x10^{-2}$
h_2(m)	$1x10^{-2}$	--	$4.806x10^{-3}$	$7.466x10^{-3}$
h_3(m)	$1x10^{-2}$	--	--	$2.970x10^{-3}$
h_4(m)	$1x10^{-2}$	--	--	$5.776x10^{-3}$
Obj. Func.(kg)	25.0	17.590	15.424	13.942
Const.(%)	38	44.7	44.7	44.7
Decrease Objective. Function (%)		29.64	8.76	9.6

Table 2
Results of the probabilistic optimization.

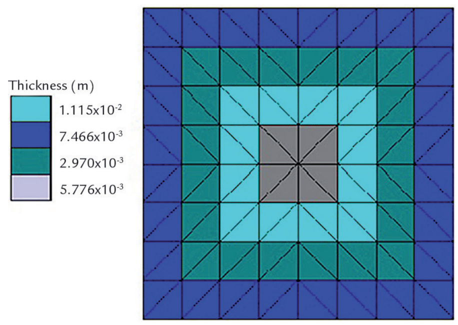

Figure 6
Thickness Distribution throughout the plate considering 4 design variables.

7. Conclusion

A comparison of deterministic and probabilistic optimization of plate submitted to dynamic load indicates the reliability of both types of optimization. If there is consistency in the definition of the deterministic and the probabilistic load, the results of both types of optimization agree quite well.

A better design in relation to the initial design is obtained in all the analyzed cases. In Example 1, with 4 design variables, the variable least altered is variable 1 (h). This is to be expected as the elements near the point of application of the load must be more rigid. This example is illustrative, as the real plate has a constant thickness

Although the design variables are different in the two types of optimization, the values of the objective functions are quite similar for the cases with 2 and 4 design variables. This conclusion is to be expected as the two problems work with different design spaces and two different analyses, one deterministic and the other, probabilistic.

8. References

ALVES, E. C., VAZ, L E . Optimun design of plates structures under random loadings. ***REM-Revista Escola de Minas***, v. 66, p. 41-47, 2013.

ALVES, E. C., VAZ, L. E., KATAOKA FILHO, M. Análise da sensibilidade da resposta de estrutura submetida a carregamentos aleatório. IBERO LATIN. AMERICAN CONGRESS OF COMPUTATION METHODS IN ENGINEERING, 21. ***Anais...*** Rio de Janeiro, Brazil, 2000. (Full tex in CD Rom).

FALCO, S. A. ***Otimização de forma de cascas submetidas a carregamento dinâmico.*** Rio de Janeiro: Pontifícia Universidade Católica do Rio de Janeiro – PUC-Rio, 2000. (Tese de Doutorado).

GOTO, S. T., NETO, E. L., KATAOKA FILHO, M. ***Um elemento triangular para placas e cascas laminadas.*** Divisão de Mecânica Espacial e Controle (DMC/ETE), Instituto Nacional de Pesquisas Espaciais – Brazil, 2000. (Proposta de Dissertação de Mestrado).

HERSKOVITS, J. ***Advances in structural optimization.*** Kluver Academic Publishers, 1995.

SZE, K. Y., Zhu, D., Chen, D. P. Quadratic triangular Co plate bending element. ***International Journal of Numerical Methods in Engineering,*** v. 40, p. 937-981, 1997.

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