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Numerical analysis of soft soil improved with stone column technique

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Abstract

The present research aims to do a numerical study on the behavior of a test embankment reinforced with stone column spacing of 1.85 m in a square grid with an average diameter of 1.0 m and 11.35 m length. The case study investigated the work in a steel company (ThyssenKrupp), located in Santa Cruz, western region of Rio de Janeiro - Brazil, where a large-sized ore yard was built for loading and unloading activities. It was built on very soft and compressible clay with support of several equipment installed to monitor the area. These data were compared with plane strain analysis results of vertical displacement, horizontal displacement and excess pore pressure obtained by two finite element programs: PLAXIS and RS^2 (Rocsience). Results showed that Mohr Coulomb model predicted well vertical displacement comparing with a Cam-Clay model and the instrumentation data. Simulated curves of excess pore pressure and horizontal displacement presented expected peculiarities that are interpreted throughout the article. Numerical analysis of yielding columns demonstrate compatibility between both models.

Keywords: soft clay, stone column, numerical modeling, constitutive models, Mohr Coulomb, numerical modeling, settlement.

1. Introduction

Population growth and the necessity to expand projects to new areas outside metropolitan centers force industrial and residential constructions to move towards remote and often uninhabited places. However, some areas may not present favorable characteristics for large constructions, since they are composed of low resistance clays, often found in Brazilian coastal regions reported by Almeida and Marques (2013), Almeida *et al.* (2014) and Hosseinpour *et al.* (2017, 2016). In order to solve the problem, several methods of construction on compressible soils have been designed to allow these previously uninhabited lands to receive huge constructions without great problems. Among the techniques to improve the strength of loose soil, stone columns often stand out for supplying the deficiencies of soils with high compressibility also taking into account the project deadline. More about basic principles and characteristics of these columns can be seen in the FHWA (1983) published report.

Advances in technology have allowed the development of geotechnical software. Computational modeling through the finite element method (FEM) has gained space, and consequently enriched the research projects and refined geotechnical analysis and estimations. Problems before solved with analytical and semi-empirical methods can be calculated with great accuracy and according to the particularities and behavior of each study. These analyses also include studies of the stone column technique for predicting soil behavior. (e.g. Dash and Bora, 2013; Indraratna *et al.*, 2013; Castro, 2014; Tan *et al.*, 2014; Ellouze *et al.*, 2016). Field load tests may also be a good alternative for understanding the behavior of the stone columns reflecting the actual response of the site through data response. However, such tests are harder to perform than numerical and analytical studies (e.g. Mestat *et al.*, 2006; Yee *et al.*, 2007; Egan *et al.*, 2008; Weber *et al.*, 2008; McCabe *et al.*, 2009).

The present article aims to compare the results of an ore stocking test area case study, located in Rio de Janeiro, Brazil, with the plane strain 2D numerical modeling of two different software (RS^2 and Plaxis). The instrumentation data were compared with numerical responses (vertical and horizontal displacement, excess pore pressure and yielded points) performed on the Mohr Coulomb model, due to its simplicity and wide dissemination at the geotechnical community. In addition, these previous analyses were compared with the Almeida *et al.* (2014) analyses on a Cam-Clay model (soft soil creep) to verify the compatibility of the constitutive models.

2. Material and methods

The case study investigated the work in a steel company (ThyssenKrupp), located in Santa Cruz, west region of Rio de Janeiro - Brazil, where a large-sized ore yard was built for loading and unloading activities. Part of this region would receive loads related to a coal deposit and iron ore pile. Because the ore pile's vertical stress is more than three times greater than the mineral coal pile, different solutions were adopted.

Vertical drains and geosynthetic encased columns (Almeida *et al.*, 2018 and Hosseinpour, 2016), were used for the area destined for the mineral coal deposit, while in the iron ore pile area, the stone column technique was preferred. This decision was made due to the high loads expected during the loading process, combined with the relative short deadline to conclude the work. The area where the stone column solution was adopted will be the one concerned in this article. Figure 1 indicates the location of the construction

Figure 1 - Satellite positioning of the construction - after finished (Google Maps, 2020).

In order to study the effects of increased loading in soft soils, common in the region contemplated, a test area was built, instrumented, and monitored in the future iron ore deposit area. The site is composed of an extensive, very compressible marine clay layer, typical of the lowland regions in Rio de Janeiro, with thicknesses ranging from 7.0 m to 15.0 m (Almeida and Marques, 2013). The stratigraphy model was elaborated in accordance with standard penetration and cone penetration tests (CPT/CPTu) previously performed at the site. According to the investigations, three soft clay layers interposed by sand layers composed most part of the ore yard area.

Due to the difficulty to work at the site, a working platform with average thickness of 2.6 meters was hydraulically installed. Underlying this working platform, a square mesh of stone columns was built with varied spacing from 1.75 m to 2.20 m, with lengths from 10 to 17 m (average 12 meters).

Inside the test area, the loading was applied with iron rails that were superimposed on a concrete plate of dimensions 6.5 m x 6.5 m x 0.4 m, covering a group of 16 columns spaced 1.85m, located in the center. The loading step applied to this group of columns consisted of loading phases from 102.3 kPa to 182.3 kPa with different durations of days. Figure 2 shows the evolutionary sequence of loading in the test area.

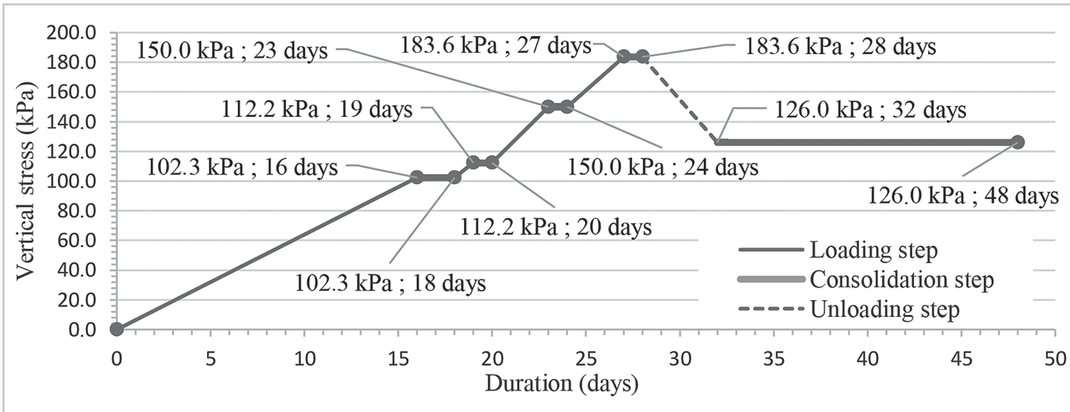

Figure 2 - Evolutionary sequence of loading in the test area.

The profile assumed for the study was prepared based on CPTu tests performed just before the installation of all field instrumentation. The water level gauge position was 80 centimeters below the ground level. Figure 3 indicates the situation of the soil profile after the installation of the working platform and the stone columns on site.

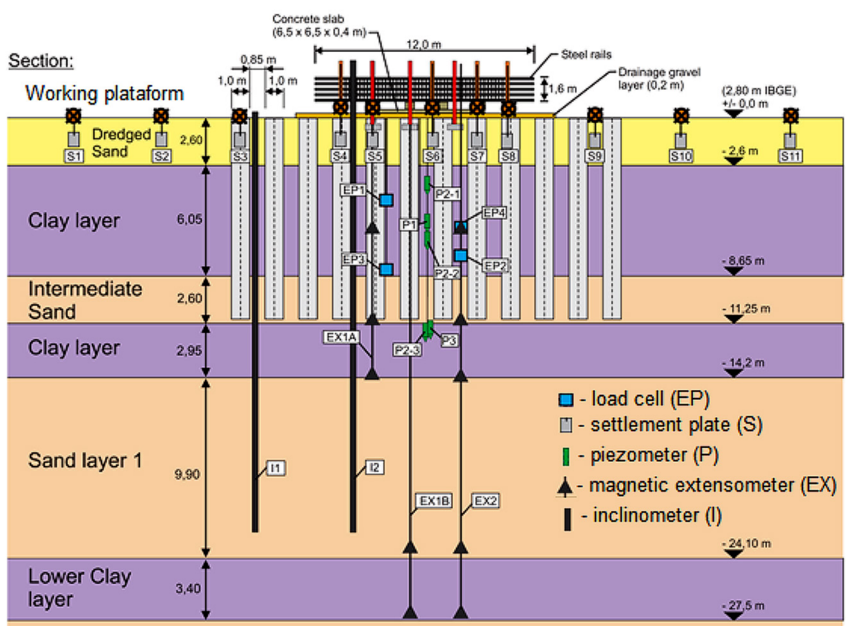

Figure 3 - Soil profile adopted after the installation of the working platform and stone columns with the instrumentation positions (Almeida *et al.*, 2014).

2.1 Geotechnical parameters

The geotechnical parameters used for the numerical analyses in this study were based on field and laboratory tests, and calibration of the geotechnical model presented at Almeida *et al.* (2014) and Roza (2012), as well as usual values in some cases. In general, the geotechnical investigation at the stockyard was composed of 14 standard penetration tests (SPT), 20 vertical cone penetration tests with pore pressure (CPTu), 3 vane shear tests, 6 Marchetti dilatometer tests and 16 undeformed samples. More information about results can be found on In Situ (2006) and Lima (2012).

The bulk density (γ) of the first clay layer was estimated based on undisturbed soil sampling while for the subsequent clay layers, due to the lack of information about these layers, samples were collected from an area located outside the yard with similar soil characteristics. Because of the consolidation produced by the installation of the working platform, Roza (2012) performed several axisymmetric analyses to estimate the final value of bulk density of the clay layers. For the stone column, sand layers (including working the platform) and concrete plate, usual values were used.

The friction angles (ϕ) of the clay layers were determined by the consolidated undrained triaxial tests (CU). For the stone columns, the friction angle was obtained from parametric analysis performed by Roza (2012), which aimed to estimate the most appropriate design value for the case studied. In addition, for the friction angle of the working platform and the sand layers, the values suggested by Terzaghi and Peck (1967) and Lambe and Whitman (1979) were used.

Model evaluation was done through comparison between numerical results and field measurements (Almeida *et al.*, 2014), providing the best fit for the horizontal and vertical permeability (k_h and k_v, respectively) and the earth pressure coefficient after column installation (K*). Table 1 presents the geotechnical parameters used in the case study.

Table 2 used correlations of the oedometric modulus (E_{oed}) resulted from the one-dimensional deformation and drainage simulation of the soil. Equation 1 estimate the oedometric modulus of clays that was used to develop Table 2.

$$E_{oed} = \frac{\sigma_{v'avg}\,(1 + e_0)}{0.435\, C_C} \qquad (1)$$

Where C_c is the compression index and $\sigma_{v'avg}$ is the average vertical stress of the soil. Where

Table 1 – Parameters used in the case study.

Material	γ_{nat} (kN/m³)	ϕ (°)	k_v (m/day)	k_h (m/day)	C_c	K* or K_0
Working plataform	18.0	30	0.86	0.86	-	1.25
Stone column	20.0	40	27000	27000	-	1.25
Clay layer 1	13.4	25	7.9×10^{-5}	3.9×10^{-5}	1.92	1.25
Clay layers 2	16.0	25	7.8×10^{-5}	7.9×10^{-5}	1.07	0.6
Clay layer 3	15.6	25	14.0×10^{-5}	7.0×10^{-5}	1.00	0.6
Sand layer 1	18.0	30	0.86	0.86	-	1.25
Sand layer 2	18.0	30	0.86	0.86	-	0.5
Sand layer 3	18.0	30	0.86	0.86	-	0.5

Due to the variation in elastic modulus values during the consolidation and the stress concentration factor (CF) (2 to 5) of the case presented in Almeida *et al.* (2014), different oedometric elastic modules were estimated for the clay layers. The initial modulus did not consider the applied load or the soil CF. The minimum modulus of elasticity used the maximum concentration factor (CF=5) and the minimum applied load (102.3 kPa), since it represents the situation of lower load absorption by the soil. For the maximum modulus, the applied load was the maximum (183.6 kPa) and the minimum concentration factor (CF=2) representing the situation which the soil absorbs the maximum load. For Layers 2 and 3 of clay, both have the elastic modulus in the initial state because they are outside the region of influence of the applied load. Table 2 presents the elastic modulus used for the various materials represented in the model.

Table 2 – Applied parameters of modules of elasticity.

Material	E_{oed} (kPa) initial	Mín (CF=5)	Máx (CF=2)
Working plataform	2000	-	-
Stone column	80000	-	-
Clay layer 1	190	440	900
Clay layer 2	460	1020	1490
Clay layer 3	1060	1800	2300
Sand layer 1	30000	-	-
Sand layer 2	30000	-	-
Sand layer 3	250000	-	-

The elastic modulus of stone columns was based on the average values presented in scientific articles related to the subject. For the sand layers, the elastics modulus was established based on the work by Lambe and Whitman (1979).

The other parameters used in the model were taken from the work of Lima (2012) and Roza (2012) and are shown in Table 1 and 2.

2.2 Numerical modeling

The numerical analyses were performed in Plaxis and RS² software using the constitutive model Mohr-Coulomb for all soils. Although it is not the best model to represent the behavior of a soft clay, due to the inability to reproduce its volumetric variation, unlike the Cam-Clay soft soil model, it is widely employed due to its simplicity, availability and diffusion in the geotechnical community.

Before the preparation of 2D numerical analysis, the second method of geometric conversion by Tan *et al.* (2008) was applied to achieve the same replacement ration in the geometric modeling. From the guidelines of the method, the thickness of the stone columns was recalculated and fixed at 0.40 meter, a value different from the original thickness (1 meter). Table 3 indicates the characteristics and conditions used for each model.

Table 3 – Characteristics and conditions for each model.

Plaxis	RS²
Mesh - 40m x 32m	Mesh - 40m x 32m
15 node triangle elements	6 node triangle elements
7462 elements	101655 elements
Biot's theory for coupled consolidation	Biot's theory for coupled consolidation
Drained behavior	Drained behavior

It should be noted that there are many more elements in the RS² mesh than in the Plaxis because the first works with 2nd order interpolation functions, while in Plaxis, the interpolation functions are 4th order. For this reason, this difference of discretization is necessary. During the modeling process, the convergence of results was verified for both models according to the mesh refinement. More information about the analysis can be found in Pereira (2018). Figure 4 shows the mesh used for both software.

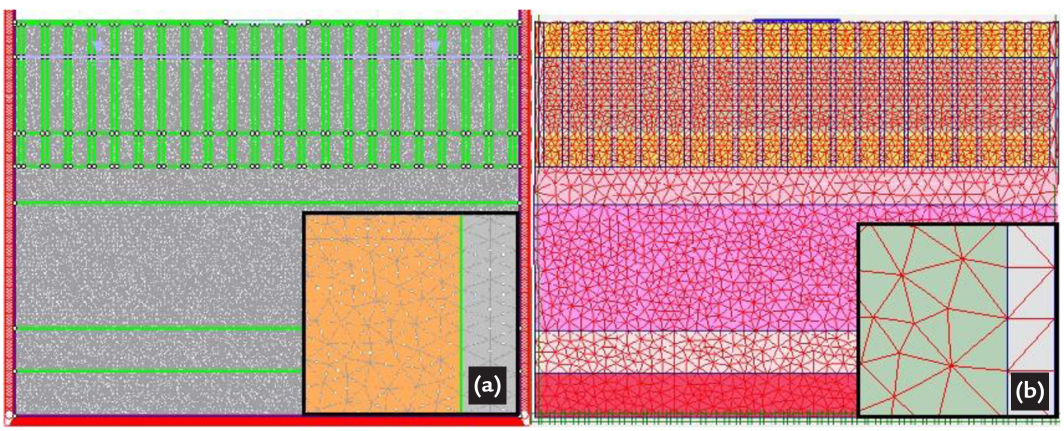

Figure 4 - Mesh used for the model in RS² (a) and Plaxis 2D (b).

It should be noted that there are many more elements in the RS² mesh than in the Plaxis because the first works with 2nd order interpolation functions, while in Plaxis, the interpolation functions are 4th order. For this reason, this difference of discretization is necessary. During the modeling process, the convergence of results was verified for both models according to the mesh refinement. More information about the analysis can be found in Pereira (2018). Figure 4 shows the mesh used for both software.

3. Results

In order to compute the results of the numerical analysis, displacement nodes and stress points were fixed at the field instrumentation original positions. Thus it was possible to perform a comparison between the instrumentation readings and the plane strain analysis. Settlement plates were installed 0.5 meters below the surface and inclinometers I-1 and I-2 with distance of 6.25 and 0.75 meters to the concrete edge, respectively. The piezometers were fixed 4 m, 6 m and 7 m depth represented by P2-1, P1 and P2-2 in Figure 3.

For Leroueil (2001), the safety factor on soft clay can be estimated by the horizontal displacement rate at the bottom of the slope that cannot exceed 10 mm/day. If this happens, it represent a probably soil rupture from approximately the 21st day on. Thus, the numerical analyses after this date represent qualitative values, since it is not possible to simulate the post-rupture effect with the software used. To deal with this situation, more sophisticated analyses would be required.

3.1 Vertical displacements

Figure 5 illustrates vertical displacements of the plane strain model associated with different elasticity modules. The results are compared with the settlement plates (S5 and S7) positioned under the concrete plate edge. The Plaxis model using initial oedometric modulus (E_{oed} initial) presented extremely high vertical displacement, probably indicating rupture.

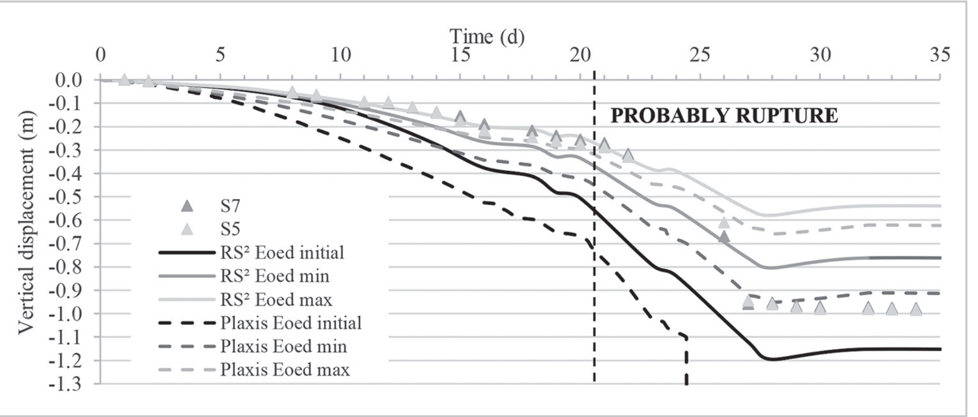

Figure 5 - Vertical displacements comparison.

It is possible to notice the difference between the results performed with different modules of elasticity. The vertical displacements considering the maximum modulus of elasticity (E_{oed}-max) got closer to measured values of the settlement plates until the day 21 (probably before soil rupture). Therefore, this was the standard modulus adopted for the remaining analyses.

Furthermore, it was also possible to analyze the results of the same geometric model performed by Almeida *et al*. (2014) using the constitutive model Cam-Clay, which best represents the behavior of very compressive soils. The vertical displacement with the Cam-Clay model were close to the Mohr-Coulomb model and the settlement plates as can be seen in Figure 6.

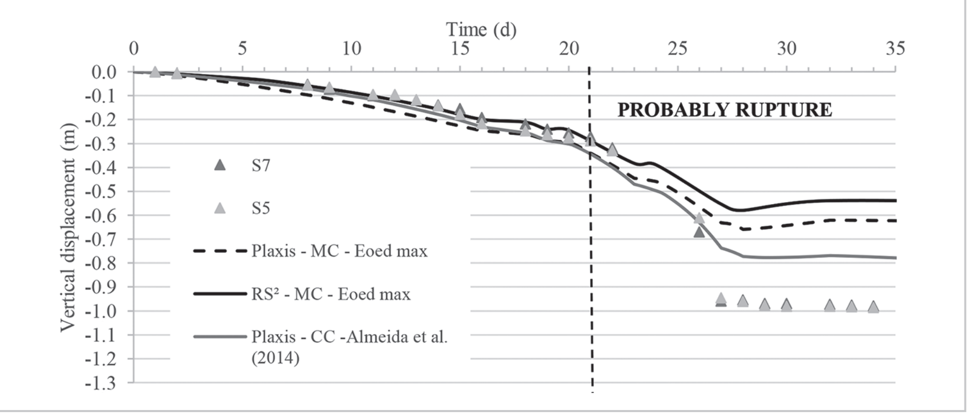

Figure 6 - Vertical displacements - Mohr Coulomb (MC) and Cam-Clay (CC) analysis.

3.2 Horizontal displacements

The horizontal displacements measured by Inclinometers 1 and 2 and the results obtained by numerical analysis (Mohr-Coulomb and Cam-Clay) are presented in Figures 7a and 7b, respectively. The numerical analyses were able to represent in a satisfactory way the soil behavior. Higher displacements can be observed in clay layers and smaller displacements in sandy regions.

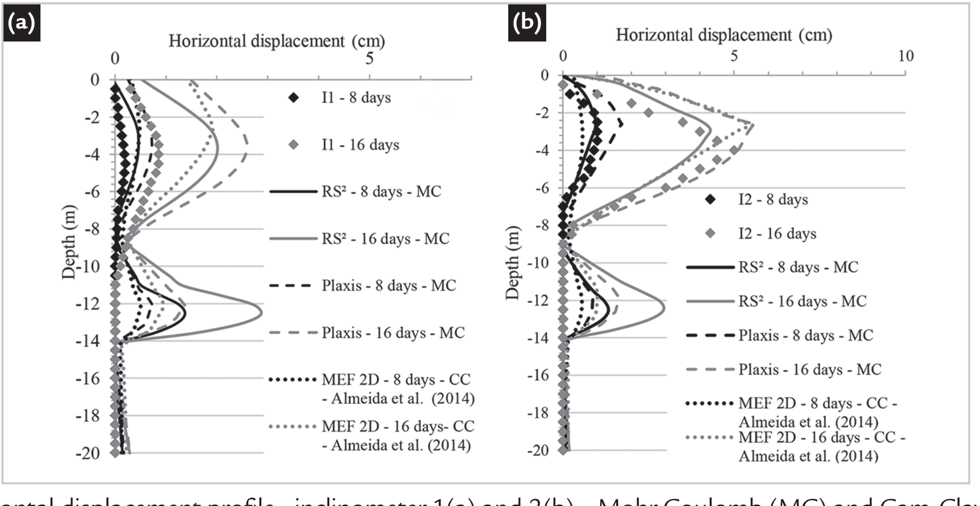

Figure 7 - Horizontal displacement profile - inclinometer 1(a) and 2(b) – Mohr Coulomb (MC) and Cam-Clay (CC) analysis.

The differences in the maximum displacements of the inclinometer profile may be associated with the presence of a less resistant layer at these depths, and/or with the rigidity of the aluminum inclinometer, which was not contemplated in the numerical analyses performed.

In terms of horizontal displacement, the field measurements showed similar characteristics to the models when compared to Inclinometer 2. As for Inclinometer 1, the results of readings were below those verified. It is important to remember that the inclinometer 1 is less affected by the stresses derived from the loading phase because it is located farther from the model's central axis. Therefore the rigidity of the inclinometer may have been a crucial factor for the lower horizontal displacement values.

3.3 Excess pore pressure

Figure 8 (a) shows the excess pore pressure in the plain strain model for the maximum modulus of elasticity at different depths (4 m, 6 m, 7 m) of the piezometer. These piezometers are located under the center of the concrete plate.

Numerical analysis nodes selected for the excess pore pressure were located exactly at the midpoint between the stone columns, assuming that they were installed perfectly vertical, with no spacing and diameter variation. However, in field, this situation may not occur. The installation of the piezometer may also have occurred in a position slightly different from the predicted. The closer the piezometer is to the stone columns, the greater the dissipation and consequently, the lower the measured value of excess pore pressure. In contrast, according to Wong (2009), the Mohr-Coulomb model tends not to reproduce pore pressure behavior very well, possibly generating values below those measured in the field. Hence, it is necessary to be aware of the different results of the model. In order to verify the validity of pore pressure analyses, Figure 8(b) compares the piezometric curves with the Cam-Clay and Mohr-Coulomb model, all at 6 meters depth. Figure 8(b) also shows that the excess pore pressures were actually lower than they should have been, as mentioned above. The Cam-Clay model tends to reproduce pore pressure levels better than the Mohr-Coulomb model. Possibly, the location of the piezometer was not perfectly respected, being closer to the column and consequently, presenting lower pore pressure values. For this reason, these values coincided with the indicated by the Mohr-Coulomb model.

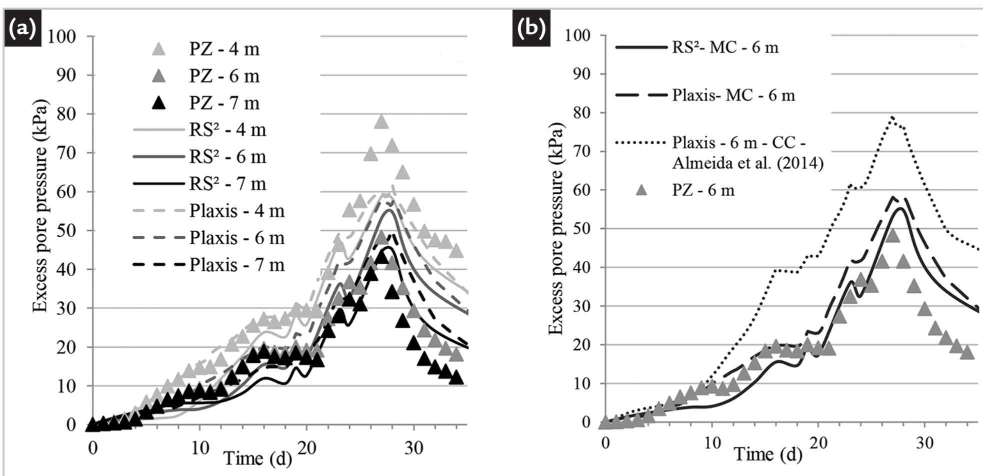

Figure 8 - Excess pore pressure over time in different depths (a) and compared with Cam-Clay (CC) analysis (b).

3.4 Stone column yielding

In Figure 9, there was almost total yield of the stone columns under the concrete plate due to the stress concentration and the arching effect, which occurs not only at the top of the columns but also along their depth (Lima, 2012). The higher load absorption of the columns instead of the clay has prevented the yielding of the clay.

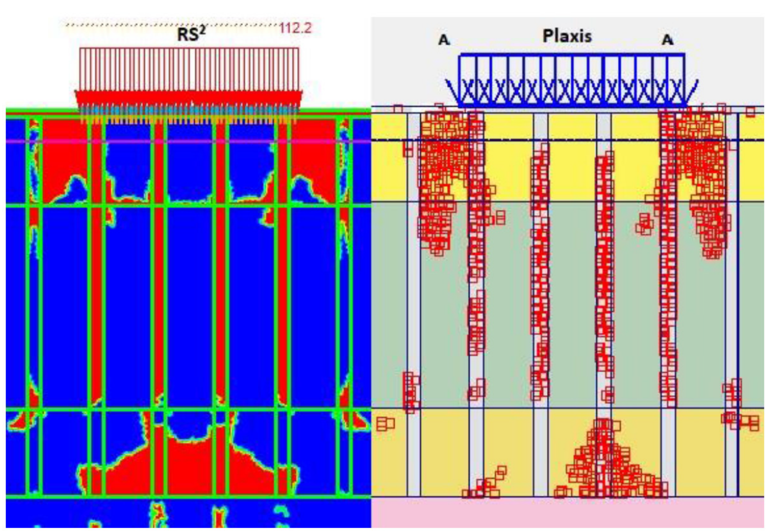

Figure 9 - Yielding points model at Plaxis and RS² software.

The perfect shape of the rupture wedge can also be observed by marking the movement of the rupture mass during the loading process, as well as realizing that both results show yielding on the same points, ensuring the compatibility of operation of the two models created. The yielding columns are represented on the 21st day, supposed rupture in the evolutionary sequence presented in Figure 2. The yielding points at Plaxis model are not symmetrical, but with minor differences, due to the non-symmetrical mesh.

4. Discussion

Vertical displacements presented good compatibility for both software with the settlement plates, indicating a reasonable magnitude of the estimated values for maximum modulus of elasticity. The results from the Cam-Clay model performed by Almeida *et al.* (2014) also showed satisfactory proximity to the above-mentioned analyses. Horizontal displacements presented some differences that can be associated with the rigidity of the aluminum inclinometer and/or the presence of a less resistant layer at these depths. However, concerning numerical analyses, Mohr Coulomb and Cam Clay models were very similar to each other. Excess pore pressure cannot be asserted, despite indicating values that are very close to the piezometer reading, due to Mohr-Coulomb model's deficiency calculating the effect of pore pressure and uncertainties about piezometer location after installation between columns.

The yielding of the columns was almost the same in RS² and Plaxis. The concentration of most part of the load by the stone columns represents a relief in the soil layers, and consequently, a lower yielding of the clays. For this reason, the Mohr-Coulomb model, which has difficulties reproducing the behavior of very compressible clays, satisfactorily represented the case studied.

4. Conclusion

The aim of this study was to reproduce the results of a numerical analysis of a test area with stone column treatment located in a very compressible clay soil. The geotechnical profile was modeled in two finite element software (Plaxis and RS²) with the 2D Mohr-Coulomb model in order to compare each one and the data from instrumentation. Furthermore, with Cam-Clay soft soil creep analysis by Almeida *et al.* (2014), also 2D model, it became possible to compare the estimated values with the numerical analysis.

It is possible to assume that there was optimal compatibility for the numerical analysis of vertical displacements in both software until rupture and during the yielding phase. It is likely that it may have occurred due to the absorption of most of the loads by the stone columns, even though the Mohr-Coulomb model does not represent compressible soils as well as the Cam-Clay model. The horizontal displacements and excess pore pressure showed slightly different values from the instrumentation due to previously expected situations. It is suggested that other analyses such as these be done to contribute to further research on constitutive models and instrumentations.

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