Volume 7, Number 3 (June 2014) p. 329-348 • ISSN 1983-4195

IBRACON

REVISTA IBRACON DE ESTRUTURAS E MATERIAIS
IBRACON STRUCTURES AND MATERIALS JOURNAL

Evaluation of second order moments in reinforced concrete structures using the γ_z and B_2 coefficients

Avaliação dos momentos de segunda ordem em estruturas de concreto armado utilizando os coeficientes γ_z e B_2

D. M. OLIVEIRA [a]
danielle@demc.ufmg.br

N. A. SILVA [b]
ney@dees.ufmg.br

P. M. OLIVEIRA [a]
priscillameireles@ufmg.br

C. C. RIBEIRO [a]
ccrstar@demc.ufmg.br

Abstract

This paper presents an alternative to estimate the magnifier of first order moments to be applied on each storey of reinforced concrete structures, from the values obtained for the γ_z and B_2 coefficients, used to evaluate second order effects in reinforced concrete structures and in steel structures, respectively. In order to develop the study, initially several reinforced concrete buildings of medium height are processed, in first order and in second order, using the ANSYS software. Next, γ_z, B_2 and the increase in first order moments, when considering the second order effects along the height of the buildings, are calculated. Finally, from the results obtained, the magnifier of the first order moments, differentiated for each storey of the structure and calculated from both γ_z and B_2 coefficients, is estimated and the efficiency of the simplified method of obtaining final moments using the magnifier proposed is evaluated.

Keywords: reinforced concrete, second order effects, γ_z coefficient, B_2 coefficient.

Resumo

Neste trabalho apresenta-se uma alternativa para estimar o majorador dos momentos de primeira ordem que deve ser aplicado em cada pavimento das estruturas de concreto armado, a partir dos valores obtidos para os coeficientes γ_z e B_2, utilizados para avaliar os efeitos de segunda ordem em estruturas de concreto armado e de aço, respectivamente. Para conduzir o estudo, inicialmente diversos edifícios de médio porte de concreto armado são processados em primeira e segunda ordem utilizando o programa ANSYS. Em seguida, são calculados os valores dos coeficientes γ_z e B_2, bem como dos acréscimos sofridos pelos momentos de primeira ordem, quando considerados os efeitos de segunda ordem, ao longo da altura dos edifícios. Finalmente, a partir dos resultados obtidos, estima-se o majorador dos momentos de primeira ordem, diferenciado para cada pavimento das estruturas e calculado a partir de ambos os coeficientes γ_z e B_2, e avalia-se a eficiência do método simplificado de obtenção dos momentos finais utilizando o majorador estimado.

Palavras-chave: concreto armado, efeitos de segunda ordem, coeficiente γ_z, coeficiente B_2.

[a] *Universidade Federal de Minas Gerais, Escola de Engenharia, Deptº de Engenharia de Materiais e Construção, Belo Horizonte, MG, Brasil;*
[b] *Universidade Federal de Minas Gerais, Escola de Engenharia, Deptº de Engenharia de Estruturas, Belo Horizonte, MG, Brasil.*

Received: 22 Dec 2013 • Accepted: 13 Apr 2014 • Available Online: 02 Jun 2014

1. Introduction

In recent decades, as in many other areas, engineering has undergone great change, particularly in project engineering and civil construction. Optimization techniques of weight and form, the development of experimental equipment, information technology and efficient numerical models have all contributed to the creation of more economical and streamlined constructions, and higher, more ambitious buildings.
Issues which once caused problems for engineers have now taken on fundamental importance in structural projects. Among these issues, two which stand out in particular are stability analysis and the evaluation of second order effects.
When one studies structural balance, taking into account the deformed configuration, there occurs an interaction between existing forces and displacements, thereby producing additional efforts. In these conditions, the so-called second order effects emerge. These effects can be extremely important in some structures, while in others they may not need to be taken into account.
If the second order effects are significant, a second order analysis should be carried out. As this analysis is often incompatible with practical demands such as time considerations, engineers have sought to develop simplified processes capable of predicting the behavior of structures in the second order.
The coefficients γ_z and B_2, commonly employed in reinforced concrete and steel structures, may be used to evaluate second order effects and provide an estimate of the final efforts of a structure – as long as their values do not go beyond certain limits.
At this point, it is worth remarking on an important feature of the coefficient γ_z: unlike the coefficient B_2, it has just one value for the whole structure, although, as various studies (Carmo [1], Lima & Guarda [2] and Oliveira [3]) have shown, second order effects undergo variations along the whole height of the building.
This study presents an alternative way of assessing the final efforts of a structure, including second order effects, using both the coefficients γ_z and B_2. A range of medium-sized buildings made from reinforced concrete are analyzed in first and second order using the software ANSYS-9.0, and the simplified process of assessing final effort is evaluated, taking into account the variation of second order effects along the whole height of the structures.

2. Classification of structures in relation to horizontal displacement

As previously mentioned, second order effects emerge when the study of a structure's equilibrium is carried out considering the deformed configuration, in other words, when displacements are taken into account in the analysis. Existing forces interact with the displacements, producing additional internal efforts. Second order efforts induced by the horizontal displacements of the structural joints, when subject to vertical and horizontal loads, are defined as global second order effects.
In some, more stiff structures, the horizontal displacements of the joints are small, and as such, global second order effects have a negligible effect on total efforts. These structures are defined as non-sway structures.
However, in more flexible structures, horizontal displacements are significant. As a result, global second order effects represent a significant proportion of final efforts and must be taken into account. These structures are defined as sway structures, and, in such cases, a second order analysis must be carried out.
According to the Brazilian standard NBR 6118:2007 [4], if global second order effects are less than 10% of the corresponding first order efforts, the structure can be classified as a non-sway structure. If not – if the global second order effects are more than 10% of the corresponding first order efforts – the structure is classified as a sway structure.
The NBR 6118:2007 [4] also states that the classification of structures can be achieved by means of the coefficient γ_z, as detailed in the following section.

3. Coefficient γ_z

The NBR 6118:2007 [4] stipulates that the coefficient γ_z, valid for reticulated structures of at least four storeys, can be determined by means of a linear, first order analysis, reducing the stiffness of the structural elements, in order to consider physical non-linearity in an approximate manner.
For any possible load combination, the value of γ_z is calculated by means of the following formula:

$$\gamma_z = \frac{1}{1 - \frac{\Delta M_{tot,d}}{M_{1,tot,d}}} \quad (1)$$

– $M_{1,tot,d}$ (first order moment) being the sum of all the horizontal force moments (with design values) of the combination considered, in relation to the base of the structure. This can be expressed as the following:

$$M_{1,tot,d} = \Sigma\,(F_{hid}\,.\,h_i) \quad (2)$$

F_{hid} being the horizontal force applied to storey i (with design value) and h_i the height of storey i.
– $\Delta M_{tot,d}$ (the increase in moments following the first order analysis): the sum of the products of all the vertical forces acting on the structure (with design values), in the considered combination, by the horizontal displacements of their respective points of application:

$$\Delta M_{tot,d} = \Sigma\,(P_{id}\,.\,u_i) \quad (3)$$

P_{id} being the vertical force acting on storey i (with design value), and u_i the horizontal displacement of storey i.
If this condition $\gamma_z \leq 1.1$ is satisfied, the structure will be classified as a non-sway structure.
The Brazilian standard NBR 6118:2007 [4] stipulates that total efforts (first order + second order) can be assessed by additionally increasing the horizontal forces of the combination of load to be considered by $0.95\gamma_z$, as long as g_z is less than 1.3. Nonetheless, according to the project of revision of the NBR 6118:2000 [5], the total efforts could be obtained from the multiplication of the first order moments by $0.95\gamma_z$, also under the condition that $\gamma_z \leq 1.3$. It can be observed, therefore, that γ_z ceased to be the first order moments magnification coefficient,

and became the horizontal loads magnification coefficient.
According to Franco & Vasconcelos [6], the use of γ_z as a first order moments magnification coefficient provides a good estimate of the results of the second order analysis; the method was applied successfully to tall buildings with γ_z in the region of 1.2 or more. Vasconcelos [7] adds that this process is valid even for values of γ_z of less than 1.10, cases in which technical norms allow second order effects to be disregarded.

4. Coefficient B_2

For the evaluation of second order effects in steel structures, AISC/LRFD [8] adopts the approximate method of amplifying the first order moments by the magnification factors B_1 e B_2. The second order bending moment, M_{Sd}, must be determined by means of the following formula:

$$M_{Sd} = B_1 . M_{nt} + B_2 . M_{lt} \quad (4)$$

M_{nt} being the design bending moment, assuming that there is no lateral displacement in the structure, and M_{lt} the design bending moment resulting from lateral displacement of the frame; both M_{nt} and M_{lt} are obtained by first order analyses. The B_1 amplification coefficient represents the effect P-δ, related to the instability of the bar, or to local second order effects; B_2 considers the effect P-Δ, related to the instability of the frame, or to global second order effects. For each storey of the structure, the coefficient B_2 can be calculated as follows:

$$B_2 = \frac{1}{1 - \frac{\Delta_{0h}}{L} \frac{\sum N_{Sd}}{\sum H_{Sd}}} \quad (5)$$

ΣN_{Sd} being the sum total of the design axial forces of compression on all the columns and other elements resistant to the storey's vertical forces, Δ_{0h} the relative horizontal displacement, L the length of the storey and ΣH_{Sd} the sum total of all the design horizontal forces on the storey that produce Δ_{0h}.
According to Silva [9], if all in all storeys the coefficient B_2 is less than 1.1, the structure can be considered largely resistant to horizontal displacements, and as such, global second order effects need not be taken into account. When the greatest B_2 is between 1.1 and 1.4, the approximate method B_1-B_2 may be used to calculate the bending moment, the other forces (axial force and shear force) being obtained directly in the first order analysis. Finally, a rigorous elasto-plastic second order analysis is recommended in the event that $B_2 >$ 1.40. Silva [9] adds that if $1.1 < B_2 \leq 1.2$, the bending moments can also be calculated by means of a first order analysis performed with the horizontal efforts magnified by the greatest B_2.
In short, as in the case of the coefficient γ_z, the coefficient B_2 constitutes an indicator of the importance of global second order effects in a structure. Oliveira [10] developed a formula capable of relating these parameters, which will be analyzed in the following section.

5. Relation between the coefficients γ_z and B_2

Figure [1] represents a structure composed of three storeys of equal length (L). The figure also shows the vertical (P_{id}) and horizontal (F_{hid}) design forces acting on each storey i, together with their respective horizontal displacements (u_i).
To calculate the γ_z, equation (1), it is necessary to determine the values of $M_{1,tot,d}$ and $\Delta M_{tot,d}$. Equations (2) and (3) provide the following, respectively:

$$M_{1,tot,d} = (F_{h1d} \cdot L + F_{h2d} \cdot 2L + F_{h3d} \cdot 3L) = F_{h1d} \cdot L + 2\,F_{h2d} \cdot L + 3\,F_{h3d} \cdot L \quad (6)$$

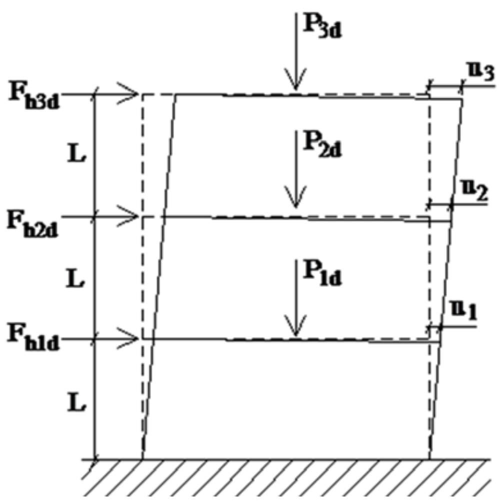
$$\Delta M_{tot,d} = P_{1d} \cdot u_1 + P_{2d} \cdot u_2 + P_{3d} \cdot u_3 \quad (7)$$

Coefficient B_2, provided by equation (5), has a different value for each storey of the structure. Thus, referring to coefficient B_2 of storey i as $B_{2,i,}$ and the parts $(L \cdot \Sigma H_{Sd})$ and $(\Delta_{0h} \cdot \Sigma N_{Sd})$ as M_i and ΔM_i, respectively, the following formulas are obtained:
– 1st storey

$$M_1 = L \cdot (F_{h1d} + F_{h2d} + F_{h3d}) = F_{h1d} \cdot L + F_{h2d} \cdot L + F_{h3d} \cdot L \quad (8)$$

$$\Delta M_1 = (u_1 - 0) \cdot (P_{1d} + P_{2d} + P_{3d}) = P_{1d} \cdot u_1 + P_{2d} \cdot u_1 + P_{3d} \cdot u_1 \quad (9)$$

Figure 1 – Structure of three storeys subject to vertical and horizontal forces



$$B_{2,1}=\frac{1}{1-\frac{\Delta M_1}{M_1}}\Rightarrow B_{2,1}=\frac{1}{\frac{M_1-\Delta M_1}{M_1}}\Rightarrow (M_1-\Delta M_1)=\frac{M_1}{B_{2,1}} \quad (10)$$

– 2nd storey:

$$M_2 = L\cdot (F_{h2d}+F_{h3d}) = F_{h2d}\cdot L + F_{h3d}\cdot L \quad (11)$$

$$\Delta M_2 = (u_2-u_1)\cdot (P_{2d}+P_{3d}) = P_{2d}\cdot u_2 + P_{3d}\cdot u_2 - P_{2d}\cdot u_1 - P_{3d}\cdot u_1 \quad (12)$$

$$B_{2,2}=\frac{1}{1-\frac{\Delta M_2}{M_2}}\Rightarrow B_{2,2}=\frac{1}{\frac{M_2-\Delta M_2}{M_2}}\Rightarrow (M_2-\Delta M_2)=\frac{M_2}{B_{2,2}} \quad (13)$$

– 3rd storey:

$$M_3 = L\cdot (F_{h3d}) = F_{h3d}\cdot L \quad (14)$$

$$\Delta M_3 = (u_3-u_2)\cdot (P_{3d}) = P_{3d}\cdot u_3 - P_{3d}\cdot u_2 \quad (15)$$

$$B_{2,3}=\frac{1}{1-\frac{\Delta M_3}{M_3}}\Rightarrow B_{2,3}=\frac{1}{\frac{M_3-\Delta M_3}{M_3}}\Rightarrow (M_3-\Delta M_3)=\frac{M_3}{B_{2,3}} \quad (16)$$

Adding M_1, M_2 and M_3, equations (8), (11) and (14), and ΔM_1, ΔM_2 and ΔM_3, equations (9), (12) and (15), results in the following:

$$M_1 + M_2 + M_3 = F_{h1d}\cdot L + 2F_{h2d}\cdot L + 3F_{h3d}\cdot L \quad (17)$$

$$\Delta M_1 + \Delta M_2 + \Delta M_3 = P_{1d}\cdot u_1 + P_{2d}\cdot u_2 + P_{3d}\cdot u_3 \quad (18)$$

A comparison of equations (17) and (18) with (6) and (7) can be expressed as follows:

$$M_{1,tot,d} = M_1 + M_2 + M_3 \quad (19)$$

$$\Delta M_{tot,d} = \Delta M_1 + \Delta M_2 + \Delta M_3 \quad (20)$$

Substituting equations (19) and (20) in equation (1), the coefficient γ_z is defined as follows:

$$\gamma_z = \frac{1}{1-\frac{\Delta M_1+\Delta M_2+\Delta M_3}{M_1+M_2+M_3}} = \frac{1}{\frac{(M_1+M_2+M_3)-(\Delta M_1+\Delta M_2+\Delta M_3)}{M_1+M_2+M_3}}$$

$$\gamma_z = \frac{M_1+M_2+M_3}{(M_1-\Delta M_1)+(M_2-\Delta M_2)+(M_3-\Delta M_3)} \quad (21)$$

Inverting equation (21) provides the following:

$$\frac{1}{\gamma_z} = \frac{(M_1-\Delta M_1)+(M_2-\Delta M_2)+(M_3-\Delta M_3)}{M_1+M_2+M_3} \quad (22)$$

Substituting equations (10), (13), (16) and (19) in equation (22) provides the following:

$$\frac{1}{\gamma_z} = \frac{\frac{M_1}{B_{2,1}}+\frac{M_2}{B_{2,2}}+\frac{M_3}{B_{2,3}}}{M_{1,tot,d}} \Rightarrow \frac{1}{\gamma_z} = \frac{M_1}{M_{1,tot,d}\cdot B_{2,1}} + \frac{M_2}{M_{1,tot,d}\cdot B_{2,2}} + \frac{M_3}{M_{1,tot,d}\cdot B_{2,3}} \quad (23)$$

Finally, equation (23) may be expressed as follows:

$$\frac{1}{\gamma_z} = \frac{c_1}{B_{2,1}} + \frac{c_2}{B_{2,2}} + \frac{c_3}{B_{2,3}} \quad (24)$$

With the constants c_1, c_2 and c_3 being provided by the following:

$$c_1 = \frac{M_1}{M_{1,tot,d}} = \frac{F_{h1d} \cdot L + F_{h2d} \cdot L + F_{h3d} \cdot L}{F_{h1d} \cdot L + 2F_{h2d} \cdot L + 3F_{h3d} \cdot L} = \frac{F_{h1d} + F_{h2d} + F_{h3d}}{F_{h1d} + 2F_{h2d} + 3F_{h3d}} \quad (25)$$

$$c_2 = \frac{M_2}{M_{1,tot,d}} = \frac{F_{h2d} \cdot L + F_{h3d} \cdot L}{F_{h1d} \cdot L + 2F_{h2d} \cdot L + 3F_{h3d} \cdot L} = \frac{F_{h2d} + F_{h3d}}{F_{h1d} + 2F_{h2d} + 3F_{h3d}} \quad (26)$$

$$c_3 = \frac{M_3}{M_{1,tot,d}} = \frac{F_{h3d} \cdot L}{F_{h1d} \cdot L + 2F_{h2d} \cdot L + 3F_{h3d} \cdot L} = \frac{F_{h3d}}{F_{h1d} + 2F_{h2d} + 3F_{h3d}} \quad (27)$$

Therefore, in a structure consisting of n storeys, the coefficient γ_z may be calculated with reference to the B_2 coefficient in the following manner:

$$\frac{1}{\gamma_z} = \sum_{i=1}^{n} \frac{c_i}{B_{2,i}} \quad (28)$$

and

$$c_i = \frac{\sum_{j=i}^{n} F_{hjd}}{\sum_{j=1}^{n} j \cdot F_{hjd}} \quad (29)$$

6. Numerical applications

In conducting the study, ten medium-sized buildings made from reinforced concrete (the storey types of which are dealt with in Oliveira [10]) were processed in first and second order. Table [1] lists the main characteristics of the structures analyzed.

The buildings were processed for vertical actions (consisting of permanent loads and accidental load), acting simultaneously with horizontal loads (corresponding to the action of the wind, in the directions parallel to the axes *X* and *Y*, and calculated according to the requirements of Brazilian standard NBR 6123:1988 [11]). The coefficients applied to the actions, defined by means of the ultimate normal combination that considers the wind as the main variable action, were determined according to the requirements of Brazilian standard NBR 6118:2007 [4].

The structures were analyzed using three-dimensional models in the software ANSYS-9.0. The bar element "beam 4" was used to represent the columns and beams. It provides six degrees of freedom in each node: three translations and three rotations, in the directions *X*, *Y* and *Z*.

It is worth reiterating that the buildings were processed in second order by means of a non-linear geometric analysis, reducing the stiffness of the structural elements in order to consider physical non-linearity in a simplified manner (the values adopted were equal to 0.8 I_c for the columns and 0.4 I_c for the beams, I_c being the moment of inertia of gross concrete section).

6.1 Results obtained

Initially, with the results of the first order analysis, the coefficients γ_z and B_2 were calculated for all the buildings, in the directions *X*

Table 1 – Main characteristics of the buildings analysed

Building	Number of storeys	Ceiling height (m)	Number of slabs	Number of beams	Number of columns	fck (MPa)
I	16	2.90	8	8	15	20
II	18	2.55	11	21	16	30
III	20	2.75	9	10	15	45
IV	30	2.85	4	6	9	20
V	22	2.75	11	20	22	65
VI	15	2.90	9	8	16	25
VII	18	2.88	10	11	16	25
VIII	18	2.70	17	31	28	25
IX	20	2.56	12	27	14	30
X	20	2.90	6	9	12	25

and *Y*. Tables [2] and [3] present the values obtained, along with classifications of the structures, in both directions. However, in the case of coefficient B_2, only the mean ($B_{2,m}$) and maximum ($B_{2,max}$) values of the storeys are presented. Again, it is noteworthy that for Silva [9], a structure can be considered largely resistant to horizontal displacement if, in all its storeys, the coefficient B_2 does not exceed the value of 1.1. If B_2 exceeds this value in one storey or more, the structure will be considered highly sensitive to horizontal displacement. Thus, the classification of buildings is carried out by analyzing the value of $B_{2,max}$ which is obtained.
In tables [2] and [3] it can be observed that in all cases, the coefficients γ_z e B_2 provide the same classification for the structures. Moreover, the values of γ_z and $B_{2,m}$ are extremely close, with the greatest difference, corresponding to direction *X* in building I, being around 3.4%. It is also remarkable that in nearly 17% of cases, $B_{2,m}$ was superior to γ_z.

6.1.1 Assessment of γ_z as magnification coefficient of first order efforts (bending moments, axial and shear forces), in obtaining final efforts

The relation between the efforts obtained by the second and first order analyses, in directions *X* and *Y*, was calculated for all storeys of each building.
The only efforts taken into account in the analysis are those which are relevant in terms of structural dimensioning. In other words, the bending moments and axial forces were considered for the columns; and the bending moments and shear forces in the case of the beams.
Table [4] contains the average results of the storeys, along with the values for coefficient γ_z for each building, in the directions *X* and *Y*. One may thus compare the increases in the first order efforts, when second order effects are considered, and the increases predicted by coefficient γ_z.
In table [4] it can be observed that, for all buildings and in both directions, the average increases obtained in the case of axial force in the columns and of shear force in the beams are very small (between 1% and 4%) – in general, far smaller than the increases predicted by γ_z. Therefore, in practical terms, the magnification of these forces by the coefficient γ_z is not necessary, even when the value it represents may be high (as in the case of building II, in direction *X*).
In table [4], it can also be observed that the average increases display close proximity in relation to γ_z for the bending moment in the columns and beams. In the case of the bending moment of the columns, the greatest difference between the average increases and those predicted by γ_z is close to 6% (building III, direction *Y*) in favor of safety. For the bending moment of the beams, the greatest difference (building I, direction *X*) is 6.7%, also in favor of safety. Nonetheless, considering only those cases

Table 2 – Values of the coefficients γ_z and B_2, and classification of the structures (buildings I, II, III, IV and V)

Building	Direction	Coefficient	Value	Classification
I	X	γ_z	1.19	Sway structure
		$B_{2,m}$	1.15	Structure highly sensitive
		$B_{2,max}$	1.28	to horizontal displacement
	Y	γ_z	1.14	Sway structure
		$B_{2,m}$	1.13	Structure highly sensitive
		$B_{2,max}$	1.20	to horizontal displacement
II	X	γ_z	1.32	Sway structure
		$B_{2,m}$	1.29	Structure highly sensitive
		$B_{2,max}$	1.47	to horizontal displacement
	Y	γ_z	1.16	Sway structure
		$B_{2,m}$	1.17	Structure highly sensitive
		$B_{2,max}$	1.22	to horizontal displacement
III	X	γ_z	1.06	Non-sway structure
		$B_{2,m}$	1.05	Structure largely resistant
		$B_{2,max}$	1.07	to horizontal displacement
	Y	γ_z	1.32	Sway structure
		$B_{2,m}$	1.29	Structure highly sensitive
		$B_{2,max}$	1.44	to horizontal displacement
IV	X = Y	γ_z	1.30	Sway structure
		$B_{2,m}$	1.26	Structure highly sensitive
		$B_{2,max}$	1.45	to horizontal displacement
V	X	γ_z	1.17	Sway structure
		$B_{2,m}$	1.15	Structure highly sensitive
		$B_{2,max}$	1.23	to horizontal displacement
	Y	γ_z	1.28	Sway structure
		$B_{2,m}$	1.28	Structure highly sensitive
		$B_{2,max}$	1.35	to horizontal displacement

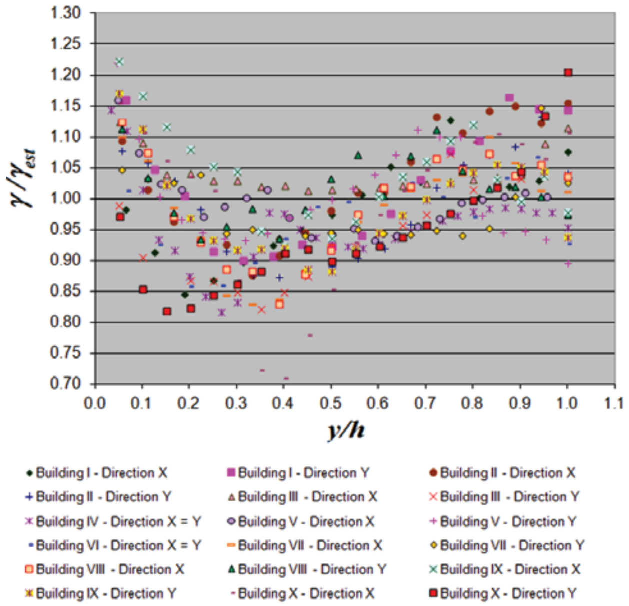
Table 3 – Values of the coefficients γ_z and B_2, and classification of the structures ((buildings VI, VII, VIII, IX and X)

Building	Direction	Coefficient	Value	Classification
VI	X = Y	γ_z	1.21	Sway structure
		$B_{2,m}$	1.18	Structure highly sensitive
		$B_{2,max}$	1.31	to horizontal displacement
	X	γ_z	1.27	Sway structure
		$B_{2,m}$	1.25	Structure highly sensitive
		$B_{2,max}$	1.40	to horizontal displacement
VII	Y	γ_z	1.14	Sway structure
		$B_{2,m}$	1.14	Structure highly sensitive
		$B_{2,max}$	1.18	to horizontal displacement
	X	γ_z	1.30	Sway structure
		$B_{2,m}$	1.28	Structure highly sensitive
		$B_{2,max}$	1.44	to horizontal displacement
VIII	Y	γ_z	1.22	Sway structure
		$B_{2,m}$	1.20	Structure highly sensitive
		$B_{2,max}$	1.30	to horizontal displacement
	X	γ_z	1.31	Sway structure
		$B_{2,m}$	1.34	Structure highly sensitive
		$B_{2,max}$	1.47	to horizontal displacement
IX	Y	γ_z	1.29	Sway structure
		$B_{2,m}$	1.30	Structure highly sensitive
		$B_{2,max}$	1.38	to horizontal displacement
X	X	γ_z	1.30	Sway structure
		$B_{2,m}$	1.30	Structure highly sensitive
		$B_{2,max}$	1.44	to horizontal displacement
	Y	γ_z	1.22	Sway structure
		$B_{2,m}$	1.18	Structure highly sensitive
		$B_{2,max}$	1.34	to horizontal displacement

Table 4 – Coefficient γ_z and average values of relation (second order effort/first order effort)

Building	Direction	γ_z	Columns		Beams	
			Axial force	Bending moment	Shear force	Bending moment
I	X	1.19	1.01	1.17	1.01	1.11
	Y	1.14	1.01	1.16	1.01	1.07
II	X	1.32	1.01	1.35	1.02	1.27
	Y	1.16	1.02	1.14	1.03	1.20
III	X	1.06	1.02	1.11	1.03	1.03
	Y	1.32	1.02	1.24	1.04	1.27
IV	X = Y	1.30	1.03	1.23	1.03	1.23
V	X	1.17	1.02	1.16	1.03	1.15
	Y	1.28	1.03	1.28	1.01	1.28
VI	X = Y	1.21	1.02	1.17	1.03	1.20
VII	X	1.27	1.02	1.24	1.04	1.24
	Y	1.14	1.03	1.12	1.04	1.15
VIII	X	1.30	1.02	1.28	1.03	1.32
	Y	1.22	1.02	1.23	1.03	1.20
IX	X	1.31	1.01	1.35	1.02	1.29
	Y	1.29	1.01	1.27	1.02	1.23
X	X	1.30	1.02	1.28	1.03	1.26
	Y	1.22	1.02	1.15	1.03	1.18

Figure 2 – Variation of the γ/γ_{est} ratio along the whole height of the buildings, in both directions, for the columns



in which the magnification by γ_z is against security, the maximum differences are less than 5% for the bending moment of the columns (building III, direction *X*) and 4% for the same moment in the beams (building II, direction *Y*).

Therefore, it can be affirmed that the calculation of the final moments (first order + second order), by means of the magnification of the first order moments by γ_z is satisfactory. However, the present study is concerned only with structures which provide maximum values for γ_z in the region of 1.3; in other words, structures for which the simplified process of evaluation of final efforts using the coefficient γ_z is still valid, according to the Brazilian standard NBR 6118:2007 [4]. Moreover, the average increases of the structures were considered as a whole, without taking into account the variation of second order effects along the whole height of the buildings, as reported in various studies (Carmo [1], Lima & Guarda [2] and Oliveira [3]). This means that if only the coefficient γ_z is used as a magnifier of first order moments, the final moments may be underestimated for some storeys, and overestimated for others.

Therefore, a better estimate for the final moments can be obtained using also the coefficient B_2, which is calculated for each storey of the structure, and the average value of which is close to γ_z. The magnifier of the first order moments would thus be different for each storey *i* of the structure, and provided by $(B_{2,i}/B_{2,m})\cdot\gamma_z$. This method thus takes into account both the ability of coefficient γ_z to obtain the final average moments of the storeys, as well as the ability of coefficient B_2 to take into account the variation of second order effects along the whole height of a building. The efficacy of this method, when applied to buildings of reinforced concrete, will be evaluated in the next section.

6.1.2 Study of the variation of second order effects with the height of the storeys in the buildings

With the results of the first order analysis, the estimated magnifier of the first order moments was calculated by means of the following formula:

$$\gamma_{est} = \frac{B_{2,i}}{B_{2,m}} \cdot \gamma_z \tag{30}$$

As mentioned above, the relation between the moments obtained by the second and first order analyses was also calculated for columns and beams, along the whole height of the buildings, and in directions *X* and *Y*.

This relation between the moments (second order moment/ first order moment) can be defined as the magnifier of the first order moments, "γ", since it represents the value by which the first order moments should be multiplied in order to obtain the final moments, which include the second order effects. In an ideal situation, in which the magnification of the first order moments by γ_{est} provides the final moments with 100% accuracy, the values of γ and γ_{est} would coincide for all the storeys of the buildings. In other words, $\gamma/\gamma_{est} = 1$ would be true along the whole height of the structure.

In view of these considerations, the graphs in figures [2] and [3] were produced. They represent the variation of the γ/γ_{est} ratio along the whole height of all the buildings, in both directions, for columns and beams, respectively. In these graphs, the axis of the abscissas corresponds to the relation *y/h*, where *y* represents

Figure 3 – Variation of the γ/γ_{est} ratio along the whole height of the buildings, in both directions, for the beams

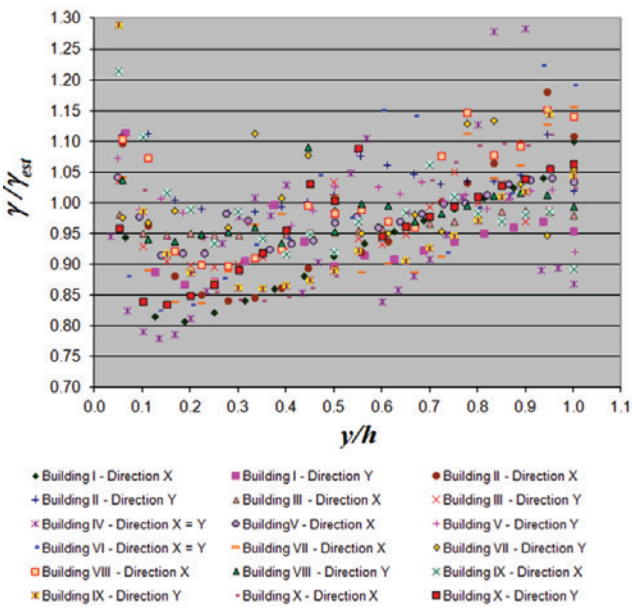


Figure 4 – Histogram for the variable γ/γ_{est}, in reference to the columns

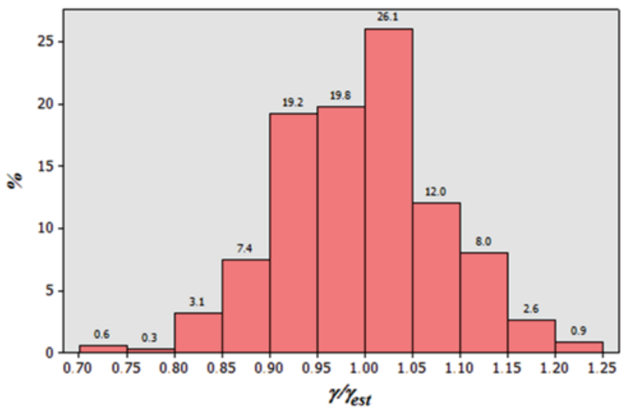
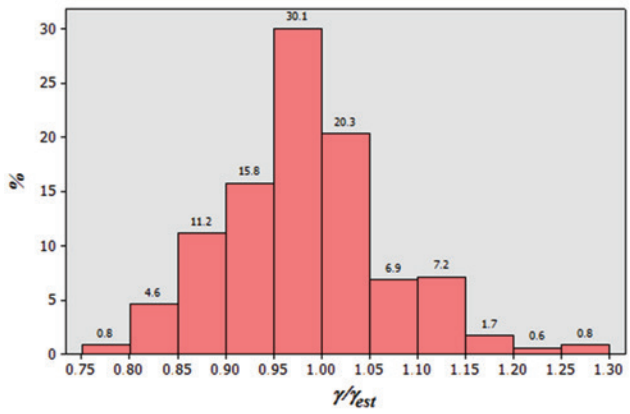


Figure 5 – Histogram for the variable γ/γ_{est}, in reference to the beams



the height of the storey to be considered and *h* the total height of the structure.

In figures [2] and [3], it is clear that most of the values for γ/γ_{est} are situated between, approximately, 0.85 and 1.10, both in the case of the columns and the beams. It is also clear that it is not possible to evaluate with great accuracy the variation and the distribution of the variable γ/γ_{est} by means of a simple observation of figures [2] and [3]. For a more precise assessment of the results obtained, a statistical analysis was carried out, using the software MINITAB-14.

Measurements of central tendency (the mean and the median) and variability (standard deviation, coefficient of variation, minimum and maximum) were calculated for the variable used in the study, the relation γ/γ_{est}. The results are presented in table [5]. For a graphic representation of the distribution of the variable γ/γ_{est}, see the histograms presented in figures [4] and [5], which correspond to columns and beams, respectively.

In table [5], it can be observed that the relation γ/γ_{est} varies from 0.71 to 1.29, with the mean being less than 1.0 in both the columns and the beams. Approximately 50% of the values for γ/γ_{est} are less than 0.992 in the case of the columns and 0.973 in the case of the beams. In addition, the variability of γ/γ_{est} can be considered small, since the coefficients of variation obtained are between 8% and 9%. The coefficient of variation is a measurement which expresses variability in relative terms, comparing the standard deviation with the mean, and can be considered small as long as it remains below 30%.

Observing the histograms in figures [4] and [5], it can be seen that the γ/γ_{est} ratio is less than 1.05 in approximately 77% of instances when considering the columns, and 83% when considering the beams. This means, that in most cases, the magnification of first order moments by γ_{est} would produce a maximum error, opposing to safety, of less than 5%. In the case of the columns, the frequencies are greater for values of γ/γ_{est} between 0.90 and 1.05. For the beams, the frequency is greatest in the range $0.95 \leq \gamma/\gamma_{est} < 1.00$.

7. Final considerations

This study has presented an alternative method of estimating the magnification coefficient of first order moments to be applied to each storey in reinforced concrete structures, by means of the values obtained by the coefficients γ_z and B_2, used to evaluate the second order effects in structures made from reinforced concrete and steel, respectively. Several medium-sized, reinforced concrete buildings were analyzed in both first and second orders using the computer program ANSYS-9.0, and the simplified process of obtaining the final moments was assessed, taking into account the variation of second order effects along the whole height of the structures. The γ/γ_{est} relation was defined, with "γ" being the magnifier of first order moments (the relation between the moments obtained by second and first order analysis, for both columns and beams, along the whole height of the buildings), and γ_{est} the estimated magnifier of first order moments, differentiated for each storey *i* of the structure, calculated by $(B_{2,i}/B_{2,m})\cdot\gamma_z$.

Table 5 – Basic descriptive measures for the variable γ/γ_{est}

Variable	Sample size (n)	Mean	Standard deviation	Coefficient of variation (%)	Minimum	Median	Maximum
γ/γ_{est} columns	349	0.991	0.084	8.442	0.709	0.992	1.222
γ/γ_{est} beams	349	0.978	0.085	8.677	0.782	0.973	1.290

It was observed that the γ/γ_{est} relation varied from 0.71 to 1.29, with the mean and median values obtained being less than 1.0, both for the columns and the beams. In addition, in approximately 77% of cases for the columns and 83% of cases for the beams, the γ/γ_{est} ratio was less than 1.05. Therefore, for most of the situations analyzed, the magnification of first order moments by the estimated magnifier γ_{est} would produce a maximum error opposing to safety of less than 5%.
Future research in the same area should analyse a greater number of buildings and structures, including larger structures or structures with geometrical irregularities, such as sudden changes of inertia and in the heights between storeys.

8. Acknowledgements

The authors thank the Institutional Research Grant Program for Recently Contracted PhD Professors of the PRPq/UFMG for the support provided.

9. Bibliographical references

[01] CARMO, R.M.S. Efeitos de segunda ordem em edifícios usuais de concreto armado. São Carlos. Dissertação (Mestrado) – Escola de Engenharia de São Carlos, Universidade de São Paulo, 1995.
[02] LIMA, J.S.; GUARDA, M.C.C. Comparação entre o parâmetro alfa e o coeficiente γ_z na análise da estabilidade global de edifícios altos. In: CONGRESSO BRASILEIRO DO CONCRETO, 41., Salvador. *Anais*, 1999.
[03] OLIVEIRA, D.M. Parâmetros de instabilidade global das estruturas de concreto armado segundo a nova NBR-6118. Belo Horizonte. Dissertação (Mestrado) – Escola de Engenharia da Universidade Federal de Minas Gerais, 2002.
[04] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. NBR 6118 – Projeto de estruturas de concreto – Procedimento. Rio de Janeiro, 2007.
[05] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. Projeto de revisão da NBR 6118 – Projeto de estruturas de concreto. Rio de Janeiro, 2000.
[06] FRANCO, M.; VASCONCELOS, A.C. Practical assessment of second order effects in tall buildings. In: COLOQUIUM ON THE CEB-FIP MC90, Rio de Janeiro. *Proceedings*, p.307-323, 1991.
[07] VASCONCELOS, A.C. Revisão da NB-1: O problema dos efeitos de 2ª ordem. Jornal TQS News, n.3, Out., p.10-11, 1996.
[08] AMERICAN INSTITUTE OF STEEL CONSTRUCTION – AISC. Load and resistance factor design specification for structural steel buildings. Chicago, 1999.
[09] SILVA, R.G.L. Avaliação dos efeitos de 2ª ordem em edifícios de aço utilizando métodos aproximados e análise rigorosa. Belo Horizonte. Dissertação (Mestrado) – Escola de Engenharia da Universidade Federal de Minas Gerais, 2004.
[10] OLIVEIRA, D.M. Estudo dos processos aproximados utilizados para a consideração das não-linearidades física e geométrica na análise global das estruturas de concreto armado. Belo Horizonte. Tese (Doutorado) – Escola de Engenharia da Universidade Federal de Minas Gerais, 2007.
[11] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. NBR 6123 – Forças devidas ao vento em edificações. Rio de Janeiro, 1988.