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Stability of reinforced concrete buildings with flat slabs: influence of frames with flat slab layers and inverted edge beams

Estabilidade em edifícios de concreto armado com lajes sem vigas: influência de pórticos formados por faixas de lajes e vigas invertidas nas bordas

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Abstract

This paper discusses several possible ways to evaluate reinforced concrete frames designed for multiple floor buildings using flat slabs, from the standpoint of instability and second-order loads. Based on floor designs with simple flat slab frames and regular distribution of columns, models with different numbers of floors are considered. The models do not involve highly rigid elements such as staircases and elevator shafts. The modeling adopts simplified criteria to design vertical loads, forming frames with slab bands representing beams of little height. Based on the results, an analysis is made of the validity of the application of the criteria without considering the second-order loads presented in the NBR6118:2003 code. A comparison is also made of the results with and without the use of inverted edge beams.

Keywords: instability, second-order loads, flat slabs.

Resumo

O presente trabalho aborda maneiras possíveis de avaliar estruturas de concreto armado, sem vigas, destinadas a edifícios de múltiplos pavimentos quanto à instabilidade e necessidade de considerações de esforços de segunda ordem. A partir de plantas simples de estruturas em lajes planas, com distribuição regular de pilares, são considerados modelos com diferentes números de pavimentos. Os modelos não utilizam elementos de grande rigidez, como poços de elevadores e escadas. A modelagem utilizada adota os critérios simplificados para dimensionamento às ações verticais, formando pórticos com faixas de lajes admitidas como vigas de pequena altura. A partir dos resultados, analisa-se a validade da aplicação dos critérios para dispensa de consideração dos esforços globais de segunda ordem apresentados na NBR6118:2003. São comparados resultados para estruturas com e sem utilização de vigas invertidas na periferia.

Palavras-chave: instabilidade, segunda ordem, lajes lisas.

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1. Introduction

1.1 Initial considerations

Building design and construction methods have evolved in the search for the rational and efficient use of resources. The civil construction sector consumes large volumes of natural resources and energy, thereby causing environmental impacts. Thus, it has become increasingly important to build rapidly, economically, safely and with quality. Slab systems without beams (flat slabs) meet these requirements by eliminating the need for beams, simplifying the design and reducing formwork and reinforcements. Flat slab systems offer other direct advantages, such as savings in materials and labor, faster construction, lower costs and improved construction quality, all of which favor the rationalization of construction.

1.2 General characteristics of flat slab systems

For centuries, buildings were constructed of stone and wood, with the floors bearing the loads and distributing them to the transverse beams, from these to the main beams, and from them to the columns. With the advent of reinforced concrete, structures began to be executed with the same principle, i.e., with a reticular concept, which prioritizes vertical load-bearing designs. As buildings evolved to more floors and greater heights, the need emerged for the design of free-standing horizontal load-bearing structures. The design of reticular structures is well defined. The connection between beams and columns, forming frame structures with suitable resistance to wind loads, allows for a good response and safe performance.
The evolution of construction techniques and of the performance of construction materials revealed the advantage of eliminating some elements by means of leaner and more economical solutions. An example of this is structures with flab slab or mushroom slab floors, whose conception is completely different from the conventional systems. In these systems, the slabs are supported directly on the columns. In the region of connection the column (capital) may be thicker, or the thickness of the slab may be greater (abacus, tablet), which the Brazilian standard NBR 6118:2003 [1] calls a mushroom slab. However, it is advantageous to avoid capitals and abacuses in order to obtain flat sooth ceilings (flat slabs, according to the Brazilian building code NBR 6118) and thus use the system to the best advantage.
As for its constructive aspects, the flat slab offers the benefits of a more rational production process, which speeds up and simplifies the execution of several construction phases (production and setup of formwork, preparation of rebars, concreting and execution of installations). From the architectural standpoint its advantages are also evident: greater floor-to-ceiling height of each floor, the presence of smooth ceilings which provide greater freedom in defining spaces, greater slimness and better conditions of ventilation and illumination.
It is easier to implement the advantages offered by the flab slab system when the columns are distributed regularly (arranged in regular lines of aligned columns on the floor plan), the spans are regular, and vertical loads show only minor variations in the values of the same panel and among the various panels that make up each floor of a building. On the other hand, the heights of the slabs are high, and the limits established by the NBR 6118:2003 standard [1], i.e., 16 cm for flat slabs at 14 cm for mushroom slabs, must be observed.
The behavior of the slab subjected to gravitational loads has been studied extensively (Montoya [2], Figueiredo [3], Melo [4], Silva [5], Sylvany [6]). The forms of treatment, and modeling and design tools help overcome difficulties when it is impossible to arrange columns regularly. On the other hand, the use of designs and construction without beams requires the analysis of three problems, as follows:

a) Transverse displacements
The absence of reticulate elements in the structural design translates into lower inertia for the remaining elements, which impairs the performance of the floor with respect to vertical loads, resulting in higher strains. This problem can be solved by employing prestressment. Another possibility is to place beams at the edges of the structure, where the problem is more critical and visible.

b) Punching of the slab
Punching is an ever present phenomenon that requires careful analysis and proper treatment (Leonhardt [7], Figueiredo [3], Montoya [2]). However, the gain in strength of current concretes and the development of industrial reinforcements or mechanisms to reinforce regions subject to such loads, based on numerous studies and tests, solve this problem satisfactorily.

c) Lateral load stiffness
When multistory structures are involved, concern about the building's global stability is greater in the case of beamless floor slabs. In the absence of elements for the formation of conventional frames, the horizontal load-bearing strength is deficient compared to that of other structures. This limitation can be overcome, in part, by rigid cores or structural walls. The work of the set composed of columns and slab bands that constitute rigid frames has been neglected, due to the low value of inertia of horizontal elements when compared to beams. Our intention here is to evaluate the efficiency of this mechanism and determine the limitations for its use.

1.3 Rationale and objectives of the study

Because beamless systems have low horizontal-load bearing stiffness, their use may often be unfeasible, especially in multistory buildings. Several factors, most of which have emerged in recent years, justify studies and evaluations to better clarify the problem. These factors include:

- A greater discussion about the parameters used in the classification of structures, in terms of horizontal load performance, and the approach of the NBR 6118:2003 standard [1] regarding the structural stability of buildings.
- The application of flat slabs in low buildings, together with new closing and dividing systems (gypsum cardboard panels, for example), is an interesting alternative. In such cases there may be no elevator shafts, and in the case of staircases, the prefabricated ones may be the solution. This eliminates two elements that constitute rigid cores and that are traditionally used to bear horizontal loads.
- Even when elevator shafts and stairwells are necessary, their location on the floor plan must be analyzed, since ar-

chitectural interests do not always meet structural requirements adequately.

- The evolution of computational tools and methods to better evaluate accessibility, second-order loads and nonlinearity of materials.

Based on these premises, and keeping in mind that this paper focuses on flat slab structures of solid reinforced concrete, the main objectives here are as follows:

a) To analyze structures in which second-order effects may be significant, item 15.4 of the NBR 6118:2003 standard [1] presents definitions and classifications of structures, indicating possible treatments, albeit in a simplified way. We intend to point out possibilities and limitations for their use.

b) Evaluate the possible ways for treating building structures composed of beamless floor slabs in terms of their stability and the need to consider second-order effects. The influence of beams placed only at the edges of the floor will be highlighted, since, although this may represent a loss of some advantages, it improves the slab's punching performance in relation to transverse displacements at the edges of the slab (where they are more noticeable), and helps increase the building's lateral load-bearing capacity.

c) There are approximate considerations for the treatment of floors designed without beams. Item 14.7.8 of the NBR 6118:2003 standard [1] foresees the possibility of designing the floor using an approximate elastic process, when the columns are placed in a regular arrangement. In these cases, and when the floors are part of the structures of multi-storey buildings, one can consider the formation of spatial frames composed of columns and slab bands working as flat slabs. An evaluation will be made of the ability of this structural design to behave adequately under horizontal loads, considering simplifications consistent with those adopted in the treatment of the floor.

d) An evaluation will also be made of the horizontal strain of these buildings and their performance in relation to the Service Limit State under horizontal loads.

2. Behavior of buildings under lateral loads

The loading of building structures subjected to simultaneous horizontal and vertical forces indicates the need to analyze their global stability. The variation of the loads that appear in the structure as a result of strains caused by horizontal loads (second-order effects) will depend mainly on the horizontal deformation of the structure and on the magnitude of these loads. The ultimate limit state of instability in reinforced concrete structures is described by the NBR 6118:2003 standard [1] as the state that is reached when, upon increasing the load intensity, and hence the strains, some of the elements are subjected to flexural compression, and the structure's load-bearing capacity is insufficient to bear the increased load.

2.1 Global stability of buildings: bracing and displacement

To design high-rise slim structures subject to instabilities there are tools to for evaluating, measuring or establishing limits to the elimination of further attention. Some examples are the instability parameter **α** and the coefficient γ_z prescribed by the NBR 6118:2003 standard [1].

Table 1 – Inertia reducer ratios for the consideration of physical nonlinearity (NBR 6118:2003 (1))

Element	Reducer ratio
Slabs	0.3
Beams (asymmetric frame)	0.4
Beams (symmetric frame)	0.5
Columns	0.8

Concepts such as bracing, bracing structures and braced elements appear in codes such as the CEB-FIP-90 [8] and NBR 6118:2003 [1]. It is up to the designer to define the horizontal load-bearing elements of a structure. Franco [9] demonstrated that the spatial performance of the structure is crucial, and that the contribution of less rigid elements to this performance should not be overlooked when it comes to wind loads.

2.1.1 First-order global analysis

Once the building contains elements that clearly define the horizontal load-bearing structure, a first-order global analysis can be performed with the combination of increased horizontal and vertical loading. In considering the physical nonlinearity of the materials, a stratagem is the adoption of stiffness reducer ratios for the concrete elements. Table 1 lists the values of these reducer ratios described in the NBR 6118:2003 standard [1].

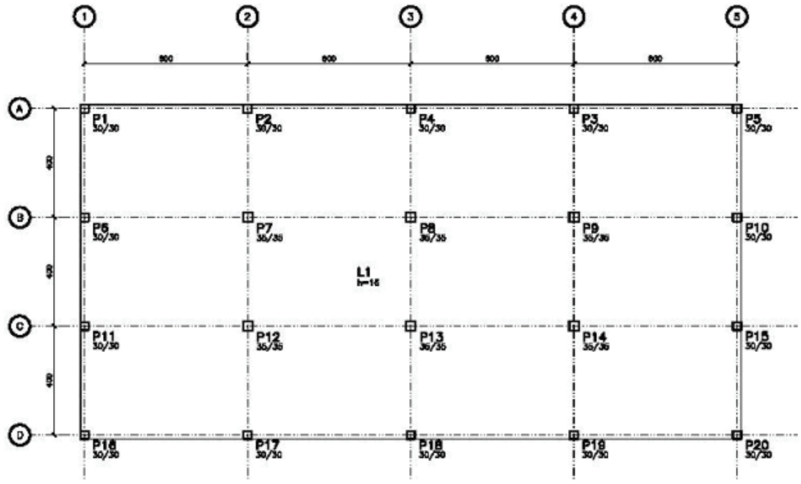
According to the NBR 6118:2003 [1] (item 15.2), the second-order effects can be neglected provided they do not represent increases of more than 10% in the values of the reactions and in the important loads of the structure resulting from the first-order analysis. This limit is the same as that prescribed by the CEB-FIP-90 code [8]. Structures with stiff nodes fit these cases.

2.1.2 Instability parameter α

The first studies that evaluated second-order effects focused only on the behavior of individual bars. Based on Eüler's theory, Beck & König [10] developed the first major studies involving structures as a whole, working in a linear elastic regime. To this end, they considered the building as being equivalent to a single column, fastened at the base and free at the top, with a constant section, and subject to a vertical load distributed uniformly along its length. Thus, the stiffness of the column was equal to the sum of the stiffnesses of the individual columns that made up the bracing of the structure. They then proposed a parameter (instability parameter **α**) which allows the structure to be classified as having either stiff or movable nodes.

The parameter is a function of the total height of the structure, the sum of all the active vertical loads with a characteristic value, and the sum of the values of stiffness of all the columns of the building in the direction considered (for the case of column structures). For framed structures, the stiffness of each frame is considered as an equivalent column.

Figure 1 – Rectangular grid floor plan (type A)



The structure is considered as having stiff nodes in the following situations:

$\alpha < 0{,}2 + 0{,}1 \cdot n$ for n ≤ 3 floors, or;

$\alpha < 0{,}6$ if $n \geq 4$ floors.

The calculation and other considerations about **α** are given in item 15.5.2 of the NBR 6118:2003 standard [1].

2.1.3 The γ_z coefficient

The $\boldsymbol{\gamma_z}$ coefficient was created by engineers Mário Franco and Augusto Carlos de Vasconcelos (Franco & Vasconcelos [11]). This coefficient measures the sensitivity of the structure to second-order effects, i.e., to the effects of geometric nonlinearity, estimating the importance of second-order loads compared to that of first-order loads. The main characteristics of the coefficient are:

- It indicates if a building is a movable or stiff nodes structure, and determines if its mobility is excessive.
- It serves to estimate the amplification of the first-order moments to consider the second-order moments, without requiring calculation of the latter.

The condition required for the structure to be considered as having stiff nodes is for γ_z to be smaller than 1.1 (γ_z < 1.1). When this occurs, a second-order analysis is not necessary. Item 15.5.3 of the NBR 6118:2003 standard [1] described the calculation and other considerations about the γ_z coefficient. In the case of buildings with $\gamma_z \leq 1.3$, the NBR 6118:2003 standard [1] (item 15.7.2) allows an approximate analysis to consider the final second-order loads, by increasing the horizontal loads of the combination considered by a factor of $0{,}95 \cdot \gamma_z$.

2.2 Consideration of nonlinearity in first-order analysis

For a simplified analysis of cracking and physical nonlinearity of the material, the stiffnesses are reduced using the coefficients listed in Table 1. For beams, Franco [12] indicates a value of 0.5 when both sides have flexural reinforcement and of 0.4 when only one side of the tensioned beam is reinforced. For bracing composed exclusively of beams and columns which present a value of $\gamma_z \leq 1.3$, item 15.7.3 of the NBR 6118:2003 standard [1] suggests a simplification to a single reducer ratio of 0.7 for both the beams and columns.

3. Analyzed structures

3.1 Introduction

The NBR 6118:2003 code [1] presents concepts and recommendations to be observed in the analysis of the global stability of buildings also subject to horizontal loads. In certain cases, the approximate processes (parameter α and coefficient γ_z) allow the global second-order loads to be disregarded. This is a widely accepted way of evaluating the structure's feasibility and dispenses with further calculations of its overall instability.

Two types of floor plan geometry will be analyzed for the structure, each one representing buildings with four, five, seven and ten floors. Thus, a total of eight models of different buildings will be analyzed. The instability parameter α and the coefficient γ_z will be determined for each model, according to the NBR 6118:2003 standard [1].

3.2 Floors totally devoid of beams

3.2.1 Floor plan geometry

Each of the two proposed floors consists of a beamless slab with a height of 16 cm (defined according to the designed vertical loads and considering slab punching), supported on 20 square columns. The columns located at the boundaries have 30-cm sides while the central ones have 35-cm sides. The difference between the two cases is in the arrangement of the plan. In the first case (type A), the columns form a rectangular grid with 4.00m spans in one direction and 6.00m spans in the other. The external dimensions are as follows: 12.30 m × 24.30 m (Figure 1). In the second case (type B), the only difference is that the columns are arranged in a square plan, forming a grid with 4.00m spans and resulting in external dimensions of: 12.30 m × 16.30 m (the plan is similar to the one

Table 2 – Heights of the structures under study; similarity between models

Structure with free edges	Structure with edge beams	Floors	Height (m)
EXAMP1A_4P	E1VA_04P	4	11.40
EXAMP1A_5P	E1VA_05P	5	14.20
EXAMP1A_7P	E1VA_07P	7	19.80
EXAMP1A_10P	E1VA_10P	10	28.20
EXAMP1B_4P	E1VB_04P	4	11.40
EXAMP1B_5P	E1VB_05P	5	14.20
EXAMP1B_7P	E1VB_07P	7	19.80
EXAMP1B_10P	E1VB_10P	10	28.20

in Figure 1 and is not shown here). Figueiredo [3] used these two geometries to study bending in beamless slabs, and analyze their structural performance in response to vertical loads.

3.2.2 Geometry of buildings in terms of vertical dimensions

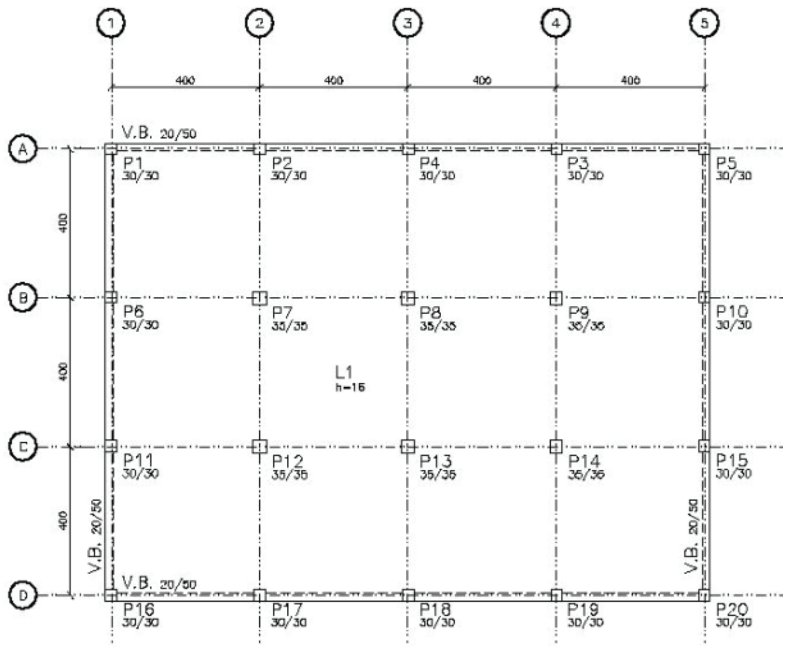
The models consist of slabs separated by a vertical distance of 2.80 m, comprising a floor-to-ceiling height of 2.64 m. For the foundation, the distance will be considered 0.20m greater, admitting columns set in the foundation. The length of the column on the first floor was taken as 3.00 m. Table 2 lists the vertical characteristics of each structure.

3.3 Floors supported only by edge beams

3.3.1 Floor plan geometry

The plans are the same as those described above, types A and B, maintaining all the other dimensions and characteristics (only type B will be shown, square grid, Figure 2). The difference is the inverted edge beams with a 20 cm × 50 cm section.

Figure 2 – Rectangular grid floor plan (type B)



3.3.2 Geometry of the vertical dimensions

The dimensions and characteristics of buildings A and B are maintained for cases without beams: slabs with floor to floor distance of 2.80 m. Table 2 lists the vertical characteristics of each structure (without beams and with edge beams). The roof is considered as a floor.

4. Characteristics of the materials, loadings, modeling

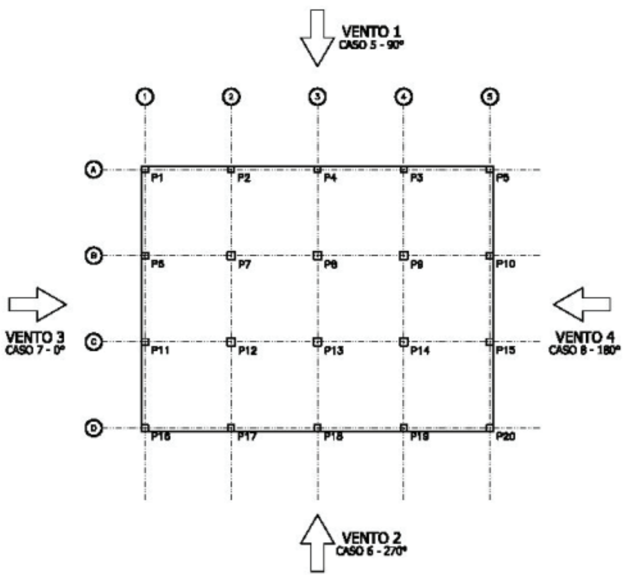
4.1 Characteristics of the materials

To evaluate the models, the following values were adopted for the mechanical properties of the concrete: f_{ck} = 30 MPa; $E_{ci} = 5600 \cdot f_{ck}^{1/2} = 3{,}07$ GPa ; $\gamma = 25$ kN/m^3.
In processing the frames to determine the coefficient γ_z for the approximate consideration of nonlinearity physical, the values of stiffness of the structural elements are reduced (NBR 6118:2003 [1], item 15.7.3). In this case, the values adopted are: $0{,}8 \cdot E_{ci} \cdot I_c$ for the columns, and $0{,}4 \cdot E_{ci} \cdot I_c$ for the beams.
The reduction of the value of the beams (40%) is justified by the behavior of the slab bands used in the models, which is similar to that of continuous beams insofar as bending moments and reinforcements are concerned, considering that the detailing is done following the criteria and recommendations of the simplified model, according to item 14.7.8 of the NBR 6118:2003 [1].

4.2 Loadings considered

The loadings adopted, which are described in the items below, correspond to the usual values for residential building structures. The vertical loads are in line with the NBR 6120:1980 standard [13] and the horizontal loads with NBR 6123:1988 [14]. The values adopted are the same for the two types of buildings, and also for situations with and without edge beams. The floor slabs of the ground floor and the other floors are considered identical. For the roof slabs, vertical loading is considered to be the same in all the buildings, with different values from those of the floor slabs. The values are adopted so as to resemble those of real design situations, but for the main purpose of allowing for comparisons.

Figure 3 – Direction of wind loads



4.2.1. Permanent vertical loads

The following values were adopted for loading on the normal floors for residential use (ground floor and remaining floor slabs), with the value presented in (b) corresponding to the filling, overlay and flooring:
a) Self weight: 4.4 kN/m²; **b)** Other loads: 1.1 kN/m²; **c)** Masonry: 1.0 kN/m².
The value of 1.0 kN/m² adopted for masonry **(c)** is based on item 2.1.2 of the NBR 6120:1980 standard, applicable to walls and room dividers (panels), whose position is not defined in the design.
The loading for the roof slab remains unchanged, i.e., the value presented in (b) corresponds to the roof and overlays:
a) Self weight: 4.4 kN/m²; **b)** Other loads: 1.1 kN/m².

4.2.2 Accidental vertical loads

The value of 1.5 kN/m² was adopted for accidental loading resulting from use, as recommended by the NBR 6120:1980 standard [13] for most rooms in residential buildings. A value of 0.5 kN/m² was adopted for the roof.

4.2.3 Accidental horizontal loading due to wind loads

Wind loads were determined according to the NBR 6123:1988 standard [14], with the following parameters for the determination of pressure and shape coefficients:
a) Characteristic wind speed: 45 m/s. This value corresponds to the speed considered by the aforementioned standard for the region of the city of São Carlos, SP, Brazil.
b) Topographic factor S1 = 1.0, for flat or slightly irregular terrain.
c) Roughness factor S2, which was determined based on the following data:
• Category I roughness (smooth surfaces of large dimensions);
• Class B building (large plan dimensions between 20 and 50 meters);
d) Statistical factor S3 = 1.0.
Four cases of wind load will be considered for each building, with directions in relation to the plan as illustrated in Figure 3. Cases 5, 6, 7 and 8 correspond to the calculations performed and to the results that will be discussed later herein.
The drag coefficients were determined according to the relationships between the plan dimensions and the height of the various buildings. Table 3 lists these relationships and the coefficients adopted. The values correspond to parallelepipedic buildings in low turbulence regime (column C1 in the table). The C1A values shown correspond to wind in high turbulence regime. They are listed solely as comparative data but are not included in the models.
In the case of floors with edge beams, because the external geometries are maintained, the same coefficients are valid for the

Table 3 – Drag coefficients of the buildings under analysis

	Zero degree wind						
Building	L1	L2	h	L1/L2	h/L1	C1	C1A
EXAMP1A_4P	12.30	24.30	11.40	0.51	0.93	0.85	0.77
EXAMP1A_5P	12.30	24.30	14.20	0.51	1.15	0.86	0.78
EXAMP1A_7P	12.30	24.30	17.00	0.51	1.38	0.88	0.79
EXAMP1A_10P	12.30	24.30	19.80	0.51	1.61	0.90	0.80
EXAMP1B_4P	12.30	16.30	11.40	0.75	0.93	1.01	0.85
EXAMP1B_5P	12.30	16.30	14.20	0.75	1.15	1.04	0.86
EXAMP1B_7P	12.30	16.30	17.00	0.75	1.38	1.06	0.88
EXAMP1B_10P	12.30	16.30	19.80	0.75	1.61	1.10	0.89
	Ninety degree wind						
Building	L1	L2	h	L1/L2	h/L1	C1	C1A
EXAMP1A_4P	24.30	12.30	11.40	1.98	0.47	1.16	0.97
EXAMP1A_5P	24.30	12.30	14.20	1.98	0.58	1.18	0.97
EXAMP1A_7P	24.30	12.30	17.00	1.98	0.70	1.21	1.01
EXAMP1A_10P	24.30	12.30	19.80	1.98	0.81	1.23	1.02
EXAMP1B_4P	16.30	12.30	11.40	1.33	0.70	1.14	0.92
EXAMP1B_5P	16.30	12.30	14.20	1.33	0.87	1.18	0.93
EXAMP1B_7P	16.30	12.30	17.00	1.33	1.04	1.20	0.95
EXAMP1B_10P	16.30	12.30	19.80	1.33	1.21	1.21	0.96

determination of horizontal loads. Strictly speaking, there is a slight increase in height due to the presence of the inverted beam of the roof slab. This fact was ignored in the determination of the drag coefficients, and therefore the values listed in Table 3 were maintained in this situation, with the geometric equivalence reported in Table 2. The direction of the wind loads are those indicated in Figure 3.

4.3. Tools and methods used for structural modeling

4.3.1 Computational system

The eight models used here were created and analyzed using version 11.9.9 of the CAD/TQS systems (TQS Informática [15]). The CAD/TQS systems are tools for calculating, dimensioning, detailing and designing concrete structures. They consist of a series of subsystems for modeling buildings, with facilities for data input and construction of the structure.
To design the floor with the proposed configuration, the most suitable tools are those that present a solution analogous to grids or finite elements. In the case of this work, because the main purpose was the processing of frames, slab bands were defined, which were taken as beams of little height, with the geometry proposed for the simplified methods. The CAD/TQS systems include a three-dimensional frame system which calculates the instability coefficients. The frames, as well as the wind loads, are defined based on the spatial structure generated by the modeler.

4.3.2. Definition of the geometry of the structural elements of the frames

4.3.2.1. Floors without beams

Upon generating the model by creating the floor and roof plans, the columns (the external ones have 30 cm sides and central ones 35 cm) will automatically be part of the frames responsible for the horizontal load-bearing work. The seven and ten-floor buildings would need more robust columns, but we decided to keep all the column sections equal in order to better evaluate the results.
The horizontal elements of the frames are defined as slab bands, which are admitted as low beams (small height). The criterion was to adopt the section that is used for the approximate elastic process, as described, for example, under item 14.7.8 of the NBR 6118:2003 standard [1]. The widths adopted

Table 4 – Dimensions of the floor beams

Type A building		bw (cm)			section	Type B building		bw (cm)			section
BEAMS	AXES	left	right	total	(cm)	BEAMS	AXES	left	right	total	(cm)
V1 = V4	A=D	15	100	115	115/16	V1 = V4	A=D	15	100	115	115/16
V2 = V3	B=C	100	100	200	200/16	V2 = V3	B=C	100	100	200	200/16
V5 = V9	1=5	15	150	165	165/16	V5 = V9	1=5	15	100	115	115/16
V6=V7=V8	2=3=4	150	150	300	300/16	V6=V7=V8	2=3=4	100	100	200	200/16

Table 5 – Dimensions of the floor beams

Type A building		b_w (cm)			section	Type B building		b_w (cm)			section
BEAMS	AXES	left	right	total	(cm)	BEAMS	AXES	left	right	total	(cm)
V1 = V4	A=D	-	-	20	20/50	V1 = V4	A=D	-	-	20	20/50
V2 = V3	B=C	100	100	200	200/16	V2 = V3	B=C	100	100	200	200/16
V5 = V9	1=5	-	-	20	20/50	V5 = V9	1=5	-	-	20	20/50
V6=V7=V8	2=3=4	150	150	300	300/16	V6=V7=V8	2=3=4	100	100	200	200/16

Figure 4 – Framework floor plans for type A building, without edge beams

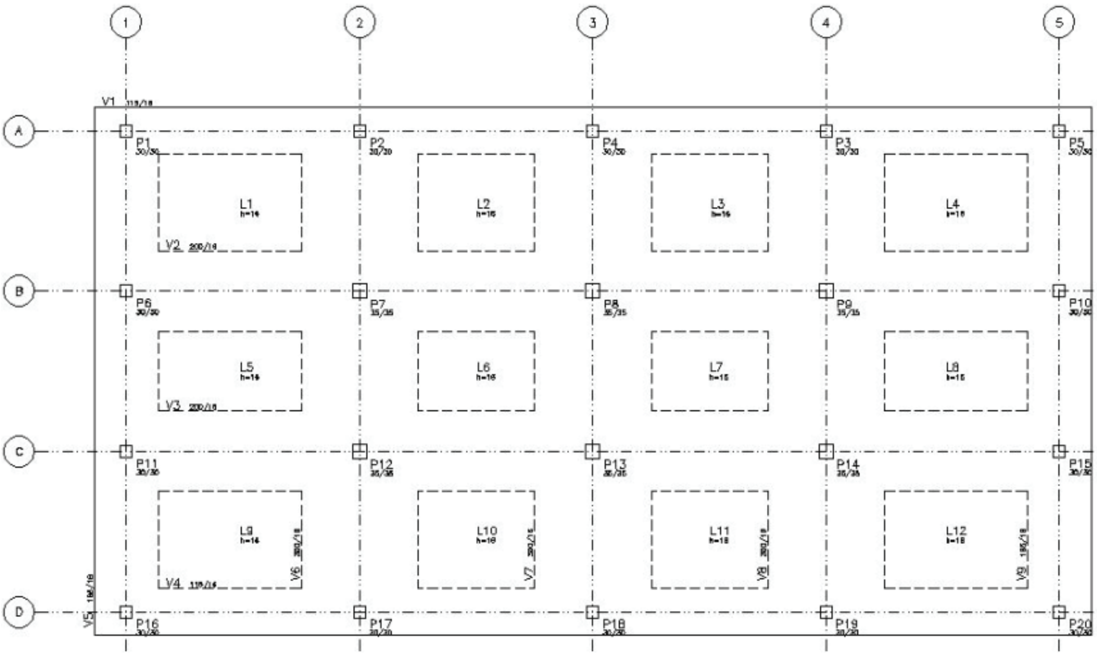
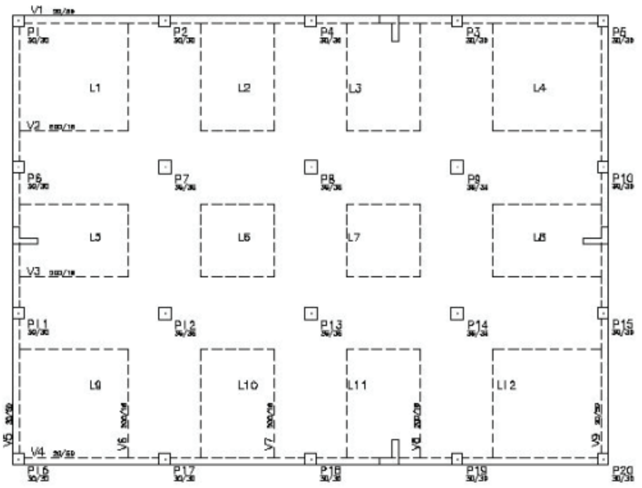


Figure 5 – Framework floor plans for type B building, with edge beams



for the beams are defined as 25% of the distance to each column, starting from the central line defined by the alignment of the columns. The geometry of each beam is defined by this criterion, and the beams' dimensions are listed in Table 4. Figure 2 illustrates the beams listed in this table, in the case of floors on the rectangular grid.

4.3.2.2. Floors with edge beams

The frames defined to verify the stability of the eight buildings are the same, the only difference being the inverted beams at the edges. The section of the beams is a function of the vertical load-bearing capacity (20 cm × 50 cm), without considering the contribution of the compression flange. The widths of the slab bands, which are admitted as internal beams, were the same as those adopted in the models without beams. The beam sections are the ones listed in Table 5. The central columns determine the dimension of b_w of the slab bands taken as beams (central bands). It should be noted that the column dimensions are unfeasible for the taller buildings.

4.3.3 Framework plans

4.3.3.1. Floors without beams

The dimensions of the type A and B buildings are listed in Table 4. Figure 4 shows the framework plan for the rectangular grid model. To simulate the behavior of the frames, leaving the beams centered on the columns, the lateral beams were shifted. The values allocated automatically by the system for the beams were adjusted to prevent this artifice from resulting in higher and unrealistic values of vertical loads. With regard to loading, it is important to note that the consideration made in the simplified method, i.e., that of the loads being taken in duplicate in the beam areas, would result in inadequate values of coefficients for comparison; care was therefore taken to ensure the values of the vertical loads were not considered in duplicate.

4.3.3.2. Floors with edge beams

The dimensions of the type A and B buildings with inverted edge beams are listed in Table 5, while Figure 5 illustrates the framework floor plan on the square grid. A typical detail of an edge, in elevation, is depicted in Figure 6. The input values of the loads were designed to maintain an equivalence with the data of the previous items, to validate the comparison.

5. Results for the cases of beamless flat-slab buildings

The results of the instability parameters a and coefficient γ_z of the eight cases were presented by Cicolin [16] for five cases of loading and several combinations, i.e., one case for vertical loads and four cases of horizontal loads (wind). Sixteen combinations of Ultimate Limit State were then considered. The value of $\gamma_{f2} = \psi_1 = 0.3$ was considered for the Service Limit State, which corresponds to the frequent combination described under item 11.7 of the NBR 6118:2003 standard [1]. Section 7 summarizes the main conclusions.

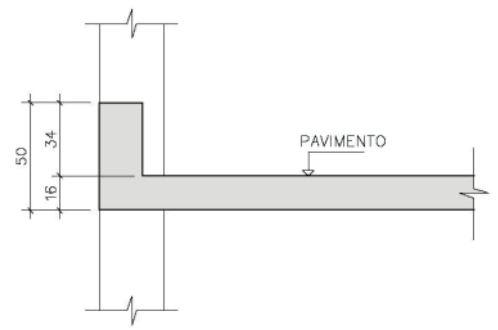
6. Results for the cases of flat-slab buildings with edge beams

This item evaluates the same structures, but with the introduction of inverted beams at the edges. This procedure should improve the performance of the frames with respect to horizontal loads, leading to smaller horizontal displacements in service. In addition, these beams facilitate the detailing by stiffening the edges and providing more efficient connections with the columns. At the edges of the slabs, which are closed with masonry, the presence of inverted beams does not represent an architectural inconvenience. Section 6.1 shows graphics of the values obtained.

6.1 Results found for the instability parameter α

Table 6 lists the results obtained for the instability parameter **α** for each of the eight models, together with those of similar buildings without edge beams. The cases of loadings and combinations are the same as those of the previous item. The graphic representa-

Figure 6 – Detail of inverted beams at the edges of the floor (measured in cm)



tions of the values are shown in Figures 7 and 8 (type A building) and Figures 9 and 10 (type B building).

6.2 Results found for the coefficient γ_z

Table 7 lists the values of γ_z, together with those obtained for the buildings without edge beams. The cases of loadings and combinations are the same as those used in the previous items.

6.3 Maximum displacement profiles and maximum displacements between floors

The displacements were evaluated based on the same criteria and were processed as described above. The values obtained are listed in Tables 8 and 9 (values of total displacement of each building) and in Tables 10 and 11 (maximum horizontal displacement between floors).
The Service Limit State is verified for the frequent combination, considering $\gamma_{f2} = \psi_1 = 0.3$.
The displacements represent relative values. These values should fall within the limits mentioned earlier (h/1700 for maximum horizontal displacements and hi/850 for maximum horizontal displacements between floors).
The values highlighted in bold indicate cases in which the values exceeded the permissible limit. With the presence of the beams, only the 10-floor buildings exceeded the established limit.

7. Conclusions

7.1 Initial considerations

To ensure a coherent analysis, the modeled structures should be representative of the types of buildings actually constructed; reflect the behavior of usual structures; and be sufficiently simple so that their behavior and results can be understood easily. Whatever the choice, there will be some degree of loss of the advantages of the intended characteristics.
It should also be kept in mind that the use of columns is considerably inefficient, notably in the case of 10-floor buildings. The sections for these cases should be increased due to the vertical load. Increasing the section, and particularly, changing the inertia, so that it is used to favor the direction where the parameters proved

Table 6 – Values of the instability parameters α

Floors with free edges			Floors with edge beams		
Building	Cases 5 and 6	Cases 7 and 8	Building	Cases 5 and 6	Cases 7 and 8
EXAMP1A_4P	0.51	0.63	E1VA_04P	0.47	0.52
EXAMP1A_5P	0.59	0.74	E1VA_05P	0.54	0.60
EXAMP1A_7P	0.73	0.93	E1VA_07P	0.66	0.74
EXAMP1A_10P	0.90	1.16	E1VA_10P	0.82	0.92
EXAMP1B_4P	0.46	0.46	E1VB_04P	0.41	0.40
EXAMP1B_5P	0.54	0.54	E1VB_05P	0.48	0.46
EXAMP1B_7P	0.67	0.67	E1VB_07P	0.59	0.57
EXAMP1B_10P	0.83	0.83	E1VB_10P	0.73	0.70

Figure 7 – Instability parameters α.
Type A buildings; cases 5 and 6

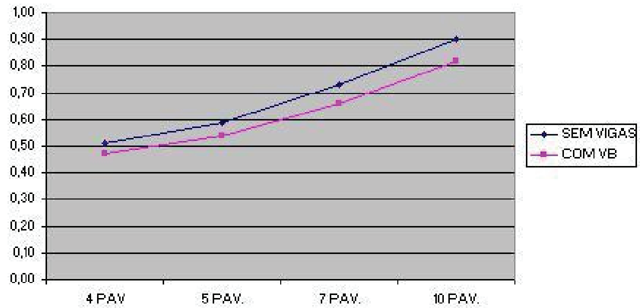


Figure 8 – Instability parameters α.
Type A buildings; cases 7 and 8

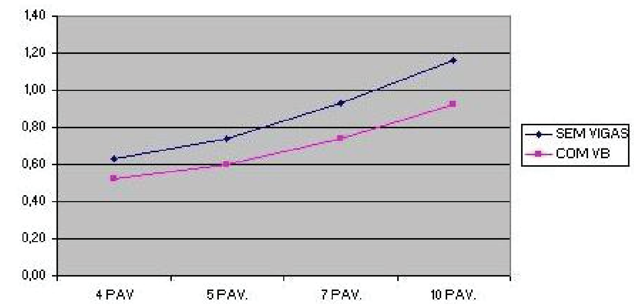


Figure 9 – Instability parameters α. Type B buildings; cases 5 and 6

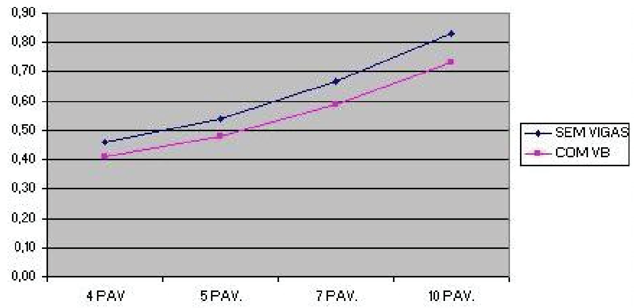


Figure 10 – Instability parameters α. Type B buildings; cases 7 and 8

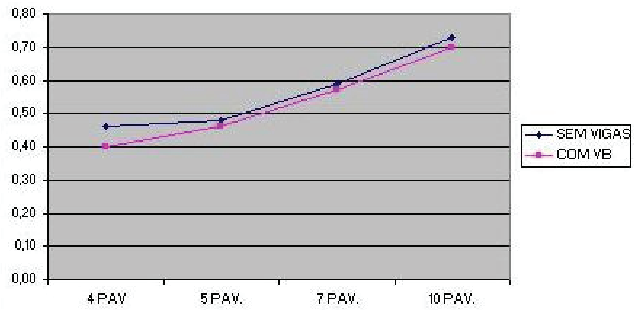


Table 7 – Values of the coefficients γ_z

Floors with free edges			Floors with edge beams		
Building	Cases 5 and 6	Cases 7 and 8	Building	Cases 5 and 6	Cases 7 and 8
EXAMP1A_4P	1.06	1.09	E1VA_04P	1.05	1.06
EXAMP1A_5P	1.08	1.13	E1VA_05P	1.07	1.08
EXAMP1A_7P	1.13	1.22	E1VA_07P	1.11	1.13
EXAMP1A_10P	1.21	1.39	E1VA_10P	1.17	1.22
EXAMP1B_4P	1.05	1.05	E1VB_04P	1.04	1.04
EXAMP1B_5P	1.07	1.07	E1VB_05P	1.05	1.05
EXAMP1B_7P	1.11	1.10	E1VB_07P	1.08	1.08
EXAMP1B_10P	1.17	1.17	E1VB_10P	1.13	1.12

Table 8 – Values of maximum displacements, in cm (cases 5 and 6)

Maximum displacements – Cases 5 and 6					
Floors with free edges			Floors with edge beams		
BUILDING	displ$_h$ (cm)	relative displ$_h$	BUILDING	displ$_h$ (cm)	relative displ$_h$
EXAMP1A_4P	0.34	h/3334	E1VA_04P	0.32	h/3530
EXAMP1A_5P	0.57	h/2479	E1VA_05P	0.53	h/2683
EXAMP1A_7P	1.25	h/1582	E1VA_07P	1.12	h/1765
EXAMP1A_10P	2.85	h/989	E1VA_10P	2.51	h/1125
EXAMP1B_4P	0.26	h/4350	E1VB_04P	0.23	h/4967
EXAMP1B_5P	0.44	h/3208	E1VB_05P	0.38	h/3609
EXAMP1B_7P	0.97	h/2048	E1VB_07P	0.80	h/2463
EXAMP1B_10P	2.19	h/1287	E1VB_10P	1.180	h/1567

to be the most deficient, would have a positive effect on the results. In addition, it should be noted that the values considered for wind loads are higher for the national territory. Buildings erected in regions subject to lower winds would show a better performance.

7.2 Disregarding global second-order loads

With regard to eliminating the need to consider global second-order loads, item 15.5 of the NBR 6118:2003 standard presents two approximate processes to classify the structure as having either stiff or movable nodes; if they are stiff, no rigorous calculation is required. In the case of the structures discussed here, where the slab bands were considered as flat beams composing frames, the limit for the structure to be considered as a stiff nodes structure is $\alpha_1 \leq 0.5$. On the other hand, when the coefficient γ_z is used for the classification, its value should be lower than 1.1.

The results were acceptable for some situations when the coefficient γ_z was used. However, the existence of cases in which the value of γ_z classifies the structure as having stiff nodes with a value of α far exceeding the limit clearly shows that the parameters are underpinned by different premises. The resulting values (α and γ_z) are grouped in Table 12.

Only the case of building B, with 4 floors, met the two criteria and can be classified in the two directions as a structure of stiff nodes.

There is a clearly visible difference between the results provided

Table 9 – Values of maximum displacements, in cm (cases 7 and 8)

Maximum displacements – Cases 7 and 8					
Floors with free edges			Floors with edge beams		
BUILDING	$displ_h$ (cm)	relative $displ_h$	BUILDING	$displ_h$ (cm)	relative $displ_h$
EXAMP1A_4P	0.18	h/6496	E1VA_04P	0.14	h/8119
EXAMP1A_5P	0.30	h/4478	E1VA_05P	0.23	h/6137
EXAMP1A_7P	0.66	h/3014	E1VA_07P	0.49	h/4031
EXAMP1A_10P	1.51	h/1873	E1VA_10P	1.09	h/2591
EXAMP1B_4P	0.17	h/6637	E1VB_04P	0.15	h/4967
EXAMP1B_5P	0.29	h/4849	E1VB_05P	0.24	h/5872
EXAMP1B_7P	0.64	h/3098	E1VB_07P	0.51	h/4378
EXAMP1B_10P	1.48	h/1900	E1VB_10P	1.14	h/2786

Table 10 – Values of maximum displacements between floors, in cm (cases 5 and 6)

Maximum displacements between floors – Cases 5 and 6					
Floors with free edges			Floors with edge beams		
BUILDING	$displ_h$ (cm)	relative $displ_h$	BUILDING	$displ_h$ (cm)	relative $displ_h$
EXAMP1A_4P	0.12	hi/2418	E1VA_04P	0.11	hi/2579
EXAMP1A_5P	0.17	hi/1659	E1VA_05P	0.16	hi/1801
EXAMP1A_7P	0.28	hi/ 990	E1VA_07P	0.25	hi/1104
EXAMP1A_10P	0.46	hi/ 604	E1VA_10P	0.41	hi/ 685
EXAMP1B_4P	0.09	hi/3193	E1VB_04P	0.08	hi/3609
EXAMP1B_5P	0.13	hi/2146	E1VB_05P	0.11	hi/2518
EXAMP1B_7P	0.22	hi/1288	E1VB_07P	0.18	hi/1542
EXAMP1B_10P	0.35	hi/791	E1VB_10P	0.29	hi/956

Table 11 – Values of maximum displacements between floors, in cm (cases 7 and 8)

Maximum displacements between floors – Cases 7 and 8					
Floors with free edges			Floors with edge beams		
BUILDING	$displ_h$ (cm)	relative $displ_h$	BUILDING	$displ_h$ (cm)	relative $displ_h$
EXAMP1A_4P	0.06	hi/4685	E1VA_04P	0.05	hi/5865
EXAMP1A_5P	0.09	hi/3210	E1VA_05P	0.07	hi/4090
EXAMP1A_7P	0.15	hi/1907	E1VA_07P	0.11	hi/2506
EXAMP1A_10P	0.24	hi/1152	E1VA_10P	0.18	hi/1557
EXAMP1B_4P	0.06	hi/4787	E1VB_04P	0.05	hi/5625
EXAMP1B_5P	0.09	hi/3239	E1VB_05P	0.07	hi/3930
EXAMP1B_7P	0.14	hi/1941	E1VB_07P	0.12	hi/2410
EXAMP1B_10P	0.24	hi/1159	E1VB_10P	0.19	hi/1496

by the two methods, as indicated by a comparison of the numbers in Table 12. The structures indicated as having stiff nodes by the parameters **α** have an equivalent in γ_z = 1.05 (EXAMP1B_4P) or γ_z = 1.06 (EXAMP1A_4P – cases 5 and 6), with the latter close to the limit (α = 0.51). On the other hand, there are situations in which the value of $\boldsymbol{\gamma_z}$ indicates the structure should be classified as having stiff nodes, while the value of **α** far exceeds the limit. The best example of this case is building B, with seven floors, cases 7 and 8. As for the values of the maximum displacement profile and maximum displacement between floors in service, these values are higher than permissible for the higher buildings: those with 7 and 10 floors. This is consistent with the classification of structures with movable nodes, given by the simplified processes. Values of maximum displacements between floors are also extrapolated. Further details are given in Cicolin [16].

Table 12 – Comparison of parameters α and coefficients γ_z

Building	Cases 5 and 6		Cases 7 and 8	
	α	γ_z	α	γ_z
EXAMP1A_4P	0.51	1.06	0.63	1.09
EXAMP1A_5P	0.59	1.08	0.74	1.13
EXAMP1A_7P	0.73	1.13	0.93	1.22
EXAMP1A_10P	0.90	1.21	1.16	1.39
EXAMP1B_4P	0.46	1.05	0.46	1.05
EXAMP1B_5P	0.54	1.07	0.54	1.07
EXAMP1B_7P	0.67	1.11	0.67	1.10
EXAMP1B_10P	0.83	1.17	0.83	1.17

7.3 Influence of edge beams

The use of edge beams significantly improved the performance of all the structures. All the parameters evaluated (strains, instability parameter α and coefficient γ_z) presented indications of improved performance in response to the presence of inverted edge beams. In general, the values of α decreased with the introduction of edge das beams, but the taller structures are still classified as having movable nodes. With the limit of α_1 = 0.5 to consider the structure as having stiff nodes, only three of the eight models fit this situation: the type A building with four floors, and type B buildings with four and five floors.
As for the coefficients γ_z, the values for the buildings with beams fall within the interval [1.05; 1.12], while the interval for the buildings without beams is [1.06; 1.39]. Based on the criterion of the NBR 6118:2003 standard [1], the structures having stiff nodes (values below 1.10), in these cases, would be type A buildings with four and five floors, and type B buildings with four, five and seven floors. The change in the values of $\boldsymbol{\gamma_z}$ is proportionally greater for the highest structures. However, there are still cases in which **α**, with a value exceeding 0.50, indicates structures having movable nodes, while the values of $\boldsymbol{\gamma_z}$ remain below 1.10, showing a structure that could be treated according to the considerations described under item 15.7.2 of the NBR 6118:2003 standard [1].
The use of edge beams reduced the values of maximum displacement profiles and maximum displacement between floors in service, and only the 10-floor buildings did not comply with the maximum permissible displacements.
The introduction of edge beams is justified, since they do not affect the building negatively from the architectural standpoint, nor do they make the frame construction and assembly, reinforcement and concreting phases significantly more difficult. Allied to these advantages, one should also consider that edge beams:

- Solve the problem of strains at the free edges of flat slabs;
- Solve the problem of punching at lateral and corner columns; and
- Improve the connection between edge columns and slabs.

The sections used for the edge beams in the models under study could be optimized to further improve the horizontal load performance. In the case of models with 6.00 meter spans between columns, the beam sections would work more efficiently using higher inertia. However, for purposes of comparison with the data of the models with free edges, the use of the 20/50 section provides an important indication.
There are specific cases in which the introduction of inverted edge beams could influence the execution of specific activities during the construction phase. An example of this is the case of buildings which use the solution of ready-made bathrooms. Except in special cases, this solution can be employed when one of the sides of the floor is left open to insert them into the completed structure. The edges of the floor must be flat at this location, at least until the cells specified by the floor plan are put in place.

7.4 Final remarks

In taller building structures which use flat slabs and regularly distributed columns, it should be stated only that the consideration of the frames cannot be neglected, and this may be a positive consideration. However, the results of high values of the parameters that measure instability, allied to the aforementioned restriction in considering the columns, does not allow for more effective conclusions about the use of only frames.
It should be noted that, because they present a perfect symmetry in relation to the two main axes, the cases treated here result in structures with excellent performance, avoiding the inconvenience of applying the horizontal load that, on the floor plan, falls outside the floor's center of torsion. The use of rigid elements such as staircases usually results in structures with a negative performance with respect to this aspect.
When one considers only the numerical values, the performance in Service Limit States is a factor for concern. However, the use of columns with reduced dimensions does not allow for a definitive conclusion.
With regard to the results of the parameters presented by the NBR 6118:2003 standard [1], the use of the coefficient γ_z is clearly advantageous. In only one case, with a γ_z value above 1.30, would it be impossible to estimate the second-order values by increasing the values. Considering that in this case the columns were used inefficiently, it can be stated that a slight improvement of the inertia of the elements would greatly increase the chances of obtaining a structure with adequate performance.
Once again, we point out that the use of edge beams resulted in a significant improvement of in the performance of all the structures and all the parameters evaluated (strains, instability parameter α and coefficient γ_z), indicating that the possibility of using edge beams should always be evaluated, especially when they are inverted.
The evolution of tools for structural analysis tends toward the treatment of three-dimensional frame structures. The complete second-order analysis of such structures, and a more precise consideration of nonlinearity, may improve their responses and render the use of beamless floor slab systems more competitive.

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