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ORIGINAL ARTICLE

Evaluation of concrete resistances: an alternative to the criteria of Brazilian standard NBR 12655 based on a Bayesian approach

Avaliação das resistências do concreto: uma alternativa aos critérios da norma brasileira NBR 12655 baseada em uma abordagem Bayesiana

Claudia Interlandi[a]
Fernando Rebouças Stucchi[b]
Luiz Fernando Campos Ramos Martha[c]
Sergio Hampshire de Carvalho Santos[d]

[a]Pontifícia Universidade Católica do Rio de Janeiro – PUC-Rio, Rio de Janeiro, RJ, Brasil
[b]Universidade de São Paulo, São Paulo, SP, Brasil
[c]Pontifícia Universidade Católica do Rio de Janeiro – PUC-Rio, Departamento de Engenharia Civil, Rio de Janeiro, RJ, Brasil
[d]Universidade Federal do Rio de Janeiro, Escola Politécnica, Departamento de Estruturas, Rio de Janeiro, RJ, Brasil

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Abstract: Frequently the required concrete resistance is not attained in Brazilian constructions. Partially to circumvent this problem, a new revision of Brazilian Standard ABNT NBR 12655 was issued in 2015. This revision maintains the insufficiently stringent criteria for evaluation of the concrete compression strength in existent and under construction structures of the revision of 1996. This evaluation can be based on very few test specimens, even in the result of a single test. It is shown herein that these criteria are clearly unsafe. Alternative criteria are proposed, based in a Bayesian approach. The proposed criteria are checked against some hundred tests done on actual structures of different characteristics, bridges and buildings.

Keywords: concrete resistance, Brazilian standard, Bayesian approach.

Resumo: Frequentemente, a resistência requerida do concreto não é atingida nas construções brasileiras. Parcialmente para solucionar este problema, uma nova revisão da Norma Brasileira ABNT NBR 12655 foi emitida em 2015. Esta revisão mantém os critérios insuficientemente rigorosos definidos na revisão anterior de 1996, para a avaliação das resistências à compressão de concreto em estruturas existentes e em construção. Essa avaliação pode ser baseada em muito poucas amostras de teste, até mesmo no resultado de um único teste. É mostrado aqui que esses critérios são claramente inseguros. Critérios alternativos são propostos, baseados em uma abordagem Bayesiana. Os critérios propostos são verificados contra resultados de algumas centenas de ensaios realizados em estruturas reais de diferentes características, pontes e edifícios.

Palavras-chave: resistência do concreto, norma brasileira, abordagem Bayesiana.

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1 INTRODUCTION

High resistance concretes are nowadays frequently used in the structural design in Brazil. This poses new technological problems for our construction industry, still in an adaptation process to this new technology. Therefore, the concrete compressive strength required in the design is frequently not attained in the construction.

Corresponding author: Claudia Interlandi. E-mail: interlandiclaudia11@gmail.com
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The Brazilian standard related to this issue, the ABNT NBR 12655 [1], maintained in its 2015 revision the same criteria defined in its 1996 previous version [2] for the evaluation of the concrete compression strength, in existent and under construction structures. This evaluation in based on a very disputable probabilistic basis and the standard allows that it should be based on very few specimens, even in the result of a single specimen. It is shown in this paper that the criteria defined in ABNT NBR 12655 [2] are clearly unsafe.

Considering the necessity of the evaluation of the resistance of the concrete in the construction, with a more solid probabilistic basis, alternative criteria are herein proposed, based on a Bayesian approach.

As exposed, for instance in Ang and Tang [3], Melchers and Beck [4] and Nowak and Collins [5], the Bayesian approach allows for the updating of previously known data, combining them with new data obtained, for instance, in new tests in a structure. It also allows for a better consideration of data, insufficient on the statistical point of view, through the combination of these data with a subjective interpretation of them (in a "engineering judgment"), done by an experimented professional.

The information and examples found in these references have a predominantly theoretical character. A practical application of the Bayesian approach can be found in Jacinto [6], that express the theory in a practical formulation and applies it in the safety evaluation of an existent bridge, analyzing the resistance of prestressed cables of this bridge, combining previously known data with results of actual tests performed in some of the cables.

The criteria herein proposed are checked against the results of some hundreds of tests done in real structures with different characteristics, as bridges and buildings.

This paper extends, summarizes and better interpret results previously presented by the authors Stucchi and Santos [7], Interlandi et al. [8] and Chaves [9]. The relevance of the presented issue recommends that it should be put for discussion among the Brazilian technical community of structural concrete and eventually could lead to revisions in the criteria presently defined in the standards.

2 EVALUATION OF CONCRETE STRENGTH – PRESENT STANDARD CRITERIA

2.1 Characteristic strength

For the design of a structure, the criteria of the modern design standards are based on probabilistic concepts, in which resistances and actions are treated as aleatory variables and, from probabilistic analyses, partial safety factors are defined, for increase loads and reduce resistances, in order to attain the required reliability level.

Regarding the concrete compression strength, subject of the study in this paper, its definition as an aleatory variable is justifiable, considering uncertainties such as imprecision in the concrete mixing, non-homogeneity of concrete and variability of its component materials. This reflects in different results obtained in tests in cylinders poured in the same concrete mix. The concrete strength shall therefore be represented by probabilistic functions.

In a probabilistic point of view, considering the existence of a sufficient number of specimens for characterizing this probability distribution, the use of the Normal Distribution for modelling the concrete strength is justifiable. This will be also discussed later on in the paper.

Considering these uncertainties, it is necessary to define a reference value for the concrete strength, for permitting the usual design calculations still done in a deterministic basis.

Then, the concept of characteristic concrete strength is defined, as the strength value, for which it is expected that only 5% of the test results should be below of. Its value, considering the Normal Distribution is given by:

$$f_{ck} = f_{cm} - 1.65 * \sigma \quad \text{(Equation 1)}$$

Where:

$f_{ck} =$ characteristic concrete strength

$f_{cm} =$ average strength of concrete specimens

$\sigma =$ standard deviation of strength of concrete specimens

In this way, the strength f_{ck} shall be defined in the structural design and attained in the construction. The verification whether the design strength was attained shall be demonstrated in tests that shall follow the methodologies defined by ABNT for pouring the cylinders, for the execution of the tests and for acceptance of the concrete strengths.

2.2 Technological concrete control

In order that the results obtained in tests could permit to decide for the acceptance or not of a given concrete volume, it is necessary to apply a process of technological concrete control.

In Brazil, the standards ABNT NBR 5738 [10], ABNT NBR 5739 [11] and ABNT NBR 12655 [1] define, respectively, how to perform the stage of molding and curing, the tests of specimens and the technological control of the concrete batches.

ABNT NBR 12655 also regulates the acceptance concrete control [1]. According it, each specimen shall be composed by two cylinders of the same mix, for each test age and being simultaneously poured. The specimen strength shall be taken as the higher value obtained in the compression tests, to be done according ABNT NBR 5739 [11].

In order to facilitate the concrete resistance control of a structure as a whole, the ABNT NBR 12655 [1] allows for the definition of "lots", which shall obey to the conditions stablished in Table 1. From each lot, a sampling shall be extracted, with a number of specimens defined according the control type (these types are defined in the next items).

Two basic types of control are defined in ABNT NBR 12655 [1]: statistical control for "total" sampling and control for "partial" sampling.

Table 1: Maximum values for the formation of concrete lots[a] (NBR 12655)

Identification (the more demanding in each case)	Main solicitations in the structural elements	
	Compression or compression and flexure	**Pure flexure [b]**
Concrete volume	50 m^3	100 m^3
Number of floors	1	1
Time of pouring	Three days of pouring [c]	

[a]In case of total sampling control, each mix shall be considered as a lot, according 6.2.3.1. [b]In the case of the complement of a column, the concrete belongs to the volume of slabs and beams. [c]This period shall be within the total maximum period of seven days, including eventual interruption for the treatment of joints

The idea of the total sampling is that the characteristic strength value f_{ck} can be evaluated with tests performed in a single specimen of a mix. However, all the concrete mixes shall be tested.

In the partial sampling, the tests are performed is some of the concrete mixes and the characteristic strength value f_{ck} of the lot is evaluated from the results of these tests.

2.3 Concrete control with partial sampling

In the concrete control with partial sampling, at least six specimens for concretes of Group I (Classes up to C50) and 12 specimens for concretes of Group II (Classes above to C50), shall be extracted for testing, for each lot.

a) If the number of specimens (n) is between 6 and 20, the expected value for the characteristic strength (denoted by $f_{ck,est}$) will be given by the highest value found with Equations 2 and 3:

$$f_{ck,est} = 2 \times \frac{f_1 + f_2 + \ldots + f_{m-1}}{m-1} - f_m \qquad \text{(Equation 2)}$$

Where:

$m = n/2$ (If n is odd, the higher f_i is discarded)

$f_1, f_2, \ldots, f_m =$ values of the specimens strength, in crescent order

$$f_{ck,est} = \Psi_6 \times f_1 \qquad \text{(Equation 3)}$$

Where:

$\Psi_6 =$ coefficient given in Table 2, to be defined according the concrete preparation condition and of the number of specimens of the sampling, linear interpolation being admitted. The preparation conditions are defined as follows:

Condition A (applicable to all the concrete classes): cement and aggregates are measured in mass, water measured in mass or volume, corrected in function of the aggregate humidity;

Table 2: Values of Ψ_6 (NBR 12655)

Preparation condition	Number of specimens (n)										
	2	**3**	**4**	**5**	**6**	**7**	**8**	**10**	**12**	**14**	**≥ 16**
A	0.82	0.86	0.89	0.91	0.92	0.94	0.95	0.97	0.99	1.00	1.02
B or C	0.75	0.80	0.84	0.87	0.89	0.91	0.93	0.96	0.98	1.00	1.02

NOTE Values of n between 2 and 5 are used in exceptional cases

Condition B (appliable to classes C10 to C20): cement is measured in mass, water is measured in volume and aggregates are measured in mass combined with volume;

Condition C (appliable only to classes C10 and C15): cement is measured in mass, aggregates are measured in volume and water is measured in volume and corrected in function of the estimative of the humidity of the aggregates.

b) If the number of specimens (n) is superior to 20, the expected value for the characteristic strength ($f_{ck,est}$) is given considering a Normal Distribution:

$$f_{ck,est} = f_{cm} - 1.65.s_d \quad \text{(Equation 4)}$$

$$s_d = \sqrt{\frac{1}{n-1}\sum_{i-1}^{n}\left(f_i - f_{cm}\right)^2} \quad \text{(Equation 5)}$$

Where:

f_{cm} is the average strength of the specimens of the lot;

s_d is the standard deviation of the strength of these n specimens

2.4 Concrete control with total sampling

The control for total sampling consists in the sampling of all the mixes, in order that each of them forms a lot (from what it is called 100% sampling). Then, the characteristic compressive strength is given by:

$$f_{ck,est} = f_{c,\ betonada} \quad \text{(Equation 6)}$$

This type of control was modified in the 2015 last revision of ABNT NBR 12644 [1]. In the 1996 version [2], the control with total sampling was defined for two situations:

a) Number of samplings (n) smaller than 20:

$$f_{ck,est} = f_1 \quad \text{(Equation 7)}$$

Where:

$f_1 =$ smaller strength found in the tested samplings;

b) Number of samplings (n) greater than que 20:

$$f_{ck,est} = f_i \quad \text{(Equation 8)}$$

Where:

$i =$ 0,05 n (whether i is fractionary, the immediately superior number should be adopted)

3 STRENGTH EVALUATION – PROPOSAL OF ALTERNATIVE CRITERIA

3.1 The proposed Bayesian approach

Herein, the proposed Bayesian approach is briefly described. The used mathematical expressions are the ones already described by Jacinto [6].

Initially, it is shown how would be an analysis without any previous knowledge of the problem. It is first shown that the adequate probabilistic distribution in the case of a small number of specimens is the T-Student distribution. It is shown that the direct consideration of this distribution, in a sampling with a small number of specimens, would lead to very small, unacceptable values for the characteristic concrete strength.

Then, it is shown how is considered the previous knowledge that the contractor possess in the concrete production. Let us suppose initially two sets of concrete cylinders. The first set (called "Tests") includes the *n* elements corresponding to the cylinders effectively tested, with their respective strengths obtained in the tests. The second set (called "Previous") contains n_0 virtual elements, possessing the set average and standard deviation corresponding to the value of f_{ck} defined by the designer for the concrete production. The value of n_0, that express with relation to n, the confidence that the analyst possess in the quality of the concrete production and in the experience of the contractor, will be qualitatively defined by an "engineering judgement". The combination of the two sets is done and the combined updated values of the concrete average and standard deviation values are obtained (called "A Posteriori").

3.2 Probabilistic analysis without previous knowledge

In the analysis of a finite (small) number of specimens, it should be considered that the Normal Distribution shall be replaced by a T-Student distribution (see Jacinto [6]). The two distribution are similar, possessing the T-Student the same parameters of the Normal, plus the parameter ν (degree of freedom, related to the number of specimens); its definition is given by Equation 9.

$$f_x\left(x|a,b,\nu\right)=\frac{c}{b}\left(1+\frac{1}{\nu}\left(\frac{x-a}{b}\right)^2\right)^{\frac{\nu-1}{2}} \qquad c=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\;.\Gamma\left(\nu/2\right)} \qquad \text{(Equation 9)}$$

Where:

Number of specimens = n; ν = n – 1; average: μ = a; variance: $\sigma = b^2.\nu/(\nu-2)$

The T-Student distribution presents a more sparse shape compared with the Normal distribution, being the more representative one in the case of a small number of specimens. As long as the number of degrees of freedom ν increases, that is to say, when $\nu \to \infty$, the T-Student distribution tends to be closer to the Normal distribution.

Applying the already presented equations to the determination of the concrete characteristic strength, for a limited number of specimens, the consideration of the T-Student distribution would lead to a reduction in the evaluated characteristic values. For this, Equation 10 can be applied:

$$\rho=\frac{1+V.\sqrt{1+\frac{1}{n}}.t\left(0.05\ ,\ \nu\right)}{1-1.645.V} \qquad \text{(Equation 10)}$$

Where:

ρ – relationship between characteristic values determined with T-Student and Normal distributions;

t (0.05, ν) - inverse T-Student distribution for a 5% quantile e ν, degree of freedom;

V – variation coefficient = $\sqrt{\frac{\sigma}{\mu}}$

Figure 1 shows the variation of ρ against *n* (number of specimens), considering an usual value for V = 0.10. Clearly, using a very limited number of specimens, the characteristic value is drastically reduced, preventing its direct application and leading to the proposition of another approach, such as the Bayesian one, using "previous knowledge", as present next.

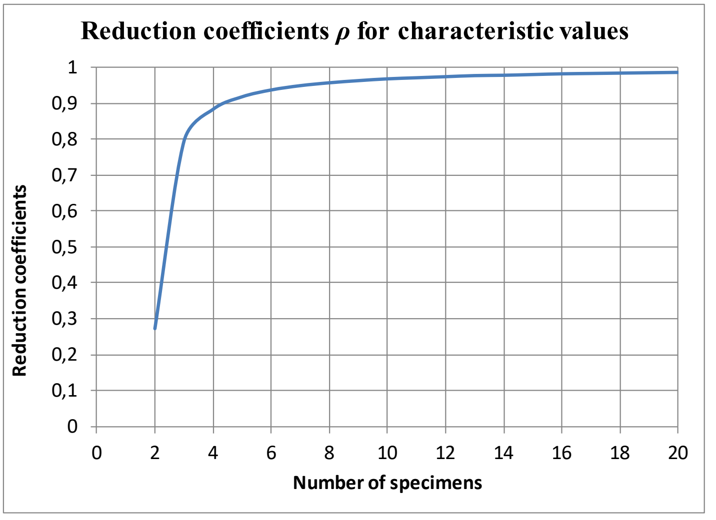

Figure 1: Relationship between characteristic values, using T-Student and Normal probabilistic distributions

3.3 Probabilistic analysis with previous knowledge

The probabilistic analysis with previous knowledge considers that the contractors possess a great previous knowledge on the production of concrete, in order to obtain the strength required by the designer, The Bayesian approach is linked to the importance that will be given to this previous knowledge.

The required formulation for the combination of the previous knowledge with the sampling data was developed by Jacinto [6]. The sampling data, modeled with a T-Student distribution, possess the parameters: $\bar{x}$ (average), s (standard deviation) e n (number of specimens).

The previous knowledge, represented by a T-Student distribution, possess average μ_0 and standard deviation s_0. The variable n_0 represents the relative weight of the contractor's previous knowledge. The value of the variable n_0 shall be defined by the analyst, based on his "engineering judgement". Auxiliary parameters α_0 and β_0 are also defined (Equations 11).

Finally, the *a posteriori* distribution combines the sampling information with the previous knowledge using a T-Student distribution.

A parametric analysis with the variable n_0 is advisable, for evaluating how the variation of n_0 values could affect the final results.

The expressions presented by Jacinto [6] are reproduced in the sequel.

a) Previous knowledge:

- number of specimens: n_0 (arbitrary in Bayesian sense); average: μ_0 ; standard deviation: s_0
- auxiliary parameters: $\alpha_0 = \left(\left(n_0 - 1\right)\right)/2;\ \beta_0 = \left(n_0/2\right)s_0^{\,2}$ (Equations 11)

b) From actual tests:

- number of specimens: n ; average: $\bar{x}$; standard deviation: s

c) "A posteriori" distribution:

- number of specimens: $n_n = n_0 + n$
- average: $\mu_n = \dfrac{n_0\mu_0 + n\mu}{n_0 + n}$; $\alpha_n = \alpha_0 + \dfrac{n}{2}$; $\beta_n = \beta_0 + \dfrac{n-1}{2}s^2 + \dfrac{n_0 n\left(\mu_0 - \bar{x}\right)}{2\left(n_0 + n\right)}$ (Equation 12)

d) T-Student (St) distribution:

$$f_x(x) = St(x \mid \mu_n, \sqrt{\left(1 + \frac{1}{n_n}\right)\frac{\beta_n}{\alpha_n}}, 2.\,\alpha_n) \quad \text{(Equation 13)}$$

3.4 Analysis with corrected total sampling

The total sampling criteria could be enhanced according to the approach proposed by Stucchi [12]. This approach considers a Monte Carlo simulation for generating virtual values for the concrete strength (using a "virtual concrete mixer"), using probabilistic parameters for the strength compatible with the concrete produced in Brazil.

From these analyses, a correction criterion is proposed, that corresponds to consider in the corrected total sampling, the value of $f_{ck} = 0.93 * f_{cm}$. It is to be observed that the value obtained in the test is considered as the f_{cm} and that in the mix the variation coefficient is 4.5%, encompassing both the variability of concrete and also measurement errors. Then:

$$f_{ck} = f_{cm} * \left(1 - 1.645 * 0.045\right) \cong 0.93 * f_{cm} \quad \text{(Equation 14)}$$

4 APPLICATION – STRENGTH EVALUATION IN A BRIDGE

4.1 Analysis according the NBR 12655

Table 3 shows a part of a report with the results of tests performed in four columns in a bridge built in Brazil. Each column was executed in four different days, leading to the consideration of four different concrete lots. Each lot corresponds to six mixes with their respective set of tests, each one including the rupture of two or three cylinders. According to NBR 12655 [1], between these two or three results, the higher strength is considered as the representative one. The nominal characteristic strength is f_{ck} = 35 MPa.

Table 3 shows the results of the analyzed tests. Table 4 shows a summary of the results of the tests in each lot and the characteristic strength values f_{ck} of each lot, with the application of Equations 2 and 3 or 4 (although this one formally cannot be applied for n <20). It can be observed that the results are equivalent.

Table 3 – Results of tests in the bridge

RMCP	SÉRIE	Data Conc.	Tipo	Local Aplicação	Itens Concretados	Vol (m³)	f_{ck}	Data Rompimento	Idade (dias)	CP1		CP2		CP3	
										Kgf	f_{ck}	*Kgf*	f_{ck}	*Kgf*	f_{ck}
008	**035**	08/06/16	Convencional	Rio Guaraí	Apoio 02	8,0	**35,0**	06/07/16	28	0,00	**36,00**	0,00	**36,40**	0,00	**0,00**
008	**036**	08/06/16	Convencional	Rio Guaraí	Apoio 03	8,0	**35,0**	06/07/16	28	0,00	**32,40**	0,00	**32,70**	0,00	**0,00**
008	**038**	08/06/16	Convencional	Rio Guaraí	Apoio 04	8,0	**35,0**	06/07/16	28	0,00	**34,30**	0,00	**34,90**	0,00	**0,00**
008	**040**	08/06/16	Convencional	Rio Guaraí	Apoio 05	8,0	**35,0**	06/07/16	28	0,00	**35,40**	0,00	**36,20**	0,00	**0,00**
008	**041**	08/06/16	Convencional	Rio Guaraí	Apoio 06	8,0	**35,0**	06/07/16	28	0,00	**36,20**	0,00	**36,50**	0,00	**0,00**
008	**042**	08/06/16	Convencional	Rio Guaraí	Apoio 07	3,0	**35,0**	06/07/16	28	0,00	**34,50**	0,00	**34,70**	0,00	**0,00**
010	**045**	20/06/16	Convencional	Rio Guaraí	Apoio 05	8,0	**35,0**	18/07/16	28	0,00	**44,10**	0,00	**44,90**	0,00	**45,20**
010	**046**	20/06/16	Convencional	Rio Guaraí	Apoio 05	8,0	**35,0**	18/07/16	28	0,00	**41,30**	0,00	**41,60**	0,00	**42,60**
010	**047**	20/06/16	Convencional	Rio Guaraí	Apoio 05	8,0	**35,0**	18/07/16	28	0,00	**41,20**	0,00	**42,70**	0,00	**44,00**
010	**048**	20/06/16	Convencional	Rio Guaraí	Apoio 05	8,0	**35,0**	18/07/16	28	0,00	**41,60**	0,00	**41,80**	0,00	**42,40**
010	**049**	20/06/16	Convencional	Rio Guaraí	Apoio 05	8,0	**35,0**	18/07/16	28	0,00	**47,10**	0,00	**47,40**	0,00	**47,70**
010	**050**	20/06/16	Convencional	Rio Guaraí	Apoio 05	8,0	**35,0**	18/07/16	28	0,00	**41,20**	0,00	**41,90**	0,00	**43,30**
013	**061**	24/06/16	Convencional	Rio Guaraí	Apoio 03	8,0	**35,0**	22/07/16	28	36,04	**45,08**	36,95	**46,22**	35,96	**44,98**
013	**062**	24/06/16	Convencional	Rio Guaraí	Apoio 03	8,0	**35,0**	22/07/16	28	33,41	**41,79**	32,96	**41,23**	35,03	**43,82**
013	**063**	24/06/16	Convencional	Rio Guaraí	Apoio 03	8,0	**35,0**	22/07/16	28	35,93	**44,94**	34,82	**43,56**	36,13	**45,19**
013	**064**	24/06/16	Convencional	Rio Guaraí	Apoio 03	8,0	**35,0**	22/07/16	28	33,90	**42,41**	34,20	**42,78**	32,97	**41,24**
013	**065**	24/06/16	Convencional	Rio Guaraí	Apoio 03	8,0	**35,0**	22/07/16	28	34,96	**43,73**	37,59	**47,02**	37,14	**46,46**
013	**066**	24/06/16	Convencional	Rio Guaraí	Apoio 03 C	8,0	**35,0**	22/07/16	28	33,26	**41,60**	35,33	**44,19**	38,08	**47,63**
016	**074**	30/06/16	Convencional	Rio Guaraí	Apoio 04	8,0	**35,0**	28/07/16	28	38,18	**47,76**	39,25	**49,10**	38,86	**48,61**
016	**075**	30/06/16	Convencional	Rio Guaraí	Apoio 04	8,0	**35,0**	28/07/16	28	34,13	**42,69**	37,45	**46,85**	33,66	**42,11**
016	**076**	30/06/16	Convencional	Rio Guaraí	Apoio 04	8,0	**35,0**	28/07/16	28	33,95	**42,47**	33,55	**41,97**	35,20	**44,03**
016	**077**	30/06/16	Convencional	Rio Guaraí	Apoio 04	8,0	**35,0**	28/07/16	28	37,07	**46,37**	35,71	**44,67**	34,59	**43,27**
016	**078**	30/06/16	Convencional	Rio Guaraí	Apoio 04	8,0	**35,0**	28/07/16	28	35,22	**44,06**	34,11	**42,67**	34,58	**43,26**
016	**079**	30/06/16	Convencional	Rio Guaraí	Apoio 04	3,0	**35,0**	28/07/16	28	35,43	**44,32**	35,17	**43,99**	36,36	**45,48**

Table 4: Summary of test results and characteristic strengths of the lots

Concrete lot	Column	Results of tests (MPa)						f_{ck} Equations 2 and 3	f_{ck} Equation 4
8	2	36.4	32.7	34.9	36.2	36.5	34.7	32.50	32.82
10	5	44.9	42.6	44.0	42.4	47.7	43.3	41.70	40.91
13	3	46.2	43.8	45.2	42.8	47.0	47.6	41.41	42.35
16	4	49.1	46.9	44.0	46.4	44.1	45.5	42.61	42.81

On the other hand, it seems evident that the total sampling criterion (directly from the results of the tests) is clearly unsafe. Therefore, for the most critical situation, the concrete lot 008 (column 02), a Bayesian updating was done.

4.2 Bayesian updating

The following data were considered, with relation to the previous knowledge:

- Considered number of specimens: $n_0 = 10$
- Strength average: $\mu_0 = 43.3$ MPa; standard deviation: $s_0 = 4.33$ MPa

(selected values for corresponding to $f_{ck} = 35$ MPa in the T-Student distribution).

Figures 2 and 3 show, respectively, the probability distribution and accumulated probability distributions corresponding to Test no. 40 (with average of 35.8 MPa and V, coefficient of variation = 4.5%).

- Blue lines (dashed): previous knowledge;
- Red lines (continuous): test results;
- Green lines (dashed): “a posteriori” probabilities.

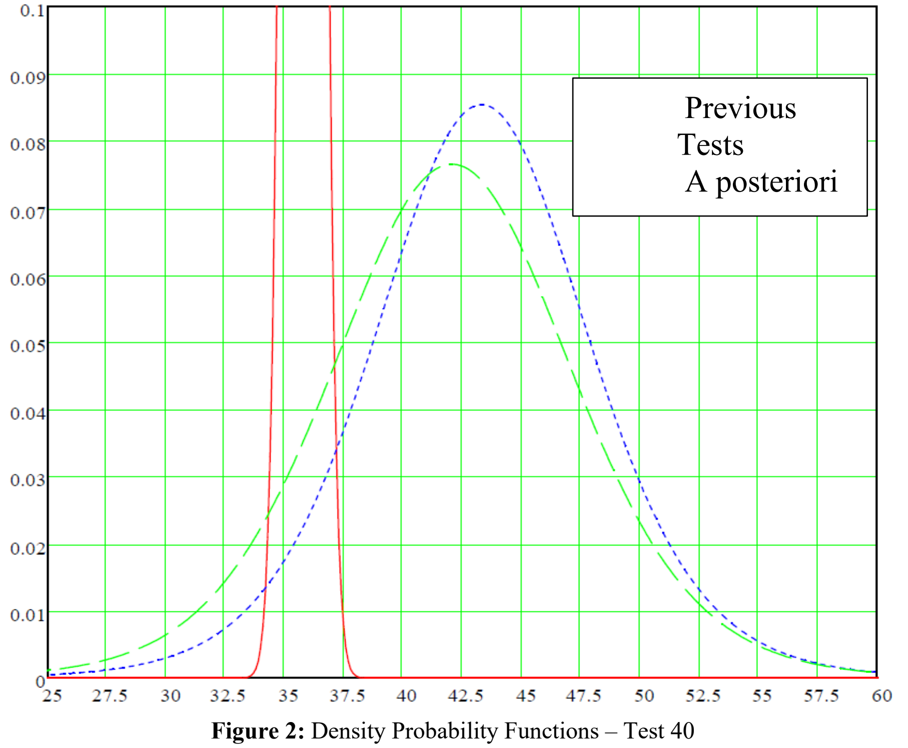

Figure 2: Density Probability Functions – Test 40

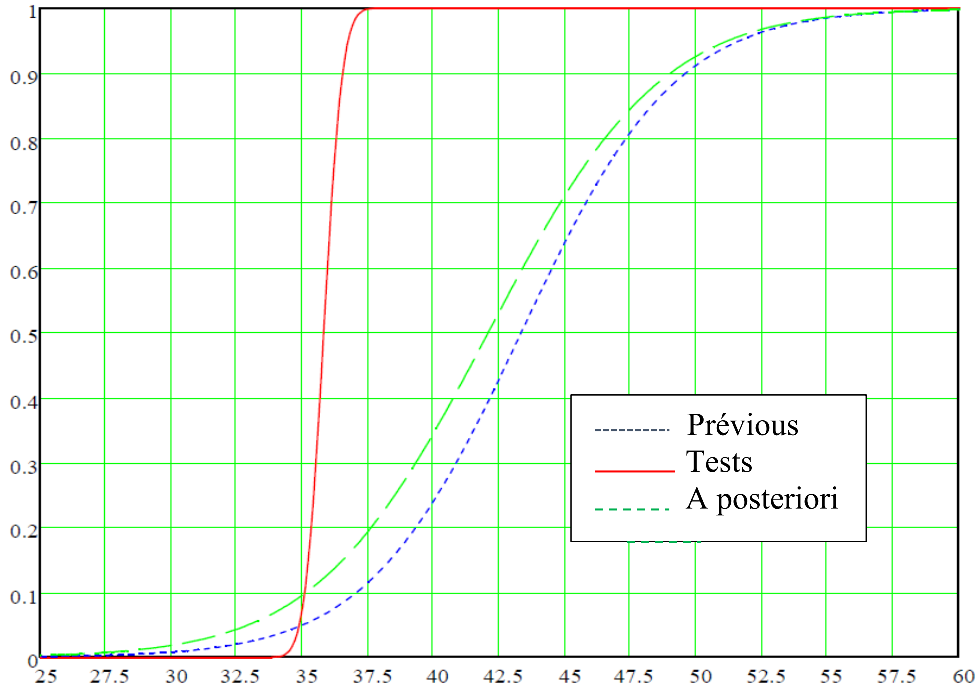

Figure 3: Accumulated Probability Functions – Test 40

Table 5 show the Bayesian updating for the test results performed for the Lot 008 (Column 02). Also in the table, the results obtained with the approach proposed by Stucchi [12] are presented. The deviation between results obtained with the two criteria are shown, indicating that the two proposed approaches lead practically to the same results. It is also evident that total sampling criteria of NBR 12655 are not safe.

Table 5: Results of the Bayesian updating for Lot 008 (column 02)

Test	NBR 12655	Bayesian approach	f_{ck} =0.93 f_{cm}	Deviation
35	36.4	33.2	33.7	1.5%
36	32.7	30.8	30.3	1.6%
38	34.9	32.2	32.2	-
40	36.2	32.9	33.3	1.2%
41	36.5	33.3	33.8	1.5%
42	34.7	32.3	32.2	-

It is to be pointed out the simplicity of the proposed approaches, which lead to acceptable and also more consistent results, in a probabilistic point of view, that the ones defined in the present standard criteria.

5 APPLICATION – STRENGTH EVALUATION IN A BUILDING

5.1 Analysis with partial sampling

Data obtained from the construction of a building in the city São Paulo are used. These data, which can be found in a more detailed form in Chaves [9], encompass: date of the production of the test cylinders; design value of f_{ck} ; volume that each pair of cylinders represent; concreted structural element and compression strength, obtained in tests performed in 28 days.

For the evaluation of the strength of each lot, according to the partial sampling control defined by NBR 12655 [1], Equation 2 and 3 were used, since all the lots possess the number of specimens between 6 and 20.

In Table 6, the values found with the two equations are presented, as well as the f_{ck} of each lot, according to NBR 12655 [1] (are considered as accepted the lots which estimated f_{ck} is higher than the defined in the design).

Table 6: Results of partial sampling

Concrete lot	f_{ck} (MPa) Equation 2	f_{ck} (MPa) Equation 3	f_{ck} (MPa) Estimated	f_{ck} Design	ACCEPTED?	f_{ck} Normal	f_{ck} T-Student
1	29.7	28.8	29.7	30	NO	29.60	30
2	28.4	27.2	28.4	30	NO	27.58	30
3	38.6	38.6	38.6	40	NO	39.40	40
4	39.9	38.9	39.9	40	NO	40.56	40
5	44.3	41.0	44.3	35	YES	44.19	35
6	41.1	40.2	41.1	40	YES	40.52	40
7	29.6	30.5	30.5	35	NO	32.45	35
8	40.6	41.3	41.3	40	YES	41.54	40
9	32.3	34.4	34.4	35	NO	35.32	35
10	34.1	32.8	34.1	30	YES	33.80	30
11	28.9	25.8	28.9	30	NO	29.14	30
12	28.1	28.6	28.6	30	NO	29.52	30
13	34.6	34.1	34.6	30	YES	34.47	30
14	39.6	38.6	39.6	45	NO	42.01	45
15	43.9	41.5	43.9	45	NO	42.56	45
16	36.3	37.6	37.6	40	NO	39.13	40
17	41.7	40.2	41.7	40	YES	42.22	40
18	41.2	39.2	41.2	45	NO	39.94	45
19	48.0	47.2	48.0	45	YES	48.18	45
20	42.6	44.7	44.7	45	NO	44.29	45
21	42.9	40.9	42.9	45	NO	43.54	45
22	48.8	46.9	48.8	45	YES	49.49	45
23	32.4	29.9	32.4	35	NO	31.32	35
24	44.3	44.9	44.9	45	NO	44.78	45
25	40.2	38.2	40.2	40	YES	39.25	40

5.2 Analysis with probability distributions

In Table 6, the evaluation of the characteristic strengths using the Normal e T-Student distributions are also presented. Figures 4 and 5 show, respectively, the probability density functions and accumulated probability density functions for the lot 14, taken as an example. The blue color lines represent the T-Student distribution (dashed) and the red color lines represent the Normal distribution (continuous). In this way, the strength values corresponding to the accumulated probability of 5% are obtained.

Only 6 of 50 obtained results differ more than 5% in relation to the ones obtained according NBR 12655 and in no case the difference is greater than 10%. This shows that the partial sampling criterion for samplings between 6 and 20 specimens is satisfactory in a statistical point of view. In two cases, results obtained with the Normal distribution present a difference greater than 5% in relation to the obtained with NBR 12655. But in both cases the Standard criterion is the more conservative one.

However, in four results, obtained with the more correct T-Student distribution, the values found with NBR 12655 [1] are greater than the ones of the T-Student distribution, indicating that in these four cases the results obtained with the Standard are slightly nonconservative.

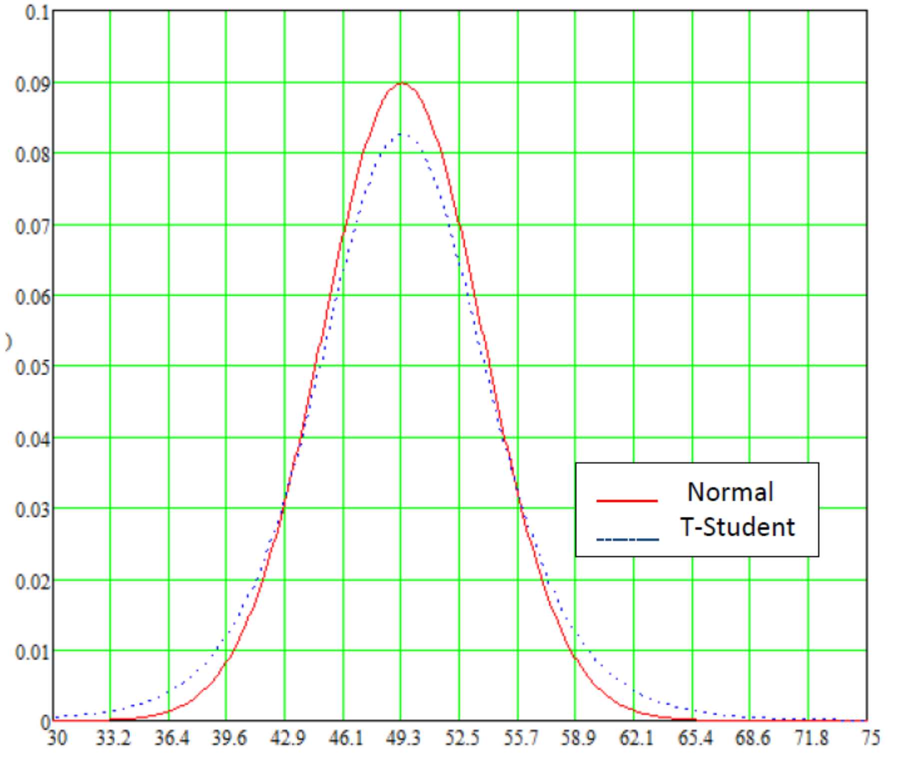

Figure 4: Density Probability Functions, Normal e T-Student – Lot 14

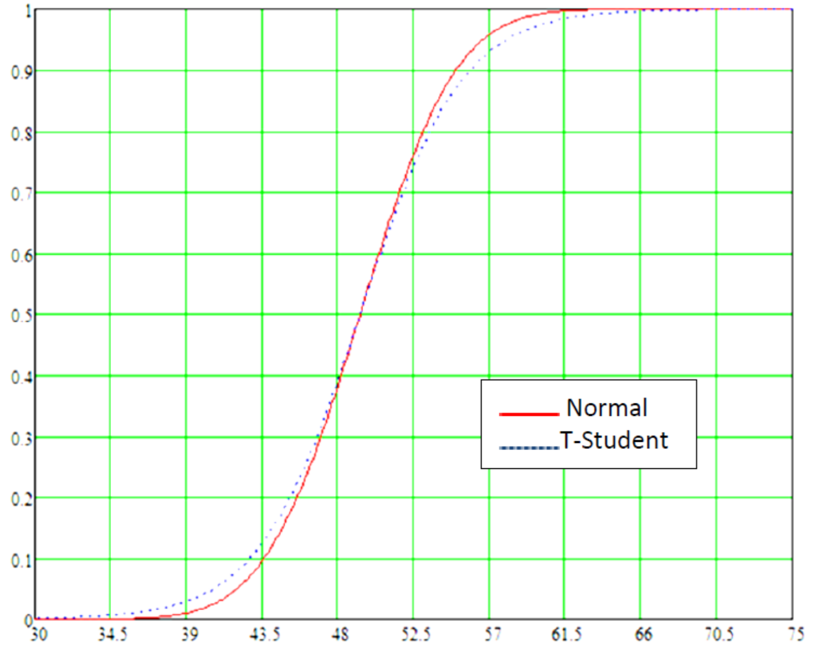

Figure 5: Accumulated Probability Functions, Normal and T-Student – Lot 14

5.3 Analysis with total sampling

Considering the total sampling criteria for the 231 specimens (each one composed by two cylinders) distributed in the 25 lots, 23 deles were rejected (marked in yellow and bold in Table 7).

5.4 Comparison between partial and global analyses

In lots 1, 4, 9, 20 e 24, all the specimens of the respective lots present strengths superior to the ones defined in the design and therefore would be approved in case they were analyzed with the total sampling criterion. However, if they would be considered as part of a lot analyzed with the partial sampling criterion, they would be rejected, since their evaluated characteristic strengths are inferior to their respective design strengths.

These situations show that the total sampling criteria, although involves 100% of the mixes, is not totally safe. Allowing for the acceptation of specimens that would be rejected in the partial sampling, this kind of control shows its incompatibility with the concept of characteristic compression strength (where the strength shall be attained in 95% of the concrete volume).

Table 7: Analysis with total sampling

Lot	n_e	f_{ck} (MPa)	n												
			1	2	3	4	5	6	7	8	9	10	11	12	13
1	9	30	30.8	30.9	30.4	30.0	30.3	30.8	32.8	31.1	31.3				
2	12	30	37.5	37.0	31.4	34.5	31.3	31.5	31.2	34.2	35.0	32.5	**28.4**	**27.5**	
3	10	40	42.2	45.1	**39.8**	49.4	44.0	44.1	42.8	44.0	47.8	47.7			
4	8	40	43.9	41.2	40.9	43.0	43.4	43.4	45.2	42.2					
5	6	35	44.7	45.0	45.6	45.9	46.4	44.6							
6	11	40	41.7	44.8	43.4	44.8	46.6	44.1	43.5	41.0	42.3	41.7	42.3		
7	6	35	38.5	37.6	27.6	**34.0**	**33.2**	40.0							
8	12	40	47.4	49.6	44.1	50.8	47.1	52.3	50.8	46.5	43.5	48.6	41.7	43.4	
9	8	35	41.2	42.9	43.0	36.4	37.5	36.2	41.1	41.8					
10	8	30	36.5	36.8	39.3	34.5	35.4	40.8	41.1	38.1					
11	12	30	36.0	34.0	38.9	35.4	34.8	39.2	36.7	36.1	37.3	**26.1**	31.3	40.3	
12	9	30	35.5	38.1	34.8	34.7	33.5	**29.8**	**29.8**	50.4	44.7	51.0	47.7		
13	11	30	35.4	39.2	42.3	37.5	37.7	34.8	40.1	40.1	38.0	37.3	36.6		
14	9	45	51.8	50.2	52.8	54.7	51.2	**40.2**	**44.5**	49.5	49.0				
15	7	45	47.1	**44.5**	**44.2**	49.5	48.4	**44.8**	51.4						
16	9	40	48.8	52.8	58.0	**39.2**	49.0	47.5	50.0	53.9	41.0				
17	11	40	52.8	47.9	45.8	54.6	41.0	48.8	48.6	50.4	44.7	51.0	47.7		
18	7	45	52.8	**41.7**	46.2	47.9	46.6	**43.2**	**43.7**						
19	13	45	50.4	53.4	52.5	52.6	50.4	52.0	47.4	52.3	48.9	51.4	50.0	54.2	50.8
20	10	45	56.8	57.0	46.2	54.8	47.8	51.1	52.0	53.1	46.1	47.3			
21	7	45	45.5	46.2	**43.5**	45.1	45.4	**44.9**	**44.3**						
22	12	45	57.1	60.8	52.8	59.5	55.9	47.4	54.7	55.8	54.8	57.0	59.5	51.9	
23	6	35	**34.1**	**32.5**	39.0	38.9	**34.2**	39.2							
24	11	45	46.6	51.1	48.8	46.9	49.5	45.8	48.9	46.9	55.0	51.1	51.1		
25	7	40	42.2	40.6	43.7	41.8	48.3	47.2	47.9						

5.5 Updated results

For each one of the specimens of lots 1, 4, 9, 20 e 24 the updating methodologies described in item 3 were used.

For the results of the tests, average values $\bar{x}$ and standard deviations s were determined from the results obtained in the two cylinders that compose each specimen, being then the number n of samplings taken as two.

For the previous knowledge, the values μ_0 and s_0 were taken in the way that the characteristic strength in a T-Student distribution corresponds to the design strength in this specimen, considering a variation coefficient of 0.10. The n_0 used values were 3, 10, 20 e 50, in the way that it would be possible to analyze in a parametric way the influence of the weight given to the previous knowledge.

Then, using the *a posteriori* T-Student distribution, the updated strength value for each specimen is obtained using the respective accumulated distribution functions. The results for different n_0 values are summarized in Tables 8 to 12.

A first glance in the Tables 8 to 12 permits to discard the values $n_0 = 3$ and $n_0 = 50$ as references for the weight to be given to the previous knowledge. In the case of $n_0 = 3$, it seems clear that the Bayesian update leads to excessively low values for f_{ck}, and for $n_0 = 50$, the f_{ck} values are excessively close to the design values, with a very small influence of the test data. These two values are then not adequate.

Table 8: Bayesian updating in specimens of lot 1

Lot	n_i	f_{ck} (Design)	f_{ck} (Total)	$n_0 = 3$	$n_0 = 10$	$n_0 = 20$	$n_0 = 50$	0.93*f_{cm}	% $(n_0 = 10)$
1	1	30.0	30.8	20.2	28.1	29.1	29.6	28.4	1.11
	2	30.0	30.9	20.3	28.2	29.1	29.6	28.5	0.91
	3	30.0	30.4	19.8	27.9	29.0	29.6	28.1	0.67
	4	30.0	30.0	19.4	27.7	28.9	29.6	27.8	0.39
	5	30.0	30.3	19.7	27.9	29.0	29.6	28.0	0.50
	6	30.0	30.8	20.3	28.2	29.1	29.6	28.5	0.91
	7	30.0	32.8	22.5	29.1	29.6	29.8	30.0	3.23
	8	30.0	31.1	20.7	28.3	29.2	29.7	28.7	1.54
	9	30.0	31.3	21.0	28.5	29.3	29.7	29.0	1.65

Table 9: Bayesian updating in specimens of lot 4

Lot	n_i	f_{ck}(Design)	f_{ck} (Total)	$n_0 = 3$	$n_0 = 10$	$n_0 = 20$	$n_0 = 50$	0.93*f_{cm}	% $(n_0 = 10)$
4	1	40.0	43.9	30.7	39.1	39.6	39.9	40.50	3.58
	2	40.0	41.2	27.3	37.7	38.9	39.5	38.08	1.02
	3	40.0	40.9	26.6	37.3	38.7	39.5	37.57	0.73
	4	40.0	43.0	29.4	38.6	39.4	39.8	39.62	2.64
	5	40.0	43.4	29.7	38.7	39.4	39.8	39.80	2.85
	6	40.0	43.4	29.8	38.7	39.4	39.8	39.9	3.09
	7	40.0	45.2	32.6	39.7	39.9	40.0	41.76	5.18
	8	40.0	42.2	28.4	38.1	39.1	39.7	38.87	2.03

Table 10: Bayesian updating in specimens of lot 9

Lot	n_i	f_{ck}(Design)	f_{ck} (Total)	$n_0 = 3$	$n_0 = 10$	$n_0 = 20$	$n_0 = 50$	0.93*f_{cm}	% $(n_0 = 10)$
9	1	35.0	41.2	30.4	35.3	35.2	35.1	37.8	6.96
	2	35.0	42.9	33.3	35.9	35.4	35.2	39.7	10.62
	3	35.0	43.0	33.4	35.9	35.4	35.2	39.8	10.75
	4	35.0	36.4	24.4	33.2	34.1	34.7	33.7	1.54
	5	35.0	37.5	25.8	33.8	34.4	34.8	34.7	2.63
	6	35.0	36.2	23.8	32.9	34.0	34.6	33.3	1.06
	7	35.0	41.1	30.7	35.4	35.2	35.1	38.0	7.32
	8	35.0	41.8	31.6	35.6	35.3	35.1	38.6	8.28

Table 11: Bayesian updating in specimens of lot 20

Lot	n_i	f_{ck} (Design)	f_{ck} (Total)	$n_0 = 3$	$n_0 = 10$	$n_0 = 20$	$n_0 = 50$	0.93*f_{cm}	% $(n_0 = 10)$
20	1	45	56.8	45.2	46.4	45.7	45.2	52.6	13.34
	2	45	57.0	45.6	46.4	45.7	45.3	52.9	14.05
	3	45	46.2	30.3	42.2	43.6	44.5	42.6	0.93
	4	45	54.8	42.3	46.0	45.5	45.2	50.7	10.18
	5	45	47.8	32.6	43.2	44.2	44.7	44.2	2.26
	6	45	51.1	37.2	44.9	45.0	45.0	47.3	5.43
	7	45	52.0	38.4	45.2	45.1	45.1	48.1	6.48
	8	45	53.1	39.9	45.6	45.3	45.1	49.1	7.68
	9	45	46.1	30.5	42.3	43.7	44.5	42.7	1.02
	10	45	47.3	32.0	42.9	44.0	44.6	43.8	2.00

Table 12: Bayesian updating in specimens of lot 24

Lot	n_i	f_{ck} (Design)	f_{ck} (Total)	$n_0 = 3$	$n_0 = 10$	$n_0 = 20$	$n_0 = 50$	0.93*f_{cm}	% $(n_0 = 10)$
24	1	45.0	46.6	31.2	42.6	43.8	44.5	43.2	1.4
	2	45.0	51.1	37.3	44.9	45.0	45.0	47.4	5.53
	3	45.0	48.8	33.8	43.7	44.4	44.8	45.1	3.11
	4	45.0	46.9	31.3	42.6	43.9	44.6	43.3	1.62
	5	45.0	49.5	35.2	44.2	44.7	44.9	46.0	4.05
	6	45.0	45.8	30.1	42.1	43.6	44.4	42.4	0.73
	7	45.0	48.9	33.8	43.7	44.4	44.8	45.0	3.00
	8	45.0	46.9	31.0	42.5	43.8	44.5	43.1	1.42
	9	45.0	55.0	42.9	46.1	45.6	45.2	51.1	10.75
	10	45.0	51.1	37.2	44.8	45.0	45.0	47.3	5.56
	11	45.0	51.1	37.2	44.8	45.0	45.0	47.3	5.56

For $n_0 = 10$ and $n_0 = 20$, it can be verified that the updated values presents a significant influence of the test data, with a more natural correction of the previous knowledge. This shows that, for the engineering judgement to be done, values of n_0 within this range are recommended. In this way, the selected n_0 value shall be proportional to the confidence of the analyst in the concrete batcher. An important "confidence estimator" in the concrete batcher is the batcher standard deviation, calculated from results of the concrete test results of the batcher during a predetermined time period.

The NBR 7212 [13] divides the batchers in four categories, according this standard deviation, as shown in Table 13.

Table 13: Levels of concrete batchers according their standard deviations

Site of concrete production	Standard deviation MPa			
	Level 1	Level 2	Level 3	Level 4
Concrete batcher	Sn < 3.0	3.0< Sn < 4.0	4.0 < Sn < 5.0	Sn > **5.0**

Tables 8 to 12 furnish the deviation between the two herein recommended criteria, the one from Stucchi [12] and the Bayesian updating with $n_0 = 10$.

6 CONCLUSIONS

With an average deviation of only 3.06% with relation to the values of the Bayesian updating with $n_0 = 10$, the consideration of the f_{ck} of each specimen as $f_{ck} = 0,93 * f_{cm}$, is an interesting alternative to the present criteria of total sampling defined by NBR 12655 [1].

With the updating of the individual results through the formula $f_{ck} = 0,93 * f_{cm}$, it could be observed that only 21 of the results of lots 1, 4, 9, 20 e 24 would be accepted by the "updated" total sampling criterion, that considers both aspects of the partial sampling, but also considers the value of each sampling.

On the other hand, regarding the total sampling control, it is clear that the criterion defined in NBR 12655 [1] ignore the probabilistic definition of characteristic strength, as well as presents incompatibility with the partial sampling control. In defining the characteristic strength of a mix has the higher value found in the test of two cylinders, there is no evidence that this value could be the one below which is the resistance of no more than 5% of the mix volume.

Two criteria are finally proposed in this paper, which are the one of Stucchi [12], proposing $f_{ck} = 0,93 * f_{cm}$ and the Bayesian updating, with $n_0 = 10$. Regarding the last one, it would be interesting the development of future studies relating the classification levels of the concrete batchers according NBR 7212 [13], with the weight to be given to the previous knowledge in the Bayesian approach. This approach will permit that concrete batchers with a small standard deviation (in other words, with better control of the productive process), could count on a greater level of confidence, according to the Bayesian approach.

It is evident that for the implementation of changes in the present standards, more studies would be necessary, considering moreover the concretes produced in different Brazilian areas. This is the proposition in the conclusion of this paper.

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Author contributions: Claudia Interlandi, Fernando Rebouças Stucchi,Luiz Fernando Campos Ramos Martha, Sergio Hampshire de Carvalho Santos: conceptualization, formal analysis, methodology, writing.

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